Ida Wolden Bache

Econometrics of exchange rate pass-through

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ECONOMETRICS OF
EXCHANGE RATE PASS-THROUGH

BY IDA WOLDEN BACHE

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PREFACE

Understanding the behaviour of import prices is a key issue for inflation targeting central banks in small open economies. Of particular importance is the responsiveness of import prices to movements in the nominal exchange rate - the degree of exchange rate pass-through. The modern literature on 'new open economy macroeconomics' has highlighted a variety of factors affecting the degree of pass-through, including market segmentation, the degree of price stickiness, the choice of invoicing currency, the distribution margin, and the degree of competition from domestic firms. Each of these factors has different implications for the transmission mechanism for monetary policy, exchange rate volatility and hence, for optimal monetary policy. An important task for empirical research is therefore to discriminate between the alternative models. In her dissertation, Ida Wolden Bache confronts the theoretical models with data on UK and Norwegian import prices, using alternative empirical methods. In addition, this dissertation provides new evidence on the small-sample properties of the various methods employed in the empirical investigations.

The dissertation was submitted in September 2006 as part of the author's examination for the PhD degree in economics at the University of Oslo. The defence took place on 19 January 2007. Norges Bank is pleased to present this work to a wider audience by publishing it as Doctoral Dissertations in Economics No. 6.

Oslo, July 2007

Research Department
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Oslo, September 2006
Ida Wolden Bache

CONTENTS

CHAPTER 1

INTRODUCTION AND OVERVIEW

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1   MOTIVATION

The responsiveness of prices to movements in the nominal exchange rate - the degree of exchange rate pass-through - has important implications for the transmission of shocks and optimal monetary policy in open economies. For example; the traditional argument for flexible exchange rates, dating back to Friedman (1953), is that exchange rate flexibility facilitates relative price adjustment in face of country-specific real shocks. The adjustment of relative prices generates an expenditure-switching effect between home and foreign goods that partly offsets the initial effect of the shock. This argument is based on the premise that domestic currency prices of imported goods respond to movements in nominal exchange rates. If the degree of exchange rate pass-through is low, that is, if import prices respond only weakly to movements in the exchange rate, the expenditure-switching effects will be small, thus limiting the short-run adjustment role of nominal exchange rates and hence the desirability of flexible exchange rates.

The issue of exchange rate pass-through has received much attention in the 'new open economy macroeconomics' (NOEM) literature. NOEM is a class of optimising dynamic stochastic general equilibrium (DSGE) models for open economies with imperfect competition and nominal rigidities (for surveys of the NOEM literature see e.g., Lane (2001), Sarno (2001), and Bowman & Doyle (2003)).[1] Over the last decade, DSGE models have become popular tools for policy analysis both in academia and in policy institutions such as central banks, and empirical evaluation of DSGE models is currently an active area of research.

The increasing popularity of DSGE models as tools for policy analysis can be viewed partly as a response to the Lucas critique. Lucas (1976) argued that coefficients in traditional data-based econometric models were unlikely to remain stable in face of changes to the policy regime. This followed from noting that in models with forward-looking agents, current decisions are influenced by expectations of future policies, which implies that, when policy is changed, expectations of future policies change, affecting current decisions. One response to the Lucas critique has been to insist that policy analysis should be based on intertemporal optimising models with explicit microfoundations, the argument being that the parameters describing preferences and technology are 'deep parameters' and more likely to be policy invariant.

The NOEM literature has identified a number of potential factors affecting the degree of exchange rate pass-through: the degree of price stickiness, the choice of price-setting currency by firms, the expected persistence of the exchange rate, the size of the distribution margin, the responsiveness of the elasticity of demand with respect to the exchange rate, and the weight on imported intermediate goods in the production function for do-

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[1] In this thesis I will refer to open economy DSGE models with imperfect competition and nominal rigidities interchangeably as NOEM models or 'New Keynesian' open-economy models.

mestic goods. Each of these factors has different implications for the transmission of shocks, exchange rate volatility and hence, for optimal monetary policy. An important task for empirical research is therefore to discriminate between the alternative models and to assess the relative importance of the different mechanisms for generating incomplete pass-through. The purpose of this thesis is to contribute towards this aim.

This chapter proceeds as follows. Section 2 provides an overview of the modelling of exchange rate pass-through in the NOEM literature. Section 3 gives an overview of the chapters in the thesis, and section 4 introduces the econometric methods used in the thesis.

2   NEW OPEN ECONOMY MACROECONOMICS AND EXCHANGE RATE PASS-THROUGH

To provide background for the discussion of the NOEM literature this section starts with a brief overview of the micro-based theoretical literature on exchange rate pass-through that evolved in the late 1980s, followed by a brief summary of the existing empirical evidence.

2.1   Previous theoretical literature

The early literature on exchange rate pass-through was spurred in part by the muted response of U.S. import prices to the strong appreciation of the dollar in the early 1980s and the subsequent depreciation (see e.g., Menon (1995a) and Goldberg & Knetter (1997) for surveys). The literature draws from the industrial organisation literature and focuses on the relationship between the exchange rate pass-through and industry characteristics such as market structure and the nature of competition. The models are partial equilibrium in nature, that is, they focus on the response of prices to an exogenous movement in the nominal exchange rate. For the most part, they are also flexible price models.

A seminal contribution to this early literature is Dornbusch (1987). Dornbusch identifies four factors that are likely to affect the degree of pass-through to destination currency import prices: (i) the degree of market integration or segmentation, (ii) the degree of product differentiation, (iii) the functional form of the demand curve, and (iv) the market structure and the degree of strategic interaction among suppliers.

As regards the importance of the degree of market integration; if markets are perfectly integrated, the law of one price (LOP) must hold. In its absolute version the LOP says that, when prices are measured in a common currency, identical products should sell for the same price everywhere (see e.g., Goldberg & Knetter, 1997). The relative version of the LOP allows for a constant wedge between the common currency prices of identical products. By contrast, if markets are segmented (e.g., due to formal or informal trade

barriers), firms may set different prices to different destination markets and the LOP may not hold.

To explore the implications of product differentiation for the degree of exchange rate pass-through, Dornbusch considers the Dixit & Stiglitz (1977) model of monopolistic competition. In this model the optimal price is a constant mark-up over marginal cost, and the mark-up is inversely related to the elasticity of demand. Hence, price discrimination is optimal if the demand elasticities differ across destination markets. However, the stark prediction from the Dixit & Stiglitz model is that, for given marginal costs, destination currency import prices respond proportionally to movements in the nominal exchange rate, that is, the exchange rate pass-through is complete. This follows from the assumption that the elasticity of demand is constant. In order to get incomplete pass-through in the monopolistic competition framework one must assume that the elasticity of demand is increasing in the firm's price. Specifically, demand must be less convex than in the constant elasticity case. In this case it will be optimal for the monopolist to adjust the mark-up in response to an exchange rate change. This has the effect of lowering the degree of exchange rate pass-through to import prices. Krugman (1987) refers to such exchange rate induced mark-up adjustment as 'pricing-to-market'.

To illustrate the importance of market structure and strategic interaction among suppliers, Dornbusch uses the example of a Cournot industry of domestic and foreign firms that supply a homogenous good in the domestic market. In the baseline case with a linear demand curve, the elasticity of the equilibrium price with respect to the exchange rate is found to be less than one, that is, the exchange rate pass-through is incomplete. The pass-through elasticity is increasing in the relative number of foreign firms to total firms in the domestic market and in the overall level of market concentration. In general, the pass-through elasticity also depends on the form of the demand curve. The Cournot model illustrates that incomplete pass-through can be an equilibrium outcome even if the goods produced by foreign and domestic firms are perfect substitutes.

The models considered by Dornbusch are all static. Krugman (1987) conjectured that a full explanation of pricing-to-market would require a dynamic model of imperfect competition. Froot & Klemperer (1989) consider a two-period duopoly competing in the domestic market and assume that the firms' second period demands depend on their market share in the first period. Possible sources of such dependence are brand-switching costs or network externalities. In this model, the expected value of the exchange rate affects the value of the market share in the second period, and hence, the optimal price in the first period. The authors show that the magnitude and sign of the exchange rate pass-through will depend on whether exchange rate changes are perceived to be temporary or permanent.

Examples of models emphasising dynamic supply-side effects are the 'hysteresis mo-

dels' of Baldwin (1988) and Baldwin & Krugman (1989). A basic assumption in these models is that firms incur significant sunk costs when entering foreign markets. The entry costs could represent investment in marketing and advertising, or investments in distribution networks. The hysteresis models predict that the exchange rate pass-through will depend both on the expected duration and the *size* of the exchange rate change. In particular, the exchange rate pass-through will depend on whether the exchange rate change is large enough to induce new firms to enter and old firms to exit the market. A testable implication of the hysteresis models is that large exchange rate changes permanently alter the market structure and lead to structural breaks in estimated trade equations.

Another model focusing on dynamic supply-side effects is the model in Kasa (1992). Kasa considers a monopolistic exporter that faces quadratic costs of adjusting supply. As in Froot & Klemperer (1989), a critical factor affecting the degree of exchange rate pass-through is the relative importance of the transitory component of exchange rate fluctuations. Exchange rate changes that are perceived to be transitory are absorbed in the monopolist's profit margin, resulting in a low degree of pass-through to import prices.

A common feature of all the models considered so far is that they are flexible price models; that is, prices are allowed to adjust instantaneously to shifts in costs or demand. As emphasised by Engel (2004), there is no role for monetary policy or nominal prices in these models. Giovannini (1988) derives the optimal pricing policy of a price discriminating monopolist when prices have to be set in advance, that is, before the realisation of the variables determining cost and demand. A main result is that, when prices are predetermined, the comovement between the exchange rate and traded goods prices depends critically on the currency denomination of export prices. If prices are set in the currency of the exporter, deviations from the LOP and incomplete pass-through indicate ex ante price discrimination and pricing-to-market. If, on the other hand, prices are set in the currency of the importing country, the observed deviations from the LOP and incomplete pass-through are the sum of a price discrimination effect and an expectations effect. The model implies that, when prices are predetermined in the currency of the importing country, the exchange rate pass-through depends on the stochastic properties of the nominal exchange rate. The emphasis on nominal rigidities and the choice of price-setting currency makes Giovannini (1988) an important precursor to the NOEM literature to which I turn below.

2.2   *Empirical evidence*

Concurrently with the theoretical literature there emerged a large literature estimating the exchange rate pass-through.[2] A popular approach in the empirical literature was, and

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[2]The early literature is surveyed in Menon (1995a) and Goldberg & Knetter (1997).

still is, to estimate variants of what may be termed a 'pass-through regression'. The pass-through regression is a regression of a price index (most commonly, an import price or an export price index) on the nominal exchange rate and other hypothesised determinants of prices. Exchange rate pass-through is usually defined as the (partial) elasticity of prices with respect to the exchange rate (or, in dynamic models, as the accumulated responses of prices to an exchange rate change), keeping other determinants of prices fixed.

Influenced by the micro-based theoretical literature, a number of studies tested for pricing-to-market using industry-level data (see e.g., Knetter, 1989; Marston, 1990; Knetter, 1993). The findings in these studies are twofold. First, there is substantial evidence that exporters adjust their mark-ups in response to exchange rate changes in order to stabilise destination-currency import prices. Second, the degree of pricing-to-market varies significantly across industries, suggesting that industry structure is a critical dimension for understanding the exchange rate pass-through process. In the studies surveyed by Goldberg & Knetter (1997), the median pass-through to import prices of manufactures over the one-year horizon is around 0.5. Another empirical regularity is that the exchange rate pass-through is gradual: pass-through is higher in the long-run than in the short-run. These findings are confirmed in a more recent study by Campa & Goldberg (2005) who estimate pass-through regressions for 23 OECD countries over the period 1975-2003. The (unweighted) average of pass-through elasticities to import prices of manufactures is 0.46 after one quarter and approximately 0.64 over the longer run.

One strand of the literature tests for pricing-to-market within a cointegration framework. The literature has focused on testing a particular implication of many pricing-to-market models, namely that the price of import-competing goods enters the exporting firm's pricing equation. The long-run exchange rate pass-through is defined as the coefficient on the exchange rate in a long-run import price equation, and a significant coefficient on domestic prices in the long-run price equation is interpreted as evidence of long-run pricing-to-market. Using this approach, several studies find evidence of long-run pricing-to-market, even in small open economies (see e.g., Menon, 1995b; Naug & Nymoen, 1996; Herzberg et al., 2003; Kongsted, 2003).

Most of the pass-through literature has focused on traded goods prices such as import- or export prices. Recently, a number of studies have estimated pass-through regressions with aggregate consumer prices as the dependent variable. The main finding in this literature is that the exchange rate pass-through to consumer prices is numerically small. Choudhri & Hakura (2006) estimate the exchange rate pass-through to consumer price inflation for 71 countries over the period 1979–2000. The average pass-through elasticity for the set of countries classified as low inflation countries is 0.04 in the first quarter, 0.14 after four quarters and 0.16 after twenty quarters. The averages mask the fact that several countries have negative short-run pass-through elasticities. Making comparisons of re-

gimes across countries and across time, the authors find evidence of a significant and positive relationship between pass-through and average inflation. Similarly, in a study which covers 20 industrial countries over the period 1971–2003, Gagnon & Ihrig (2004) find that countries with low and stable inflation rates also tend to have low estimated rates of pass-through to consumer prices.

The issue of whether the exchange rate pass-through has declined since the 1980s has been much debated in the recent literature. Campa & Goldberg (2005) find evidence that a shift in the commodity composition of manufactured imports contributed to a fall in the pass-through to aggregate import prices in many countries in the 1990s. Marazzi et al. (2005) document a significant decline in the pass-through to U.S. import prices. As possible explanations they point to changes in the composition of imports, the increasing market shares of Chinese imports and changes in the pricing behaviour of Asian firms in the wake of the Asian financial crisis in 1997-98. Gagnon & Ihrig (2004) find evidence that the exchange rate pass-through to consumer prices declined in many countries after the beginning of the 1990s.

As an alternative to pass-through regressions, structural vector autoregressions (VARs) have become increasingly popular as a method to estimate the exchange rate pass-through (e.g., McCarthy, 2000; Hahn, 2003; Choudhri et al., 2005; Faruqee, 2006). A motivation for using the structural VAR approach is that it takes explicit account of the endogeneity of the exchange rate and permits the estimation of pass-through to a set of prices, such as import prices, producer prices and consumer prices, simultaneously. Another motivation is that structural VARs can be a useful tool to evaluate and estimate DSGE models (see e.g., Rotemberg & Woodford, 1997; Christiano et al., 2005).

The VARs used to estimate the degree of exchange rate pass-through typically include a nominal exchange rate, one or several price indices (typically, import prices, producer prices and consumer prices) and sometimes additional variables such as oil prices, a measure of the output gap, wages and interest rates. Recognising that the comovement between prices and the exchange rate depends on the source of the shock, most studies define the exchange rate pass-through as the impulse responses of prices to a particular shock, namely an exogenous exchange rate shock. The findings in the structural VAR literature can be summarised as follows. First, the exchange rate pass-through is incomplete, even in the long-run. Second, the size and speed of pass-through decline along the distribution chain: import prices respond stronger and faster to exchange rate shocks than producer- and consumer prices. Finally, consumer prices are largely unresponsive to exchange rate shocks. These are features that the NOEM models aim to explain.

2.3 New open economy macroeconomics

In the seminal *Redux* model (Obstfeld & Rogoff, 1995), the LOP holds for all goods, and prices are set in the currency of the producer (so-called producer currency pricing, PCP). Under these assumptions, local currency import prices respond proportionally to unexpected exchange rate movements, that is, the exchange rate pass-through is complete and immediate. This is in keeping with traditional open-economy macromodels such as the Mundell-Fleming-Dornbusch model and with the recent New Keynesian small open economy model considered by Galí & Monacelli (2005). Betts & Devereux (1996, 2000) generate incomplete pass-through and short-run deviations from the LOP by allowing for international market segmentation and by assuming that import prices are temporarily rigid in the currency of the importing country (so-called local currency pricing, LCP). In their model, prices are set one period in advance and hence are predetermined every period. Local currency price stickiness then implies that the short-run exchange rate pass-through is zero. Due to the assumption that foreign and domestic households have identical constant elasticity of substitution (CES) preferences over differentiated goods, the LOP holds and the exchange rate pass-through is complete in the flexible price equilibrium.[3]

Subsequent papers have combined the LCP framework with more general models of time-dependent pricing such as Calvo's (1983) model of random price adjustment (e.g., Smets & Wouters, 2002; Monacelli, 2005), the linear quadratic adjustment cost model of Rotemberg (1982) (e.g., Adolfson, 2001; Laxton & Pesenti, 2003; Bergin, 2006), or a staggered contracts model (e.g., Bergin & Feenstra, 2001; Chari et al., 2002).[4] A key feature of these models is that the optimal price-setting rules are forward-looking: import prices depend on the expected future path of the driving variables. The models predict that the exchange rate pass-through to import prices will be gradual, and moreover, that the size and speed of pass-through will depend on the expected persistence of the exchange rate change. One implication of the forward-looking nature of the price-setting rules is that the degree of exchange rate pass-through will be endogenous to the monetary policy regime (see e.g., Taylor, 2000; Gagnon & Ihrig, 2004). The link between the inflation environment and pass-through has also been explored by Devereux & Yetman (2003). They argue that in an environment with low and stable inflation, firms will adjust prices less frequently, implying that, if at least some firms engage in LCP, the short-run exchange rate pass-through will be lower.

A recent strand of the literature analyses the choice of price-setting currency (i.e., the choice between LCP and PCP) in the context of the NOEM framework. The opti-

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[3]If the preferences of foreign and domestic consumers exhibited different elasticities of substitution, LOP in its absolute form would not hold, however, the exchange rate pass-through would still be complete.

[4]In a recent paper, Flodén & Wilander (2006) analyse the exchange rate pass-through in a model with state dependent pricing.

mal choice of price-setting currency is found to depend on several factors, including the exporting firm's market share in the foreign market (Bacchetta & van Wincoop, 2005), and the degree of substitutability between foreign and domestic goods (Goldberg & Tille, 2005).[5] The model in Devereux et al. (2004) predicts that the exchange rate pass-through will be lower in countries with relatively stable monetary conditions because foreign exporters have an incentive to stabilise local currency import prices in these countries. Another contribution emphasising the joint endogeneity of the exchange rate pass-through and the monetary policy regime is Corsetti & Pesenti (2005). In the model in that paper, foreign exporters decide how much of an exchange rate change should be passed-through to local currency import prices prior to the realisation of the exchange rate. LCP and PCP arise as special cases. The expected profits from exports and hence, the optimal degree of pass-through, depend on the monetary policy rule and the nature of the shocks hitting the economy.

The first-generation NOEM models do not distinguish between the consumer ('retail') prices of imports and import prices 'at the docks'. By contrast, Smets & Wouters (2002) assume that importing firms buy a homogenous good at a given price from the world market and transform it into differentiated goods for sale in the domestic market. Similarly, Monacelli (2005) assumes that domestic retailers import differentiated goods for which the LOP holds. In these models, the exchange rate pass-through to import prices at the docks is immediate and complete. However, because of local currency price stickiness, the exchange rate pass-through to import prices at the consumer stage is incomplete in the short run.

Corsetti & Dedola (2005) extend the basic NOEM framework by assuming that the distribution of traded goods to final consumers requires the input of local, non-traded goods and services. This assumption is consistent with the notion that traded goods prices at the consumer level contain a significant non-traded component. Distribution costs create a wedge between the import prices at the docks and the consumer price of imports. This has the direct effect of lowering the degree of exchange rate pass-through to import prices at the consumer level. In addition, the existence of a wedge between producer and consumer prices implies that the price elasticity of demand perceived by the exporter, and hence the exporter's optimal mark-up, will be a function of the price of non-traded goods in the importing country. This creates scope for price discrimination between the domestic and foreign markets and implies that the exchange rate pass-through to import prices will be incomplete, even in the absence of local currency price stickiness. In this thesis, I follow Bergin & Feenstra (2001) and refer to models with this feature as 'pricing-to-market' models. This is in line with the definition of pricing-to-market in the

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[5]Goldberg & Tille (2005) also discuss the circumstances under which it might be optimal to invoice in a third-country vehicle currency.

micro-based, partial equilibrium literature discussed above.

Bergin & Feenstra (2001) and Gust & Sheets (2006) introduce 'pricing-to-market' by replacing the standard assumption that households have CES preferences over differentiated goods with preference specifications that have the property that the elasticity of demand facing a firm depends on the firm's price relative to the prices set by its competitors. In these models, an exporter contemplating raising her price will take into account that, if the prices of import-competing goods remain constant, an increase in the firm's price will cause demand to become more elastic, leading to a reduction in the desired mark-up. Hence, it is optimal for an exporter to absorb part of an exchange rate movement in the mark-up and so the exchange rate pass-through to local currency import prices will be incomplete.

A direct channel through which the exchange rate affects *domestic* firms' prices, is via the prices of imported intermediate goods. When imported goods enter the production function for domestic goods, marginal costs will depend on the prices of imported intermediate goods. This is potentially an important transmission channel for exchange rate changes in a small open economy (see e.g., McCallum & Nelson, 2000). The direct effect of import prices on the aggregate consumer price index depends on the degree of openness and on the degree of home bias in consumption. Obviously, in a general equilibrium framework, the reduced form comovement between exchange rates and prices depends not only on the optimal response of price setters to movements in the exchange rate, but on the entire structure of the model and the source of the shocks hitting the economy (see e.g., Ambler et al. (2003) for an illustration of this point).

There is a burgeoning literature estimating NOEM models with incomplete pass-through.[6] Choudhri et al. (2005) focus explicitly on the ability of different versions of a small open economy NOEM model to explain the degree of exchange rate pass-through to a set of prices in non-US G7 countries.[7] The NOEM models are estimated by minimising a measure of the distance between the impulse responses of prices to an exchange rate shock obtained from an identified VAR and the corresponding responses in the theoretical models. The best-performing model incorporates many of the mechanisms for generating incomplete or slow pass-through proposed in the literature, including nominal price- and wage rigidities, a combination of LCP and PCP, and distribution costs.

3   OVERVIEW OF THE THESIS

The thesis covers two broad themes: the econometrics of the New Keynesian import price equation (chapters 2 and 3) and the structural VAR approach to estimating the exchange

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[6] A non-exhaustive list of references includes Smets & Wouters (2002), Bergin (2003), Bergin (2006) and Adolfson et al. (2005).

[7] Faruqee (2006) conducts a similar analysis on euro area data.

rate pass-through (chapters 4 and 5).

Chapter 2: Estimating New Keynesian import price models

A key feature of import price equations in New Keynesian open-economy models is that they are forward-looking: import prices depend on the expected future path of the driving variables. Despite this feature, the exchange rate pass-through has usually been estimated by regressing import prices on current and lagged values of the exchange rate and other variables believed to affect import prices. If indeed price setters are forward-looking, the coefficients in such regressions will depend on the parameters in the price-setting rules and on the parameters in the expectations mechanisms. These mechanisms will in turn depend on the regime of monetary policy. The New Keynesian models thus predict that the coefficients in conventional pass-through regressions will vary with changes in the expectations mechanisms and with changes in the monetary policy regime; that is, the regressions are susceptible to the Lucas (1976) critique.

In chapter 2 of this thesis (co-written with Bjørn E. Naug), we estimate and evaluate a range of New Keynesian import price equations using generalised method of moments (GMM). GMM has been widely used to estimate individual equations in New Keynesian DSGE models, including the New Keynesian Phillips Curve, the Euler equation for output and forward-looking monetary policy rules.

We use the Calvo (1983) model of random price adjustment as a unifying framework for deriving New Keynesian import price equations. We first derive and discuss a standard (purely forward-looking) LCP model where current import price growth depends on the expected future price growth and the level of import prices relative to foreign marginal costs measured in the importing country's currency. Consumers are assumed to have constant elasticity of substitution (CES) preferences over differentiated goods; that is, the elasticities of demand for individual goods are assumed to be constant. We extend the model to allow firms that do not re-optimise prices in a given period to index their prices to past import price growth and to allow a subset of foreign exporters to engage in PCP. Finally, we consider two pricing-to-market models: a model with translog preferences and a model with distribution costs. The pricing-to-market models imply that the exporters' desired mark-ups are a function of domestic prices or costs in the importing country.

The models are estimated on data from 1980Q1 to 2003Q1 for two small open economies: the UK and Norway. The GMM estimates obtained for the UK do not lend much support to the hypothesis that the price-setting rules are forward looking: the coefficient on expected future import price growth is either statistically insignificant, economically implausible, or both. The evidence of forward-looking price-setting is stronger for Norway: the coefficient on the forward-term is positive and, in most cases, statistically si-

gnificant. For both countries, the estimation results favour a specification that allows for both PCP and LCP. By contrast, we find little evidence of indexation to past import price growth. For Norway, the estimated coefficients on foreign costs and the pricing-to-market variables are statistically insignificant and close to zero in most cases. This contrasts with the results obtained for the UK: the coefficients on the foreign cost variables are statistically significant and, moreover, the pricing-to-market models suggest a role for domestic prices or costs in explaining import prices.

Chapter 3: Assessing GMM and ML estimates of New Keynesian import price equations

There is increasing evidence that weak identification problems cause GMM estimates to exhibit substantial bias in small samples. At the same time, several authors have found that maximum likelihood (ML) performs better than GMM in forward-looking rational expectations models (see e.g., Fuhrer et al., 1995; Fuhrer & Rudebusch, 2004; Lindé, 2005). This is the motivation for chapter 3 of this thesis, which uses Monte Carlo techniques to examine the small-sample properties of GMM and ML estimates of New Keynesian import price equations.

The data generating process in the simulation experiments is the New Keynesian import price equation augmented by a data-consistent VAR model for the driving variables. The same VAR is used as the completing model for the driving variables in the ML estimation. The VAR is estimated using UK data for the period 1980Q1–2003Q1. I conduct experiments for different specifications of the import price equation, different auxiliary VARs, different sample sizes, different instrument sets and different values of the structural parameters. Throughout, the estimated model is assumed to be correctly specified.

The main result that emerges from the simulation exercise is that the GMM estimates exhibit a significant small-sample bias. Small-sample estimation bias could thus be part of the explanation behind the economically implausible parameter estimates we obtained on actual UK data. A key finding is that the GMM estimate of the coefficient on expected future import price inflation is insensitive to the true value of this parameter in the data generating process.

The ML estimates are fairly accurate, even in small samples, and are in general more precise than the GMM estimates. Motivated by these findings, the last part of the paper uses ML to estimate New Keynesian import price equations for the UK. The preferred specification is a purely forward-looking model which combines LCP and PCP. The coefficient estimates are statistically significant and within the ranges suggested by theory. The historical fit of the restricted equilibrium-correction model for import prices implied by the rational expectations model is comparable to that of a data-based equilibrium-correction model over the sample period. Moreover, the two models imply similar esti-

mates of the exchange rate pass-through.

Chapter 4: Unit roots and exchange rate pass-through to UK prices

The fourth chapter of the thesis provides structural VAR evidence on the degree of exchange rate pass-through to UK prices. The price indices included in the VAR are import prices, export prices, producer prices and consumer prices. The model is estimated on quarterly data for the period 1980Q1–2003Q2. The chapter focuses on two issues that have received relatively little attention in the previous literature: small-sample estimation bias and the sensitivity of the estimates to different ways of dealing with the apparent non-stationarity in the data. The paper thus adds to and complements the previous studies by McCarthy (2000) and Choudhri et al. (2005) who provide structural VAR evidence on the degree of exchange rate pass-through for several countries, including the UK.

Univariate and multivariate unit root tests suggest that the levels of prices and the exchange rate are well described by unit root processes over the sample period. The cointegration tests suggest that there is one, or possibly two, cointegrating relations among the variables in the UK data. The cointegration restrictions implied by many open-economy DSGE models, namely that relative prices and inflation rates are stationary, are strongly rejected by the data.

I proceed by computing the impulse responses of prices to an exchange rate shock from three different specifications of the VAR: a VAR in levels, a VAR in first differences and a vector equilibrium correction model that imposes stationarity of relative prices and inflation rates. To take account of small-sample bias in the estimated impulse responses, the confidence bands for the impulse response estimator are computed using the bias-corrected bootstrap procedure suggested by Kilian (1998). The main conclusion from this exercise is that the structural VAR estimates of the exchange rate pass-through are highly sensitive to the treatment of the apparent non-stationarity in the data, even at relatively short horizons.

Simulation evidence suggests that when the data generating process and the model are a first-differenced VAR, there is essentially no bias in the impulse responses. By contrast, if the data generating process and the model are a VAR in levels, the impulse responses display a downward bias. For both specifications, the coverage rates of the confidence intervals are lower than the nominal level at short horizons, but close to the nominal level at longer horizons.

In another set of simulation experiments I ask the following question: what would an econometrician find on average if she estimated a VAR in levels, but the data were generated by a first-differenced VAR? And conversely; what would the econometrician find if she estimated a VAR in first-differences when the data generating process was a VAR in levels? The findings suggest that when the data generating process is stationary,

the estimated responses of the exchange rate, import prices and export prices obtained from a VAR estimated in first differences exhibit a strong upward bias. When the VAR is non-stationary, but the econometrician estimates a VAR in levels, the opposite holds: the estimated responses are biased downwards.

Chapter 5: Assessing the structural VAR approach to exchange rate pass-through

A common approach to the evaluation of DSGE models is to compare the impulse response functions from the DSGE model and the impulse responses obtained from identified VARs. Recently, several papers have examined the reliability of the structural VAR approach using Monte Carlo simulations (e.g., Chari et al., 2005; Erceg et al., 2005; Christiano et al., 2006; Kapetanios et al., 2005). The basic idea in this literature is to generate artificial data from a DSGE model, construct impulse responses from a VAR estimated on the artificial data and ask whether the VAR recovers the DSGE model's responses. One conclusion that can be drawn from these studies is that the reliability of the VAR approach depends on the specification of the shocks, the characteristics of the underlying model and the specification of the VAR.

Chapter 5 of this thesis assesses the reliability of the structural VAR approach to estimating the exchange rate pass-through. The motivating question is: are impulse responses of prices to a UIP shock a useful tool to evaluate and estimate DSGE models with incomplete exchange rate pass-through? To address this question I generate a large number of artificial datasets from a small open economy DSGE model, estimate a VAR on the artificial data and compare the responses of prices to a UIP shock in the VAR and the DSGE model. The DSGE model that serves as the data generating process incorporates many of the mechanisms for generating imperfect pass-through that have been proposed in the NOEM literature, including market segmentation, local currency price stickiness, nominal wage stickiness and distribution costs.

The specification of the DSGE model implies that the nominal exchange rate and nominal prices are non-stationary unit root processes, but that relative prices and the real exchange rate are stationary. Given that the exchange rate pass-through is usually defined in terms of the levels of prices and the nominal exchange rate, a conjecture is that the magnitude of the bias in the estimated VAR responses will depend on whether the correct cointegration rank has been imposed during estimation. To test this conjecture I compare the performance of two different VAR specifications: a pure first-differenced VAR and a VAR that includes the cointegration relations implied by the DSGE model. The former is the most common specification in the structural VAR literature on exchange rate pass-through. As a second exercise, I investigate whether an econometrician would be able to infer the true cointegration rank and identify the cointegration relations using Johansen's (1988) maximum likelihood procedure. The results suggest that (i) the estimates of the

exchange rate pass-through obtained from a VAR estimated in first differences exhibit a systematic downward bias, even when the VAR is specified with a large number of lags; (ii) by contrast, estimates derived from a low order vector equilibrium-correction model are fairly accurate; but (iii) standard cointegration tests have low power to detect the cointegration relations implied by the DSGE model.

4 THE ECONOMETRICS OF EXCHANGE RATE PASS-THROUGH

The empirical literature on exchange rate pass-through reflects the plurality of econometric methodologies and estimation techniques currently used in applied macroeconometrics (see e.g., Favero, 2001). This section provides an overview of the econometric approaches that are used in subsequent chapters of the thesis. The methods are illustrated by means of a simple example. To establish notation and introduce important concepts the section starts with some preliminaries on VARs.

4.1 Vector autoregressions

Let y_t be an $n \times 1$ vector of variables observed at time t. Ignoring deterministic terms, the unrestricted k-th order VAR for y_t is

$$A(L)y_t = \varepsilon_t, \tag{1}$$

where $A(L) = I - \sum_{i=1}^{k} A_i L^i$ is an $n \times n$ matrix polynomial in the lag operator L ($L^j y_t \equiv y_{t-j}$), $A_1, A_2, \ldots, A_k$ are $n \times n$ matrices of autoregressive coefficients, and ε_t is an $n \times 1$ vector of innovations. The innovations are assumed to be independently and normally distributed with mean zero and variance-covariance matrix Ω, $\varepsilon_t \sim IN(0, \Omega)$. The initial values $y_{-k+1}, \ldots, y_0$ are fixed.

If all the roots of the characteristic polynomial $\left| I - \sum_{i=1}^{k} A_i z^i \right| = 0$ are outside the unit circle, the process for y_t is covariance stationary. In this case, $A(L)$ is invertible, and y_t has a moving average (MA) representation

$$y_t = A(L)^{-1} \varepsilon_t = C(L)\varepsilon_t, \tag{2}$$

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is a convergent matrix polynomial in the lag operator, and $C_0 = I$. The $\{j, i\}$ element in C_s identifies the impulse response of $y_{j,t+s}$ to a one-unit increase in $\varepsilon_{i,t}$, $\partial y_{j,t+s} / \partial \varepsilon_{i,t}$.

For stationary processes, the responses die out as the horizon increases. However, macroeconomic time series are often found to be well described by unit root processes. A unit root process that becomes stationary after differencing once is said to be integrated of order one, denoted $y_t \sim I(1)$. When y_t is $I(1)$ it is convenient to reparameterise the

VAR as a vector equilibrium-correction model (VEqCM):

$$\Gamma(L)\Delta y_t = \Pi y_{t-1} + \varepsilon_t, \tag{3}$$

where Δ is the difference operator ($\Delta y_t \equiv y_t - y_{t-1}$), $\Gamma(L) = I - \sum_{i=1}^{k-1} \Gamma_i L^i$, where $\Gamma_i = -\sum_{j=i+1}^{k} A_j$, and $\Pi = -A(1)$. If $y_t \sim I(1)$, but there exists a linear combination of the series that is stationary, the variables in y_t are said to be *cointegrated* (see Engle & Granger, 1987). In this case, the matrix Π has reduced rank (denoted r, $0 \le r < n$) equal to the number of cointegrating relations. Specifically,

$$\Pi = \alpha\beta', \tag{4}$$

where β is an $n \times r$ matrix of cointegration coefficients, and α is an $n \times r$ matrix of adjustment coefficients. If the variables are not cointegrated $\Pi = 0$, and the model collapses to a VAR in Δy_t.

4.1.1 Conditional and marginal models

Suppose that y_t is decomposed into an $n_1 \times 1$ vector x_t and an $n - n_1$ vector z_t: $y_t' = \{x_t', z_t'\}$ and assume that the coefficient matrices are partitioned conformably with y_t. For notational simplicity, assume that the lag length is $k = 2$. The reduced form model in (3) can then be written as

$$\begin{bmatrix} \Delta x_t \\ \Delta z_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \beta' y_{t-1} + \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \Delta x_{t-1} \\ \Delta z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}. \tag{5}$$

Using the normality of ε_t, the VAR in (5) can be expressed in terms of a conditional model for Δx_t

$$\begin{aligned} \Delta x_t = {} & \omega\Delta z_t + (\alpha_1 - \omega\alpha_2)\beta' y_{t-1} \\ & + (\Gamma_{11} - \omega\Gamma_{21})\Delta x_{t-1} + (\Gamma_{12} - \omega\Gamma_{22})\Delta z_{t-1} + \varepsilon_{1t} - \omega\varepsilon_{2t}, \end{aligned} \tag{6}$$

where $\omega = \Omega_{12}\Omega_{22}^{-1}$, and a marginal model for Δz_t

$$\Delta z_t = \alpha_2\beta' y_{t-1} + \Gamma_{21}\Delta x_{t-1} + \Gamma_{22}\Delta z_{t-1} + \varepsilon_{2t}. \tag{7}$$

The disturbance term in the conditional model ($\varepsilon_{1t} - \omega\varepsilon_{2t}$) is, by construction, orthogonal to ε_{2t}. If x_t is a scalar (i.e., $n_1 = 1$) then (6) is a conditional single-equation equilibrium-correction model (EqCM) for x_t.

Efficient conditional inference, in the sense that inference from the conditional model alone is without loss of relevant information, requires that the conditioning variables

are *weakly exogenous* for the parameters of interest (see Engle et al. (1983) for a formal definition). A sufficient condition for the conditioning variables z_t to be weakly exogenous for the cointegration coefficients β is that the cointegration relations do not enter the marginal model for Δz_t (i.e., $\alpha_2 = 0$) (see e.g., Harbo et al., 1998). Valid conditional forecasting (and impulse response analysis) requires *strong exogeneity*. Strong exogeneity is defined as the joint occurrence of weak exogeneity and absence of Granger causality from x_t to z_t. Sufficient conditions for z_t to be strongly exogenous in (6) are that $\alpha_2 = 0$ and $\Gamma_{21} = 0$.

4.1.2 Structural VARs

The VAR in (1) can be interpreted as the reduced form of a *structural* VAR

$$B(L)y_t = u_t, \tag{8}$$

where $B(L) = \sum_{i=0}^{k} B_i L^i$ and $u_t \sim IN(0, \Sigma)$. If B_0 is non-singular, the relationship between the parameters of the reduced form and the structural form can be expressed as

$$A_i = -B_0^{-1} B_i, \quad \Omega = B_0^{-1} \Sigma \left(B_0^{-1}\right)'.$$

The reduced form disturbances ε_t are thus a linear combination of the structural shocks u_t, $\varepsilon_t = B_0^{-1} u_t$.

The impulse responses to the structural shocks u_t are traced out by the MA representation

$$y_t = C(L)B_0^{-1} u_t. \tag{9}$$

In general, an infinite number of structural models will be consistent with the same reduced form representation. Hence, knowledge of the parameters in the reduced form VAR (i.e., the A_i's and Ω) does not imply knowledge of the parameters in the structural VAR (i.e., the B_i's and Σ). To recover the structural form parameters, the econometrician has to impose a set of identifying restrictions. A common assumption is that the structural shocks are uncorrelated (i.e., Σ is diagonal). Identification of the structural shocks then requires $n \times (n-1)/2$ additional restrictions. Sims (1980) argues against imposing zero restrictions on the lag coefficients A_i on the grounds that models incorporating expectations rarely imply such restrictions. The structural VAR approach instead achieves identification by imposing restrictions on the matrix of contemporaneous coefficients B_0 and, in the case of non-stationary VARs, on the long-run impulse responses (see Blanchard & Quah, 1989). More recently, some authors (e.g., Faust, 1998; Canova & Nicoló, 2002; Uhlig, 2005) have proposed to impose identifying restrictions on the sign and shape of the impulse responses.

In the structural VAR literature on exchange rate pass-through, identification is typically achieved by imposing that B_0 is lower triangular. In this case, the parameters in B_0 can be recovered from the Choleski decomposition of Ω, that is, by setting B_0 equal to Λ^{-1}, where Λ is the unique lower triangular matrix satisfying $\Omega = \Lambda\Lambda'$. The assumption that B_0 is lower triangular imposes a recursive structure on the variables. Letting λ_{ij} denote the $\{i, j\}$ element of Λ, the relationship between the reduced form innovations and the structural shocks is

$$
\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 \\ \lambda_{21} & \lambda_{22} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{nt} \end{bmatrix}. \tag{10}
$$

The first variable in the ordering is contemporaneously affected only by the shock to the first equation, the second variable is affected by the shocks to the first and second equation and so on. The last variable in the ordering is contemporaneously affected by all the shocks in the system. It is clear that, unless the reduced form innovations are uncorrelated, the impulse response functions will not be invariant to the ordering of the variables in the VAR.

4.2 Structural VAR and single-equation estimates of exchange rate pass-through

The single-equation and structural VAR approaches to estimating the exchange rate pass-through can be illustrated through a simple example. Suppose that the purpose of the empirical exercise is to estimate the degree of exchange rate pass-through to import prices. The variables included in the analysis are import prices (p_t), a nominal exchange rate (s_t) and a measure of foreign exporters' marginal costs (mc_t). Lower-case letters denote variables in natural logs.

For simplicity, the variables are assumed to follow a first-order VAR:

$$
\begin{bmatrix} s_t \\ mc_t \\ p_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} s_{t-1} \\ mc_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{mc,t} \\ \varepsilon_{p,t} \end{bmatrix}, \tag{11}
$$

where

$$
\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{mc,t} \\ \varepsilon_{p,t} \end{bmatrix} \sim IN \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \sigma_{s,mc} & \sigma_{s,p} \\ \sigma_{s,mc} & \sigma_{mc}^2 & \sigma_{mc,p} \\ \sigma_{s,p} & \sigma_{mc,p} & \sigma_p^2 \end{bmatrix} \right). \tag{12}
$$

The VAR can be written as a conditional model for import prices

$$p_t = \omega_s s_t + \omega_{mc} mc_t + (a_{31} - \omega_{mc} a_{21} - \omega_s a_{11}) s_{t-1} \qquad (13)$$
$$+ (a_{32} - \omega_{mc} a_{22} - \omega_s a_{12}) mc_{t-1} + (a_{33} - \omega_{mc} a_{23} - \omega_s a_{13}) p_{t-1}$$
$$+ \varepsilon_{p,t} - \omega_s \varepsilon_{s,t} - \omega_{mc} \varepsilon_{mc,t},$$

where

$$\omega_s = \frac{\sigma_{mc,p} \sigma_{s,mc} - \sigma_{s,p} \sigma_{mc}^2}{\sigma_{s,mc}^2 - \sigma_s^2 \sigma_{mc}^2}, \qquad \omega_{mc} = \frac{\sigma_{s,p} \sigma_{s,mc} - \sigma_{mc,p} \sigma_s^2}{\sigma_{s,mc}^2 - \sigma_s^2 \sigma_{mc}^2},$$

and marginal models for the exchange rate and marginal costs

$$s_t = a_{11} s_{t-1} + a_{12} mc_{t-1} + a_{13} p_{t-1} + \varepsilon_{s,t} \qquad (14)$$
$$mc_t = a_{21} s_{t-1} + a_{22} mc_{t-1} + a_{23} p_{t-1} + \varepsilon_{mc,t}. \qquad (15)$$

The conditional single-equation model for import prices p_t in (13) has the form of a conventional 'pass-through regression'. Traditionally, exchange rate pass-through is defined as the dynamic multiplier on the exchange rate; that is, the dynamic effects on import prices of a one unit exchange rate change, keeping other determinants of prices (here; marginal costs) fixed.

The structural VAR literature defines the exchange rate pass-through as the impulse responses of import prices to an exogenous shock to the exchange rate. Suppose that the exchange rate shock is identified by placing the exchange rate first in a recursive ordering of the variables. The inverse of the matrix of contemporaneous responses B_0^{-1} can then be recovered from a Choleski decomposition of the variance covariance matrix of the error terms in the reduced form VAR, that is

$$B_0^{-1} = \begin{bmatrix} \sqrt{\sigma_s^2} & 0 & 0 \\ \frac{\sigma_{s,mc}}{\sqrt{\sigma_s^2}} & \sqrt{\sigma_{mc}^2 - \frac{\sigma_{s,mc}^2}{\sigma_s^2}} & 0 \\ \frac{\sigma_{s,p}}{\sqrt{\sigma_s^2}} & \frac{\sigma_{p,mc} - \frac{\sigma_{s,p} \sigma_{s,mc}}{\sigma_s^2}}{\sqrt{\sigma_{mc}^2 - \frac{\sigma_{s,mc}^2}{\sigma_s^2}}} & \sqrt{\sigma_p^2 - \frac{\sigma_{s,p}^2}{\sigma_s^2} - \frac{\left(\sigma_{p,mc} - \frac{\sigma_{s,p} \sigma_{s,mc}}{\sigma_s^2}\right)^2}{\sigma_{mc}^2 - \frac{\sigma_{s,mc}^2}{\sigma_s^2}}} \end{bmatrix}. \qquad (16)$$

Inverting B_0^{-1} and normalising the diagonal elements of B_0 to one, the structural form equation for p_t can be written as

$$p_t = \omega_s s_t + \omega_{mc} mc_t + (a_{31} - \omega_{mc} a_{21} - \omega_s a_{11}) s_{t-1} \qquad (17)$$
$$+ (a_{32} - \omega_{mc} a_{22} - \omega_s a_{12}) mc_{t-1} + (a_{33} - \omega_{mc} a_{23} - \omega_s a_{13}) p_{t-1}$$
$$+ u_{p,t},$$

where $u_{p,t} = \varepsilon_{p,t} - \omega_s \varepsilon_{s,t} - \omega_{mc} \varepsilon_{mc,t}$. This equation is identical to the conditional single-equation model in (13). Thus, the recursive identification scheme with the import price index ordered last corresponds to the conditional/marginal factorisation of the VAR.

Valid impulse response analysis from the conditional single-equation model requires that import prices do not Granger-cause the exchange rate or marginal costs (i.e., $a_{13} = a_{23} = 0$).[8] However, even if this condition is satisfied, the structural VAR and the single-equation estimates of pass-through, as traditionally defined, will differ. The reason is that single-equation estimates of pass-through are conditional on fixed values of marginal costs, while the structural VAR estimates take into account the response of marginal costs to the exchange rate shock. Alternatively, the single-equation estimates can be interpreted as the response of prices to a particular sequence of shocks, namely the sequence which makes the exchange rate increase by one unit in the first period and return to its original level in the second period, while marginal costs remain constant, that is

$$\begin{bmatrix} u_{s,1} \\ u_{mc,1} \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{\sigma_{s,mc}}{\sigma_{mc}^2} \end{bmatrix}$$

$$\begin{bmatrix} u_{s,2} \\ u_{mc,2} \end{bmatrix} = \begin{bmatrix} -a_{11} \\ -a_{21} + a_{11}\frac{\sigma_{s,mc}}{\sigma_{mc}^2} \end{bmatrix}$$

$$\begin{bmatrix} u_{s,j} \\ u_{mc,j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ for } j \geq 2$$

Sufficient conditions for the single-equation and the structural VAR estimates to coincide are thus that the contemporaneous correlation between marginal costs and exchange rates is zero ($\sigma_{s,mc} = 0$) and the exchange rate does not Granger-cause marginal costs ($a_{21} = 0$).

So far, no explicit assumptions have been made about the time-series properties of the variables in the VAR. The levels of nominal prices and the exchange rate are often found to be well described by unit root processes. The choice facing the researcher is whether to ignore the non-stationarity and estimate a VAR in levels, or to obtain a stationary representation of the VAR prior to computing the impulse responses. First-differenced specifications are common both in the single-equation and the structural VAR literature on exchange rate pass-through.[9] In these models, exchange rate changes have a permanent effect on the level of prices, and there are no restrictions on the long-run responses.

An alternative approach to obtaining a stationary representation is to impose restrictions on the cointegrating properties of the variables. In the simple example above,

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[8] See Ericsson et al. (1998, p. 379–380) for a more general discussion of exogeneity and impulse response analysis.

[9] Some recent examples are the single-equation models in Campa & Goldberg (2005) and Marazzi et al. (2005), and the structural VARs in Choudhri et al. (2005) and Faruqee (2006).

suppose that there is a single cointegration relation and that exchange rates and marginal costs are weakly exogenous for the cointegration parameters. The coefficient on the exchange rate in the cointegrating relation can then be interpreted as an estimate of long-run exchange rate pass-through.[10] For example, if $p_t - s_t - mc_t \sim I(0)$, long-run exchange rate pass-through is complete. Note that, in general, this measure of long-run pass-through will differ from the long-run impulse response of prices to an exchange rate shock in the cointegrated VAR. The reason is the same as above, namely that the structural VAR estimate takes into account the response of marginal costs (or, more generally, the response of the other variables entering the cointegrating relation) to the exchange rate shock.

4.3 Estimating rational expectations models

This section discusses the mapping from forward-looking rational expectations models to VARs and conditional single-equation models of exchange rate pass-through and introduces the methods to estimate rational expectations models that are used in subsequent chapters of this thesis. Throughout, I focus on models that have a linear state space representation, either directly or after taking a linear or log-linear approximation of a non-linear model about a non-stochastic steady-state. In the latter case the variables are measured as log deviations from the model-dependent non-stochastic steady state (i.e., $x_t \equiv \ln(X_t/X)$, where X_t is the original (untransformed) variable and X denotes the non-stochastic steady-state value).

4.3.1 Model solution and mapping to a VAR

A large class of rational expectations models can be formulated as follows

$$FE_t\xi_{t+1} + G\xi_t + H\xi_{t-1} + Ju_t = 0, \tag{18}$$

where ξ_t is an $m \times 1$ vector of endogenous and exogenous state variables, and u_t is an $l \times 1$ vector of uncorrelated economic shocks (e.g., shocks to preferences or technologies) satisfying $Eu_t = 0$, $Eu_tu_t' = \Sigma$ and Eu_tu_s' for $s \neq t$, where Σ is diagonal. The coefficient matrices F, G, H are $m \times m$, while J has dimension $m \times l$. The system (18) could represent the collection of log-linearised equilibrium decision rules in a DSGE model, or it could represent a single-equation rational expectations model augmented with reduced form equations for the forcing variables.

Several solution algorithms are available for linearised rational expectations systems

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[10]Johansen (2005) describes the counterfactual experiment which supports the interpretation of the coefficient on the exchange rate in the cointegration relation as the long-run effect on prices of keeping the other variables in the cointegration relation fixed.

such as (18) (e.g., Blanchard & Kahn, 1980; Anderson & Moore, 1985; Klein, 2000; Sims, 2002). Depending on the eigenvalues of the system there are three possibilities: there are no stable rational expectations solutions, there exists a unique stable solution, or there are multiple stable solutions. According to Blanchard & Kahn (1980, prop. 1), there exists a unique stable solution if the number of eigenvalues outside the unit circle equals the number of non-predetermined ('forward-looking') variables.

The solution takes the form of a *recursive equilibrium law of motion*

$$\xi_t = A\xi_{t-1} + Bu_t, \tag{19}$$

where the eigenvalues of A are inside the unit circle.[11] Adding a set of measurement equations relating the elements of ξ_t to an $n \times 1$ vector of observable variables y_t, we obtain the state-space representation

$$
\begin{aligned}
\xi_t &= A\xi_{t-1} + Bu_t \\
y_t &= C\xi_{t-1} + Du_t.
\end{aligned}
\tag{20}
$$

The impulse responses from the economic shocks u_t to the observables y_t can then be computed from the MA representation:

$$y_t = Du_t + C \sum_{j=0}^{\infty} A^j Bu_{t-j-1}. \tag{21}$$

Whether or not y_t has a VAR representation hinges on whether the MA representation is *invertible*, that is, whether the economic shocks u_t can be constructed as a function of current and lagged values of the observables (Watson, 1994, p. 2901). In general, however, we cannot rule out the possibility that a DSGE model has a non-invertible MA representation for a given set of observables[12], in which case a VAR representation fails to exist.

If the number of observables equals the number of shocks (i.e., $n = l$) and D^{-1} exists, a necessary and sufficient condition for invertibility is that the eigenvalues of $A - BD^{-1}C$ are strictly smaller than one in modulus (Fernández-Villaverde et al., 2005). If this con-

---

[11]This requires that the model is cast in stationary form. For example, in a DSGE model where monetary policy acts to stabilise inflation around a non-zero target level, the price level is non-stationary. In the absence of other sources of non-stationarity, stationarity is induced by deflating all nominal quantities by the price level. The model has testable implications for the cointegration properties of the data: relative prices are stationary when measured in a common currency.

[12]Lippi & Reichlin (1994) and Fernández-Villaverde et al. (2005) provide examples of economic models with non-invertible MA components.

dition is satisfied, y_t has a restricted VAR representation

$$y_t = C \sum_{j=0}^{\infty} \left(A - BD^{-1}C\right)^j BD^{-1}y_{t-j-1} + Du_t \qquad (22)$$

As emphasised by Fernández-Villaverde et al. (2005), the VAR exhausts the implications of the theoretical model for the first and second order moments of the observables y_t. If y_t contains all the endogenous state variables (i.e., if all the endogenous state variables are observable and included in the VAR), the VAR representation is of finite order (see e.g., Ravenna, 2005). In general, however, the VAR representation is of infinite order. The rate at which the autoregressive coefficients converge to zero is controlled by the largest eigenvalue of $A - BD^{-1}C$. If one or more of the eigenvalues of $A - BD^{-1}C$ are exactly equal to one in modulus, y_t does not have a VAR representation; the autoregressive coefficients do not converge to zero as the number of lags tends to infinity. Fernández-Villaverde et al. (2005) refer to this as a 'benign borderline case'.

The mapping from the rational expectations model to a conditional single-equation model for import prices can be illustrated through a simple example inspired by Nickell (1985). The theoretical import price equation is a 'New Keynesian' import price equation derived from the Calvo (1983) model of staggered price setting:

$$\Delta p_t = \beta E_t \Delta p_{t+1} - \frac{(1-\beta\eta)(1-\eta)}{\eta}(p_t - s_t - mc_t) + u_{p,t}, \quad u_{p,t} \sim iid(0,\sigma_u^2), \quad (23)$$

where $\beta \in (0,1)$ is a discount factor, and $0 \leq 1 - \eta < 1$ is the constant probability that a firm is allowed to reset prices in any given period. The error term $u_{p,t}$ has the interpretation of an exogenous 'structural' shock unobservable to the econometrician.[13]

The import price equation is augmented by first-order autoregressive processes for the first-difference of exchange rates and exporters' marginal costs, that is

$$\Delta s_t = \gamma_s \Delta s_{t-1} + u_{s,t} \qquad (24)$$

$$\Delta mc_t = \gamma_{mc} \Delta mc_{t-1} + u_{mc,t}, \qquad (25)$$

where $0 < \gamma_s < 1$ and $0 < \gamma_{mc} < 1$, and the disturbances $u_{s,t}$ and $u_{mc,t}$ are white noise. The specification implies that all the variables in the model are $I(1)$, but that there exists a single cointegrating relation between the variables; $p_t - s_t - mc_t \sim I(0)$.

The lack of feedback from import prices to exchange rates or marginal costs implies that the nature of the solution to the model consisting of equations (23)-(25) can be determined from the roots of the characteristic equation associated with the import price

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[13]Chapter 2 of this thesis provides details on the derivation of (23) and several extensions to it that have been proposed in the literature.

equation alone, that is, the roots of

$$\lambda^2 - \left(1 + \frac{1}{\beta} + \frac{(1-\beta\eta)(1-\eta)}{\beta\eta}\right)\lambda + \frac{1}{\beta} = 0. \tag{26}$$

The roots are $\lambda_1, \lambda_2 = \{\eta, 1/\beta\eta\}$ and are positive, lying on either side of unity. Thus, there is one eigenvalue outside the unit circle and one forward-looking variable, hence the 'forward solution' is the unique stable solution to the model.

Using, for example, the method described in Sargent (1987, pp. 200–204), equation (23) can be solved forward to yield a forward-looking EqCM

$$\Delta p_t = -(1-\eta)(p_{t-1} - s_{t-1} - mc_{t-1}) + (1-\eta)\sum_{j=0}^{\infty}(\beta\eta)^j(E_t\Delta s_{t+j} + E_t\Delta mc_{t+j}) + \eta u_{p,t}. \tag{27}$$

By substituting in for $E_t\Delta s_{t+j} = (\gamma_s)^j\Delta s_t$ and $E_t\Delta mc_{t+j} = (\gamma_{mc})^j\Delta mc_t$, we obtain the closed form solution for Δp_t:

$$\Delta p_t = -\phi_1(p_{t-1} - s_{t-1} - mc_{t-1}) + \phi_2\Delta s_t + \phi_3\Delta mc_t + \widetilde{u}_t, \tag{28}$$

where

$$\phi_1 = 1 - \eta, \ \phi_2 = \frac{1-\eta}{1-\gamma_s\beta\eta}, \ \phi_3 = \frac{1-\eta}{1-\gamma_{mc}\beta\eta}, \text{ and } \widetilde{u}_t = \eta u_{p,t}.$$

The closed form solution is seen to be a restricted EqCM for import prices (or, in the terminology of Tinsley (2002), a 'rational' error correction mechanism). The rational expectations model thus implies a set of testable over-identifying restrictions on the coefficients in an EqCM. The parameters in the restricted EqCM in (28) are a mixture of the parameters of the price setting rule (β, η) and the process governing the driving variables s_t and mc_t (γ_s, γ_{mc}). In particular, short-run exchange rate pass-through as measured by ϕ_2 will be a function of the parameters of the stochastic process governing exchange rates.

4.3.2 Estimation techniques

This section discusses three different methods to estimate linear rational expectations models: generalised method of moments (GMM), maximum likelihood (ML) and minimum distance estimation based on a measure of the distance between the theoretical model's impulse responses and the VAR responses (the 'impulse response matching' approach).

Generalised method of moments (GMM) The equilibrium conditions (the 'Euler equations') of the rational expectations model (18) yield population moment conditions that

can be exploited by GMM. The idea behind GMM is to choose the parameters to minimise the distance between the population moment condition and the corresponding sample moment condition.

Suppose that the vector of endogenous variables ξ_t is partitioned into $\xi_{1,t}$ of dimension m_1 and $\xi_{2,t}$ of dimension m_2: $\xi_t' = \left\{ \xi_{1,t}', \xi_{2,t}' \right\}$, and assume that the first block of equations in (18) can be written as

$$F_{11}E_t\xi_{1,t+1} + G_{11}\xi_{1,t} + G_{12}\xi_{2,t} + H_{11}\xi_{1,t-1} + H_{12}\xi_{2,t-1} + J_1 u_t = 0, \qquad (29)$$

where the coefficient matrices are partitioned conformably with ξ_t and $F_{12} = 0$. Next, let $\mathcal{F}_t$ denote the information set of the economic agents at time t. The information set is assumed to contain at least lagged values of the variables in the model, that is, $\mathcal{F}_t \subseteq \left\{ \xi_{t-j} \right\}_{j=1}^{\infty}$. Under the assumptions of the model,

$$E_t\left[(F_{11}\xi_{1,t+1} + G_{11}\xi_{1,t} + G_{12}\xi_{2,t} + H_{11}\xi_{1,t-1} + H_{12}\xi_{2,t-1}) Z_t \right] = 0 \qquad (30)$$

for any $1 \times q$ vector of variables $Z_t \in \mathcal{F}_t$ that satisfies $E_t\left[u_t Z_t \right] = 0$.

The moment conditions (30) provide the basis for GMM estimation of the parameters of interest θ. Defining

$$g(\xi_t, \theta_0) = F_{11}\xi_{1,t+1} + G_{11}\xi_{1,t} + G_{12}\xi_{2,t} + H_{11}\xi_{1,t-1} + H_{12}\xi_{2,t-1}, \qquad (31)$$

the population moment condition can be written

$$E\left[g(\xi_t, \theta_0) Z_t \right] = 0, \qquad (32)$$

and the GMM estimator is

$$\widehat{\theta}_{GMM} = \arg\min_{\theta} \left(\frac{1}{T} \sum_{t=1}^{T} g(\xi_t, \theta) Z_t \right) W_T \left(\frac{1}{T} \sum_{t=1}^{T} g(\xi_t, \theta) Z_t \right)', \qquad (33)$$

where W_T is a positive definite weighting matrix. Under the regularity conditions stated in Hansen (1982), the GMM estimator is consistent and asymptotically normal.

A fundamental condition for consistency of the GMM estimator is that the population moment condition is satisfied at only one value in the parameter space, that is, $E\left[g(\xi_t, \theta) Z_t \right] \neq 0$ for all $\theta \neq \theta_0$ (see e.g., Hall, 2005, p. 51). If this condition is satisfied, we say that the parameter vector θ_0 is *identified*. In the linear instrumental variables case, identification requires that the covariance matrix between the instruments and the endogenous variables has full rank. It follows that a necessary condition for identification is that there are at least as many instruments as endogenous regressors. However, the lite-

rature on *weak identification* (see Stock et al. (2002) for a survey) has demonstrated that generic identification is not sufficient to ensure reliable inference using GMM in finite samples. In linear instrumental variable regressions, weak identification occurs when the instruments are only weakly correlated with the endogenous variables. If the parameters are weakly identified, conventional point estimates and confidence intervals based on the asymptotic normal approximation will be misleading, even in what will usually be considered a large sample.

An important aspect of the GMM approach to estimating rational expectations models is that it does not involve solving for the model-consistent expectations, and hence, does not require the investigator to specify a completing model for the forcing variables $\xi_{2,t}$. Potentially, this makes GMM more robust to misspecification than full-information ML, which is based on the restricted reduced form of the model and imposes all the cross-equation restrictions implied by the rational expectations assumption in the estimation. However, if the completing model for the forcing variables is correctly specified, the full-information based method will be more efficient. Moreover, Fuhrer & Rudebusch (2004) argue that the failure to impose the rational expectations restrictions during estimation is at the root of the weak instruments problem in conventional GMM estimation of rational expectations models.

The GMM approach can be illustrated using the simple model considered in the previous section. The import price equation is[14]

$$\Delta p_t = \beta E_t \Delta p_{t+1} - \frac{(1-\beta\eta)(1-\eta)}{\eta}(p_t - s_t - mc_t) + u_{p,t}, \tag{34}$$

where the parameters of interest are $\theta = \{\beta, \eta\}$. The forcing variables are strictly exogenous and evolve according to

$$\Delta s_t = \gamma_s \Delta s_{t-1} + u_{s,t} \tag{35}$$
$$\Delta mc_t = \gamma_{mc} \Delta mc_{t-1} + u_{mc,t}, \tag{36}$$

where

$$\begin{bmatrix} u_{p,t} \\ u_{s,t} \\ u_{mc,t} \end{bmatrix} \sim IID \left(0, \begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_s^2 & 0 \\ 0 & 0 & \sigma_{mc}^2 \end{bmatrix} \right).$$

Equation (34) contains an unobserved expectations variable and hence, cannot be estimated directly. The GMM *estimating equation* is obtained by replacing the unobserved

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[14]The exposition is based on Gregory et al. (1993).

expectation with its actual value

$$\Delta p_t = \beta \Delta p_{t+1} - \frac{(1-\beta\eta)(1-\eta)}{\eta}(p_t - s_t - mc_t) + \omega_t, \qquad (37)$$

where the error term $\omega_t \equiv u_{p,t} - \beta\varepsilon_{t+1}$ is a linear combination of the structural shock $u_{p,t}$, and the rational expectations forecast error defined as $\varepsilon_{t+1} \equiv \Delta p_{t+1} - E_t\Delta p_{t+1}$. Since Δp_{t+1} is correlated with the error term, OLS will not yield consistent estimates of the parameters. This motivates the use of an instrument variable method such as GMM. The price setter's information set is assumed to contain at least current and past values of the forcing variables and past values of the endogenous variable, that is $\widetilde{F}_t \subseteq \{\Delta s_{t-j+1}, \Delta mc_{t-j+1}, \Delta p_{t-j}, p_{t-j} - s_{t-j} - mc_{t-j}\}_{j=1}^{\infty}$. Under the assumptions of the model, these variables are orthogonal to the error term in the estimating equation and hence, are admissible instruments.

The requirement for identification of θ is that the covariance matrix between the instruments and the endogenous variables, that is,

$$\begin{bmatrix} E\{\Delta s_t \Delta p_{t+1}\} & E\{\Delta s_t(p-s-mc)_t\} \\ E\{\Delta mc_t \Delta p_{t+1}\} & E\{\Delta mc_t(p-s-mc)_t\} \\ E\{(p-s-mc)_{t-1}\Delta p_{t+1}\} & E\{(p-s-mc)_{t-1}(p-s-mc)_t\} \end{bmatrix}$$

is of full rank. Suppose that the forcing variables s_t and mc_t are random walk processes (i.e., $\gamma_s = \gamma_{mc} = 0$) and assume that the import price equation in (34) is estimated using $\Delta s_t, \Delta mc_t$ and $(p-s-mc)_{t-1}$ as instruments. This instrument set contains all the relevant instruments; further lags of the forcing variables or lags of the endogenous variable contain no additional information about import prices in period $t+1$. The covariance matrix between the instruments and the endogenous variables converges to

$$\begin{bmatrix} (1-\eta)\eta\sigma_s^2 & \eta\sigma_s^2 \\ (1-\eta)\eta\sigma_{mc}^2 & \eta\sigma_{mc}^2 \\ \frac{(1-\eta)\eta^3(\sigma_s^2+\sigma_{mc}^2+\sigma_u^2)}{1-\eta^2} & \frac{\eta^3(\sigma_s^2+\sigma_{mc}^2+\sigma_u^2)}{1-\eta^2} \end{bmatrix}.$$

The rank of this matrix is one. Hence, if the forcing variables follow random walks, the parameters in θ are not identified, and the GMM estimates are inconsistent. Identification requires higher order dynamics in at least one of the forcing variables (e.g., $\gamma_s \neq 0$).[15] This example illustrates that, although GMM does not require the investigator to specify a stochastic process for the forcing variables, whether the parameters are identifiable, or more generally, the strength of identification, will depend on the properties of this

---

[15]The result that identification requires higher-order dynamics in the forcing variables holds more generally (see e.g., Pesaran, 1987; Nason & Smith, 2005; Mavroeidis, 2004).

process. To the extent that the exchange rate follows a random walk, identification of the parameters in the forward-looking New Keynesian import price equation requires sufficient dynamics in the other forcing variables such as for example, marginal costs.

A final point is that the error term in the GMM estimating equation will be serially correlated by construction. The closed form solution implies that the rational expectations error is

$$\varepsilon_{t+1} = \frac{1-\eta}{1-\gamma_s\beta\eta}u_{s,t+1} + \frac{1-\eta}{1-\gamma_{mc}\beta\eta}u_{mc,t+1} + \eta u_{p,t+1},\tag{38}$$

which implies that the error term ω_t will be an MA(1) process:

$$\omega_t = u_{p,t} - \beta\left(\frac{1-\eta}{1-\gamma_s\beta\eta}u_{s,t+1} + \frac{1-\eta}{1-\gamma_{mc}\beta\eta}u_{mc,t+1} + \eta u_{p,t+1}\right).\tag{39}$$

Thus, first-order residual autocorrelation is not in itself a valid cause for rejecting the rational expectations model.[16]

Maximum likelihood (ML)   In contrast to GMM estimation, ML estimation is based on the restricted reduced form of the model and requires the investigator to specify the complete stochastic structure of the model, including the distribution of the disturbances. In the absence of measurement errors, the state space representation (20) can be written

$$\begin{aligned}\xi_t &= A\xi_{t-1} + Bu_t \\ y_t &= H'\xi_t.\end{aligned}\tag{40}$$

Allowing for unobservable state variables, the likelihood function can be evaluated using the Kalman filter (see e.g., Hamilton, 1994, chap. 13). Assuming that the structural shocks $\{u_t\}_{t=1}^T$ are multivariate normal with mean zero and variance-covariance matrix Σ, the prediction error decomposition of the log-likelihood function is

$$\begin{aligned}\ell(\theta) &= -\frac{Tn}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^T \log\left|H'P_{t|t-1}H\right| \\ &\quad -\frac{1}{2}\sum_{t=1}^T \left(y_t - H'\xi_{t|t-1}\right)'\left(H'P_{t|t-1}H\right)^{-1}\left(y_t - H'\xi_{t|t-1}\right),\end{aligned}\tag{41}$$

where

$$\xi_{t|t-1} \equiv \mathcal{P}_t\left(\xi_t|y_{t-1}\right)$$

---

[16]At the same time, however, first-order autocorrelation should not be interpreted as evidence supporting the forward-looking model. Autocorrelation could be due to model misspecification (see e.g., Bårdsen et al., 2005, app. A.2.5).

denotes the linear projection of the state vector ξ_t on a constant and data observed through time $t-1$ (i.e., $y_{t-1} \equiv \{1, y'_{t-1}, y'_{t-2}, \ldots\}$). The matrix

$$P_{t|t-1} \equiv E\left[\left(\xi_t - \xi_{t|t-1}\right)\left(\xi_t - \xi_{t|t-1}\right)'\right]$$

is the associated mean squared error matrix. For given initial values, $\xi_{t|t-1}$ and $P_{t|t-1}$ can be obtained from the Kalman filter recursions

$$\xi_{t+1|t} = A\xi_{t|t-1} + AP_{t|t-1}H\left(H'P_{t|t-1}H\right)^{-1}\left(y_t - H'\xi_{t|t-1}\right) \tag{42}$$

$$P_{t+1|t} = A\left(P_{t|t-1} - P_{t|t-1}H\left(H'P_{t|t-1}H\right)^{-1}H'P_{t|t-1}\right)A' + B\Sigma B'. \tag{43}$$

The ML estimate $\widehat{\theta}_{ML}$ is the value of θ that maximises the log-likelihood function. In general, the estimation procedure starts from an initial guess for the values of the parameters, and then uses a numerical optimisation algorithm to find $\widehat{\theta}_{ML}$, solving for the model consistent expectations at each iteration.

The number of observable variables that can be included in the estimation is limited by the number of structural shocks and measurement errors. If the number of observables exceeds the number of shocks then the variance-covariance matrix of the prediction errors $H'P_{t|t-1}H$ will be singular. This is the *stochastic singularity problem* discussed by e.g., Ingram et al. (1994).

Returning to the New Keynesian import price equation; assuming that the disturbances $u_t = \{u_{p,t}, u_{s,t}, u_{mc,t}\}'$ are normally distributed, the log-likelihood can be written as in (41) with $\xi_t = \{\Delta p_t, \Delta s_t, \Delta mc_t, (p-s-mc)_t\}'$ and $y_t = \{\Delta p_t, \Delta s_t, \Delta mc_t\}$. Since, in this case, all the variables are observable, the recursive forecasts of the state vector are simply $\xi_{t+1|t} = A\xi_t$, and the mean squared error matrix collapses to $P_{t+1|t} = B\Sigma B'$.

The coefficient matrices are

$$
A = \begin{bmatrix}
0 & \frac{\gamma_s(1-\eta)}{1-\gamma_s\beta\eta} & \frac{\gamma_{mc}(1-\eta)}{1-\gamma_{mc}\beta\eta} & \eta-1 \\
0 & \gamma_s & 0 & 0 \\
0 & 0 & \gamma_{mc} & 0 \\
0 & \frac{\gamma_s(1-\eta)}{1-\gamma_s\beta\eta}-\gamma_s & \frac{\gamma_{mc}(1-\eta)}{1-\gamma_{mc}\beta\eta}-\gamma_{mc} & \eta
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
\eta & \frac{1-\eta}{1-\gamma_s\beta\eta} & \frac{1-\eta}{1-\gamma_{mc}\beta\eta} \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\eta & \frac{1-\eta}{1-\gamma_s\beta\eta}-1 & \frac{1-\eta}{1-\gamma_{mc}\beta\eta}-1
\end{bmatrix}
$$

$$
H = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix},
$$

from which it is seen that, in contrast to the GMM approach, ML estimation exploits the cross-equation restrictions implied by the rational expectations model.[17]

Impulse response matching approach   The 'impulse response matching' approach to estimating DSGE models is based on the minimisation of a measure of the distance between the DSGE model's impulse responses and those obtained from an identified VAR. The idea is that by leaving most of the structural shocks unspecified and matching the responses to a limited set of shocks, the estimators will be more robust to misspecification than full-information, likelihood-based estimators.

Let $\Psi(\theta)$ denote the mapping from the parameters of interest θ to the impulse responses of the DSGE model and let $\widehat{\Psi}$ denote the corresponding VAR estimates. The impulse response matching estimator is then

$$
\widehat{\theta}_{IR} = \arg\min_{\theta} \left(\widehat{\Psi} - \Psi(\theta)\right) W_T \left(\widehat{\Psi} - \Psi(\theta)\right)',
$$

where W_T is a positive definite weighting matrix. Christiano et al. (2005) set W_T equal to the inverse of the diagonal matrix containing the variances of the impulse response coefficients.

The impulse response matching approach requires that the identification scheme used to recover the structural shocks from the reduced form VAR innovations is consistent with the DSGE model assumptions. It also requires that the DSGE model has a VAR

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[17]Fuhrer & Olivei (2004) propose a GMM procedure that imposes the rational expectations restrictions on the instrument set. This 'optimal instruments' approach differs from ML in that it does not rely on the assumption of normality of the disturbances.

representation and, moreover, that the VAR representation is well approximated by a finite-order VAR. If this is not the case, the impulse response functions obtained from the finite-order VAR will be inconsistent. Whether the infinite order VAR can be approximated by a finite-order VAR is an empirical question and is likely to depend on the structure and parameterisation of the DSGE model as well as on which variables are included in the VAR. This point can be illustrated by deriving the VAR representation of the simple New Keynesian import price model for two different sets of observables.

In the first example, the econometrician estimates a VAR in

$$y_t = \{\Delta p_t, \Delta s_t, (p-s-mc)_t\}'.$$

This VAR is isomorphic to a VEqCM in $\widetilde{y}_t = \{p_t, s_t, mc_t\}'$ when the correct cointegration restrictions are imposed. For this set of observables the matrix $A - BD^{-1}C$ has four eigenvalues equal to zero, implying that the model is invertible and a VAR representation exists. Specifically, the VAR representation is

$$
\begin{bmatrix} \Delta p_t \\ \Delta s_t \\ (p-s-mc)_t \end{bmatrix} =
$$

$$
\begin{bmatrix} \frac{\gamma_{mc}(1-\eta)}{1-\beta\eta\gamma_{mc}} & \frac{(1-\eta)(\gamma_s-\gamma_{mc})}{(1-\beta\eta\gamma_{mc})(1-\beta\eta\gamma_s)} & -\frac{(1-\eta)(1+\gamma_{mc}-\beta\eta\gamma_{mc})}{1-\beta\eta\gamma_{mc}} \\ 0 & \gamma_s & 0 \\ \frac{\gamma_{mc}(1-\eta)}{1-\beta\eta\gamma_{mc}}-\gamma_{mc} & \gamma_{mc}-\gamma_s+\frac{(1-\eta)(\gamma_s-\gamma_{mc})}{(1-\beta\eta\gamma_{mc})(1-\beta\eta\gamma_s)} & \frac{\eta(\gamma_{mc}(1-\beta\gamma_{mc}-\beta\eta)+1)}{1-\beta\eta\gamma_{mc}} \end{bmatrix} \begin{bmatrix} \Delta p_{t-1} \\ \Delta s_{t-1} \\ (p-s-mc)_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 & 0 & \frac{\gamma_{mc}(1-\eta)}{1-\beta\eta\gamma_{mc}} \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\eta\gamma_{mc}(1-\beta\gamma_{mc})}{1-\beta\eta\gamma_{mc}} \end{bmatrix} \begin{bmatrix} \Delta p_{t-2} \\ \Delta s_{t-2} \\ (p-s-mc)_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{p,t} \\ \varepsilon_{s,t} \\ \varepsilon_{p-s-mc,t} \end{bmatrix},
$$

where the mapping between the reduced form disturbances and the structural shocks, $\varepsilon_t = D u_t$, is

$$
\begin{bmatrix} \varepsilon_{p,t} \\ \varepsilon_{s,t} \\ \varepsilon_{p-s-mc,t} \end{bmatrix} = \begin{bmatrix} \eta & \frac{1-\eta}{1-\beta\eta\gamma_s} & \frac{1-\eta}{1-\beta\eta\gamma_{mc}} \\ 0 & 1 & 0 \\ \eta & -\frac{\eta(1-\beta\gamma_s)}{1-\beta\eta\gamma_s} & -\frac{\eta(1-\beta\gamma_{mc})}{1-\beta\eta\gamma_{mc}} \end{bmatrix} \begin{bmatrix} u_{p,t} \\ u_{s,t} \\ u_{mc,t} \end{bmatrix}.
$$

The structural VAR literature defines the exchange rate pass-through as the impulse responses of prices to an exchange rate shock. The correct identification of the exchange rate shock depends only on the identification of the second column of D. If we normalise the variances of the structural shocks to unity, the variance-covariance matrix of the VAR innovations is DD'. If the exchange rate is ordered first in the VAR, a Choleski

decomposition of DD' yields the following estimate of B_0^{-1}:

$$B_0^{-1} = \begin{bmatrix} 1 & 0 & 0 \\[2mm] \frac{1-\eta}{1-\beta\eta\gamma_s} & \sqrt{\eta^2 + \left(\frac{1-\eta}{1-\beta\eta\gamma_{mc}}\right)^2} & 0 \\[4mm] -\frac{\eta(1-\beta\gamma_s)}{1-\beta\eta\gamma_s} & \frac{\eta^2 - \frac{\eta(1-\eta)(1-\beta\gamma_{mc})}{(1-\beta\eta\gamma_{mc})^2}}{\sqrt{\eta^2+\left(\frac{1-\eta}{1-\beta\eta\gamma_{mc}}\right)^2}} & \sqrt{\eta^2 + \left(\frac{\eta(1-\beta\gamma_{mc})}{1-\beta\eta\gamma_{mc}}\right)^2 - \frac{\eta^2 - \frac{\eta(1-\eta)(1-\beta\gamma_{mc})}{(1-\beta\eta\gamma_{mc})^2}}{\eta^2+\left(\frac{1-\eta}{1-\beta\eta\gamma_{mc}}\right)^2}} \end{bmatrix}.$$

The impact effects of the exchange rate shock contained in the first column of B_0^{-1} coincide with the corresponding elements in the second column of D. Thus, in this model, the recursive identification scheme succeeds in recovering the exchange rate shock, and the impulse response functions from the second-order VAR coincide with the theoretical impulse response functions.

In the second example, the econometrician is assumed to estimate a first-differenced VAR, that is, $y_t = \{\Delta p_t, \Delta s_t, \Delta mc_t\}'$. With this set of observables the matrix $A - BD^{-1}C$ has one unit eigenvalue and three eigenvalues equal to zero. This is the 'benign' borderline case referred to by Fernández-Villaverde et al. (2005); strictly speaking, a VAR representation does not exist. If we nevertheless were to calculate the VAR coefficients based on (22) we would obtain

$$\begin{bmatrix} \Delta p_t \\ \Delta s_t \\ \Delta mc_t \end{bmatrix} = \begin{bmatrix} \eta-1 & \frac{(\eta-1)(\beta\eta\gamma_s-\gamma_s-1)}{1-\beta\eta\gamma_s} & \frac{(\eta-1)(\beta\eta\gamma_{mc}-\gamma_{mc}-1)}{1-\beta\eta\gamma_{mc}} \\ 0 & \gamma_s & 0 \\ 0 & 0 & \gamma_{mc} \end{bmatrix} \begin{bmatrix} \Delta p_{t-1} \\ \Delta s_{t-1} \\ \Delta mc_{t-1} \end{bmatrix}$$
$$+ \sum_{j=2}^{\infty} \begin{bmatrix} -(1-\eta) & 1-\eta & 1-\eta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta p_{t-j} \\ \Delta s_{t-j} \\ \Delta mc_{t-j} \end{bmatrix} + \begin{bmatrix} \varepsilon_{p,t} \\ \varepsilon_{s,t} \\ \varepsilon_{mc,t} \end{bmatrix},$$

where the mapping between the reduced form disturbances and the structural shocks is

$$\begin{bmatrix} \varepsilon_{p,t} \\ \varepsilon_{s,t} \\ \varepsilon_{mc,t} \end{bmatrix} = \begin{bmatrix} \eta & \frac{1-\eta}{1-\beta\eta\gamma_s} & \frac{1-\eta}{1-\beta\eta\gamma_{cm}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{p,t} \\ u_{s,t} \\ u_{mc,t} \end{bmatrix}.$$

In this case, the autoregressive coefficients do not converge to zero as the number of lags tends to infinity. Thus, for this set of observables, the impulse response functions from a finite-order VAR and the impulse responses from the rational expectations model do not coincide, even if the rational expectations model is the true data generating process and the identification scheme is consistent with this model. The failure of a VAR representation to exist is due to 'overdifferencing' (see e.g., Plosser & Schwert, 1977): if the data

are generated by a stationary (or cointegrated) process, first-differencing induces a unit root in the MA representation of the transformed process, in which case the latter cannot be approximated by a finite-order VAR.

Figure 1 compares the normalised impulse response of import prices to a unit exchange rate shock in the theoretical model and the first-differenced population VAR for different lag orders k. The normalised impulse responses can be interpreted as a measure of the exchange rate pass-through.[18] The coefficients in the population VAR are the probability limits of the OLS estimates in a VAR estimated on data generated by the theoretical rational expectations model.[19] The exchange rate shock is identified by placing the exchange rate first in a recursive ordering. The structural parameters are $\beta = 0.99, \eta = 0.75, \gamma_s = 0.1, \gamma_{mc} = 0.5$ and $\sigma_u^2 = \sigma_s^2 = \sigma_{mc}^2 = 1$. The value of the price stickiness parameter implies that price setters change prices on average every four periods (quarters). As is evident from the graph, the impulse responses are accurate up to the imposed lag order. For horizons larger than the imposed lag order, the estimates of the exchange rate pass-through from the first-differenced VAR are biased downwards.

This simple example illustrates that the accuracy of the VAR representation to the DSGE model, and hence the reliability of the impulse response matching approach, depends critically on whether the cointegration relations are included in the VAR. Over-differencing induces a unit root in the MA representation which is eliminated by including the cointegration relations in the VAR.

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[18]The normalised impulse responses are the impulse responses of import prices divided by the exchange rate response. This normalisation facilitates a comparison with single-equation estimates of pass-through defined as the dynamic responses of prices to a one per cent permanent exchange rate change.

[19]Formulas for these coefficients as functions of the matrices A, B, C and D in the state space representation are provided in Fernández-Villaverde et al. (2005). I am grateful to Jesús Fernández-Villaverde for sharing the Matlab program `ssvar.m` which calculates the coefficients of the population version VAR.

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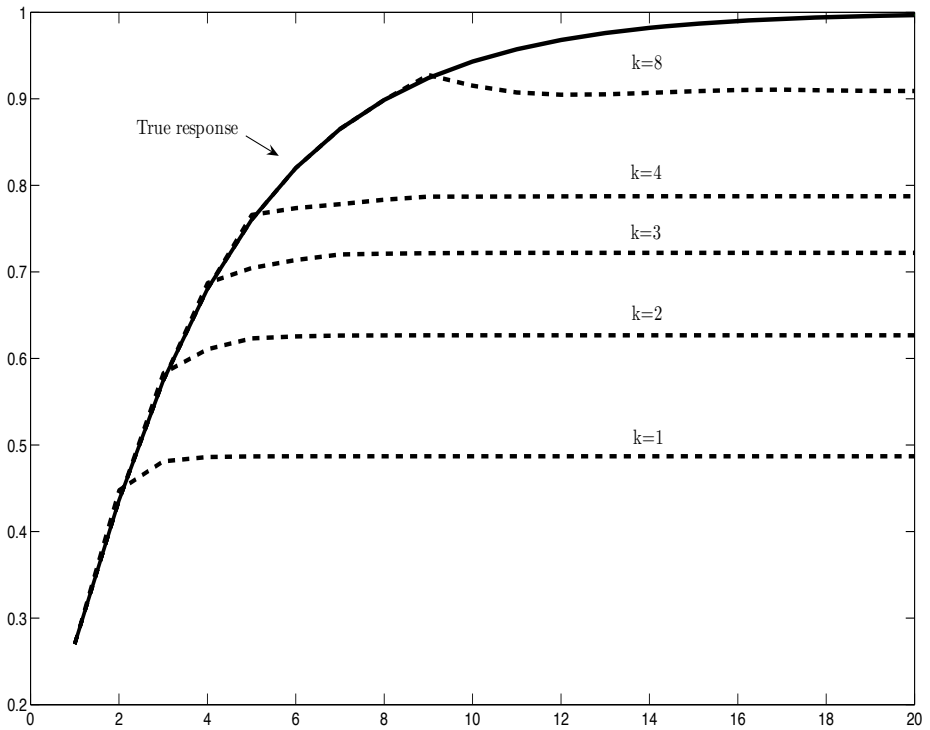
Figure 1: Normalised VAR impulse responses of import prices to one standard deviation exchange rate shock in first-differenced VAR(k).

ESTIMATING NEW KEYNESIAN IMPORT PRICE MODELS

Written with Bjørn E. Naug

1   INTRODUCTION

The development of import prices is important for policy-makers in small open economies and for inflation-targeting central banks in particular. Of special interest for monetary policy is the responsiveness of import prices to changes in the nominal exchange rate; that is, the degree of exchange rate pass-through.

New Keynesian open-economy models have become popular tools for analysing exchange rate pass-through and the effects of monetary policy. These models are dynamic stochastic general equilibrium (DSGE) models that allow for imperfect competition and nominal rigidities, typically in the form of sticky prices.[1] A number of seminal models in the New Keynesian literature (e.g., those in Obstfeld & Rogoff (1995) and Galí & Monacelli (2005)) assume that prices are set (and sticky) in the currency of the producer (so-called producer currency pricing, PCP). They also assume that the law of one price (LOP) holds at all times for traded goods. This implies that the exchange rate pass-through to import prices is complete and immediate, in keeping with the Mundell-Fleming model.

The LOP-PCP framework is rejected by empirical studies, however: they typically find that import prices respond incompletely to changes in the exchange rate (at least in the short run); see e.g., Campa & Goldberg (2005). Hence many New Keynesian models now allow for incomplete pass-through. Following Betts & Devereux (1996, 2000), the most common approach is to assume that international product markets are segmented and that prices are set and sticky in the currency of the importing country (local currency pricing, LCP).[2] This implies that import prices respond only gradually to changes in the exchange rate.

The standard LCP model assumes that the exporters' mark-ups are constant in the long-run (flexible price) equilibrium. This means that the long-run pass-through is complete. The model also implies that import prices are independent of the prices and costs in the importing country (at a constant exchange rate). Recently, several authors have allowed for a non-constant mark-up in the flexible-price equilibrium by assuming that (i) the demand elasticities facing a firm depend on the firm's price relative to those of its competitors (see e.g., Bergin & Feenstra, 2001; Gust & Sheets, 2006) or (ii) the distribution of traded goods to consumers requires local goods and services (see e.g., Corsetti & Dedola, 2005). These models imply that import prices depend on domestic prices or costs in the importing country. Following Bergin & Feenstra (2001), we will refer to models with this feature as 'pricing-to-market' models. Pricing-to-market provides an

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[1]New Keynesian open-economy models are also referred to as 'new open economy macroeconomics' (NOEM) models.

[2]A partial list of papers that have adopted the LCP framework includes Bacchetta & van Wincoop (2000), Devereux & Engel (2003), Chari et al. (2002), and Laxton & Pesenti (2003).

additional source of incomplete pass-through besides local currency price stickiness.

A key feature of import price equations in the New Keynesian models is that they are forward-looking: current import prices depend on expected future import prices and thus (implicitly) on the expected future path of the driving variables. Despite this feature, exchange rate pass-through has usually been estimated by regressing import prices on current and lagged values of the exchange rate and other variables believed to affect import prices.[3] If indeed price setters are forward-looking, the coefficients in such regressions will depend on the parameters in the price-setting rules and on the parameters in the expectations mechanisms. These mechanisms will in turn depend on the regime of monetary policy. The New Keynesian models thus predict that the coefficients in conventional pass-through regressions will vary with changes in the expectations mechanisms and with changes in the monetary policy regime; that is, the regressions are susceptible to the Lucas (1976) critique.

In this paper, we estimate and evaluate a range of New Keynesian import price equations using generalised method of moments (GMM). Contrary to standard pass-through regression analysis, this approach allows us to make a clear distinction between the parameters in the price-setting rules and the parameters in the expectations mechanisms. GMM has been widely used to estimate individual equations in New Keynesian DSGE models, including the New Keynesian Phillips Curve (e.g., Galí & Gertler, 1999; Galí et al., 2001; Batini et al., 2005), the Euler equation for output (e.g., Fuhrer & Rudebusch, 2004) and forward-looking monetary policy rules (e.g., Clarida et al., 1998).

An alternative approach is to estimate the parameters of New Keynesian import price equations as part of fully-specified DSGE models; see for example Bergin (2006), Adolfson et al. (2005) and Choudhri et al. (2005). An advantage of the general equilibrium approach is that cross-equation restrictions implied by the DSGE model are exploited in estimation. If the model is correctly specified, this increases efficiency relative to single-equation GMM estimation. At the same time, however, imposing the cross-equation restrictions could make the estimates of the parameters in the import price equation sensitive to misspecification in other parts of the model. Single-equation analysis does not rely on a specific completing model for the driving variables.

We use the Calvo (1983) model of random price adjustment as a unifying framework for deriving New Keynesian import price equations. This eases the comparison across model specifications. The overlapping contracts model of Taylor (1980) and the linear quadratic adjustment cost framework of Rotemberg (1982) would give rise to similar price dynamics (see Roberts, 1995). Building on previous empirical studies, we only consider models that allow for incomplete pass-through. We first derive and dis-

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[3]A recent contribution is Campa & Goldberg (2005). See Goldberg & Knetter (1997) for a general discussion of pass-through regressions.

cuss a standard (purely forward-looking) LCP model where current import price growth depends on the expected future price growth and the level of import prices relative to foreign marginal costs measured in the importing country's currency. Consumers are assumed to have constant elasticity of substitution (CES) preferences over differentiated goods; that is, the elasticities of demand for individual goods are assumed to be constant. We extend the model to allow firms that do not re-optimise prices in a given period to index their prices to past import price growth and to allow a subset of foreign exporters to engage in PCP. Finally, we consider two pricing-to-market models: a model with translog preferences and a model with distribution costs. To our knowledge, no previous studies have estimated all these versions of the New Keynesian import price equation.[4]

The models are estimated on data from 1980Q1 to 2003Q1 for two small open economies: the UK and Norway. The GMM estimates obtained for the UK do not lend much support to the hypothesis that the price-setting rules are forward looking: the coefficient on expected future import price growth is either statistically insignificant, economically implausible, or both. The evidence of forward-looking price-setting is stronger for Norway: the coefficient on the forward-term is positive and, in most cases, statistically significant. For both countries, the estimation results favour a specification that allows for both PCP and LCP. By contrast, we find little evidence of indexation to past import price growth.

For Norway, the estimated coefficients on foreign costs and the pricing-to-market variables are statistically insignificant and close to zero in most cases. This contrasts with the results obtained for the UK: the coefficients on the foreign cost variables are statistically significant and, moreover, the pricing-to-market models suggest a role for domestic prices or costs in explaining import prices.

The remainder of the paper is organised as follows. Section 2 provides a survey of New Keynesian import price equations. Section 3 discusses the data and the econometric methodology, and section 4 presents the empirical results. Section 5 concludes the paper.

2   NEW KEYNESIAN IMPORT PRICE EQUATIONS

This section derives the import price equations that provide the theoretical starting point for the empirical analysis. Throughout, the world is assumed to consist of two countries: home and foreign. We model the price-setting decisions of foreign exporters that produce differentiated goods for sale in the home country. Product markets are characterised by monopolistic competition. We also assume that international product markets are segmented; that is, we allow firms to set distinct prices for the home and foreign markets. The price that firms would choose if prices had been perfectly flexible is referred to as the

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[4]Freystätter (2003) estimates forward-looking import price equations using GMM on Finnish data. The equations allow for PCP and LCP, but they do not allow for inflation indexation or pricing-to-market.

'frictionless' price. The associated mark-up is referred to as the 'frictionless' mark-up. In section 2.1 we consider models where the frictionless mark-up is constant. In section 2.2 we consider models where the frictionless mark-up is a function of domestic prices or costs in the importing country, so-called pricing-to-market models.

2.1 Models with a constant frictionless mark-up

This section derives three New Keynesian import price equations with a constant frictionless mark-up: a purely forward-looking model where all exporters engage in LCP (section 2.1.1), a 'hybrid' LCP model where firms that do not re-optimise prices in a given period index their prices to past import price growth (section 2.1.2) and a model where a share of exporters engages in PCP and a share engages in LCP (section 2.1.3).

2.1.1   A purely forward-looking model with local currency pricing

Foreign firms are indexed by $i \in [0,1]$. Households in the home economy derive utility from the consumption of a composite foreign good $Y_{F,t}$, defined as a CES aggregate of differentiated goods[5]

$$Y_{F,t} \equiv \left[\int_0^1 Y_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $Y_{F,t}(i)$ is the imported quantity of good i in period t and $\varepsilon > 1$ is the constant elasticity of substitution between the individual goods. The corresponding ideal price index is

$$P_{F,t} \equiv \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

where $P_{F,t}(i)$ is the import price of good i, measured in the currency of the importing country. Cost-minimisation yields a conditional demand function for an individual imported good of the form

$$Y_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t}. \tag{3}$$

Price setting is staggered as in Calvo (1983); the probability that a firm is allowed to re-optimise its price in any given period is $1 - \eta$. The expected average time between price changes is thus $1/(1 - \eta)$. All exporters engage in LCP; that is, they set prices in the currency of the importing country. A firm that is allowed to re-optimise its price in

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[5]The aggregate consumption index is a CES aggregate of the composite foreign good and a composite domestic good defined as a CES index of differentiated domestic goods. An alternative approach is to model a perfectly competitive firm that combines differentiated foreign goods using a CES technology.

period t sets the price $\widetilde{P}_{F,t}(i)$ to maximise

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\frac{\widetilde{P}_{F,t}(i)}{S_\tau} - MC_{F,\tau}(i) \right) \left(\frac{\widetilde{P}_{F,t}(i)}{P_{F,\tau}} \right)^{-\varepsilon} Y_{F,\tau}, \tag{4}$$

where $D_{t,\tau}$ is a stochastic discount factor ($D_{t,t} = 1$), S_τ is the nominal exchange rate and $MC_{F,\tau}(i)$ denotes the foreign firm's marginal costs. In the following, we assume that all firms have access to the same technology and that all factors of production can be costlessly and instantaneously reallocated across firms. These assumptions imply that the marginal cost of firms that are allowed to reset prices is the same as the average marginal cost across all firms, that is, $MC_{F,t}(i) = MC_{F,t}$. Moreover, since all firms solve the same optimisation problem, $\widetilde{P}_{F,t}(i) = \widetilde{P}_{F,t}$ for all firms that re-optimise in period t. The first-order condition can be written as

$$0 = E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} Y_{F,\tau} \left(\frac{\widetilde{P}_{F,t}}{P_{F,\tau}} \right)^{-\varepsilon} \left[(1-\varepsilon)\frac{1}{S_\tau} + \varepsilon \frac{MC_{F,\tau}}{\widetilde{P}_{F,t}} \right]. \tag{5}$$

If firms change prices on average every period (i.e., $\eta = 0$), the first-order condition collapses to the standard mark-up rule

$$P_{F,t} = \frac{\varepsilon}{\varepsilon-1} S_t MC_{F,t}. \tag{6}$$

Hence the frictionless price is a constant mark-up over foreign marginal costs, measured in the currency of the importing country. In this case, therefore, a change in the exchange rate is completely passed-through to import prices in the same period.

We now consider the general case where $\eta > 0$. Taking a log-linear approximation of (5) around a zero inflation deterministic steady-state we obtain

$$\widetilde{p}_{F,t} - p_{F,t} = (1-\beta\eta)E_t \sum_{\tau=t}^{\infty} (\beta\eta)^{\tau-t} (s_\tau + mc_{F,\tau} - p_{F,\tau}), \tag{7}$$

where β is the steady-state value of the stochastic discount factor ($0 < \beta < 1$) and lower case letters denote the percentage deviation of the original variable from its deterministic steady-state value. The optimal price thus depends on a weighted average of expected future marginal costs measured in the importer's currency. It follows that changes in costs or the exchange rate have stronger short-run effects on import prices if the shocks are expected to be long-lasting than if they are expected to be reversed soon. A testable implication of this model is thus that the parameters in 'backward-looking' pass-through regressions will not be invariant to changes in the stochastic processes for the exchange rate and foreign marginal costs.

Quasi-differentiation of (7) yields

$$\widetilde{p}_{F,t} - p_{F,t} = (1 - \beta\eta)\left(s_t + mc_{F,t} - p_{F,t}\right) + \beta\eta E_t\left(\widetilde{p}_{F,t+1} - p_{F,t+1}\right). \tag{8}$$

The aggregate price index can be written as (see Woodford, 2003, p. 178)

$$P_{F,t}^{1-\varepsilon} = (1 - \eta)\widetilde{P}_{F,t}^{1-\varepsilon} + \eta P_{F,t-1}^{1-\varepsilon}. \tag{9}$$

Log-linearisation around a zero inflation steady-state yields

$$0 = (1 - \eta)\left(\widetilde{p}_{F,t} - p_{F,t}\right) - \eta\Delta p_{F,t}, \tag{10}$$

where Δ is the difference operator ($\Delta x_t \equiv x_t - x_{t-1}$). By substituting in from (8) we obtain

$$\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} - \frac{(1 - \beta\eta)(1 - \eta)}{\eta}\left(p_{F,t} - s_t - mc_{F,t}\right). \tag{11}$$

The equilibrium condition relates current import price inflation to expected import price inflation in the next period and to deviations of current import prices $p_{F,t}$ from the local currency value of foreign marginal costs $s_t + mc_{F,t}$. The effect of the forcing term ($p_{F,t} - s_t - mc_{F,t}$) is decreasing in η: a higher degree of price stickiness implies lower pass-through of changes in the exchange rate and marginal costs in the short run. In the long-run, however, there is complete pass-through of permanent changes in the exchange rate and marginal costs.

Figure 1 illustrates the response of import prices to an unexpected 1% depreciation of the nominal exchange rate for different values of the price stickiness parameter η. The period length is assumed to be one quarter. The responses are conditional on given values of foreign marginal costs, and the discount factor β is set to 0.99. The exchange rate is assumed to follow a random walk. Hence, the exchange rate shock is perceived to be permanent and the long-run exchange rate pass-through is 100%. A higher degree of price stickiness reduces the short-run pass-through and also makes it more gradual. If firms change prices on average every two quarters ($\eta = 0.5$), the model predicts that the pass-through is 50% in the first quarter and near complete after one year. If the average time between price changes is four quarters ($\eta = 0.75$), on the other hand, the immediate pass-through is about 25%, increasing to 75% after about four quarters. In the case where firms only adjust prices on average every eight quarters ($\eta = 0.875$), the short-run pass-through is 12.5%.

Figure 2 illustrates the dependence of pass-through on the (expected) persistence of the exchange rate. The exchange rate is now assumed to follow a first-order autoregressive process: $s_t = \tau s_{t-1} + \varepsilon_{s,t}$, where $\varepsilon_{s,t}$ is white noise. We plot the responses to a purely

temporary shock ($\tau = 0$), a temporary but highly persistent shock ($\tau = 0.95$) and a permanent shock ($\tau = 1$), where the latter corresponds to the random walk assumption above. The values of β and η are kept fixed at 0.99 and 0.75, respectively. The figure illustrates that the degree of pass-through is increasing in the expected persistence of the exchange rate. In the case of a purely temporary shock, the impact effect on import prices is negligible. In response to the temporary but persistent shock, exchange rate pass-through is around 20% in the first quarter, compared to 25% in the case of a permanent exchange rate shock. This illustrates that, in general, the comovement of exchange rates and import prices depends on the nature of the shock that causes the variables to move. The long-run pass-through of the exchange rate shock is complete in all cases in the sense that the long-run effects on the exchange rate and import prices are the same (i.e., zero unless the exchange rate shock is perceived to be permanent).

2.1.2 A hybrid model with local currency pricing

In the literature on the New Keynesian Phillips Curve it is common to consider 'hybrid' specifications with both forward-looking and backward-looking components. In the model derived above, firms that are not allowed to re-optimise prices in period t charge a price equal to the price charged in period $t-1$. Here, following Smets & Wouters (2002), we assume instead that firms that are not allowed to re-optimise prices in period t update their prices according to the partial indexation rule

$$P_{F,t}(i) = \left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\chi} P_{F,t-1}(i), \tag{12}$$

where $\chi \in [0,1]$ is the indexation parameter. If $\chi = 1$ (full indexation), this scheme collapses to the dynamic indexation scheme considered by Christiano et al. (2005). The problem facing a firm that is allowed to re-optimise in period t is now

$$\max_{\widetilde{P}_{F,t}(i)} E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\frac{\widetilde{P}_{F,t}(i) \left(\frac{P_{F,\tau-1}}{P_{F,t-1}} \right)^{\chi}}{S_{\tau}} - MC_{F,\tau} \right) \left(\frac{\widetilde{P}_{F,t}(i) \left(\frac{P_{F,\tau-1}}{P_{F,t-1}} \right)^{\chi}}{P_{F,\tau}} \right)^{-\varepsilon} Y_{F,\tau}, \tag{13}$$

and the aggregate price index is

$$P_{F,t}^{1-\varepsilon} = (1-\eta)\widetilde{P}_{F,t}^{1-\varepsilon} + \eta \left(\left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\chi} P_{F,t-1} \right)^{1-\varepsilon}. \tag{14}$$

The log-linearised equilibrium condition for aggregate import price growth becomes

$$\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} - \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)} (p_{F,t} - s_t - mc_{F,t}). \tag{15}$$

Hence the lagged growth in import prices enters the equation. The weight on lagged import price growth is increasing in the degree of indexation. However, the maximum weight on lagged price growth (obtained for $\chi = 1$) is $1/(1+\beta) \simeq 0.5$ for values of β close to unity.

Figure 3 shows the response of import prices to a 1% permanent exchange rate shock for different values of the indexation parameter χ when $\beta = 0.99$ and $\eta = 0.75$. Varying the degree of indexation has a relatively small effect on the short-run pass-through. The differences become more pronounced after about four quarters, however. In the medium run, the pass-through is higher for higher values of the indexation parameter; that is, pass-through is higher the larger is the weight on lagged import price growth in the import price equation. With full indexation ($\chi = 1$) the import price overshoots the flexible price level and pass-through exceeds 100% in the medium run.

2.1.3 Models with local- and producer currency pricing[6]

Evidence on the currency denomination of foreign trade suggests that a substantial share of imports are invoiced in the exporter's currency (see e.g., Bekx, 1998).[7] This motivates extending the model to allow a subset ϕ of foreign firms to engage in PCP and a subset $1 - \phi$ to engage in LCP, as in e.g., Betts & Devereux (1996, 2000), Bergin (2006) and Choudhri et al. (2005). Following Bergin (2006) and Choudhri et al. (2005), we assume that PCP firms are able to segment markets, but choose to set prices in their own currency.[8]

Admittedly, a limitation of our framework is that the fraction of price setters engaging in PCP is assumed to be constant and independent of the other parameters in the model. A recent literature considers the optimal choice of invoicing currency in the context of NOEM models (e.g., Devereux et al., 2004; Bacchetta & van Wincoop, 2005; Goldberg & Tille, 2005). The choice between LCP and PCP is found to depend on several factors, including the exporting firm's market share in the foreign market, the degree of substitutability between foreign and domestic goods and relative monetary stability. Thus, the parameter ϕ could vary over time.

Let $P_{F,t}^{P}$ and $P_{F,t}^{L}$ denote the prices set by PCP firms and LCP firms respectively. The aggregate import price index can then be written

$$P_{F,t} = \left[\phi \left(S_t P_{F,t}^{P} \right)^{1-\varepsilon} + (1-\phi)(P_{F,t}^{L})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{16}$$

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[6]See appendix A.1 for details on the derivations in this subsection.

[7]The share of UK imports invoiced in sterling in the years 1999 to 2002 was approximately 40%. See http://customs.hmrc.gov.uk/.

[8]Betts & Devereux (1996, 2000) assume that PCP firms are unable to segment markets internationally and hence cannot price discriminate.

We assume that the frequency of price adjustment η is the same for LCP and PCP firms.[9] The LCP firms' price-setting problem is the same as in equation (4) above, while the optimisation problem facing a PCP firm that re-optimises in period t is

$$\max_{\widetilde{P}_{F,t}^P(i)} E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\widetilde{P}_{F,t}^P(i) - MC_{F,\tau} \right) \left(\frac{S_\tau \widetilde{P}_{F,t}^P(i)}{P_{F,\tau}} \right)^{-\varepsilon} Y_{F,\tau}. \tag{17}$$

The log-linearised equilibrium conditions for LCP firms and PCP firms are

$$\Delta p_{F,t}^L = \beta E_t \Delta p_{F,t+1}^L - \frac{(1-\beta\eta)(1-\eta)}{\eta} \left(p_{F,t}^L - s_t - mc_{F,t} \right) \tag{18}$$

$$\Delta p_{F,t}^P = \beta E_t \Delta p_{F,t+1}^P - \frac{(1-\beta\eta)(1-\eta)}{\eta} \left(p_{F,t}^P - mc_{F,t} \right) \tag{19}$$

Using the definition of the aggregate price index in (16) we obtain the following expression for aggregate import price inflation

$$\Delta p_{F,t} = \phi(\Delta s_t + \Delta p_{F,t}^P) + (1-\phi)\Delta p_{F,t}^L \tag{20}$$

$$= \beta E_t \Delta p_{F,t+1} - \frac{(1-\beta\eta)(1-\eta)}{\eta}(p_{F,t} - s_t - mc_{F,t}) + \phi(\Delta s_t - \beta E_t \Delta s_{t+1})$$

When some firms engage in PCP ($\phi > 0$), the aggregate price equation is augmented with the current change in the exchange rate and the expected change in the exchange rate in the next period. The latter term reflects that PCP firms set prices according to expected future price growth measured in their own currency ($\Delta p_{F,t+1}^P$) rather than import price growth measured in the importing country's currency ($\Delta p_{F,t+1}^P + \Delta s_{t+1}$) which appears in the definition of aggregate import price growth ($\Delta p_{F,t+1}$).

Figure 4 shows the response of import prices to a permanent 1% exchange rate shock for different values of the share of PCP price setters ϕ. The remaining parameters take the values $\beta = 0.99$ and $\eta = 0.75$. The short-run pass-through is increasing in the share of exporters that engages in PCP. If all firms engage in PCP ($\phi = 1$), the pass-through is complete at all horizons (this holds regardless of the degree of price stickiness). The short-run pass-through is (as noted above) about 25% in the absence of PCP firms ($\phi = 0$). When the share of PCP firms is 0.5 ($\phi = 0.5$), the short-run pass-through increases to 62.5%.

The model can be extended to allow for inflation indexation in the same manner as above. We assume that PCP and LCP firms index their prices to last period's aggregate inflation rate, measured in the exporting and the importing country's currency, respec-

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[9]This assumption has some empirical support. Using micro data for traded goods prices at the docks for the US, Gopinath & Rigobon (2006) find that the stickiness of prices invoiced in foreign currencies in terms of foreign currency is similar to the stickiness of prices invoiced in dollars in terms of dollars.

tively. That is, firms that do not re-optimise in period t update their prices according to the rules

$$P_{F,t}^P(i) = \left(\frac{P_{F,t-1}}{P_{F,t-2}} \frac{S_{t-2}}{S_{t-1}} \right)^{\chi} P_{F,t-1}^P(i) \text{ and } P_{F,t}^L(i) = \left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\chi} P_{F,t-1}^L(i) \qquad (21)$$

The equation describing the evolution of aggregate import price inflation is now[10]

$$\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} - \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)} \left(p_{F,t} - s_t - mc_{F,t} \right) \quad (22)$$
$$+ \phi \left(\Delta s_t - \frac{\beta}{1+\beta\chi} E_t \Delta s_{t+1} - \frac{\chi}{1+\beta\chi} \Delta s_{t-1} \right)$$

Thus, the model with both LCP- and PCP firms and inflation indexation ascribes separate roles for both lagged import price growth and the lagged change in the exchange rate in determining import prices.

2.2 Pricing-to-market models

In the models considered so far, the frictionless mark-up is constant. Hence, the only mechanism generating incomplete pass-through is local currency price stickiness. In this section we consider two New Keynesian open-economy models that have the feature that the elasticity of demand perceived by the exporter is non-constant: a model with translog preferences due to Bergin & Feenstra (2001) and the distribution cost model due to Corsetti & Dedola (2005). In these models, the frictionless mark-up is a function of domestic prices in the importing country. This creates a scope for price discrimination and acts to reduce the degree of pass-through. Adopting the terminology in Bergin & Feenstra (2001), we refer to models with this property as pricing-to-market models. Several previous empirical studies have found evidence of long-run pricing-to-market, also in small open economies (see e.g., Menon, 1995; Naug & Nymoen, 1996; Herzberg et al., 2003; Kongsted, 2003).

2.2.1 A model with translog preferences[11]

Following Bergin & Feenstra (2001), we assume that there are a large number N of varieties of goods available in the domestic market. Of these, goods indexed $i = 1,\ldots,N_H$ are produced by domestic firms, and goods indexed $i = N_H + 1,\ldots,N$ are produced by foreign firms. The aggregate ideal price index, P_t, implied by the translog expenditure

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[10]Since we are assuming that the indexation parameter is the same for PCP- and LCP firms, we would obtain the same expression for aggregate import price inflation if we instead allowed PCP- and LCP firms to index their prices to last period's aggregate PCP- and LCP inflation rate, respectively.

[11]See appendix A.2 for details on the derivation.

function is[12]

$$\ln P_t \equiv \sum_{i=1}^{N} \alpha_i \ln P_t(i) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln P_t(i) \ln P_t(j), \tag{23}$$

where $\gamma_{ij} = \gamma_{ji}$. The prices of the imported goods are import prices (i.e., measured 'at the docks'). In the special case where all goods enter the expenditure function symmetrically, the parameters become

$$\alpha_i = \frac{1}{N}, \gamma_{ii} = -\frac{\gamma}{N}, \gamma_{ij} = \frac{\gamma}{N(N-1)} \text{ for } i \neq j, \tag{24}$$

where $\gamma > 0$. With these restrictions, the expenditure function is homogenous of degree one: $\sum_{i=1}^{N} \alpha_i = 1$ and $\sum_{i=1}^{N} \gamma_{ij} = \sum_{j=1}^{N} \gamma_{ij} = 0$. The demand for good i is given by

$$Y_t(i) = \psi_t(i) \frac{P_t Y_t}{P_t(i)}, \tag{25}$$

where Y_t is the aggregate demand for goods in the home country and $\psi_t(i)$ is the expenditure share on good i, defined as

$$\psi_t(i) = \frac{\partial \ln P_t}{\partial \ln P_t(i)} = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln P_t(j). \tag{26}$$

The elasticity of demand for each good is then

$$\varepsilon_t(i) = 1 - \frac{\partial \ln \psi_t(i)}{\partial \ln P_t(i)} = 1 - \frac{\gamma_{ii}}{\psi_t(i)}, \quad \gamma_{ii} < 0. \tag{27}$$

The demand elasticity depends negatively on the price of competing products. Hence, a fall in the competitors' prices will lead to a reduction in the desired mark-up.[13] If prices are flexible, the optimal price set by a foreign firm satisfies

$$P_t(i) = \frac{\varepsilon_t(i)}{\varepsilon_t(i) - 1} S_t MC_{F,t}, \qquad i = N_H + 1, \ldots, N, \tag{28}$$

which, using the expression for the demand elasticity in (27), can be written as

$$\frac{S_t MC_{F,t}}{P_t(i)} \left(1 - \frac{\psi_t(i)}{\gamma_{ii}}\right) - 1 = 0. \tag{29}$$

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[12]There is no closed form solution for the direct utility function. See Feenstra (2003) for details.

[13]This is also a property of the more general preference specification proposed by Kimball (1995) that has become popular in the literature on the New Keynesian Phillips Curve (see e.g., Eichenbaum & Fisher, 2004; Woodford, 2005). See Gust & Sheets (2006) for an open-economy application.

Following Bergin & Feenstra (2001), the left-hand side of the equation can be approximated as[14]

$$\frac{S_\tau MC_{F,\tau}}{P_t(i)}\left(1-\frac{\psi_\tau(i)}{\gamma_{ii}}\right)-1 \approx \ln\left(\frac{S_\tau MC_{F,\tau}}{P_t(i)}\left(1-\frac{\psi_\tau(i)}{\gamma_{ii}}\right)\right) \tag{30}$$
$$\approx -\frac{\psi_\tau(i)}{\gamma_{ii}}+\ln\left(\frac{S_\tau MC_{F,\tau}}{P_t(i)}\right).$$

This allows us to rewrite the first-order condition as

$$\ln P_t(i) = \frac{1}{2}\gamma + \frac{1}{2}\left(\ln S_t + \ln MC_{F,t}\right) + \frac{1}{2}\sum_{j\neq i}^{N}\frac{1}{N-1}\ln P_t(j), \tag{31}$$

where we have substituted in for the expenditure share $\psi_t(i)$ in (26). The optimal price puts a weight of one half on marginal costs measured in the importing country's currency and one half on the competitors' prices. Holding all other prices fixed, therefore, a 1% depreciation will increase the optimal price by 0.5%. This is an important characteristic of the translog preference specification. Imposing symmetry we can rewrite (31) as

$$p_{F,t} = \frac{N-1}{N+N_H-1}\left(s_t + mc_{F,t}\right) + \frac{N_H}{N+N_H-1}p_{H,t}, \tag{32}$$

where lower case letters represent percent deviations from the deterministic steady state, and $p_{H,t}$ and $p_{F,t}$ denote the common price set by the domestic and foreign firms, respectively.

Equation (32) shows that the optimal frictionless price is a function of the price of import-competing goods. Translog preferences are thus a source of strategic complementarity in price-setting (see Woodford, 2003, p. 161). Holding marginal costs and the prices of import-competing goods fixed, the exchange rate pass-through is incomplete. Intuitively, a foreign firm will take into account that, if the prices of importing-competing goods are kept constant, an increase in its price will cause demand to become more elastic. This lowers the desired mark-up and reduces the incentive to raise the price following a depreciation of the exchange rate relative to the CES case. Rather than passing it through completely to the buyer, the exporter will absorb part of an exchange rate depreciation in her mark-up. The degree of (conditional) pass-through is inversely related to the share of domestic firms in the importing country. Note, however, that the import price equation is linearly homogenous in marginal costs and the price of import-competing goods. In a fully-specified general equilibrium model that satisfies

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[14]The first approximation holds if $\frac{S_t MC_{F,t}}{P_t(i)}\left(1-\frac{\psi_t(i)}{\gamma_{ii}}\right)$ is close to unity (meaning that the price is not too different from the optimal frictionless price), and the second approximation holds if $\psi_t(i)$ is small (see Bergin & Feenstra, 2001).

nominal neutrality, the unconditional pass-through of an exogenous shock to the nominal exchange rate (i.e., taking into account the effects of the exchange rate change on domestic prices) is complete.

In Bergin & Feenstra (2001), prices are set in two-period overlapping contracts. Here, we combine the translog preference specification with the Calvo pricing model. As before, we assume that a fraction $1 - \eta$ of firms are allowed to re-optimise prices in a given period. Following Bergin & Feenstra (op.cit) we assume that all firms engage in LCP. The price-setting problem of a foreign firm that is allowed to reset prices in period t is

$$\max_{\widetilde{P}_t(i)} E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\frac{\widetilde{P}_t(i)}{S_\tau} - MC_{F,\tau} \right) Y_\tau(i), \qquad (33)$$

and the optimal price $\widetilde{P}_t(i)$ satisfies

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{P_\tau Y_\tau}{S_\tau} \left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \left(1 - \frac{\psi_\tau(i)}{\gamma_{ii}} \right) - 1 \right) = 0. \qquad (34)$$

Taking a log-linear approximation of the first-order condition around a zero inflation steady state, we obtain

$$\widetilde{p}_t(i) = \frac{1}{2}(1 - \beta\eta)E_t \sum_{\tau=t}^{\infty} (\beta\eta)^{\tau-t} \left(s_\tau + mc_{F,\tau} + \frac{N_H}{N-1} p_{H,\tau} + \frac{N-N_H-1}{N-1} p_{F,\tau} \right). \qquad (35)$$

Imposing symmetry and assuming that the number of foreign exporters ($N - N_H$) is large, the aggregate import price index can be written

$$p_{F,t} = (1 - \eta)\widetilde{p}_t + \eta p_{F,t-1}, \qquad (36)$$

and the equation for aggregate import price inflation becomes

$$\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} \qquad (37)$$
$$- \frac{(1-\beta\eta)(1-\eta)}{\eta} \frac{1}{2} \frac{N+N_H-1}{N-1} \left(p_{F,t} - \frac{N-1}{N+N_H-1}(s_t + mc_{F,t}) - \frac{N_H}{N+N_H-1} p_{H,t} \right).$$

The effect of the forcing term is smaller than in the CES case. A foreign firm that contemplates raising the price of its product will take into account that, since not all foreign firms are allowed to change prices in the short run, an increase in its price will cause demand to become more elastic.[15] This reduces the incentive to raise the price. The model with translog preferences thus requires a smaller amount of nominal rigidities to generate slow exchange rate pass-through than the models with CES preferences. Notice

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[15]The effect on the demand elasticity of the fact that domestic firms' prices are kept fixed is captured by the forcing term.

that this holds also when there are no import-competing firms in the domestic market (i.e., $N_H = 0$), in which case the coefficient on the forcing term is reduced by a factor of two relative to the CES case.

Figure 5 shows the response of import prices to a permanent 1% shock to the exchange rate for different values of the share of import-competing firms (N_H/N). The price of import-competing goods is held fixed, and the remaining parameters take the values $\beta = 0.99$ and $\eta = 0.75$. The figure illustrates that, with translog preferences and a positive share of import-competing firms, the long-run pass-through is incomplete as long as domestic prices $p_{H,t}$ are constant. If foreign and domestic firms have an equal market share ($N_H/N = 0.5$), the long-run pass-through is 67%; the long-run pass-through is 53% if the market share of domestic firms is 0.9 ($N_H/N = 0.9$). In the case of no import-competing firms ($N_H/N = 0$), the long-run pass-through is complete. However, even in the absence of import-competing firms, short- and medium-run pass-through is still lower than in the models that assume a constant demand elasticity.

2.2.2 A model with distribution costs

The final model we consider is the distribution cost model in Corsetti & Dedola (2005). The key assumption is that the distribution of traded goods requires the input of local goods and services such as e.g., transportation, marketing and retail services. The distribution technology is Leontief; the distribution of one unit of imported goods to domestic households requires the input of μ units of local goods and services. If the distribution sector is perfectly competitive, the price paid by home consumers for a unit of the imported good (the 'retail' price), $\overline{P}_{F,t}$, is

$$\overline{P}_{F,t} = P_{F,t} + \mu P_{N,t}, \tag{38}$$

where $P_{F,t}$ denotes the import price and $P_{N,t}$ is the price of the local goods and services that are required to distribute the good in the importing country.

Consumers are assumed to have CES preferences over differentiated goods. When setting prices, the exporter takes into account that the price paid by the consumers depends on the distribution costs. In the flexible price case the exporter's price-setting problem is

$$\max_{P_{F,t}(i)} \left(\frac{P_{F,t}(i)}{S_t} - MC_{F,t} \right) \left(\frac{P_{F,t}(i) + \mu P_{N,t}}{\overline{P}_{F,t}} \right)^{-\varepsilon} Y_{F,t}, \tag{39}$$

and the optimal price satisfies

$$P_{F,t} = \frac{\varepsilon}{\varepsilon - 1} S_t MC_{F,t} + \frac{\mu}{\varepsilon - 1} P_{N,t}. \tag{40}$$

This equation shows that, in the presence of distribution costs, the optimal price varies across destination markets. Moreover, the desired mark-up is a function of the exchange rate and the distribution costs in the importing country. This can be seen more clearly if we rewrite (40) as

$$P_{F,t} = \frac{\varepsilon}{\varepsilon - 1} S_t MC_{F,t} \left(1 + \frac{\mu}{\varepsilon} \frac{P_{N,t}}{S_t MC_{F,t}} \right). \tag{41}$$

The desired mark-up is a decreasing function of the exchange rate. As in the model with translog preferences, the exporter absorbs part of an exchange rate movement in her mark-up. This lowers the degree of exchange rate pass-through to import prices. Taking a log-linear approximation around a deterministic steady-state we obtain

$$p_{F,t} = \frac{1}{1 + \zeta(mk_F - 1)} (s_t + mc_{F,t}) + \frac{\zeta(mk_F - 1)}{1 + \zeta(mk_F - 1)} p_{N,t}, \tag{42}$$

where ζ is the steady-state share of distribution costs in the retail price of imports and mk_F is the steady-state mark-up, that is

$$\zeta = \frac{\mu P_N}{\overline{P}_F} \quad \text{and} \quad mk_F = \frac{\varepsilon}{\varepsilon - 1} \left(1 + \frac{\mu}{\varepsilon} \frac{P_N}{SMC_F} \right). \tag{43}$$

The long-run exchange rate pass-through is decreasing in the share of distribution costs in the retail price of imports. Note that the weights on foreign marginal costs and domestic distribution prices in the expression for the optimal frictionless price in (42) sum to unity. Hence, the model satisfies long-run price homogeneity. In the benchmark calibration in Corsetti et al. (2005), $\zeta = 0.5$ and $mk_F \simeq 1.15$, which implies that the long-run pass-through coefficient is equal to 0.93. Thus, the long-run pass-through is close to being complete, even when the share of distribution costs in the retail price of imports is large.

As a next step, we derive a dynamic import price equation using the Calvo assumption.[16] An LCP exporter who is allowed to reset price in period t faces the following optimisation problem

$$\max_{\widetilde{P}_{F,t}(i)} E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\frac{\widetilde{P}_{F,t}(i)}{S_\tau} - MC_{F,\tau} \right) \left(\frac{\widetilde{P}_{F,t}(i) + \mu P_{N,\tau}}{\overline{P}_{F,\tau}} \right)^{-\varepsilon} Y_{F,\tau}. \tag{44}$$

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[16]Choudhri et al. (2005) combine distribution costs with the Calvo assumption, but specify distribution costs in terms of labour services rather than non-traded goods. Corsetti et al. (2005) derive an import price equation similar to ours assuming quadratic adjustment costs in pricing.

Imposing symmetry, the first-order condition is

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} Y_{F,\tau} \left(\frac{\widetilde{P}_{F,t} + \mu P_{N,\tau}}{\overline{P}_{F,\tau}} \right)^{-\varepsilon} \left(\frac{1}{S_\tau} - \varepsilon \frac{\left(\frac{\widetilde{P}_{F,t}}{S_\tau} - MC_{F,\tau} \right)}{\widetilde{P}_{F,t} + \mu P_{N,\tau}} \right) = 0. \qquad (45)$$

The aggregate price index is the same as before, and the log-linearised import price equation becomes

$$\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (46)$$
$$- \frac{(1-\eta\beta)(1-\eta)}{\eta} \left(p_{F,t} - \frac{1}{1+\zeta(mk_F-1)} (s_t + mc_{F,t}) - \frac{\zeta(mk_F-1)}{1+\zeta(mk_F-1)} p_{N,t} \right).$$

The import price equation takes the same general form as above: current import price inflation depends on expected future inflation and the deviation of the current price from the frictionless price. The coefficient on the forcing term is the same as in the LCP model with CES preferences.

Figure 6 shows the response of import prices to a 1% permanent exchange rate shock for different values of ζ. The price of domestic goods and services is held fixed, and $\beta = 0.99$ and $\eta = 0.75$. As is evident from the graph, the degree of pass-through is not very sensitive to the size of the share of distribution costs in the retail price of imports. For the parameter values considered here, the pass-through is still close to 90% after twenty periods, even when the share of distribution costs is as high as 0.75.

We can extend the distribution cost model to allow for PCP price setters and inflation indexation.[17] The equation for aggregate import price growth takes the same form as equation (22) above, except that the forcing term is now $p_{F,t} - \frac{1}{1+\zeta(mk_F-1)} (s_t + mc_{F,t}) -$ $\frac{\zeta(mk_F-1)}{1+\zeta(mk_F-1)} p_{N,t}$. An interesting feature of the pricing-to-market models is that when the share of PCP firms is sufficiently large, the short-run pass-through of a permanent exchange rate depreciation will exceed the long-run (conditional) pass-through. The intuition is as follows: with pricing-to-market, the frictionless mark-up falls in response to an exchange rate depreciation. Because prices are sticky in the exporter's currency, it takes time before this mark-up adjustment is fully reflected in the export price. Import prices respond instantly to the exchange rate depreciation, however. By similar reasoning, in a pure PCP model with pricing-to-market, the short-run pass-through will be *decreasing* in the frequency of price adjustment: the lower is the degree of price stickiness, the stronger is the short-run effect of an exchange rate change on export prices and hence, the weaker

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[17]The model in Choudhri et al. (2005) has distribution costs and a combination of LCP- and PCP price setters. Laxton & Pesenti (2003) combine distribution costs and a specification of adjustment costs in pricing that implies a linearised equation for aggregate import price inflation that is observationally equivalent to a Calvo model with full indexation.

is the effect on import prices.

3   EMPIRICAL IMPLEMENTATION

We estimate the New Keynesian import price equations using data for two small open economies: the UK and Norway. In this section we discuss the data used in the empirical analysis (section 3.1), the econometric model specification (section 3.2), the GMM estimator (section 3.3) and the implications of the New Keynesian import price model for the cointegration properties of the variables (section 3.4).

3.1   Data

The data are seasonally adjusted, quarterly series covering the period 1980Q1–2003Q1 (see appendix B for details on the variable definitions and sources). The import price series is an index of import prices of manufactures and the exchange rate is a broad trade-weighted nominal exchange rate.

To implement the price-setting rules empirically, we need a measure of the marginal costs of foreign firms. Following Batini et al. (2005), we assume that the marginal cost of producing value added output depends on unit labour costs and the price of raw materials input. Specifically, we assume that the log-linearised equation for foreign marginal costs is given by

$$mc_{F,t} = (1-\delta)ulc_{F,t} + \delta p_{COM,t}, \qquad (47)$$

where $ulc_{F,t}$ denotes foreign unit labour costs, $p_{COM,t}$ denotes the price of raw materials and δ is a parameter to be estimated. The dataseries for foreign unit labour costs are constructed using data for domestic unit labour costs and trade-weighted relative unit labour costs in manufacturing industries. As a proxy for the price of raw materials we use an index of the world price of metals constructed by the IMF. The commodity price index is converted into foreign currency using the trade-weighted nominal exchange rate.

To estimate the pricing-to-market models we need a measure of domestic prices; in the distribution cost model import prices depend on the price of local goods and services, and the model with translog preferences predicts that import prices depend on the prices of import competing products. We use domestic unit labour costs as a proxy for distribution costs in the importing country. As a proxy for the price of import-competing goods we use a producer price index for manufactures sold in the domestic market.

3.2 Econometric model specification

The import price equations derived above can be obtained as restricted versions of the following general specification:

$$\Delta p_{F,t} = \alpha_1 E_t \Delta p_{F,t+1} + \alpha_2 \Delta p_{F,t-1} + \alpha_3 \left(\Delta s_t - \alpha_1 E_t \Delta s_{t+1} - \alpha_2 \Delta s_{t-1} \right) \tag{48}$$
$$+ \alpha_4 \left(s_t + ulc_{F,t} - p_{F,t} \right) + \alpha_5 \left(s_t + p_{COM,t} - p_{F,t} \right) + \alpha_6 (p_{H,t} - p_{F,t}) + u_t,$$

where the error term u_t is assumed to be a mean zero, serially uncorrelated process and can be interpreted as arising from e.g., exogenous variations in the desired mark-up (see e.g., Adolfson et al., 2005). $\Upsilon = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ depends on the underlying structural parameters:

$$\alpha_1 = \frac{\beta}{1 + \beta\chi}$$
$$\alpha_2 = \frac{\chi}{1 + \beta\chi}$$
$$\alpha_3 = \phi$$
$$\alpha_4 = \frac{1}{1 + \beta\chi} \frac{(1 - \beta\eta)(1 - \eta)}{\eta} \xi\rho(1 - \delta)$$
$$\alpha_5 = \frac{1}{1 + \beta\chi} \frac{(1 - \beta\eta)(1 - \eta)}{\eta} \xi\rho\delta$$
$$\alpha_6 = \frac{1}{1 + \beta\chi} \frac{(1 - \beta\eta)(1 - \eta)}{\eta} \xi(1 - \rho)$$

When the frictionless mark-up is constant $\xi = \rho = 1$. In the distribution cost model, $p_{H,t}$ is the price of local goods and services used in the distribution of imported goods, $\xi = 1$ and $\rho = 1/(1 + \zeta(mk_F - 1))$, where the latter is inversely related to the distribution cost parameter μ. In the model with translog preferences, $p_{H,t}$ represents the price of import-competing goods, $\xi = \frac{1}{2} \frac{N + N_H - 1}{N - 1}$ and $\rho = \frac{N - 1}{N + N_H - 1}$. The latter is inversely related to the share of domestic firms in the domestic market.

To increase the generality of the results, we do not impose all the restrictions implied by the Calvo model with indexation. For example, an equation like (48) could be derived from a Calvo model without indexation if a share of the firms that are allowed to change their prices did not set their prices optimally, but applied a rule of thumb based on the recent pricing behaviour of their competitors (see Galí & Gertler, 1999). The exact interpretation of the coefficients in the import price equation would be different, however. Equation (48) could also be derived from a model with quadratic costs of price adjustment as in Rotemberg (1982). The presence of lagged import price growth in the equation could then be motivated by quadratic costs of adjusting the level of import price

growth relative to the previous period's import price growth (see e.g., Price, 1992; Ireland, 2001). Again, although the form of the linearised import price equation would be the same, the interpretation of the coefficients would be different. Finally, the model is potentially consistent with alternative models of pricing-to-market that imply that the optimal frictionless price can be written as a linear combination of the exporters' marginal costs and domestic prices in the importing country.

3.3   The GMM estimator

We estimate the models using GMM. Limited information methods such as GMM do not rely on a specific completing model for the driving variables. This is an advantage given the lack of a satisfactory structural model for the exchange rate and the challenge involved in recovering a stable reduced form model for the exchange rate over a period that covers several monetary policy regimes.[18]

Let $\varepsilon_{t+1} \equiv \Delta p_{F,t+1} - E_t \Delta p_{F,t+1}$ and $v_{t+1} \equiv \Delta s_{t+1} - E_t \Delta s_{t+1}$, and let $\mathcal{F}_t$ denote the exporting firm's information set at time t. Then, according to the rational expectations hypothesis, $E_t[\varepsilon_{t+1}|\mathcal{F}_t] = E_t[v_{t+1}|\mathcal{F}_t] = 0$. Replacing the expected values with the actual realisations of the variables and adding a constant term we obtain the following estimating equation

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2 \Delta p_{F,t-1} + \alpha_3 \left(\Delta s_t - \alpha_1 \Delta s_{t+1} - \alpha_2 \Delta s_{t-1}\right) \tag{49}$$
$$+ \alpha_4 \left(s_t + ulc_{F,t} - p_{F,t}\right) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \alpha_6 (p_{H,t} - p_{F,t}) + \omega_t,$$

where $\omega_t \equiv u_t - \alpha_1\left(\varepsilon_{t+1} - \alpha_3 v_{t+1}\right)$ is a linear combination of the rational expectations forecast errors and the 'structural' disturbance term u_t. By construction, the disturbance ω_t is correlated with the regressors, which implies that ordinary least squares will not yield consistent estimates of the parameters in the model.

Let $z_t \in \mathcal{F}_t$ denote a $q \times 1$ vector of variables satisfying $E_t[u_t z_t] = 0$. Then, it follows from the definition of the disturbance term ω_t that z_t is a vector of valid instruments, that is

$$E_t[\omega_t z_t] = 0, \quad t = 1, \ldots, T \tag{50}$$

These moment conditions provide the basis for the GMM estimation. The GMM estima-

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[18]This advantage may come at a cost of lower efficiency as not all cross-restrictions implied by the rational expectations hypothesis are imposed during estimation. Moreover, as emphasised by Pesaran (1987) and Mavroeidis (2004), the strength of identification in GMM depends on properties of the processes governing the driving variables.

tor is given by

$$\widehat{\Upsilon} = \underset{\Upsilon}{\arg\min} \left(\frac{1}{T} \sum_{t=1}^{T} \omega_t (\Upsilon) z_t \right)' W_T \left(\frac{1}{T} \sum_{t=1}^{T} \omega_t (\Upsilon) z_t \right), \quad (51)$$

where W_T is a positive semi-definite weighting matrix. Under certain regularity conditions (see Hall, 2005, p. 50), the GMM estimator is consistent and asymptotically normal. The asymptotic variance of $\widehat{\Upsilon}$ is minimised by setting the weighting matrix W_T equal to a consistent estimate of the inverse of the long-run covariance matrix of the sample moments:

$$S = \lim_{T \to \infty} Var \left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \omega_t (\Upsilon) z_t \right]$$
$$= \Gamma_0 + \sum_{i=1}^{\infty} \left(\Gamma_i + \Gamma_i' \right),$$

where Γ_i is the ith autocovariance matrix of the sample moments.

The composite disturbance term ω_t can be shown to have a first-order moving-average representation (see Pesaran, 1987, p. 191).[19] This compels us to use a heteroskedasticity and autocorrelation consistent (HAC) estimate of S,

$$\widehat{S}_T = \widehat{\Gamma}_0 + \sum_{i=1}^{l} \upsilon_i \left(\widehat{\Gamma}_i + \widehat{\Gamma}_i' \right), \quad (52)$$

where $\widehat{\Gamma}_i$ are the sample autocovariances, and l denotes the bound on how many autocovariances are used to form the estimate. To ensure that $\widehat{S}_T$ is positive semi-definite in finite samples, the autocovariances are weighted using the kernel υ_i. Below we use the Bartlett kernel as proposed by Newey & West (1987)

$$\upsilon_i = \begin{cases} 1 - \frac{i}{b+1} & \text{for } \frac{i}{b+1} \leq 1 \\ 0 & \text{for } \frac{i}{b+1} > 1 \end{cases} \quad (53)$$

where b is the bandwidth parameter chosen by the investigator. Since the behaviour of the HAC estimator of the covariance matrix can be highly sensitive to the choice of bandwidth parameter, den Haan & Levin (1996) recommend using more than one approach when estimating the covariance matrix. Below we therefore consider three different choices of bandwidth: a fixed bandwidth equal to 1, a fixed bandwidth equal to 3, and the bandwidth selected by the data-based method proposed by Newey & West (1994). The

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[19]Thus, first-order residual autocorrelation is not in itself a valid cause for rejecting the New Keynesian import price equation. However, autocorrelation could also be due to model misspecification caused by e.g, omitted variables.

latter depends on the autocovariances of the moment conditions and typically results in a quite large bandwidth for our data.

A test of the over-identifying restrictions can in principle be based on the J-test statistic of Hansen (1982)

$$J = \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \omega_t \left(\widehat{\Upsilon} \right) z_t \right)' \widehat{S}_T^{-1} \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \omega_t \left(\widehat{\Upsilon} \right) z_t \right) \xrightarrow{d} \chi^2(q-r), \tag{54}$$

where r is the number of parameters to be estimated and $q - r$ denotes the number of over-identifying restrictions. However, Monte Carlo evidence in Mavroeidis (2005) suggests that the finite-sample power of the $J-$test to detect misspecification in forward-looking inflation equations is low, particularly when the number of instruments is large or the HAC estimate of the covariance matrix allows for a very general correction for autocorrelation.

Finally, a fundamental condition for consistency of the GMM estimator is that the population moment condition in (50) is satisfied at only one value in the parameter space (see e.g., Hall, 2005, p. 51). If this condition is satisfied, we say that the parameter vector Υ is identified. However, the literature on weak identification in GMM estimation (see Stock et al. (2002) for a survey) has demonstrated that generic identification is not sufficient to ensure reliable inference in finite samples. If the parameters are weakly identified; that is, if the instruments are only weakly correlated with the endogenous variables, conventional point estimates and confidence intervals based on the asymptotic normal approximation will be misleading, even in large samples. In the models with both PCP and LCP, we need instruments for the rate of exchange rate depreciation in period $t + 1$. The fact that it has proven difficult to beat the random walk forecast of exchange rates suggests that weak identification might be of particular concern when estimating these models.

3.4   *The cointegration implications of the New Keynesian import price models*

The asymptotic properties of the GMM estimator are derived under the assumption that the variables in the model are stationary. GMM estimation of equation (49) implicitly assumes that $s_t + ulc_{F,t} - p_{F,t}$, $s_t + p_{COM,t} - p_{F,t}$ and $p_{H,t} - p_{F,t}$ are stationary or cointegrated.

Cointegration is a testable implication of the theoretical model. Focusing on the case where the variables in the model are at most integrated of order one, $I(1)$, the New Keynesian import price models imply that import prices are cointegrated with the optimal frictionless price. The models with a constant frictionless mark-up imply that import prices should be cointegrated with foreign marginal costs measured in domestic currency,

that is

$$p_{F,t} - mc_{F,t} - s_t \sim I(0). \tag{55}$$

In this case, the long-run exchange rate pass-through, measured as the long-run elasticity of import prices with respect to the exchange rate, keeping marginal costs fixed, is complete. With our measure of marginal costs, the models with a constant frictionless mark-up imply that

$$p_{F,t} - s_t - (1-\delta)ulc_{F,t} - \delta p_{COM,t} \sim I(0), \tag{56}$$

or, equivalently, that $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ are cointegrated with cointegration parameter $\delta/(1-\delta)$. We note that this would hold also if $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ themselves were stationary, in which case there would be two cointegrating relations among the variables.

The pricing-to-market models predict that import prices should be cointegrated with foreign marginal costs and the price of domestic goods, that is

$$p_{F,t} - \rho\,(mc_{F,t} + s_t) - (1-\rho)p_{H,t} \sim I(0). \tag{57}$$

A version of (57) has served as the theoretical starting point of many empirical studies of exchange rate pass-through.[20] It is common to interpret a significant coefficient on domestic prices (i.e., $\rho < 1$) in the cointegrating regression as evidence of long-run pricing-to-market. Notice, however, that (57) would hold if relative prices and costs themselves were stationary, that is, if

$$p_{F,t} - mc_{F,t} - s_t \sim I(0) \text{ and } p_{H,t} - p_{F,t} \sim I(0). \tag{58}$$

In this case, the theory predicts that there should be two (or three) cointegrating vectors relating the variables. Hence, a finding that $p_{F,t} - s_t - mc_{F,t} \sim I(0)$ is consistent with the pricing-to-market hypothesis.

To investigate the cointegration properties of the data, we first inspect the variables graphically and then conduct formal unit root and cointegration tests. Figure 7 plots foreign unit labour costs, commodity prices and domestic prices/costs relative to the import price index, $s_t + ulc_{F,t} - p_{F,t}$, $s_t + p_{COM,t} - p_{F,t}$ and $p_{H,t} - p_{F,t}$, for the UK and Norway. It is evident from the graphs that import prices increased less than foreign unit labour costs over the sample period. One possible explanation is that the price of raw materials increased less than unit labour costs. The graphs show that this is indeed the case. It cannot be the full explanation, however; there is no apparent downward

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[20]See e.g., Naug & Nymoen (1996) and Herzberg et al. (2003) for analyses of Norwegian and UK import prices respectively.

trend in the ratio of commodity prices to import prices. The plots thus indicate that one implication of the models with a constant frictionless mark-up, namely that import prices are set as a constant mark-up on foreign marginal costs in the long run, does not hold in the data.

According to the pricing-to-market models, a fall in import prices relative to foreign unit labour costs could be explained by a fall in domestic unit labour costs or domestic producer prices relative to import prices. However, for both countries, the domestic cost- and price indices increased more than import prices over the sample period. Thus, the long-run implications from the theoretical import price equations in section 2 are seemingly rejected by the data.

One possible interpretation of the decline in import prices relative to foreign costs is that it captures a decline in tariffs and transportation costs over the sample. It may also reflect a shift in imports from high-cost to low-cost countries spurred by trade liberalisation. This fall in the price level is not picked up in our measure of unit labour costs, which, since it is a weighted average of unit labour costs indices with a common base-year value, will only pick up differences in cost inflation (see Høegh-Omdal & Wilhelmsen (2002), Røstøen (2004) and Nickell (2005) for a discussion of this point). These factors could also help explain why import prices have fallen relative to domestic prices and costs.

The effects of trade-liberalisation are not captured by the theoretical models considered in this paper. In the empirical analysis we approximate these effects by means of a linear trend. Specifically, we detrend the variables $p_{H,t} - p_{F,t}$, $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ prior to the GMM estimation by regressing each variable on a constant and a deterministic trend. The detrended variables are plotted in figure 8. The visual impression from the graphs is that $s_t + ulc_{F,t} - p_{F,t}$, $s_t + p_{COM,t} - p_{F,t}$ and $p_{H,t} - p_{F,t}$ could be trend-stationary.

Table 1 reports the results of two different unit root tests on the detrended series: the augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979; Said & Dickey, 1984) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al., 1992).[21] The null hypothesis in the ADF test is that the variable has a unit root, while the null hypothesis in the KPSS test is that the variable is stationary. The KPSS test does not reject the null hypothesis of stationarity for any of the detrended series. Moreover, the ADF test rejects the unit root hypothesis at the 5% level for all series except the series for Norwegian unit labour costs relative to import prices. It is well-known that it is difficult to distinguish empirically between non-stationary processes and highly persistent yet stationary processes. However, on the basis of the unit root tests and the visual impression of the series, the assumption that the detrended series are stationary does not

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[21]The results are obtained using EViews version 5.

seem unreasonable.

We have also tested for cointegration using Johansen's (1988) maximum likelihood approach. The results of the cointegration analysis are reported in appendix C. From the unit root tests and the graphical inspection of the series we expect to find two or three cointegration relations between the variables. For both countries, the cointegration tests suggest that the cointegration rank is one, or possibly two. The hypotheses that $s_t + ulc_{F,t} - p_{F,t}$, $s_t + p_{COM,t} - p_{F,t}$ and $p_{H,t} - p_{F,t}$ are trend-stationary are rejected based on asymptotical critical values. However, when interpreting these results, we should keep in mind that the ability of the trace test to determine the correct cointegration rank can be low when the true cointegration rank is large relative to the number of variables in the VAR (see e.g., Jacobson et al. (1998) and chapter 5 of this thesis). Simulation evidence also suggests that the LR tests of the restrictions on the cointegration relations are over-sized in small samples (see e.g., Jacobson, 1995; Gredenhoff & Jacobson, 2001). These considerations form the basis for our decision not to impose any restrictions on the cointegration properties of the model prior to the GMM estimation. Instead, we estimate all the parameters in the model jointly, in one step.

4 GMM ESTIMATION RESULTS

This section presents the GMM estimates of the New Keynesian import price equations for the UK and Norway. We first report estimation results for the models with a constant frictionless mark-up: a purely forward-looking LCP model, a hybrid LCP model and a model that allows for both PCP and LCP. Then, we report estimates of the two pricing-to-market models.

For the models with a constant frictionless mark-up the GMM estimation is based on the following sets of instruments:

$$
z_{1,t} = \left\{
\begin{array}{l}
\sum_{i=0}^{2} \Delta s_{t-i}, \ \sum_{i=0}^{2} \Delta ulc_{F,t-i}, \ \sum_{i=0}^{2} \Delta p_{COM,t-i}, \ \sum_{i=1}^{2} \Delta p_{F,t-i}, \\
s_{t-1} + ulc_{F,t-1} - p_{F,t-1}, s_{t-1} + p_{COM,t-1} - p_{F,t-1}
\end{array}
\right\}
$$

$$
z_{2,t} = \left\{
\begin{array}{l}
\sum_{i=1}^{2} \Delta s_{t-i}, \ \sum_{i=1}^{2} \Delta ulc_{F,t-i}, \ \sum_{i=1}^{2} \Delta p_{COM,t-i}, \ \sum_{i=1}^{2} \Delta p_{F,t-i} \\
s_{t-1} + ulc_{F,t-1} - p_{F,t-1}, s_{t-1} + p_{COM,t-1} - p_{F,t-1}
\end{array}
\right\}
$$

The set $z_{1,t}$ contains current values and two lags of the first difference of the driving variables $\Delta s_t, \Delta ulc_{F,t}$ and $\Delta p_{COM,t}$, two lags of import price growth $\Delta p_{F,t}$ and lagged values of real unit labour costs $s_t + ulc_{F,t} - p_{F,t}$ and real commodity prices $s_t + p_{COM,t} - p_{F,t}$. For the current values of the driving variables to be valid instruments it must be the case that (i) the variables can be observed by the exporter before she sets prices in period t, and (ii) the variables are exogenous in the sense that $E_t[u_t \Delta s_t] = E_t[u_t \Delta ulc_{F,t}] =$

$E_t [u_t \Delta p_{COM,t}] = 0$. These conditions are strict. First, because of time lags in gathering and processing information, the exporters may base their pricing decisions on expectations dated at time $t-1$ rather than at time t. Second, measurement errors in (our proxy of) marginal costs could make it correlated with the error term in the import price equation. For these reasons, we also report results for the instrument set $z_{2,t}$, which only contains variables dated $t-1$ or earlier.[22]

As mentioned above, in the models with both PCP and LCP we need instruments for the rate of exchange rate depreciation in period $t+1$. For these models, we also considered an extended instrument set which included current and lagged values of the short-term interest rate differential between the importing country and its trading partners, as well as estimates of the output gap in the importing country and the output gap of total OECD.[23] However, the main conclusions in this section were not affected by this extension of the instrument set.

Tables 2 and 3 report the GMM estimates of the parameters in the purely forward-looking LCP model with a constant frictionless mark-up.[24] For the UK, the coefficients on the levels terms are positive and, in most cases, statistically significant. The $J-$test does not reject the validity of the over-identifying restrictions. The coefficient on the forward-term is negative or close to zero in all cases, however. We also note that the estimates are highly sensitive to the choice of instrument set and the choice of bandwidth parameter in the HAC estimate of the covariance matrix. This could be a symptom of weak identification (see Nason & Smith, 2005).[25] An estimate of δ, the weight on commodity prices in marginal costs, can be computed from the ratio of the coefficients on the level terms (see section 3.2). The estimate of δ varies from 0.41 to 0.44 when estimation is based on the instrument set $z_{1,t}$ and from 0.34 to 0.37 when the instrument set is $z_{2,t}$. These estimates are in line with what we obtained using cointegration analysis.

The evidence of forward-looking price-setting is stronger for Norway: the coefficient on the forward-term is positive and, in most cases, statistically significant. The coefficient is also not significantly different from one. Moreover, the over-identifying restrictions are not rejected using the J-test. The coefficients on the level terms are statistically insignificant, however. This holds irrespective of the choice of instrument set or the

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[22]Using only lagged instruments is common in the literature on the New Keynesian Phillips Curve (see e.g., Galí & Gertler, 1999; Galí et al., 2001).

[23]The interest rate and output gap series were taken from OECD's Economic Outlook database. The following countries were included in the measure of the trading partners' interest rate: Australia, Canada, Japan, the euro area, Sweden, the US, Switzerland and in the case of Norway; the UK. The weights are based on the trade-weights used to construct the effective exchange rate (fixed 1995 weights).

[24]The results are obtained using the simultaneous-updating GMM estimator in EViews 5.

[25]In the purely forward-looking models, the coefficient on the forward-term corresponds to the subjective discount factor β. It has been noted by several authors that it is difficult to obtain accurate estimates of this parameter in single-equation rational expectations models (see e.g., the discussion in Gregory et al. (1993)). Some authors therefore fix the value of β prior to estimation.

choice of bandwidth parameter. When estimation is based on the instrument set $z_{1,t}$, the implicit estimate of δ varies from 0.21 to 0.28 depending on the choice of bandwidth parameter. However, when the instrument set is $z_{2,t}$, the estimate of δ varies from 0.03 when the bandwidth is based on the data-based method to 0.51 when the bandwidth is one.

The results for the 'hybrid' model with local currency pricing are reported in tables 4 and 5. For the UK, the coefficient on lagged import price growth is small and imprecisely estimated. The other coefficient estimates are similar to what was obtained in the purely forward-looking LCP model. In particular, the coefficient on the forward-term is still negative. For Norway, the coefficient on lagged import price growth is somewhat larger than for the UK. The estimates are far from being statistically significant, however. The coefficient on future import price growth is slightly smaller compared with the purely forward-looking model. Overall, the estimation results lend little support to the LCP model with indexation as a model of UK or Norwegian import prices of manufactures. This is consistent with the findings reported by Smets & Wouters (2002), who do not find evidence of strong indexation in euro-area import prices.

Next, we turn to the model that allows a subset of exporters to engage in PCP. The results are reported in tables 6 and 7. The key result emerging from these tables is the following: the coefficient on the exchange rate term is positive and both numerically and statistically significant. This holds for both datasets and across instrument sets and bandwidth parameters. The estimated share of PCP firms depends strongly on the choices of instrument set and bandwidth parameter, however. For the UK the estimated share of PCP firms ranges from 0.42 to 0.74; for Norway it ranges from 0.31 to 0.96. The remaining parameters are also affected by the inclusion of the exchange rate term. For the UK, the coefficient on the forward-term is positive (but still statistically insignificant) when estimation is based on the instrument set $z_{2,t}$.[26] For Norway, the coefficient on the forward-term is somewhat smaller than in the pure LCP model. The coefficients on the level terms are still small and statistically insignificant, however.

The results illustrate that the J-test has low power to detect misspecification: in the pure LCP model, the $J-$test did not reject the validity of the over-identifying restrictions when the instrument set contained current values of the exchange rate. The results do not offer support for the exact specification of the LCP-PCP model, but nevertheless constitute strong evidence against the pure LCP model.[27] This finding is consistent with the results reported by Choudhri et al. (2005), who estimate open-economy DSGE models

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[26]When we use the Newey-West method to select the bandwidth in the HAC estimate of the weighting matrix, the coefficient on the forward-term is statistically significant at the 5% significance level if we use a one-sided test. Given our prior about the sign of the effect of expected future import price growth, it could be argued that the one-sided test is more relevant than a two-sided test.

[27]These conclusions are robust to extending the LCP-PCP model to allow for inflation indexation.

on data for non-US G7 countries using an impulse response matching approach and find that the best-fitting model incorporates a combination of PCP and LCP.

Tables 8 to 11 report estimation results for the pricing-to-market models for the two different measures of domestic prices and costs. The estimation is based on the following instrument sets:

$$z_{3,t} = \left\{ z_{1,t}, \sum_{i=0}^{2} \Delta p_{H,t-i}, p_{H,t-1} - p_{F,t-1} \right\}$$

$$z_{4,t} = \left\{ z_{2,t}, \sum_{i=1}^{2} \Delta p_{H,t-i}, p_{H,t-1} - p_{F,t-1} \right\}$$

For the UK, the coefficient on domestic producer prices is positive and statistically significant, except in the case where the instrument set is $z_{3,t}$ and the bandwidth in the HAC estimate of the covariance matrix is set equal to one. Ignoring the latter case, the estimate of the coefficient on domestic prices in the implied expression for the optimal frictionless price $(1 - \rho)$ lies in the range 0.33-0.40. The estimate of long-run (conditional) exchange rate pass-through thus lies in the range 0.60-0.67. This is in line with the estimate we obtained using cointegration analysis. The implied estimate of δ varies from 0.30 to 0.34. The coefficient on domestic unit labour costs is positive in all of the distribution cost models. Five of the six estimates are significant at the 10% level. The implied estimate of $1 - \rho$ now varies from 0.15 to 0.22, and the estimate of δ lies in the range 0.33-0.41. Thus, the long-run (conditional) pass-through is somewhat larger in this model: the estimates lie in the range 0.78-0.85.

The estimates of the purely forward-looking LCP model with pricing-to-market do seem to suggest a role for domestic prices and costs in explaining UK import prices. This conclusion is robust to extending the model to allow for indexation to past import price growth. The estimated coefficient on lagged import price inflation is now negative. The evidence of pricing-to-market is somewhat weaker if we extend the model to allow for both PCP and LCP, however. In this case, the coefficient on domestic producer prices or domestic unit labour costs is statistically insignificant and in some cases, negative. As in the models without pricing-to-market variables, the estimated share of PCP firms is positive and statistically significant. Moreover, the coefficient on the forward term is negative in most cases.

For Norway, the coefficient on domestic prices and costs is statistically insignificant. The coefficient on domestic producer prices is in many cases negative. The coefficient on domestic unit labour costs is positive, however. Again, the evidence of forward-looking price-setting is stronger for Norway than for the UK: the coefficient on the forward-term is positive and, in most cases, statistically significant. Moreover, the estimate of the dis-

count factor β is economically plausible. For example, when the estimation of the distribution cost model is based on the instrument set $z_{3,t}$ and the bandwidth is selected using the Newey-West method, the estimate of the discount factor is 0.99 and is statistically significant at the 1% level. In this case, the implied estimate of long-run (conditional) exchange rate pass-through (ρ) is 0.58 and the estimate of δ is 0.14. If we interpret the effect of domestic unit labour costs as the effect of distribution costs (see section 2.2.2), this estimate of long-run pass-through seems unreasonably low. More plausibly, the domestic unit labour cost variable is acting as a proxy for the price of import-competing products.

Extending the pricing-to-market models to allow for indexation to lagged import price growth, we reach similar conclusions as above: the coefficient on lagged import price growth is fairly small and statistically insignificant. The coefficient on the forward-term is still positive and statistically significant in most cases, whereas the coefficients on the levels terms, including the coefficients on the pricing-to-market variables, are statistically insignificant. Finally; extending the model to allow for both PCP and LCP has the effect of lowering the coefficient on the forward-term in most cases, although the coefficient remains statistically significant. The coefficient on domestic prices and costs is still statistically insignificant. As in the LCP-PCP model without pricing-to-market variables, the coefficient on the exchange rate term is positive and statistically significant.

5   CONCLUDING REMARKS

A key issue in the empirical literature on the New Keynesian Phillips Curve has been to determine whether price setters are 'forward-looking', in the sense that expected future prices matter for the determination of current prices. By contrast, most of the empirical work on import prices has taken the form of reduced-form pass-through regressions, with no attempt to distinguish between expectational dynamics and dynamics arising from other sources. This paper makes a first attempt to fill this gap by estimating New Keynesian import price equations derived from the Calvo model of staggered price setting.

Taken at face value, the GMM estimates obtained for the UK do not lend much support to the hypothesis that the price-setting rules are forward looking: the coefficient on expected future import price growth is either statistically insignificant, economically implausible, or both. The evidence of forward-looking price-setting is stronger for Norway: the coefficient on the forward-term is positive and, in most cases, statistically significant. For both countries, the estimation results favour a specification that allows for both PCP and LCP. By contrast, there seems to be little evidence of indexation to past import price growth.

For Norway, the estimated coefficients on foreign costs and the pricing-to-market

variables are statistically insignificant and close to zero in most cases. This contrasts with the results obtained for the UK: the coefficients on the foreign cost variables are statistically significant and, moreover, the pricing-to-market models suggest a role for domestic prices or costs in explaining import prices.

The fact that the estimation results for the UK and Norway are so different is somewhat puzzling. The differences in the results could be related to differences in the country- or commodity composition of imports. The estimation of the pricing-to-market models requires proxies for variables which are inherently hard to measure: the price of local goods and services used in distribution and the price of import-competing goods. The commodity composition of manufacturing imports and the domestic production of manufactures is likely to be different. In particular, Norway imports manufactured goods (e.g., motor vehicles) for which there do not exist domestic substitutes. Such measurement problems could be part of the explanation why we do not obtain a significant effect of domestic prices for Norway.

By using a linear trend to capture the effects of trade liberalisation, we have implicitly assumed that these effects have been constant over the sample period. This is a strict assumption. The plots of the Norwegian dataseries indicate that the downward trend in import prices relative to domestic prices and costs became more pronounced in the last part of the sample. A more flexible approach would be to allow for an unobservable stochastic trend in the model.

Finally, the GMM estimates may exhibit small-sample estimation bias due to weak identification. A symptom of weak identification is the fact that the GMM estimates were highly sensitive to the choice of instruments and the choice of weighting matrix in the GMM procedure. Whether the findings in this paper can be attributed to small sample bias is the subject of chapter 3 of this thesis.

A   DETAILS ON THE DERIVATION OF THE IMPORT PRICE EQUATIONS

A.1   The LCP-PCP model

A.1.1   The sectoral production- and price indices

The aggregate import quantity index is

$$Y_{F,t} \equiv \left[\phi^{\frac{1}{\varepsilon}} \left(Y_{F,t}^P \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\phi)^{\frac{1}{\varepsilon}} (Y_{F,t}^L)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A1}$$

and the sectoral production indices are

$$Y_{F,t}^P \equiv \left[\left(\frac{1}{\phi} \right)^{\frac{1}{\varepsilon}} \int_0^\phi Y_{F,t}^P(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A2}$$

$$Y_{F,t}^L \equiv \left[\left(\frac{1}{1-\phi} \right)^{\frac{1}{\varepsilon}} \int_\phi^1 Y_{F,t}^L(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A3}$$

The corresponding price indices are

$$P_{F,t} \equiv \left[\phi(S_t P_{F,t}^P)^{1-\varepsilon} + (1-\phi)(P_{F,t}^L)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{A4}$$

$$P_{F,t}^P \equiv \left[\frac{1}{\phi} \int_0^\phi P_{F,t}^P(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{A5}$$

$$P_{F,t}^L \equiv \left[\frac{1}{1-\phi} \int_\phi^1 P_{F,t}^L(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}, \tag{A6}$$

and the demand for imports from PCP and LCP firms is

$$Y_{F,t}^P(i) = \frac{1}{\phi} \left(\frac{P_{F,t}^P(i)}{P_{F,t}^P} \right)^{-\varepsilon} Y_{F,t}^P = \left(\frac{S_t P_{F,t}^P(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t} \tag{A7}$$

$$Y_{F,t}^L(i) = \frac{1}{1-\phi} \left(\frac{P_{F,t}^L(i)}{P_{F,t}^L} \right)^{-\varepsilon} Y_{F,t}^L = \left(\frac{P_{F,t}^L(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t}. \tag{A8}$$

A.1.2   The LCP-PCP model with inflation indexation

PCP firms   Firms that do not re-optimise in period t set prices according to

$$P_{F,t}^P(i) = \left(\frac{P_{F,t-1}}{P_{F,t-2}} \frac{S_{t-2}}{S_{t-1}} \right)^\chi P_{F,t-1}^P(i). \tag{A9}$$

A firm that is allowed to re-optimise in period t sets the price $\widetilde{P}_{F,t}^P(i)$ to maximise

$$E_t \sum_{\tau=t}^\infty \eta^{\tau-t} D_{t,\tau} \left(\widetilde{P}_{F,t}^P(i) \left(\frac{S_{t-1} P_{F,\tau-1}}{S_{\tau-1} P_{F,t-1}} \right)^\chi - MC_{F,\tau} \right) \left(\frac{S_\tau \widetilde{P}_{F,t}^P(i) \left(\frac{S_{t-1} P_{F,\tau-1}}{S_{\tau-1} P_{F,t-1}} \right)^\chi}{P_{F,\tau}} \right)^{-\varepsilon} Y_{F,\tau}. \tag{A10}$$

Imposing symmetry, the first-order condition is

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} Y_{F,\tau} \left(\frac{S_\tau \widetilde{P}_{F,t}^P \left(\frac{S_{t-1} P_{F,\tau-1}}{S_{t-1} P_{F,t-1}} \right)^\chi}{P_{F,\tau}} \right)^{-\varepsilon} \left((1-\varepsilon) \left(\frac{S_{\tau-1} P_{F,\tau-1}}{S_{\tau-1} P_{F,t-1}} \right)^\chi + \varepsilon \frac{MC_{F,\tau}}{\widetilde{P}_{F,t}^P} \right) = 0. \quad \text{(A11)}$$

Log-linearising the first-order condition around a zero inflation steady-state we obtain

$$0 \simeq E_t \sum_{\tau=t}^{\infty} (\eta\beta)^{\tau-t} \left(-\chi(s_{t-1} + p_{F,\tau-1} - s_{\tau-1} - p_{F,t-1}) + mc_{F,\tau} - \widetilde{p}_{F,t}^P \right), \quad \text{(A12)}$$

which can be rearranged to get

$$\widetilde{p}_{F,t}^P - p_{F,t}^P \simeq \eta\beta\chi \left(p_{F,t-1} - s_{t-1} \right) + (1-\eta\beta)mc_{F,t} \quad \text{(A13)}$$
$$+ \eta\beta E_t(\widetilde{p}_{F,t+1}^P - \chi \left(p_{F,t} - s_t \right)) - p_{F,t}^P.$$

The aggregate PCP price index is

$$\left(P_{F,t}^P \right)^{1-\varepsilon} = (1-\eta) \left(\widetilde{P}_{F,t}^P \right)^{1-\varepsilon} + \eta \left(\left(\frac{P_{F,t-1}}{P_{F,t-2}} \frac{S_{t-2}}{S_{t-1}} \right)^\chi P_{F,t-1}^P \right)^{1-\varepsilon}, \quad \text{(A14)}$$

which implies

$$\widetilde{p}_{F,t}^P - p_{F,t}^P \simeq \frac{\eta}{1-\eta} \Delta p_{F,t}^P - \frac{\eta}{1-\eta} \chi \left(\Delta p_{F,t-1} - \Delta s_{t-1} \right). \quad \text{(A15)}$$

Combining (A13) and (A15) we get

$$\frac{\eta}{1-\eta} \Delta p_{F,t}^P - \frac{\eta}{1-\eta} \chi \left(\Delta p_{F,t-1} - \Delta s_{t-1} \right) = \eta\beta\chi \left(p_{F,t-1} - s_{t-1} \right) + (1-\eta\beta)mc_{F,t} \quad \text{(A16)}$$
$$+ \eta\beta E_t(\widetilde{p}_{F,t+1}^P - \chi \left(p_{F,t} - s_t \right)) - p_{F,t}^P,$$

which, after some algebraic manipulation, can be written as

$$\Delta p_{F,t}^P = \beta E_t \Delta p_{F,t+1}^P + \chi \left(\Delta p_{F,t-1} - \Delta s_{t-1} \right) - \beta\chi \left(\Delta p_{F,t} - \Delta s_t \right) \quad \text{(A17)}$$
$$+ \frac{(1-\eta)(1-\eta\beta)}{\eta} (mc_{F,t} - p_{F,t}^P).$$

LCP firms   Firms that do not re-optimise in period t set prices according to

$$P_{F,t}^L(i) = \left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^\chi P_{F,t-1}^L(i) \quad \text{(A18)}$$

The problem facing a firm that is allowed to re-optimise in period t is then

$$\max_{\widetilde{P}_{F,t}^L(i)} E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\frac{\widetilde{P}_{F,t}^L(i) \left(\frac{P_{F,\tau-1}}{P_{F,t-1}} \right)^\chi}{S_\tau} - MC_{F,\tau} \right) \left(\frac{\widetilde{P}_{F,t}^L(i) \left(\frac{P_{F,\tau-1}}{P_{F,t-1}} \right)^\chi}{P_{F,\tau}} \right)^{-\varepsilon} Y_{F,\tau}. \quad \text{(A19)}$$

Imposing symmetry, the first-order condition can be written as

$$0 = E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} Y_{F,\tau} \left(\frac{\widetilde{P}_{F,t}^L \left(\frac{P_{F,\tau-1}}{P_{F,t-1}} \right)^{\chi}}{P_{F,\tau}} \right)^{-\varepsilon} \left((1-\varepsilon) \frac{1}{S_\tau} \left(\frac{P_{F,\tau-1}}{P_{F,t-1}} \right)^{\chi} + \varepsilon \frac{MC_{F,\tau}}{\widetilde{P}_{F,t}^L} \right). \qquad \text{(A20)}$$

Log-linearising the first-order condition around a zero inflation steady-state, we obtain

$$0 \simeq E_t \sum_{\tau=t}^{\infty} (\eta\beta)^{\tau-t} \left(s_\tau - \chi(p_{F,\tau-1} - p_{F,t-1}) + mc_{F,\tau} - \widetilde{p}_{F,t}^L \right), \qquad \text{(A21)}$$

which can be rearranged to get

$$\widetilde{p}_{F,t}^L - p_{F,t}^L \simeq \eta\beta\chi p_{F,t-1} + (1-\eta\beta)(s_t + mc_{F,t}) + \eta\beta E_t (\widetilde{p}_{F,t+1}^L - \chi p_{F,t}) - p_{F,t}^L. \qquad \text{(A22)}$$

The aggregate LCP price index is

$$\left(P_{F,t}^L \right)^{1-\varepsilon} = (1-\eta) \left(\widetilde{P}_{F,t}^L \right)^{1-\varepsilon} + \eta \left(\left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\chi} P_{F,t-1}^L \right)^{1-\varepsilon}, \qquad \text{(A23)}$$

which implies

$$\widetilde{p}_{F,t}^L - p_{F,t}^L \simeq \frac{\eta}{1-\eta} \Delta p_{F,t}^L - \frac{\eta}{1-\eta} \chi \Delta p_{F,t-1}. \qquad \text{(A24)}$$

Combining (A22) and (A24) we get

$$\frac{\eta}{1-\eta} \Delta p_{F,t}^L - \frac{\eta}{1-\eta} \chi \Delta p_{F,t-1} = \eta\beta\chi p_{F,t-1} + (1-\eta\beta)(s_t + mc_{F,t}) \qquad \text{(A25)}$$
$$+ \eta\beta E_t (\widetilde{p}_{F,t+1}^L - \chi p_{F,t}) - p_{F,t}^L,$$

which can be re-written as

$$\Delta p_{F,t}^L = \beta E_t \Delta p_{F,t+1}^L + \chi \Delta p_{F,t-1} - \beta\chi \Delta p_{F,t} + \frac{(1-\eta)(1-\eta\beta)}{\eta} (s_t + mc_{F,t} - p_{F,t}^L). \qquad \text{(A26)}$$

Aggregate import prices   The equation describing the evolution of aggregate import prices is

$$\Delta p_{F,t} = \phi \Delta p_{F,t}^P + \phi \Delta s_t + (1-\phi) \Delta p_{F,t}^L, \qquad \text{(A27)}$$

which, after substituting in for $\Delta p_{F,t}^P$ and $\Delta p_{F,t}^L$, can be written as

$$\Delta p_{F,t} = \phi \left(\begin{array}{c} \beta E_t \Delta p_{F,t+1}^P + \chi \left(\Delta p_{F,t-1} - \Delta s_{t-1} \right) - \beta\chi \left(\Delta p_{F,t} - \Delta s_t \right) \\ + \frac{(1-\eta)(1-\eta\beta)}{\eta} (mc_{F,t} - p_{F,t}^P) \end{array} \right) \qquad \text{(A28)}$$
$$+ \phi \Delta s_t + (1-\phi) \left(\begin{array}{c} \beta E_t \Delta p_{F,t+1}^L + \chi \Delta p_{F,t-1} - \beta\chi \Delta p_{F,t} \\ + \frac{(1-\eta)(1-\eta\beta)}{\eta} (s_t + mc_{F,t} - p_{F,t}^L) \end{array} \right)$$

Rearranging, we obtain equation (22) in the main text:

$$\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \frac{(1-\eta)(1-\eta\beta)}{(1+\beta\chi)\eta} \left(s_t + mc_{F,t} - p_{F,t}\right) \qquad (A29)$$
$$+ \phi \left(\Delta s_t - \frac{\beta}{1+\beta\chi} E_t \Delta s_{t+1} - \frac{\chi}{1+\beta\chi} \Delta s_{t-1} \right).$$

A.2   The model with translog preferences

The price-setting problem of an LCP firm that resets its price in period t is

$$\max_{\widetilde{P}_t(i)} E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\frac{\widetilde{P}_t(i)}{S_\tau} - MC_{F,\tau} \right) Y_\tau(i). \qquad (A30)$$

The first-order condition is

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \left(\left(\frac{\widetilde{P}_t(i)}{S_\tau} - MC_{F,\tau} \right) \frac{\partial Y_\tau(i)}{\partial \widetilde{P}_t(i)} + \frac{Y_\tau(i)}{S_\tau} \right) = 0 \qquad (A31)$$

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{Y_\tau(i)}{S_\tau} \left(\left(1 - \frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \right) \frac{\partial Y_\tau(i)}{\partial \widetilde{P}_t(i)} \frac{\widetilde{P}_t(i)}{Y_\tau(i)} + 1 \right) = 0 \qquad (A32)$$

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{Y_\tau(i)}{S_\tau} \left(-\left(1 - \frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \right) \left(1 - \frac{\gamma_{ii}}{\psi_\tau(i)} \right) + 1 \right) = 0 \qquad (A33)$$

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{Y_\tau(i)}{S_\tau} \left(\frac{\gamma_{ii}}{\psi_\tau(i)} + \frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \left(1 - \frac{\gamma_{ii}}{\psi_\tau(i)} \right) \right) = 0 \qquad (A34)$$

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{Y_\tau(i)}{S_\tau} \frac{\gamma_{ii}}{\psi_\tau(i)} \left(1 + \frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \frac{\psi_\tau(i)}{\gamma_{ii}} \left(1 - \frac{\gamma_{ii}}{\psi_\tau(i)} \right) \right) = 0 \qquad (A35)$$

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{P_\tau Y_\tau}{S_\tau} \left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \left(1 - \frac{\psi_\tau(i)}{\gamma_{ii}} \right) - 1 \right) = 0. \qquad (A36)$$

Using the approximation

$$\left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \left(1 - \frac{\psi_\tau(i)}{\gamma_{ii}} \right) - 1 \right) \approx \ln \left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \left(1 - \frac{\psi_\tau(i)}{\gamma_{ii}} \right) \right) \qquad (A37)$$
$$\approx -\frac{\psi_\tau(i)}{\gamma_{ii}} + \ln \left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \right),$$

we can rewrite the first-order condition as

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{P_\tau Y_\tau}{S_\tau} \left(-\frac{\alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln P_\tau(j)}{\gamma_{ii}} + \ln \left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \right) \right) = 0 \qquad (A38)$$

$$E_t \sum_{\tau=t}^{\infty} \eta^{\tau-t} D_{t,\tau} \frac{P_\tau Y_\tau}{S_\tau} \left(\frac{1}{\gamma} - \ln \widetilde{P}_t(i) + \sum_{j\neq i}^{N} \frac{1}{N-1} \ln P_\tau(j) + \ln \left(\frac{S_\tau MC_{F,\tau}}{\widetilde{P}_t(i)} \right) \right) = 0. \qquad (A39)$$

Log-linearising around a zero inflation steady state we obtain

$$0 \simeq E_t \sum_{\tau=t}^{\infty} (\beta\eta)^{\tau-t} \left(-\widetilde{p}_t(i) + \sum_{j\neq i}^{N} \frac{1}{N-1} p_\tau(j) + s_\tau + mc_{F,\tau} - \widetilde{p}_t(i) \right). \tag{A40}$$

Solving out for $\widetilde{p}_t(i)$ we get

$$\widetilde{p}_t(i) = \frac{1}{2}(1-\beta\eta)E_t \sum_{\tau=t}^{\infty} (\beta\eta)^{\tau-t} \left(s_\tau + mc_{F,\tau} + \sum_{j\neq i}^{N} \frac{1}{N-1} p_\tau(j) \right), \tag{A41}$$

which, after imposing symmetry, can be written as

$$\widetilde{p}_t(i) = \frac{1}{2}(1-\beta\eta)E_t \sum_{\tau=t}^{\infty} (\beta\eta)^{\tau-t} \left(s_\tau + mc_{F,\tau} + \frac{N_H}{N-1} p_{H,\tau} + \frac{N-N_H-1}{N-1} p_{F,\tau} \right). \tag{A42}$$

Quasi-differentiation of the linearised first-order condition yields

$$\widetilde{p}_t(i) = \frac{1}{2}(1-\beta\eta) \left(s_t + mc_{F,t} + \frac{N_H}{N-1} p_{H,t} + \frac{N-N_H-1}{N-1} p_{F,t} \right) + \beta\eta E_t \widetilde{p}_{t+1}(i). \tag{A43}$$

Assuming that the number of foreign exporters (i.e., $N-N_H$) is large and imposing symmetry, the definition of the aggregate import price index implies that

$$\Delta p_{F,t} = \frac{(1-\eta)}{\eta} \left(\widetilde{p}_t - p_{F,t} \right), \tag{A44}$$

and hence, that

$$\Delta p_{F,t} = \frac{(1-\eta)}{\eta} \left(\widetilde{p}_t - p_{F,t} \right) \tag{A45}$$

$$= \frac{1-\eta}{\eta} \left(\frac{1}{2}(1-\beta\eta) \left(s_t + mc_{F,t} + \frac{N_H}{N-1} p_{H,t} + \frac{N-N_H-1}{N-1} p_{F,t} \right) + \beta\eta E_t \widetilde{p}_{t+1}(i) - p_{F,t} \right)$$

$$= \frac{1-\eta}{\eta} \left(\begin{array}{c} \frac{1}{2}(1-\beta\eta) \left(s_t + mc_{F,t} + \frac{N_H}{N-1} p_{H,t} + \frac{N-N_H-1}{N-1} p_{F,t} \right) \\ + \beta\eta \left(E_t \widetilde{p}_{t+1}(i) - E_t p_{F,t+1} \right) + \beta\eta E_t p_{F,t+1} - p_{F,t} \end{array} \right)$$

$$= \frac{1-\eta}{\eta} \left(\begin{array}{c} \frac{1}{2}(1-\beta\eta) \left(s_t + mc_{F,t} + \frac{N_H}{N-1} p_{H,t} + \frac{N-N_H-1}{N-1} p_{F,t} \right) + \beta\eta \left(\frac{\eta}{1-\eta} E_t \Delta p_{F,t+1} \right) \\ + \beta\eta E_t \Delta p_{F,t+1} - (1-\beta\eta) p_{F,t} \end{array} \right)$$

$$= \beta E_t \Delta p_{F,t+1} + \frac{1-\eta}{\eta} \left(\frac{1}{2}(1-\beta\eta) \left(s_t + mc_{F,t} + \frac{N_H}{N-1} p_{H,t} + \frac{N-N_H-1}{N-1} p_{F,t} \right) - (1-\beta\eta) p_{F,t} \right)$$

$$= \beta E_t \Delta p_{F,t+1} - \frac{(1-\beta\eta)(1-\eta)}{\eta} \frac{1}{2} \frac{N+N_H-1}{N-1} \left(p_{F,t} - \frac{N-1}{N+N_H-1} (s_t + mc_{F,t}) - \frac{N_H}{N+N_H-1} p_{H,t} \right).$$

B VARIABLE DEFINITIONS AND SOURCES

- P_F: Import price of manufactured goods (local currency).[28] Seasonally adjusted (SA). Source: OECD International Trade and Competitiveness Indicators (ITCI)

- S: Nominal effective exchange rate. Sources: Bank of England (Broad effective exchange rate index [XUQABK82]) and Norges Bank (Trade-weighted exchange rate [TWI]).

- ULC_F: Unit labour costs in foreign manufacturing (foreign currency). Trade-weighted. SA. Source: OECD Economic Outlook. Constructed as $\frac{ULCM}{EXCHEB \times ULCMDR}$ where $ULCM$ is unit labour costs in domestic manufacturing industries [Q.GBR.ULCM/Q.NOR.ULCM], $EXCHEB$ is the nominal effective exchange rate [Q.GBR.EXCHEB/Q.NOR.EXCHEB] and $ULCMDR$ is relative unit labour costs [Q.GBR.ULCMDR/Q.NOR.ULCMDR].

- P_{COM}: Index of world metal prices. The original series is measured in US dollars. We convert the index to the currency of the trading partners using the official effective exchange rate S. Source: IMF International Financial Statistics [00176AYDZF...].

- P_H: Producer price index (home sales). Seasonally adjusted using the X12-ARIMA routines implemented in EViews 5. Sources: OECD Main Economic Indicators [GBR.PPIAMP01.IXOB.Q] and Statistics Norway [Commodity price index for the industrial sectors (VPPI). Total industry. Domestic market.] Or; Unit labour costs for the total economy. Source: OECD Economic Outlook [Q.GBR.ULC/Q.NOR.ULC].

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[28]All variables are converted to a common baseyear 1995=1.

C   COINTEGRATION ANALYSIS

In this appendix we report the results of testing for cointegration using the Johansen (1988) approach. The starting point for the Johansen procedure is an unrestricted vector autoregression (VAR) of order k

$$\Delta x_t = \Pi_0 x_{t-1} + \sum_{i=1}^{k-1} \Pi_i \Delta x_{t-i} + \phi D_t + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{C1}$$

where x_t is a $p \times 1$ vector of variables observed at time t, $\Pi_0, \Pi_1, \ldots, \Pi_{k-1}$ are $p \times p$ matrices of parameters, D_t is a vector of deterministic terms, ε_t is a $p \times 1$ vector of innovations, $\varepsilon_t \sim IN(0, \Omega)$, and T is the number of observations. The matrix Π_0 has rank $0 \leq r \leq p$, where r is the number of cointegrating vectors. Π_0 can be rewritten as

$$\Pi_0 = \alpha \beta',$$

where α is a $p \times r$ matrix of short-run adjustment parameters, and β is a $p \times r$ matrix of cointegration coefficients. Testing for cointegration amounts to determining the rank of Π_0. A test of the null hypothesis that there are at most r cointegration vectors can be based on the trace statistic ϑ_r

$$\vartheta_r = -T \sum_{i=r+1}^{p} \ln(1 - \lambda_i), \quad r = 0, 1, 2, \ldots, p-1 \tag{C2}$$

where $1 \geqslant \lambda_1 \geqslant \ldots \geqslant \lambda_p \geqslant 0$ are the eigenvalues from a reduced rank regression of Δx_t on x_{t-1} corrected for $\Delta x_{t-1}, \ldots, \Delta x_{t-k-1}$ and D_t (see Johansen, 1995, chap. 6). Under the null hypothesis that there are r cointegrating relations, the distribution of ϑ_r is nonstandard and involves Brownian motions. The cointegrating rank is selected as zero if ϑ_0 is not significant and $r + 1$ if the last significant statistic is ϑ_r. The estimates of β are obtained as the eigenvectors corresponding to the r largest eigenvalues. Conditional on the correct cointegration rank, tests of linear restrictions on β are asymptotically distributed as χ^2 (see Johansen, 1995, chap. 7).

A vector of variables z_t is said to be weakly exogenous for the cointegration parameters β if the corresponding rows of α are zero, that is, if there is no feedback from $\beta' x_{t-1}$ to z_t. Let x_t be decomposed into $x_t = \{y_t', z_t'\}'$ where y_t is a p_1 dimensional vector of endogenous variables, and z_t is a $p - p_1$ dimensional vector of weakly exogenous variables. In this case, efficient inference on the cointegration rank can be based on the following

conditional model for y_t

$$\Delta y_t = \Pi_{y,0}x_{t-1} + \sum_{i=1}^{k-1} \Pi_{y,i}\Delta x_{t-i} + \Lambda\Delta z_t + \phi_y D_t + \varepsilon_{y,t}, \qquad \text{(C3)}$$

where the parameters are partitioned conformably with $x_t = \{y_t', z_t'\}'$. Harbo et al. (1998) show that the appropriate critical values for the trace test depend on the number of weakly exogenous conditioning variables $(p - p_1)$, as well as the specification of the deterministic part of the model.

We conduct the cointegration analysis in two steps. We first test for cointegration in the system containing $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}\}$. Then, we redo the analysis in a VAR that includes domestic prices/costs, that is, we analyse the cointegration properties of the system $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}$.[29] Throughout, we assume that foreign unit labour costs ($ulc_{F,t}$) and the commodity price index ($p_{COM,t}$) are weakly exogenous for the cointegration parameters. This does not seem restrictive, given that the UK and Norwegian economies are small relative to the world economy. The VARs include an unrestricted constant and a restricted trend. This means that we allow for linear trends in all directions of the data, including the cointegration relations, but rule out the possibility of quadratic trends.[30]

A necessary preliminary step in VAR analyses is to determine the lag length, k. We base the choice of lag length on the information criteria and F-tests for successive removal of lags reported in tables 12 and 13, and the additional requirement that the VAR should be statistically well-specified. The information criteria and the tests of successive lag deletions point to either one or two lags as being appropriate. However, in most cases, the first-order VARs display signs of misspecification. As evidenced in tables 14 and 15, the second-order VARs are statistically well-specified.[31] In the following we therefore set the lag length to two.

Results for the UK data   The results of the cointegration tests on UK data are reported in table 16. In the VAR without pricing-to-market variables the trace test suggests that there is one or two cointegrating vectors in the data. Conditional on $r = 1$, the hypothesis of long-run homogeneity is accepted with a p-value of 0.10 when using the LR test. When we impose homogeneity, the estimated cointegrating relation is

$$CI_{UK,t} = p_{F,t} - s_t - \underset{(0.080)}{0.64}\, ulc_{F,t} - 0.36 p_{COM,t} + \underset{(0.0007)}{0.003}\, t, \qquad \text{(C4)}$$

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[29]The approach of gradually adding more information to the cointegration analysis has been advocated by Juselius (2006).

[30]The results are obtained using PcGive 10 (see Hendry & Doornik, 2001).

[31]An exception is the UK VAR which includes domestic producer prices. This VAR has residual autocorrelation in the equation for $p_{H,t}$. This autocorrelation is not removed by increasing the lag-length.

where the numbers in parentheses below the estimated parameters are asymptotic standard errors. The estimate of δ, the share of commodity prices in foreign marginal costs, is 0.36 and the coefficient on the exchange rate is one, implying that the long-run exchange rate pass-through is complete. The cointegrating relation contains a significant deterministic trend term, reflecting the fall in import prices relative to our measures of foreign costs over the estimation period.

If the cointegration rank is two, the theory predicts that $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ should be (trend-)stationary. Imposing $r = 2$, this hypothesis is rejected at the 1% significance level.

On the basis of the above results we would expect to find a cointegration rank of two or three when we extend the information set to include domestic producer prices. However, none of the trace test-statistics are significant at the 5% significance level in the extended system. More information about the cointegration rank can be gained by inspecting the eigenvalues of the companion matrix associated with the VAR[32] for different values of the cointegration rank. If a non-stationary relation is wrongly included in the model, then the largest unrestricted eigenvalue will be close to one (see Juselius, 2006, chap. 8). When we set the cointegration rank equal to one, the eigenvalues of the companion matrix are $[1.00, 1.00, 0.68, 0.68, 0.22, 0.22]$. The largest unrestricted eigenvalue is 0.68, and so including the first cointegrating relation in the model does not seem to induce non-stationarity. Including a second cointegration relation, the largest unrestricted eigenvalue increases to 0.81 which is still quite far from the unit circle, consistent with there being two cointegration vectors in the system.

Conditional on $r = 1$, the test-statistic for the test of long-run price homogeneity is accepted with a p-value of 0.20. Imposing homogeneity the estimated cointegration relation is

$$CI_{UK,t} = p_{F,t} - \underset{(0.036)}{0.61} s_t - \underset{}{0.41} ulc_{F,t} - 0.21 p_{COM,t} - \underset{(0.138)}{0.39} p_{H,t} + \underset{(0.0008)}{0.002} t. \qquad \text{(C5)}$$

Consistent with the pricing-to-market models, this cointegration relation suggests a role for domestic prices in explaining UK import prices. The estimate of the (conditional) long-run exchange rate pass-through is now 0.61.[33]

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[32] That is, the eigenvalues of

$$\begin{bmatrix} \Pi_0 + \Pi_1 + I & -\Pi_1 \\ I & 0 \end{bmatrix}$$

[33] The finding that domestic prices enter the long-run UK import price equation is consistent with the findings reported by Herzberg et al. (2003). Their estimate of the long-run pass-through is lower than ours, however (0.36). A possible reason why the estimates differ is that Herzberg et al. (2003) use a different measure of the price of import-competing products. The authors argue that, because the commodity composition of manufacturing imports and the home production of manufactures in the UK are different, the domestic producer price index is a biased measure of the price of import-competing products.

If we impose cointegration rank equal to two, the joint hypothesis that $p_{F,t} - s_t - (1 - \delta)ulc_{F,t} - \delta p_{COM,t}$ and $p_{F,t} - p_{H,t}$ are trend-stationary is rejected at the 5% significance level. Finally, if we impose cointegration rank equal to three, trend-stationarity of $p_{F,t} - s_t - ulc_{F,t}$, $p_{F,t} - s_t - p_{COM,t}$ and $p_{F,t} - p_{H,t}$ is rejected at the 1% level.

When we instead use domestic unit labour costs as the pricing-to-market variable, the trace test suggests the existence of two or three cointegrating relations in the model. However, the eigenvalues (modulus) of the companion matrix associated with the VAR(2) for different values of cointegration rank are

$$
\begin{array}{ccccccc}
r = 3 & 1.071 & 0.769 & 0.769 & 0.123 & 0.123 & 0.109 \\
r = 2 & 1.068 & 1.000 & 0.872 & 0.287 & 0.287 & 0.034 \\
r = 1 & 1.063 & 1.000 & 1.000 & 0.266 & 0.266 & 0.026
\end{array}
$$

We notice the presence of one large (and unstable) root in the unrestricted model ($r = 3$). The unstable root remains in the system when we impose cointegration rank two or one. This is an indication that the model contains processes that are integrated of order two, $I(2)$, or possibly even explosive (see Johansen, 1995, p. 53).[34] A formal $I(2)$ analysis is, however, beyond the scope of this paper.[35]

Results for Norwegian data   Table 17 reports the results of the cointegration tests for the Norwegian data. Testing for cointegration in a VAR for $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}\}$, the trace test suggests that there is one cointegrating relation between the variables. Conditional on $r = 1$, the hypothesis of long-run homogeneity is accepted with a $p-$value of 0.25. Imposing homogeneity, we end up the following estimate of the cointegration relation

$$CI_{NOR,t} = p_{F,t} - s_t - \underset{(0.034)}{0.78}\, ulc_{F,t} - 0.22p_{COM,t} + \underset{(0.0003)}{0.005}\, t \tag{C6}$$

The estimate of δ from the cointegration analysis is thus 0.22. Like in the UK data, the deterministic drift term is highly significant, both numerically and statistically. When we impose a cointegration rank of two, the hypothesis that $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ are trend-stationary is accepted with a $p-$value of 0.27.

When we extend the information set to include domestic producer prices, the trace test still indicates that the cointegration rank is one when using a 5% significance level. This is supported by the fact that when we impose a cointegration rank of one, the largest unrestricted eigenvalue of the companion matrix is 0.57, whereas when the cointegration rank is two, the largest unrestricted eigenvalue is 0.97. Conditional on $r = 1$, the test

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[34]Another possibility is that the large root disappears if we allow for structural breaks in the deterministic components of the model.

[35]Bowdler & Nielsen (2006) analyse UK inflation within a cointegrated $I(2)$ framework. See also chapter 4 in this thesis.

of long-run homogeneity is now rejected at the 1% level. If we nevertheless impose long-run homogeneity, we obtain the following restricted cointegration relation:

$$CI_{NOR,t} = p_{F,t} - 0.97s_t - 0.71ulc_{F,t} - \underset{(0.048)}{0.26} p_{COM,t} - \underset{(0.134)}{0.03} p_{H,t} + \underset{(0.0008)}{0.002} t. \quad (C7)$$

The coefficient on domestic prices is insignificant, both numerically and statistically. The estimates of the coefficients on foreign unit labour costs and commodity prices are similar to what we found in the smaller system.

In the VAR that includes domestic unit labour costs the trace test suggests the presence of one, or possibly two, cointegrating relations between the variables at the 5% level. The eigenvalues of the companion matrix associated with the VAR(2) for different choices of r are

$r = 3$	0.986	0.784	0.455	0.432	0.432	0.276
$r = 2$	1.000	0.907	0.449	0.449	0.448	0.210
$r = 1$	1.000	1.000	0.774	0.400	0.174	0.174

We see that when we include a second cointegration relation, a root fairly close to the unit circle (0.91) remains in the system, indicating that the second cointegration relation is borderline non-stationary. The results are similar to those obtained in the model with domestic producer prices. In particular, long-run price homogeneity is rejected at the 1% significance level. Once we impose long-run homogeneity, the coefficient on domestic unit labour costs becomes statistically insignificant.

When the cointegration rank is set equal to two, the joint hypothesis that $p_{F,t} - s_t - (1 - \delta)ulc_{F,t} - \delta p_{COM,t}$ and $p_{F,t} - p_{H,t}$ are trend-stationary is rejected regardless of whether $p_{H,t}$ is proxied by producer prices or by unit labour costs. Conditional on cointegration rank equal to three, trend-stationarity of $p_{F,t} - s_t - ulc_{F,t}$, $p_{F,t} - s_t - p_{COM,t}$ and $p_{F,t} - p_{H,t}$ is also strongly rejected.

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Table 1: Univariate unit root tests. Detrended series. 1980Q1-2003Q2[a]

	UK		Norway	
	ADF	KPSS	ADF	KPSS
$s_t + ulc_{F,t} - p_{F,t}$	−2.32*	0.16	−2.57*	0.13
$s_t + p_{COM,t} - p_{F,t}$	−3.99**	0.06	−3.74**	0.11
$p_{H,t} - p_{F,t}$[b]	−2.36*	0.18	−2.07*	0.23
$p_{H,t} - p_{F,t}$[c]	−2.58*	0.15	−1.25	0.21

a The numbers in the table are the ADF t−statistics and the KPSS LM−statistics. Single asterisks (*) and double asterisks (**) denote statistical significance at the 5% level and the 1% level, respectively. The ADF test equation does not include any deterministic terms. The selection of lag-order for the ADF test is based on the Akaike Information Criterion (AIC) with the maximum number of lagged differenced terms set to four. The critical values for the ADF test are taken from MacKinnon (1996). The KPSS test equation includes a constant. The KPSS tests were run using a Bartlett kernel and the bandwidth is selected using the Newey & West (1994) method. The 1% and the 5% asymptotic critical values are 0.739 and 0.463, respectively (see table 1 in Kwiatkowski et al., 1992).

b $p_{H,t}$ denotes producer price index for manufactures.

c $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Table 2: GMM estimates of a purely forward-looking model with local currency pricing. UK data 1980Q4–2003Q1.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	$J - stat$
$z_{1,t}$	1	0.00 (0.002)	0.08 (0.212)	0.08* (0.039)	0.06** (0.018)	$\chi^2(10) = 13.06$ [0.22]
$z_{1,t}$	3	0.00 (0.002)	−0.09 (0.196)	0.13** (0.039)	0.09** (0.020)	$\chi^2(10) = 10.61$ [0.39]
$z_{1,t}$	Newey-West	−0.00 (0.002)	−0.02 (0.138)	0.13** (0.035)	0.10** (0.019)	$\chi^2(10) = 8.46$ [0.58]
$z_{2,t}$	1	0.00 (0.003)	−0.45 (0.299)	0.14** (0.051)	0.08** (0.024)	$\chi^2(7) = 7.95$ [0.34]
$z_{2,t}$	3	0.00 (0.003)	−0.52 (0.285)	0.19** (0.048)	0.10** (0.024)	$\chi^2(7) = 6.98$ [0.43]
$z_{2,t}$	Newey-West	0.00 (0.003)	−0.50 (0.277)	0.19** (0.042)	0.11** (0.021)	$\chi^2(7) = 5.89$ [0.55]

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values.

Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 3: GMM estimates of a purely forward-looking model with local currency pricing. Norwegian data 1980Q4–2002Q4.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	$J - stat$
$z_{1,t}$	1	0.00 (0.002)	0.71** (0.268)	0.09 (0.087)	0.03 (0.023)	$\chi^2(10) = 8.16$ [0.61]
$z_{1,t}$	3	0.00 (0.002)	0.82** (0.216)	0.03 (0.059)	0.01 (0.018)	$\chi^2(10) = 8.06$ [0.62]
$z_{1,t}$	Newey-West	0.00 (0.001)	0.85** (0.176)	0.04 (0.048)	0.01 (0.015)	$\chi^2(10) = 6.91$ [0.73]
$z_{2,t}$	1	0.00 (0.002)	0.67 (0.419)	0.09 (0.089)	0.03 (0.027)	$\chi^2(7) = 7.31$ [0.40]
$z_{2,t}$	3	0.00 (0.002)	0.85* (0.361)	0.01 (0.065)	0.01 (0.023)	$\chi^2(7) = 6.57$ [0.48]
$z_{2,t}$	Newey-West	0.00 (0.001)	0.88** (0.300)	0.02 (0.046)	0.00 (0.016)	$\chi^2(7) = 5.27$ [0.63]

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values.

Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 4: GMM estimates of a hybrid model with local currency pricing. UK data 1980Q4–2003Q1.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2 \Delta p_{F,t-1} + \alpha_4(s_t + ulc_{F,t} - p_{F,t}) + \alpha_5(s_t + p_{COM,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_2	$J-stat$
$z_{1,t}$	1	0.00 (0.002)	0.08 (0.212)	0.08 (0.045)	0.06** (0.019)	0.02 (0.111)	$\chi^2(9) = 13.01\ [0.16]$
$z_{1,t}$	3	0.00 (0.002)	−0.09 (0.209)	0.14** (0.049)	0.10** (0.022)	−0.03 (0.106)	$\chi^2(9) = 10.62\ [0.30]$
$z_{1,t}$	Newey-West	−0.00 (0.002)	−0.05 (0.140)	0.13** (0.041)	0.10** (0.020)	0.03 (0.087)	$\chi^2(9) = 8.38\ [0.50]$
$z_{2,t}$	1	0.00 (0.003)	−0.43 (0.325)	0.13* (0.065)	0.07** (0.026)	0.02 (0.113)	$\chi^2(6) = 8.01\ [0.24]$
$z_{2,t}$	3	0.00 (0.003)	−0.54 (0.318)	0.20** (0.064)	0.10** (0.027)	−0.04 (0.116)	$\chi^2(6) = 6.86\ [0.33]$
$z_{2,t}$	Newey-West	0.00 (0.003)	−0.52 (0.298)	0.22** (0.059)	0.12** (0.025)	−0.07 (0.095)	$\chi^2(6) = 5.82\ [0.44]$

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks **
and * denote statistical significance at the 1% and 5% level, respectively

Table 5: GMM estimates of a hybrid model with local currency pricing. Norwegian data 1980Q4–2002Q4.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2 \Delta p_{F,t-1} + \alpha_4(s_t + ulc_{F,t} - p_{F,t}) + \alpha_5(s_t + p_{COM,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_2	$J-stat$
$z_{1,t}$	1	0.00 (0.002)	0.64* (0.288)	0.12 (0.090)	0.04 (0.023)	0.13 (0.110)	$\chi^2(9) = 7.39\ [0.60]$
$z_{1,t}$	3	0.00 (0.002)	0.69** (0.238)	0.07 (0.067)	0.02 (0.019)	0.12 (0.099)	$\chi^2(9) = 7.86\ [0.55]$
$z_{1,t}$	Newey-West	0.00 (0.001)	0.74** (0.179)	0.05 (0.048)	0.01 (0.015)	0.04 (0.084)	$\chi^2(9) = 6.66\ [0.67]$
$z_{2,t}$	1	0.00 (0.002)	0.61 (0.427)	0.12 (0.093)	0.04 (0.027)	0.13 (0.111)	$\chi^2(6) = 6.36\ [0.38]$
$z_{2,t}$	3	0.00 (0.002)	0.77* (0.370)	0.05 (0.073)	0.02 (0.023)	0.10 (0.105)	$\chi^2(6) = 6.17\ [0.40]$
$z_{2,t}$	Newey-West	0.00 (0.001)	0.84** (0.307)	0.03 (0.051)	0.00 (0.018)	0.04 (0.095)	$\chi^2(6) = 5.24\ [0.51]$

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks
** and * denote statistical significance at the 1% and 5% level, respectively

Table 6: GMM estimates of a model with producer- and local currency pricing. UK data 1980Q4–2003Q1

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_3 (\Delta s_t - \alpha_1 \Delta s_{t+1}) + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_3	$J-stat$
$z_{1,t}$	1	0.00 (0.002)	−0.13 (0.207)	0.13** (0.033)	0.07** (0.014)	0.42** (0.056)	$\chi^2(9) = 9.86\ [0.36]$
$z_{1,t}$	3	0.00 (0.002)	−0.02 (0.158)	0.13** (0.033)	0.08** (0.014)	0.44** (0.052)	$\chi^2(9) = 9.26\ [0.41]$
$z_{1,t}$	Newey-West	0.00 (0.002)	0.01 (0.132)	0.15** (0.029)	0.09** (0.013)	0.42** (0.041)	$\chi^2(9) = 7.12\ [0.62]$
$z_{2,t}$	1	0.00 (0.002)	0.14 (0.191)	0.08* (0.036)	0.05** (0.018)	0.74** (0.141)	$\chi^2(6) = 7.28\ [0.30]$
$z_{2,t}$	3	0.00 (0.002)	0.20 (0.177)	0.09* (0.036)	0.06** (0.017)	0.70** (0.109)	$\chi^2(6) = 6.86\ [0.33]$
$z_{2,t}$	Newey-West	0.00 (0.001)	0.23 (0.148)	0.09** (0.031)	0.07** (0.016)	0.65** (0.084)	$\chi^2(6) = 5.79\ [0.45]$

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 7: GMM estimates of a model with producer- and local currency pricing. Norwegian data 1980Q4–2002Q4.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_3 (\Delta s_t - \alpha_1 \Delta s_{t+1}) + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_3	$J-stat$
$z_{1,t}$	1	0.00 (0.002)	0.43 (0.293)	0.09 (0.081)	0.03 (0.026)	0.46** (0.162)	$\chi^2(9) = 7.97\ [0.54]$
$z_{1,t}$	3	0.00 (0.002)	0.53* (0.243)	0.06 (0.061)	0.02 (0.023)	0.45** (0.138)	$\chi^2(9) = 6.83\ [0.65]$
$z_{1,t}$	Newey-West	0.00 (0.001)	0.56* (0.239)	0.06 (0.058)	0.02 (0.021)	0.44** (0.131)	$\chi^2(9) = 6.46\ [0.69]$
$z_{2,t}$	1	0.00 (0.002)	0.72 (0.453)	0.07 (0.120)	0.03 (0.043)	0.96* (0.450)	$\chi^2(6) = 5.26\ [0.51]$
$z_{2,t}$	3	0.00 (0.002)	0.69 (0.380)	0.08 (0.085)	0.04 (0.032)	0.71 (0.370)	$\chi^2(6) = 5.22\ [0.52]$
$z_{2,t}$	Newey-West	0.00 (0.002)	0.74 (0.374)	0.07 (0.079)	0.03 (0.031)	0.65 (0.352)	$\chi^2(6) = 4.93\ [0.55]$

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 8: GMM estimates of a pricing-to-market model with local currency pricing. Pricing-to-market variable: Domestic producer prices. UK data 1980Q4–2003Q1.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \alpha_6 (p_{H,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_6	$J - stat$
$z_{3,t}$	1	0.00 (0.002)	0.46* (0.181)	0.07 (0.062)	0.06* (0.024)	−0.00 (0.066)	$\chi^2(13) = 16.57\ [0.22]$
$z_{3,t}$	3	−0.00 (0.002)	−0.57** (0.197)	0.31** (0.060)	0.16** (0.030)	0.23* (0.096)	$\chi^2(13) = 11.43\ [0.58]$
$z_{3,t}$	Newey-West	−0.00 (0.002)	−0.40* (0.184)	0.42** (0.062)	0.19** (0.029)	0.30** (0.103)	$\chi^2(13) = 9.02\ [0.77]$
$z_{4,t}$	1	0.01 (0.003)	−0.87 (0.347)	0.39** (0.102)	0.17** (0.046)	0.37** (0.132)	$\chi^2(9) = 7.31\ [0.61]$
$z_{4,t}$	3	0.00 (0.003)	−0.83** (0.261)	0.40** (0.082)	0.18** (0.043)	0.34** (0.114)	$\chi^2(9) = 7.04\ [0.63]$
$z_{4,t}$	Newey-West	0.00 (0.003)	−0.70** (0.211)	0.41** (0.077)	0.19** (0.040)	0.34** (0.110)	$\chi^2(9) = 5.90\ [0.75]$

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 9: GMM estimates of a pricing-to-market model with local currency pricing. Pricing-to-market variable: Domestic producer prices. Norwegian data 1980Q4–2002Q4.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \alpha_6 (p_{H,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_6	$J - stat$
$z_{3,t}$	1	0.00 (0.002)	0.81** (0.236)	0.05 (0.078)	0.02 (0.021)	−0.01 (0.051)	$\chi^2(13) = 8.55\ [0.81]$
$z_{3,t}$	3	0.00 (0.002)	0.83** (0.193)	0.02 (0.055)	0.01 (0.017)	−0.02 (0.036)	$\chi^2(13) = 8.67\ [0.80]$
$z_{3,t}$	Newey-West	0.00 (0.001)	0.84** (0.150)	0.03 (0.046)	0.01 (0.014)	0.01 (0.032)	$\chi^2(13) = 7.87\ [0.85]$
$z_{4,t}$	1	0.00 (0.002)	0.91* (0.361)	0.03 (0.084)	0.01 (0.025)	−0.02 (0.068)	$\chi^2(9) = 6.97\ [0.64]$
$z_{4,t}$	3	0.00 (0.002)	0.91** (0.300)	−0.01 (0.062)	−0.00 (0.020)	−0.01 (0.046)	$\chi^2(9) = 6.61\ [0.68]$
$z_{4,t}$	Newey-West	0.00 (0.001)	0.97** (0.237)	0.01 (0.050)	−0.01 (0.016)	0.02 (0.038)	$\chi^2(9) = 5.34\ [0.80]$

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 10: GMM estimates of a pricing-to-market model with local currency pricing. Pricing-to-market variable: Domestic unit labour costs. UK data 1980Q4–2003Q1.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \alpha_6 (p_{H,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_6	$J-stat$
$z_{3,t}$	1	0.00 (0.002)	−0.07 (0.205)	0.13** (0.045)	0.09** (0.020)	0.04 (0.033)	$\chi^2(13) = 14.94$ [0.31]
$z_{3,t}$	3	−0.00 (0.003)	−0.02 (0.208)	0.30** (0.038)	0.18** (0.024)	0.09 (0.047)	$\chi^2(13) = 12.95$ [0.45]
$z_{3,t}$	Newey-West	−0.00 (0.003)	0.01 (0.209)	0.35** (0.037)	0.20** (0.026)	0.10 (0.051)	$\chi^2(13) = 8.13$ [0.84]
$z_{4,t}$	1	0.00 (0.003)	−0.41 (0.302)	0.29** (0.060)	0.14** (0.029)	0.12* (0.048)	$\chi^2(9) = 8.17$ [0.52]
$z_{4,t}$	3	0.00 (0.003)	−0.16 (0.301)	0.36** (0.052)	0.18** (0.029)	0.14* (0.057)	$\chi^2(9) = 7.39$ [0.60]
$z_{4,t}$	Newey-West	−0.00 (0.003)	0.04 (0.275)	0.38** (0.045)	0.20** (0.031)	0.14* (0.060)	$\chi^2(9) = 6.33$ [0.71]

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 11: GMM estimates of a pricing-to-market model with local currency pricing. Pricing-to-market variable: Domestic unit labour costs. Norwegian data 1980Q4–2002Q4.

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_4 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_5 (s_t + p_{COM,t} - p_{F,t}) + \alpha_6 (p_{H,t} - p_{F,t}) + \omega_t$$

Instruments	Bandwidth	α_0	α_1	α_4	α_5	α_6	$J-stat$
$z_{3,t}$	1	0.00 (0.002)	0.84** (0.226)	0.08 (0.095)	0.02 (0.023)	0.03 (0.036)	$\chi^2(13) = 8.59$ [0.80]
$z_{3,t}$	3	0.00 (0.002)	0.92** (0.185)	0.03 (0.068)	0.01 (0.018)	0.02 (0.030)	$\chi^2(13) = 8.16$ [0.83]
$z_{3,t}$	Newey-West	−0.00 (0.001)	0.99** (0.153)	0.04 (0.060)	0.01 (0.015)	0.03 (0.024)	$\chi^2(13) = 7.01$ [0.90]
$z_{4,t}$	1	0.00 (0.002)	0.75 (0.406)	0.12 (0.103)	0.04 (0.029)	0.03 (0.042)	$\chi^2(9) = 8.28$ [0.51]
$z_{4,t}$	3	0.00 (0.002)	0.92** (0.344)	0.03 (0.074)	0.01 (0.025)	0.03 (0.034)	$\chi^2(9) = 7.49$ [0.59]
$z_{4,t}$	Newey-West	−0.00 (0.001)	0.95** (0.347)	0.05 (0.063)	0.00 (0.019)	0.04 (0.031)	$\chi^2(9) = 5.93$ [0.75]

Note: Numbers in parenthesis denote standard errors, numbers in square brackets denote *p*-values. Asterisks ** and * denote statistical significance at the 1% and 5% level, respectively

Table 12: Lag-order selection criteria. Unrestricted VAR. Norwegian data 1981Q2-2003Q1.[a]

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}\}$

	Information criteria			Likelihood ratio tests			
k	SC	HQ	AIC	*k|5*	*k|4*	*k|3*	*k|2*
5	-8.7456	-9.5525	-10.097				
4	-8.9137	-9.5861	-10.040	0.05			
3	-9.2138	-9.7517	-10.115	0.11	0.50		
2	-9.5701	-9.9736	**-10.246**	0.30	0.80	0.89	
1	**-9.7116**	**-10.014**	-10.218	0.12	0.40	0.32	0.05

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}^{\text{b}}$

	Information criteria			Likelihood ratio tests			
k	SC	HQ	AIC	*k|5*	*k|4*	*k|3*	*k|2*
5	-14.419	-15.881	-16.868				
4	-14.922	-16.133	-16.949	0.42			
3	-15.415	-16.373	-17.020	0.33	0.29		
2	-16.049	-16.755	-17.231	0.58	0.64	0.88	
1	**-16.521**	**-16.975**	**-17.281**	0.36	0.36	0.46	0.12

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}^{\text{c}}$

	Information criteria			Likelihood ratio tests			
k	SC	HQ	AIC	*k|5*	*k|4*	*k|3*	*k|2*
5	-13.530	-14.993	-15.980				
4	-13.967	-15.178	-15.994	0.20			
3	-14.462	-15.420	-16.067	0.18	0.30		
2	-15.019	-15.725	**-16.201**	0.24	0.38	0.50	
1	**-15.350**	**-15.804**	-16.110	0.03	0.04	0.03	0.01

a The information criteria are the Schwarz (SC), Hannan-Quinn (HQ) and Akaike (AIC) information criteria (see e.g., Lütkepohl, 1991). Boldface letters are used to indicate the smallest values of the information criteria. The likelihood ratio tests $k|n$ are the F-transforms of the tests of the null hypothesis that the last $n-k$ lags are insignificant. The reported numbers are the associated p-values.

b $p_{H,t}$ denotes producer price index for manufactures.

c $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Table 13: Lag-order selection criteria. Unrestricted VAR. UK data 1981Q2-2003Q2.[a]

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}\}$

	Information criteria			Likelihood ratio tests			
k	SC	HQ	AIC	k|5	k|4	k|3	k|2
5	-8.4582	-9.2593	-9.8003				
4	-8.7515	-9.4191	-9.8700	0.51			
3	-9.0284	-9.5626	-9.9232	0.46	0.36		
2	-9.3778	-9.7784	**-10.049**	0.69	0.68	0.86	
1	**-9.5310**	**-9.7981**	-9.9784	0.20	0.14	0.11	0.01

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}^{b}$

	Information criteria			Likelihood ratio tests			
k	SC	HQ	AIC	k|5	k|4	k|3	k|2
5	-15.365	-16.817	-17.798				
4	-15.878	-17.080	-17.891	0.47			
3	-16.411	-17.363	-18.005	0.50	0.48		
2	-17.007	-17.708	**-18.181**	0.64	0.67	0.73	
1	**-17.295**	**-17.746**	-18.050	0.11	0.07	0.03	0.00

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}^{c}$

	Information criteria			Likelihood ratio tests			
k	SC	HQ	AIC	k|5	k|4	k|3	k|2
5	-14.296	-15.748	-16.728				
4	-14.779	-15.981	-16.792	0.35			
3	-15.346	-16.297	-16.940	0.51	0.65		
2	-15.896	-16.597	-17.071	0.53	0.63	0.49	
1	**-16.367**	**-16.817**	**-17.122**	0.32	0.36	0.21	0.12

a The information criteria are the Schwarz (SC), Hannan-Quinn (HQ) and Akaike (AIC) information criteria (see e.g., Lütkepohl, 1991). The likelihood ratio tests $k|n$ are the F-transforms of the tests of the null hypothesis that the last $n-k$ lags are insignificant. The reported numbers are the associated p-values.

b $p_{H,t}$ denotes producer price index for manufactures.

c $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Table 14: Misspecification tests. Unrestricted VAR(2). Norwegian data.[a]

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, pcom_t\}$

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
$p_{F,t}$	$F(5,71) = 0.74 [0.60]$	$F(4,68) = 0.32 [0.86]$	$\chi^2(2) = 0.49 [0.78]$	$F(14,61) = 1.37 [0.19]$
s_t	$F(5,71) = 1.36 [0.25]$	$F(4,68) = 0.33 [0.86]$	$\chi^2(2) = 3.22 [0.20]$	$F(14,61) = 0.64 [0.82]$
Vector tests	$F(20,130) = 0.67 [0.85]$		$\chi^2(4) = 3.85 [0.43]$	$F(42,175) = 1.03 [0.43]$

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, pcom_t, p_{H,t}\}^b$

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
$p_{F,t}$	$F(5,69) = 1.12 [0.36]$	$F(4,66) = 0.19 [0.94]$	$\chi^2(2) = 0.11 [0.95]$	$F(18,55) = 1.09 [0.38]$
s_t	$F(5,69) = 1.38 [0.24]$	$F(4,66) = 0.29 [0.88]$	$\chi^2(2) = 3.38 [0.19]$	$F(18,55) = 0.87 [0.62]$
$p_{H,t}$	$F(5,69) = 0.32 [0.90]$	$F(4,66) = 0.28 [0.89]$	$\chi^2(2) = 3.73 [0.15]$	$F(18,55) = 1.74 [0.06]$
Vector tests	$F(45,170) = 0.76 [0.86]$		$\chi^2(6) = 1.64 [0.95]$	$F(108,293) = 1.16 [0.17]$

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, pcom_t, p_{H,t}\}^c$

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
$p_{F,t}$	$F(5,69) = 0.21 [0.96]$	$F(4,66) = 0.35 [0.84]$	$\chi^2(2) = 0.99 [0.61]$	$F(18,55) = 1.43 [0.15]$
s_t	$F(5,69) = 1.86 [0.11]$	$F(4,66) = 0.32 [0.86]$	$\chi^2(2) = 2.21 [0.33]$	$F(18,55) = 0.86 [0.63]$
$p_{H,t}$	$F(5,69) = 0.80 [0.55]$	$F(4,66) = 0.47 [0.76]$	$\chi^2(2) = 1.26 [0.53]$	$F(18,55) = 1.00 [0.48]$
Vector tests	$F(45,170) = 0.68 [0.93]$		$\chi^2(6) = 5.52 [0.48]$	$F(108,293) = 0.87 [0.81]$

a The misspecification tests are the LM tests for residual autocorrelation ($F_{AR(1-5)}$) and autoregressive conditional heteroskedasticity ($F_{ARCH(1-5)}$) up to order 5, a test for normality of the residuals ($\chi^2_{NORMALITY}$), and a test for residual heteroskedasticity (F_{HETERO}). See Hendry & Doornik (2001) for details. The numbers in brackets are the corresponding *p*-values.

b $p_{H,t}$ denotes producer price index for manufactures.

c $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Table 15: Misspecification tests. Unrestricted VAR(2). UK data.[a]

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, pCOM,t\}$

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
$p_{F,t}$	$F(5,72) = 0.71\ [0.62]$	$F(4,69) = 1.02\ [0.40]$	$\chi^2(2) = 0.12\ [0.94]$	$F(14,62) = 1.33\ [0.21]$
s_t	$F(5,72) = 0.30\ [0.91]$	$F(4,69) = 0.13\ [0.97]$	$\chi^2(2) = 3.59\ [0.17]$	$F(14,62) = 0.76\ [0.70]$
Vector tests	$F(20,132) = 0.86\ [0.64]$		$\chi^2(4) = 6.32\ [0.18]$	$F(42,178) = 0.80\ [0.80]$

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, pCOM,t, p_{H,t}\}$[b]

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
$p_{F,t}$	$F(5,70) = 0.65\ [0.66]$	$F(4,67) = 0.73\ [0.58]$	$\chi^2(2) = 0.28\ [0.87]$	$F(18,56) = 0.84\ [0.65]$
s_t	$F(5,70) = 0.27\ [0.93]$	$F(4,67) = 0.09\ [0.99]$	$\chi^2(2) = 5.46\ [0.07]$	$F(18,56) = 0.70\ [0.80]$
$p_{H,t}$	$F(5,70) = 4.75\ [0.00]$	$F(4,67) = 0.44\ [0.78]$	$\chi^2(2) = 0.45\ [0.80]$	$F(18,56) = 0.85\ [0.64]$
Vector tests	$F(45,173) = 1.07\ [0.37]$		$\chi^2(6) = 6.76\ [0.34]$	$F(108,299) = 0.84\ [0.85]$

Information set: $x_t = \{p_{F,t}, s_t, ulc_{F,t}, pCOM,t, p_{H,t}\}$[c]

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
$p_{F,t}$	$F(5,70) = 0.72\ [0.61]$	$F(4,67) = 0.98\ [0.43]$	$\chi^2(2) = 0.14\ [0.93]$	$F(18,56) = 1.28\ [0.24]$
s_t	$F(5,70) = 0.24\ [0.94]$	$F(4,67) = 0.10\ [0.98]$	$\chi^2(2) = 3.91\ [0.14]$	$F(18,56) = 0.62\ [0.87]$
$p_{H,t}$	$F(5,70) = 3.26\ [0.01]$	$F(4,67) = 1.70\ [0.16]$	$\chi^2(2) = 4.41\ [0.11]$	$F(18,56) = 0.67\ [0.83]$
Vector tests	$F(45,173) = 1.21\ [0.19]$		$\chi^2(6) = 13.08\ [0.04]$	$F(108,299) = 0.74\ [0.96]$

a The misspecification tests are the LM tests for residual autocorrelation ($F_{AR(1-5)}$) and autoregressive conditional heteroskedasticity ($F_{ARCH(1-5)}$) up to order 5, a test for normality of the residuals ($\chi^2_{NORMALITY}$), and a test for residual heteroskedasticity (F_{HETERO}). See Hendry & Doornik (2001) for details. The numbers in brackets are the corresponding p-values.

b $p_{H,t}$ denotes producer price index for manufactures.

c $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Table 16: Trace tests for cointegration. UK data.

Information set $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}\}$

Rank	Trace test-statistic	Trace test-statistic (adjusted)[a]	Critical value[b]
0	44.91	42.89	35.96
1	17.10	16.33	18.16

Information set $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}$[c]

Rank	Trace test-statistic	Trace test-statistic (adjusted)	Critical value
0	51.61	48.13	57.32
1	28.09	26.20	35.96
2	8.23	7.67	18.16

Information set $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}$[d]

Rank	Trace test-statistic	Trace test-statistic (adjusted)	Critical value
0	79.60	74.24	57.32
1	44.63	41.62	35.96
2	16.95	15.81	18.16

a This is the degrees-of-freedom adjusted test-statistic suggested by Reinsel & Ahn (1988).

b The asymptotic critical values are the 95 per cent quantiles taken from table 13 in Doornik (2003) which accompanies Doornik (1998).

c $p_{H,t}$ denotes producer price index for manufactures.

d $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Table 17: Trace tests for cointegration. Norwegian data.

Information set $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}\}$

Rank	Trace test-statistic	Trace test-statistic (adjusted)[a]	Critical value[b]
0	42.99	41.03	35.96
1	9.44	9.01	18.16

Information set $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}$[c]

Rank	Trace test-statistic	Trace test-statistic (adjusted)	Critical value
0	74.34	69.27	57.32
1	29.93	27.89	35.96
2	9.54	8.89	18.16

Information set $x_t = \{p_{F,t}, s_t, ulc_{F,t}, p_{COM,t}, p_{H,t}\}$[d]

Rank	Trace test-statistic	Trace test-statistic (adjusted)	Critical value
0	92.71	86.39	57.32
1	40.40	37.64	35.96
2	11.10	10.35	18.16

a This is the degrees-of-freedom adjusted test-statistic suggested by Reinsel & Ahn (1988).

b The asymptotic critical values are the 95 per cent quantiles taken from table 13 in Doornik (2003) which accompanies Doornik (1998).

c $p_{H,t}$ denotes producer price index for manufactures.

d $p_{H,t}$ denotes domestic unit labour costs for the total economy.

Varying the degree of price stickiness
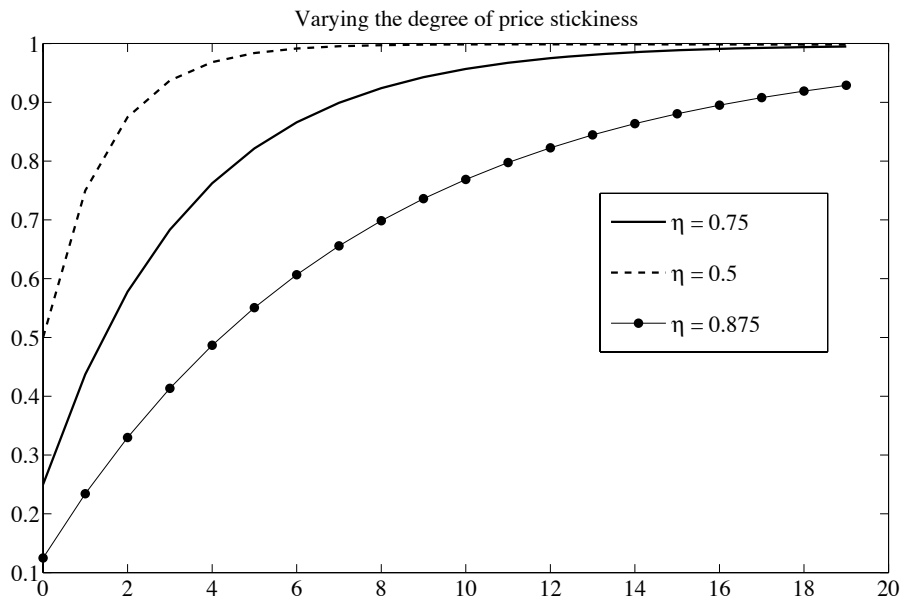
Figure 1: The response of import prices to 1% permanent exchange rate shock for different degrees of price stickiness η in the purely forward-looking LCP model. $\beta = 0.99$.

Varying the persistence of the exchange rate
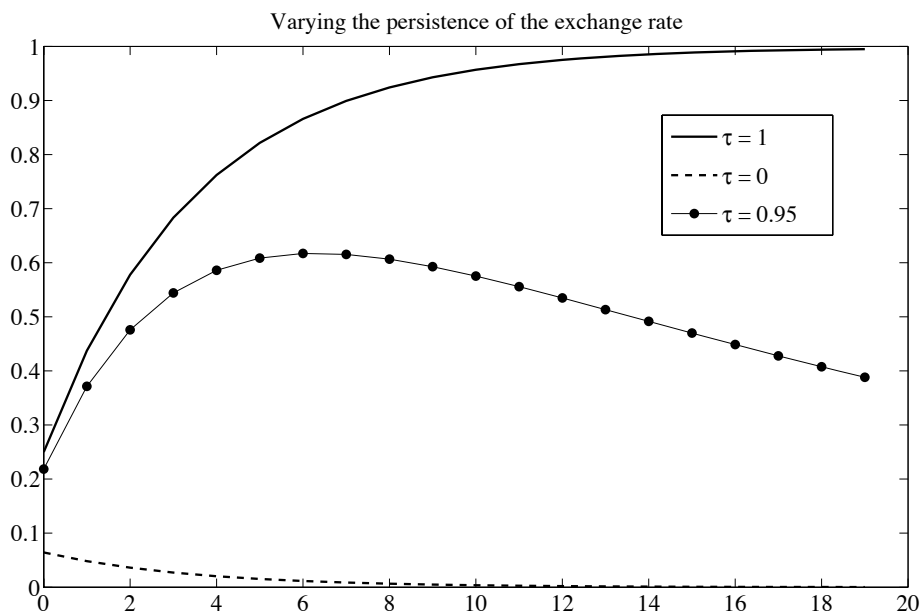
Figure 2: The response of import prices to 1% exchange rate shock for different degrees of persistence τ in the exchange rate in the purely forward-looking LCP model. $\beta = 0.99, \eta = 0.75$.

Varying the degree of indexation
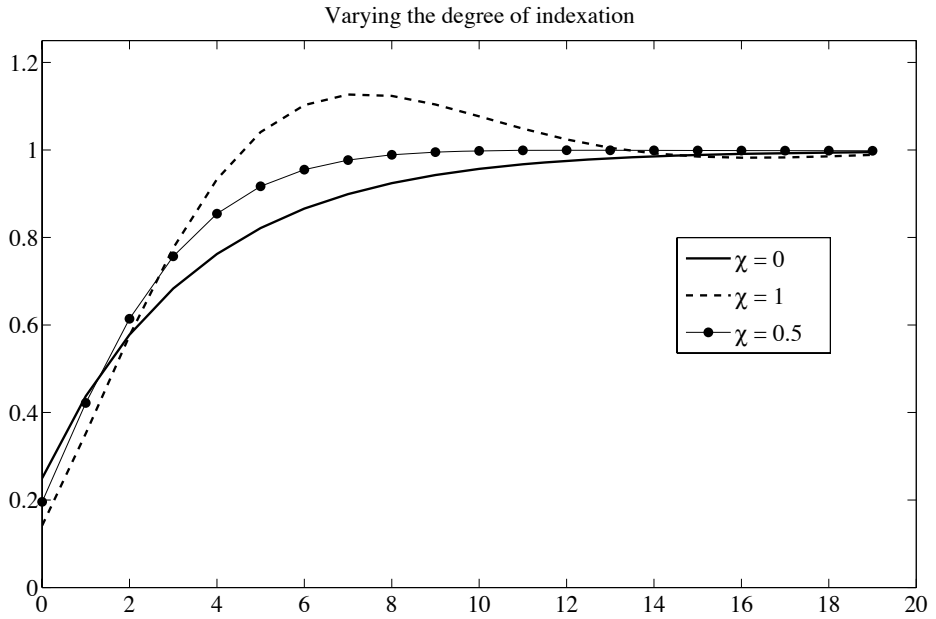
Figure 3: The response of import prices to 1% permanent exchange rate shock for different values of the indexation parameter in the hybrid LCP model. $\beta = 0.99, \eta = 0.75$.

Varying the share of LCP firms
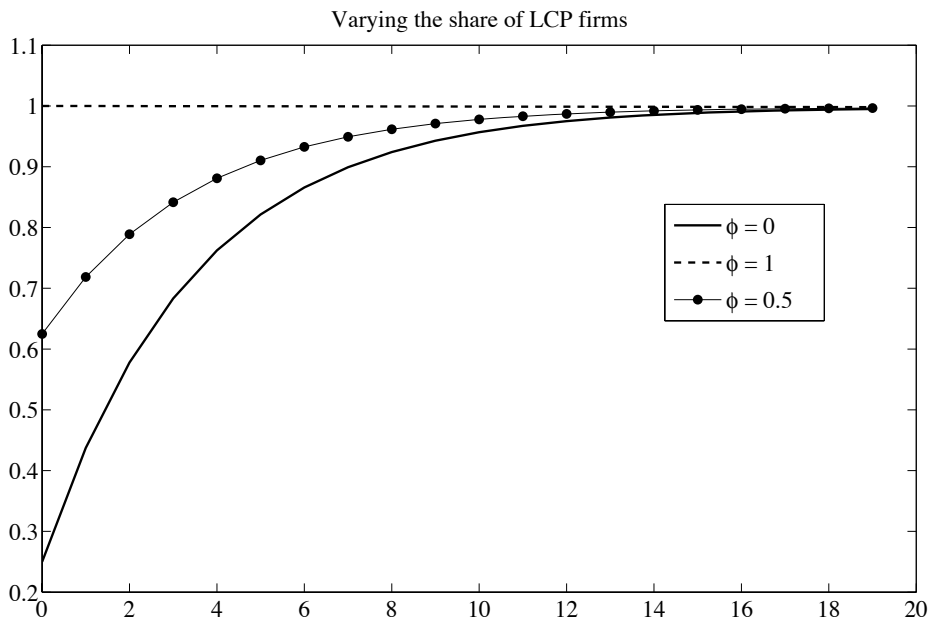
Figure 4: The response of import prices to 1% permanent exchange rate shock for different values of the share of PCP firms ϕ *in the LCP-PCP model.* $\beta = 0.99, \eta = 0.75$.

Varying the share of domestic firms

Figure 5: The response of import prices to 1% permanent exchange rate shock for different shares of import-competing firms N_H/N in the model with translog preferences. $\beta = 0.99, \eta = 0.75$.

Varying the share of distribution costs in the retail price of imports

Figure 6: The response of import prices to 1 % permanent exchange rate shock for different values of the steady-state share of distribution costs in the retail price of imports ζ in the distribution cost model. $\beta = 0.99, \eta = 0.75$.

Figure 7: Data series 1980Q1–2003Q1

Figure 8: Detrended data series 1980Q1–2003Q1

ASSESSING GMM AND ML ESTIMATES OF NEW KEYNESIAN IMPORT PRICE EQUATIONS

1   INTRODUCTION

Chapter 2 of this thesis estimates New Keynesian import price equations for the UK and Norway using generalised method of moments (GMM). Overall, the GMM estimates do not offer much support in favour of the theoretical models; the coefficient estimates are often economically implausible and statistically insignificant. Moreover, the estimates are sensitive to the exact choice of instrument set and the choice of weighting matrix in the GMM procedure.

The purpose of this paper is to assess the extent to which these findings can be attributed to finite-sample bias in GMM estimation. The exercise is motivated by the increasing Monte Carlo evidence that GMM estimators can exhibit substantial bias in small samples (see e.g., Hall, 2005, chap. 6, and references therein). The poor finite-sample performance of GMM is often attributed to weak identification. In a linear instrumental variables (IV) regression model, weak identification occurs when the instruments are only weakly correlated with the endogenous regressors. The litera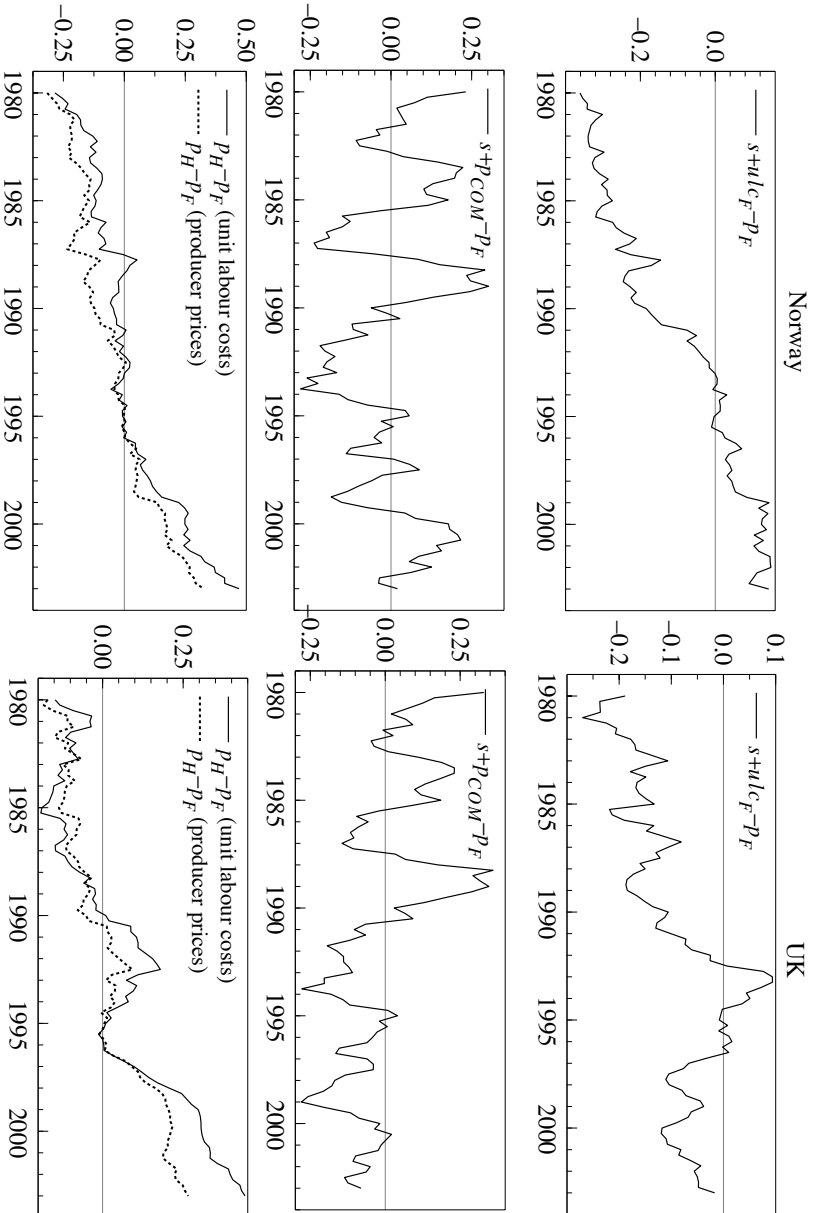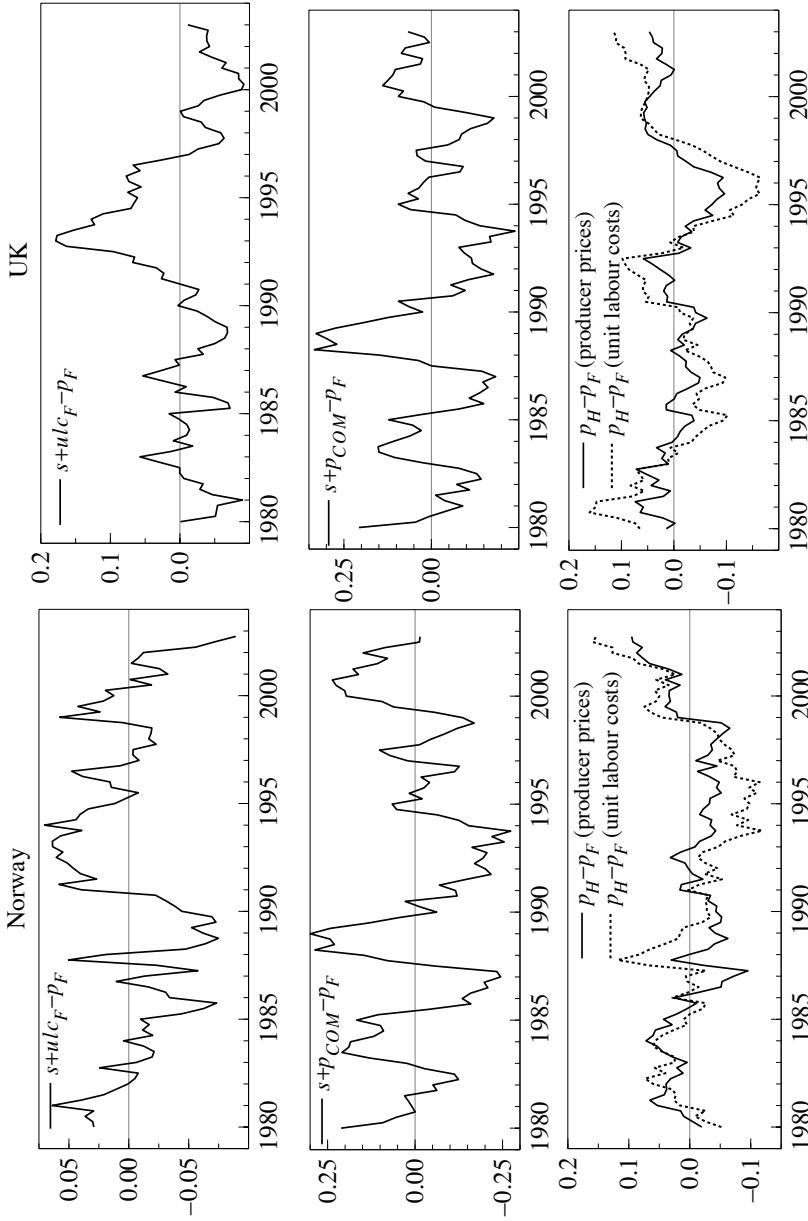ture on weak identification in GMM has demonstrated that, if the parameters (or a subset of the parameters) are weakly identified, conventional point estimates and confidence intervals based on the asymptotic normal approximation will be misleading, even in what will typically be considered a large sample (see Stock et al. (2002) for a survey). Mavroeidis (2005) and Fuhrer & Rudebusch (2004) find weak identification to be a concern when estimating single-equation rational expectations models such as the New Keynesian Phillips Curve and the output Euler equation using GMM.

To examine the small-sample properties of the GMM estimates, I construct a simple Monte Carlo experiment. Using the New Keynesian import price equation and a data-consistent vector autoregression (VAR) for the driving variables as the data generating process, I generate a large number of artificial datasets and estimate the coefficients in the import price equation using GMM. I conduct experiments for different specifications of the import price equation, different auxiliary VARs, different sample sizes, different instrument sets and different values of the structural parameters.

The main result that emerges from the simulation exercise is that the GMM estimates exhibit a significant small-sample bias. One key finding is that the GMM estimate of the coefficient on expected future import price inflation is insensitive to the true value of this parameter in the data generating process. Another key finding is that the small-sample bias is increasing in the number of lagged instruments used in estimation. However, when few lagged instruments are used, the coefficient estimates are typically imprecise and often statistically insignificant.

The paper proceeds to compare the performance of GMM to the performance of an alternative estimation technique, namely maximum likelihood (ML). ML estimation is

based on the closed form solution of the model, meaning that the cross-equation restrictions implied by the rational expectations assumption are imposed during estimation. When the model is correctly specified, both the GMM and the ML estimators are consistent. However, evidence in previous literature suggests that the ML estimator has superior finite-sample properties in single-equation rational expectations models (see e.g., Fuhrer et al., 1995; Fuhrer & Rudebusch, 2004; Lindé, 2005). My paper extends this evidence to the New Keynesian import price equation. In line with the previous Monte Carlo studies, I find that the ML estimates are fairly accurate, even in small samples, and are in general more precise than the GMM estimates.

Motivated by these findings, the last part of the paper uses ML to estimate New Keynesian import price equations for the UK. The preferred specification is a purely forward-looking model which combines local currency pricing (LCP) and producer currency pricing (PCP). The coefficient estimates are statistically significant and within the ranges suggested by the theory. In particular, the estimate of the share of exporters engaging in PCP is broadly consistent with data on invoicing currency for UK imports. The historical fit of the restricted equilibrium-correction model (EqCM) for import prices implied by the rational expectations model is comparable to that of a data-based EqCM over the sample period. Moreover, the two models imply similar estimates of the exchange rate pass-through.

The remainder of the paper is organised as follows. Section 2 describes the data generating process. Section 3 discusses the GMM and ML estimators, and section 4 presents the simulation results. Then, section 5 reinterprets the previous empirical findings and presents ML estimates of the New Keynesian import price equation for the UK. Section 6 concludes.

2   THE DATA GENERATING PROCESS

The data generating process in the Monte Carlo experiments is a New Keynesian import price equation augmented with a data-consistent reduced-form model for the driving variables. A detailed derivation of the import price equations with references to the literature is provided in chapter 2 of the thesis.

2.1   *The New Keynesian import price equation*

The import price equation is derived from the Calvo (1983) model of time-dependent pricing. The probability that a foreign firm is allowed to re-optimise its price in a given period is $1 - \eta$. The composite import good is a constant elasticity of substitution (CES) aggregate of differentiated foreign goods. The CES assumption implies that in the flexible-price equilibrium, the optimal price is a constant mark-up over marginal

costs, and hence that, conditional on the exporters' marginal costs, the exchange rate pass-through to import prices is complete.[1] The foreign exporters' marginal cost $mc_{F,t}$ is assumed to be given by

$$mc_{F,t} = (1-\delta)ulc_{F,t} + \delta p_{COM,t}, \tag{1}$$

where $0 \leq \delta \leq 1$, $ulc_{F,t}$ denotes foreign unit labour costs and $p_{COM,t}$ is a commodity price index, both measured in foreign currency. Lower case letters denote variables in logs.

In the baseline model, all exporters set prices in the currency of the importing country, that is, all exporters engage in LCP. The structural equation for aggregate import prices is

$$\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta}\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + \nu_t, \tag{2}$$

where Δ is the difference operator (i.e., $\Delta x_t \equiv x_t - x_{t-1}$), $p_{F,t}$ denotes import prices (in the importing country's currency), s_t is the nominal exchange rate and β is the subjective discount factor of households in the importing country ($0 < \beta < 1$). Finally, ν_t is a Gaussian white noise process with mean zero and variance σ_ν^2.

The second model I consider is a 'hybrid' LCP model with indexation to past import price growth. This model is derived from the assumption that a representative exporter i who is not allowed to re-optimise her price in period t, updates her price $P_{F,t}(i)$ according to the partial indexation rule

$$P_{F,t}(i) = \left(\frac{P_{F,t-1}}{P_{F,t-2}}\right)^\chi P_{F,t-1}(i), \tag{3}$$

where $\chi \in [0,1]$ is the indexation parameter. The structural equation for aggregate import prices now becomes

$$\Delta p_{F,t} = \frac{\beta}{1+\beta\chi}E_t\Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi}\Delta p_{F,t-1} \tag{4}$$
$$+ \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)}\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + \nu_t.$$

The model implies that the maximum weight on lagged import price inflation is $1/(1+\beta)$, which is approximately 0.5 for values of β close to one.

The third model assumes that a subset ϕ of exporters sets prices in their own currency (i.e., engages in PCP) and a subset $1 - \phi$ engages in LCP. Under this assumption the

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[1]Chapter 2 of this thesis also considers pricing-to-market models, that is, models that abandon the assumption of a constant frictionless mark-up.

import price equation becomes

$$\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \phi(\Delta s_t - \beta E_t \Delta s_{t+1}) \quad (5)$$
$$+ \frac{(1-\beta\eta)(1-\eta)}{\eta}(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}) + v_t.$$

The final model allows for both LCP- and PCP price setters as well as indexation to past import price growth:

$$\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} \quad (6)$$
$$+ \phi\left(\Delta s_t - \frac{\beta}{1+\beta\chi} E_t \Delta s_{t+1} - \frac{\chi}{1+\beta\chi} \Delta s_{t-1}\right)$$
$$+ \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)}(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}) + v_t.$$

2.2 Auxiliary model for the driving variables

The import price equation is augmented with a data-consistent reduced-form model for the driving variables $s_t, ulc_{F,t}$, and $p_{COM,t}$. This approach is chosen to ensure that the data generating process is empirically relevant, thus allowing the Monte Carlo evidence to speak directly to the estimation results reported in chapter 2.[2]

In the benchmark experiments the completing model is a fourth-order VAR in $\Delta s_t, \Delta ulc_{F,t}$ and $\Delta p_{COM,t}$. To check the sensitivity of the Monte Carlo results with respect to the exact specification of the completing model, I also conduct experiments for the purely forward-looking LCP model where the completing model for the driving variables is a VAR with feedback from lagged import price growth $\{\Delta p_{F,t-j}\}_{j=1}^{4}$ and relative prices $(s+ulc_F-p_F)_{t-1}$ and $(s+p_{COM}-p_F)_{t-1}$.

The parameters in the auxiliary VAR models are estimated using OLS on quarterly UK data covering the period 1981Q2–2003Q2.[3] The estimated parameters are reported in table 1. The VARs are estimated with a constant term, three centered seasonal dummies and an impulse dummy for the fourth quarter of 1992 to capture the effects of sterling's exit from the European Exchange Rate Mechanism. The deterministic terms are not included in the data generating process in the Monte Carlo experiments, however. Table 2 reports the residual correlation matrix and residual misspecification tests for the two models. Both models appear to be well-specified; the diagnostic tests reveal no

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[2]Fuhrer & Rudebusch (2004) use the same approach when assessing GMM and ML estimates of the Euler equation for output. An alternative approach would be to build a structural general equilibrium model for all the variables (see e.g., Lindé (2005) for an application of this approach to the New Keynesian Phillips Curve).

[3]See chapter 2 for a description of the data.

evidence of residual autocorrelation, heteroscedasticity or non-normality.[4]

2.3   Closed form solution

The New Keynesian import price equation and the auxiliary VAR define a system of linear expectational difference equations that can be solved, for example, by the method outlined in Klein (2000). The solution involves substituting in for the model-consistent expectations of the driving variables in the forward-looking import price equation. Provided the number of explosive roots in the system equals the number of non-predetermined ('forward-looking') variables, the rational expectations model has a unique solution (Blanchard & Kahn, 1980, prop.1).

Define

$$Z_t = \{\Delta p_{F,t}, \Delta s_t, \Delta ulc_{F,t}, \Delta p_{COM,t}, s_t + ulc_{F,t} - p_{F,t}, s_t + p_{COM,t} - p_{F,t}\}'$$

and

$$\varepsilon_t = \{v_t, \varepsilon_{s,t}, \varepsilon_{ulc_F,t}, \varepsilon_{p_{COM},t}\}',$$

where $\varepsilon_{s,t}$, $\varepsilon_{ulc_F,t}$ and $\varepsilon_{p_{COM},t}$ are the reduced-form residuals from the auxiliary VAR. The transition equation describing the model solution can then be written as

$$Z_t = D_1 Z_{t-1} + D_2 Z_{t-2} + D_3 Z_{t-3} + D_4 Z_{t-4} + D_\varepsilon \varepsilon_t, \tag{7}$$

where ε_t is a Gaussian white-noise process with variance-covariance matrix Ω. The elements in the coefficient matrices D_1, D_2, D_3, D_4 and D_ε are non-linear functions of the underlying structural parameters, including the parameters in the processes generating the driving variables.

When the completing model for the driving variables is a first-differenced VAR without feedback from import prices, the existence and nature of the solution to the rational expectations model is determined entirely by the roots of the characteristic equation associated with the import price equation.[5] Another feature of this data generating process is that one of the roots of the characteristic polynomial $\left| I - D_1 z - D_2 z^2 - D_3 z^3 - D_4 z^4 \right| = 0$ is on the unit circle, implying that the model is non-stationary. Specifically, real unit labour costs ($s_t + ulc_{F,t} - p_{F,t}$) and real commodity

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[4]A reduction in the lag-length or exclusion of the seasonal dummies induces residual misspecification in the equation for foreign unit labour costs.

[5]E.g., in the purely forward-looking LCP model the characteristic equation is

$$z^2 - \left(1 + \frac{1}{\beta} + \frac{(1 - \beta\eta)(1 - \eta)}{\beta\eta}\right) z + \frac{1}{\beta} = 0,$$

and the roots are $z_1, z_2 = \left\{\eta, \frac{1}{\beta\eta}\right\}$. The roots are positive, lying on either side of unity, hence the model has a unique forward solution.

prices $(s_t + p_{COM,t} - p_{F,t})$ are integrated of order one. However, $s_t + (1 - \delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}$ is stationary; that is, $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ are cointegrated with cointegration parameter $\delta/(1 - \delta)$. The variance of the shock in the import price equation is set to $\sigma_v^2 = 0.01975$ and is calibrated to make the model roughly match the variance of quarterly UK import price inflation over the period 1981Q2–2003Q2. The remaining elements of Ω are taken from the variance-covariance matrix of the estimated VAR, $\widehat{\Sigma}$. That is,

$$\Omega = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \widehat{\Sigma} \end{bmatrix}. \tag{8}$$

The structural shock in the import price equation is uncorrelated with the current values of the driving variables; that is, $E[v_t s_t] = E[v_t ulc_{F,t}] = E[v_t p_{COM,t}] = 0$. Hence, the first-differenced driving variables $\Delta s_t, \Delta ulc_{F,t}$ and $\Delta p_{COM,t}$ are strictly exogenous and are thus valid instruments for $\Delta p_{F,t+1}$.

When the data generating process for the driving variables is a VAR with feedback from lagged import price growth and relative prices, the nature of the solution to the rational expectations model cannot be determined from knowledge of the parameters of the import price equation alone, but will depend on the parameters in the auxiliary VAR. In this case, all the roots of the characteristic polynomial associated with (7) are outside the unit circle, and the model is stationary. To calibrate the variance-covariance matrix Ω for this data generating process I proceed as follows. First, I estimate an unrestricted fourth-order vector equilibrium correction model (VEqCM) in $\Delta p_{F,t}, \Delta s_t, \Delta ulc_{F,t}$ and $\Delta p_{COM,t}$, with the two cointegrating relations $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$. From this VEqCM I obtain an estimate of the residual variance-covariance matrix $\widetilde{\Sigma}$. Then, I find Ω by solving the matrix equation

$$\widetilde{\Sigma} = D_\varepsilon \Omega D_\varepsilon', \tag{9}$$

where D_ε is obtained from the solution to the baseline purely forward-looking LCP model with $\beta = 0.99$, $\eta = 0.75$ and $\delta = 0.25$. Thus, the variance-covariance matrix of the restricted VEqCM implied by the rational expectations model is calibrated to match the variance-covariance matrix of the unrestricted VEqCM estimated on actual UK data.[6] In this case, the current values of the driving variables are correlated with the structural shock in the import price equation. This means that $\Delta s_t, \Delta ulc_{F,t}$, and $\Delta p_{COM,t}$ are not strictly exogenous and hence, are not valid instruments for $\Delta p_{F,t+1}$.

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[6]Kapetanios et al. (2005) use a similar approach when generating pseudo-data from a larger-scale DSGE model.

3   ESTIMATION PROCEDURES

Before turning to the details of the GMM and ML estimation procedures, a comment on
the scope of the simulation study is in order. The standard argument in favour of a limited
information method such as GMM is that it is likely to be more robust to misspecification
of the completing model for the driving variables. The issue of which is the better esti-
mator in the presence of misspecification is not addressed in this paper, however. Rather,
the purpose of the simulation study is to compare the finite-sample properties of GMM
and ML under the assumption that the model is correctly specified. In this setting, both
estimators are consistent and converge to the same probability limits. However, because
it imposes the cross-equation restrictions implied by the rational expectations hypothesis,
the ML estimator is asymptotically more efficient than the GMM estimator.

3.1   Generalised method of moments

The GMM estimating equations are restricted versions of the following general specifi-
cation

$$\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_3 (s_t + p_{COM,t} - p_{F,t}) \quad (10)$$
$$+ \alpha_4 \Delta p_{F,t-1} + \alpha_5 (\Delta s_t - \alpha_1 \Delta s_{t+1} - \alpha_4 \Delta s_{t-1}) + \omega_t,$$

where $\Upsilon = \{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ are nonlinear functions of the underlying structural
parameters. Specifically,

$$\alpha_1 = \frac{\beta}{1 + \beta\chi}$$
$$\alpha_2 = \frac{1}{1 + \beta\chi} \frac{(1 - \eta)(1 - \beta\eta)}{\eta}(1 - \delta)$$
$$\alpha_3 = \frac{1}{1 + \beta\chi} \frac{(1 - \eta)(1 - \beta\eta)}{\eta}\delta$$
$$\alpha_4 = \frac{\chi}{1 + \beta\chi}$$
$$\alpha_5 = \phi$$

Like in chapter 2 of this thesis, the estimating equation imposes some (but not all) of the
non-linear parameter restrictions implied by the theoretical model. The error term ω_t is
defined as $\omega_t = v_t - \alpha_1 (\varepsilon_{t+1} - \alpha_5\xi_{t+1})$ and is thus a linear combination of the structural
shock v_t and the rational expectations forecast errors, $\varepsilon_{t+1} \equiv \Delta p_{F,t+1} - E_t\Delta p_{F,t+1}$ and
$\xi_{t+1} \equiv \Delta s_{t+1} - E_t\Delta s_{t+1}$.

  Let z_t be a vector of variables in the exporter's information set at time t satisfying

$E_t[v_t z_t] = 0$ and define

$$
\begin{aligned}
\omega_t(\Upsilon) \quad = \quad & \Delta p_{F,t} - \alpha_1 \Delta p_{F,t+1} - \alpha_2 (s_t + ulc_{F,t} - p_{F,t}) - \alpha_3 (s_t + p_{COM,t} - p_{F,t}) \\
& -\alpha_4 \Delta p_{F,t-1} - \alpha_5 (\Delta s_t - \alpha_1 \Delta s_{t+1} - \alpha_4 \Delta s_{t-1}).
\end{aligned}
$$

Then, according to the theoretical model

$$
E_t[\omega_t(\Upsilon) z_t] = 0, \quad t = 1, \dots, T, \tag{11}
$$

where T is the number of observations. These moment conditions provide the basis for the GMM estimation. The GMM estimator is given by

$$
\widehat{\Upsilon} = \arg\min_{\Upsilon} \left(\frac{1}{T} \sum_{t=1}^{T} \omega_t(\Upsilon) z_t \right)' W_T \left(\frac{1}{T} \sum_{t=1}^{T} \omega_t(\Upsilon) z_t \right), \tag{12}
$$

where W_T is a positive semi-definite weighting matrix. The weighting matrix used in the simulation experiments is a heteroscedasticity and autocorrelation consistent (HAC) estimate of the inverse of the long-run covariance matrix of the sample moments. The autocovariances are weighted using the Bartlett kernel as proposed by Newey & West (1987), and the lag–truncation parameter is set to one.[7] A test of the over-identifying restrictions can be based on the J-test statistic of Hansen (1982)

$$
J = \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \omega_t(\widehat{\Upsilon}) z_t \right)' \widehat{S}_T^{-1} \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \omega_t(\widehat{\Upsilon}) z_t \right) \xrightarrow{d} \chi^2(q), \tag{13}
$$

where $\widehat{S}_T$ is the HAC estimate of the long-run covariance matrix of the sample moments and q denotes the number of over-identifying restrictions.

Which variables are relevant instruments for use in the GMM estimation depends on the data generating process. If the data generating process for the driving variables is a first-differenced fourth-order VAR then, assuming only lagged instruments are available, the set of relevant instruments comprises four lags of each of the first-differenced driving variables and the lagged relative prices $(s + ulc_F - p_F)_{t-1}$, and $(s + p_{COM} - p_F)_{t-1}$. In the hybrid models with inflation indexation, $\Delta p_{F,t-1}$ is an exogenous regressor and hence also a relevant instrument. However, further lags of import price growth, $\Delta p_{F,t-2}, \Delta p_{F,t-3}$ and $\Delta p_{F,t-4}$, are redundant; that is, they do not contain any additional information about next period's import price inflation $\Delta p_{F,t+1}$ that is not already contained in the other variables in the instrument set. By contrast, when the data generating process for the

---

[7]To check the sensitivity of the results with respect to the choice of lag truncation parameter, I conducted an experiment on the baseline LCP model using a Newey-West weighting matrix with lag truncation parameter 4. The results were similar to those obtained with lag-truncation parameter equal to 1.

driving variables is the fourth-order VAR with feedback from import prices, the set of relevant instruments also includes four lags of the first difference of import prices.[8]

The instrument set used in the benchmark simulation experiments is:

$$z_{1,t} \quad = \quad \left[\begin{array}{c} \left\{\Delta s_{t-j}\right\}_{j=1}^{4}, \left\{\Delta ulc_{F,t-j}\right\}_{j=1}^{4}, \left\{\Delta p_{COM,t-j}\right\}_{j=1}^{4}, \left\{\Delta p_{F,t-j}\right\}_{j=1}^{4}, \\ (s+ulc_F-p_F)_{t-1}, (s+p_{COM}-p_F)_{t-1} \end{array} \right]$$

Evidence in previous literature (see e.g., Hall, 2005, chap. 6, and references therein) suggests that the finite-sample bias in GMM estimation is increasing in the number of instruments, or more precisely, in the number of over-identifying restrictions. This is the motivation for also considering the smaller instrument sets $z_{2,t}$ and $z_{3,t}$:

$$z_{2,t} \quad = \quad \left[\begin{array}{c} \left\{\Delta s_{t-j}\right\}_{j=1}^{2}, \left\{\Delta ulc_{F,t-j}\right\}_{j=1}^{2}, \left\{\Delta p_{COM,t-j}\right\}_{j=1}^{2}, \left\{\Delta p_{F,t-j}\right\}_{j=1}^{2}, \\ (s+ulc_F-p_F)_{t-1}, (s+p_{COM}-p_F)_{t-1} \end{array} \right]$$

$$z_{3,t} \quad = \quad \left[\begin{array}{c} \Delta s_{t-1}, \Delta ulc_{F,t-1}, \Delta p_{COM,t-1}, \Delta p_{F,t-1}, \\ (s+ulc_F-p_F)_{t-1}, (s+p_{COM}-p_F)_{t-1} \end{array} \right]$$

The instrument sets $z_{2,t}$ and $z_{3,t}$ contain two lags and one lag of the first-differenced variables, respectively. The GMM results in this chapter are obtained using the iterated GMM estimator implemented in Michael T. Cliff's GMM routines for Matlab.[9]

3.2   *Maximum likelihood*

The ML estimates are obtained using the algorithms for estimation of rational expectations models implemented in Dynare (see Juillard, 2005). The first step in the estimation procedure is to compute the closed form solution (or the 'observable structure') of the model, see equation (7). This requires that we specify a completing model for the driving variables $s_t, ulc_{F,t}$ and $p_{COM,t}$. The completing model (and the data generating process) in the ML experiments is the first-differenced VAR without feedback from import prices discussed in the previous section.

Supposing we have $T+4$ observations of the N variables in Z_t, we can condition on the first four observations and base the estimation on the last T observations. Under the assumption that the innovations $\{\varepsilon_t\}_{t=1}^{T}$ are distributed as $N(0,\Omega)$, the prediction-error decomposition of the conditional log-likelihood function can be written (see e.g.,

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[8]If current values of the driving variables are used as instruments, variables dated $t-4$ are redundant.

[9]Software and documentation are available to download from http://www.mgmt.purdue.edu/faculty/mcliff/progs.html.

Canova, 2005, chap. 6)

$$\ln \ell(D_1, D_2, D_3, D_4, D_\varepsilon, \Omega) = -(TN/2)\ln(2\pi) - (T/2)\ln\left|D_\varepsilon \Omega D'_\varepsilon\right| \tag{14}$$

$$-(1/2)\sum_{t=1}^{T}\left(Z_t - Z_{t|t-1}\right)'\left(D_\varepsilon \Omega D'_\varepsilon\right)^{-1}\left(Z_t - Z_{t|t-1}\right),$$

where $Z_{t|t-1} = E\left(Z_t | Z_{t-1}, \dots, Z_1\right) = D_1 Z_{t-1} + D_2 Z_{t-2} + D_3 Z_{t-3} + D_4 Z_{t-4}$. The estimation procedure starts from an initial guess for the values of the parameters and then uses a numerical optimisation algorithm to find the parameter values that maximise (14), at each iteration computing the solution to the rational expectations model.[10]

The likelihood is maximised with respect to a subset of the parameters that appear in the model. As noted by Roberts (2005), it can be difficult to achieve convergence when the model has multiple highly collinear parameters as is typical for VAR models. Therefore, the coefficients in the auxiliary models for the driving variables, including the elements of the residual covariance matrix, are held fixed at their population values. To ensure comparability with the GMM estimates, the likelihood is maximised with respect to the parameters that appear in the GMM estimating equation and the variance of the shock to the import price equation. That is, the likelihood is maximised with respect to $\theta = \left\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \sigma_v^2\right\}$. The variance-covariance matrix of the estimates $\widehat{\theta}$ is approximated by the inverse of the Hessian of the log-likelihood function,

$$E\left[\left(\widehat{\theta} - \theta\right)\left(\widehat{\theta} - \theta\right)'\right] \simeq \left[-\left.\frac{\partial^2 \ell(\theta)}{\partial\theta\partial\theta'}\right|_{\theta=\widehat{\theta}}\right]^{-1}. \tag{15}$$

4   MONTE CARLO EVIDENCE

In each experiment I generate 5000 artificial datasets from the transition equation (7) with random shocks drawn from a multivariate normal distribution with mean zero and variance-covariance matrix Ω.[11] To limit the influence of the initial conditions, the first 1000 observations in each dataset are discarded. Throughout, the values of β and δ are kept fixed at 0.99 and 0.25, respectively. Unless stated otherwise, the estimated models are correctly specified (with the exception that the GMM estimation allows for a non-zero constant term) and all the variables in the instrument set used in the GMM estimation are valid instruments. For each experiment I compute the Monte Carlo mean, the median and the standard deviation of the estimates, in addition to the median standard error of the parameter estimates.

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[10]The results in this paper are obtained using Christopher Sims' `csminwel` algorithm.

[11]The model is solved and simulated using the algorithms implemented in Dynare.

4.1   Generalised method of moments

This section reports the simulation evidence for the GMM estimator in the purely forward-looking LCP model (section 4.1.1), the hybrid LCP model (section 4.1.2) and the model that allows for both PCP and LCP (section 4.1.3).

4.1.1   The purely forward-looking model with local currency pricing

Table 3 summarises the results of the Monte Carlo experiments for the purely forward-looking LCP model for different values of the price stickiness parameter $\eta = \{0.5, 0.75, 0.875\}$ and different sample sizes $T = \{100, 200, 1000\}$. The data generating process for the driving variables is a first-differenced VAR without feedback from import prices and the instrument set is $z_{1,t}$. The key result emerging from the table is that the GMM estimates exhibit a distinct small-sample bias. For example, when the true value of the price stickiness parameter η is 0.75 and the sample size is $T = 100$, the median estimate of the coefficient on the forward term (α_1) is 0.473. The bias decreases when the sample size is increased, however, the GMM estimate still understates the true value when $T = 1000$.

There is a negative relationship between the bias in the coefficient on the forward term and the bias in the coefficients on the levels terms: the GMM estimates overstate the true values of the coefficients on real unit labour costs and real commodity prices (α_2 and α_3). With a sample size of $T = 100$, the median estimates of α_2 and α_3 are 0.133 and 0.046, compared to the true values 0.064 and 0.022, respectively. The estimator biases are evident in figure 1, which displays the distribution of the parameter estimates together with a fitted normal density. The true values of the parameters are indicated with a vertical line. The graph reveals that the empirical distribution of the estimates of α_1 is slightly left-skewed, whereas the distributions of the estimates of α_2 and α_3 are slightly right-skewed. The results also indicate that the estimated GMM standard errors understate the uncertainty surrounding the estimates when the sample size is small ($T = 100$ and $T = 200$): the median estimated standard errors are smaller than the Monte Carlo standard deviations of the estimates.

GMM estimation of New Keynesian import price equations on data for the UK often yields statistically insignificant estimates of the coefficient on the forward term (see chapter 2 of this thesis). In the Monte Carlo experiments for the purely forward-looking LCP model with $\eta = 0.75$ and $T = 100$, the rejection frequencies of the t-tests on α_1, α_2 and α_3 are 0.60, 0.64 and 0.65, respectively, when using a 5% nominal significance level.

In the purely forward-looking LCP model it is possible to back out estimates of the underlying parameters in the Calvo model. The implied estimates of the frequency of

price adjustment η and the marginal cost parameter δ are fairly accurate: the median estimates are 0.78 and 0.25, respectively.[12] Thus, small-sample estimation bias mainly affects the estimate of the discount factor β. The difficulties involved in estimating the discount factor have been noted by many authors (see e.g., the discussion in Gregory et al., 1993). The results presented here would seem to offer a justification for the common practice of fixing the value of this parameter when estimating single-equation rational expectations models. However, as noted by Canova & Sala (2006) (albeit in a slightly different context), fixing the value of the discount factor at the wrong value may induce estimation biases in the other model parameters.

A similar pattern emerges for lower and higher values of the price stickiness parameter η: the GMM estimates of α_1 are biased downwards, whereas the estimates of α_2 and α_3 exhibit an upward bias. The largest biases are obtained when the frequency of price adjustment is low: when $T = 100$ and $\eta = 0.875$ (i.e., when firms re-optimise prices on average once every eight quarters), the median estimate of the coefficient on expected future import price inflation is 0.341. In what follows, the value of the price stickiness parameter is held constant at $\eta = 0.75$.

As discussed above, when the driving variables follow a pure first-differenced VAR process, the relative prices $s_t + ulc_{F,t} - p_{F,t}$ and $s_t + p_{COM,t} - p_{F,t}$ are non-stationary, but cointegrated. Dolado et al. (1991) suggest the following two-step procedure for estimating linear rational expectations models in this case.[13] First, estimate the cointegration parameters using a procedure such as e.g., Johansen's (1988) maximum likelihood approach. Then, estimate the remaining parameters using GMM, taking the estimated cointegration parameters obtained in the first step as given. Ignoring the sampling uncertainty of the cointegration parameters when estimating the parameters in the second step is justified by the fact that the former are super-consistent, that is, they converge at a rate faster than $\sqrt{T}$ (see e.g., Banerjee et al., 1993, pp. 158–159). To assess the potential of the two-step estimation procedure to improve the small-sample performance of GMM, I constructed an experiment where δ is held fixed at its true value, and the remaining parameters are estimated using GMM. As is evident from table 4, the estimator biases and the median standard errors are of similar magnitude to those obtained using the one-step GMM procedure.

Table 5 reports the results obtained when the data generating process for the driving variables is the VAR with feedback from $\left\{\Delta p_{F,t-j}\right\}_{j=1}^{4}$, $(s + ulc_F - p_F)_{t-1}$ and $(s + p_{COM} - p_F)_{t-1}$. As is evident from the table, the results are similar to those obtained

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[12]Solving for η involves finding the smallest root of a second-order polynomial. When computing the median estimate of η, I excluded replications that produced complex roots.

[13]Applications of the two-step procedure to non-linear rational expectations models are found in Cooley & Ogaki (1996) and Croix & Urbain (1998). For a general discussion of GMM with non-stationary variables see Ogaki (1999).

when the data generating process is a VAR without feedback from import prices: the bias in the estimate of the coefficient on the forward term α_1 is of the same magnitude (when $T = 100$ the median estimate is 0.481), whereas the upward biases in the estimates of the coefficients on the levels terms, α_2 and α_3, are slightly smaller (when $T = 100$ the median estimates are 0.114 and 0.032, respectively).

A feature of this data generating process is that the driving variables are not strictly exogenous, and hence, the current values of the driving variables are not valid instruments. As a second exercise, I examined the consequences of wrongly including the current values of the driving variables ($\Delta s_t, \Delta ulc_{F,t}$ and $\Delta p_{COM,t}$) in the instrument set. In this case, the orthogonality conditions in (11) are violated, implying that a bias will persist in the GMM estimates even asymptotically. Specifically, the probability limits of the GMM estimates of α_1, α_2 and α_3 (obtained from a simulation with 500000 observations) are 0.977, 0.154 and 0.037, respectively. The question is whether the violation of the moment conditions can be detected by the J-test of the over-identifying restrictions in small samples. To examine this, I compute the rejection frequency for the J-test (i.e., the fraction of the replications in which the J-statistic exceeded the 5% critical value of the chi-squared distribution) in Monte Carlo experiments for a sample size $T = 100$. In the first experiment the instrument set comprises the current values of the driving variables in addition to the variables in $z_{1,t}$. The rejection frequency of the J-test in this experiment is 0.00. In the second experiment, I exclude variables dated $t - 2$ or earlier from the instrument set. Now, the rejection frequency increases to 0.32. In conclusion, the $J-$test has very low power to detect this particular form of model misspecification, especially when many lagged instruments are used.[14]

I proceed to consider to what extent the small-sample bias in GMM depends on the number of lagged instruments used in the estimation. Table 6 shows the properties of the GMM estimates when the estimation is based on the instrument sets $z_{2,t}$ and $z_{3,t}$. It is clear that reducing the number of instruments reduces the small-sample bias. For example; when the data generating process for the driving variables is the VAR without feedback from import prices and estimation is based on the instrument set $z_{3,t}$, the median estimates of α_1, α_2 and α_3 are, respectively, 0.673, 0.124 and 0.042.

The main effect of limiting the number of lagged instruments is thus to reduce the bias in the estimate of the coefficient on the forward term. The finding that the small-sample bias is increasing in the number of lagged instruments is not surprising; with many lagged instruments, some of the instruments are redundant or only weakly correlated with the endogenous regressor. We note, however, that even when the instrument set only contains instruments dated at time $t - 1$, a significant bias remains in the estimates. In general, the

---

[14]Mavroeidis (2005) documents that the J-test has low power to detect misspecification in the context of the New Keynesian Phillips Curve. He also finds that the power of the test deteriorates when too many instruments are used.

reduction in bias comes at a cost of less precision in the estimates. When the estimation is based on the instrument set $z_{3,t}$, the median standard errors are almost twice as large as when the estimation is based on the larger instrument set $z_{1,t}$ (see table 3) and the rejection frequencies of the t-tests on $\widehat{\alpha}_1$, $\widehat{\alpha}_2$ and $\widehat{\alpha}_3$ fall to 0.27, 0.29, and 0.30 (using a 5% significance level).

4.1.2 The 'hybrid' local currency pricing model with indexation

In the next set of experiments the import price equation is derived from the 'hybrid' LCP model with indexation. The data generating process for the driving variables is a first-differenced VAR without feedback from import prices.

Table 7 reports the results of simulation experiments for different values of the indexation parameter $\chi = \{0.25, 0.5, 0.75, 1.00\}$ when the sample size is $T = 100$. The table shows that, in general, the sign and the size of the biases depend on the true value of the indexation parameter. In all cases, the median estimate of the coefficient on lagged import price inflation (α_4) is close to its true value; it exhibits a slight upward bias when the degree of indexation is low ($\chi = 0.25$) and a slight downward bias when the degree of indexation is high ($\chi = 1.00$). By contrast, the median estimate of the coefficient on expected future import price inflation (α_1) is biased downwards for low degrees of indexation ($\chi = 0.25$ and $\chi = 0.50$) and biased upwards for high degrees of indexation ($\chi = 0.75$ and $\chi = 1.00$). As in the purely forward-looking LCP model, the biases in the estimates of the coefficients on the levels terms are of opposite sign to the bias in the estimate of the coefficient on the forward term: the median estimates of α_2 and α_3 overstate the true values when the degree of indexation is low and understate the true values when the degree of indexation is high. Furthermore, the coefficients on the levels term are imprecisely estimated and often statistically insignificant. When $\chi = 0.5$, the rejection frequencies of the t-tests of the null hypotheses that α_2 and α_3 are zero are, respectively, 0.18 and 0.17 (using a 5% significance level).

A key finding emerging from the table is that the median estimate of α_1 is fairly insensitive to the degree of indexation in the data generating process.[15] For example; when $\chi = 0.25$ the median estimate of α_1 is 0.556, increasing only slightly to 0.585 when $\chi = 1.00$. Thus, the GMM estimate of the coefficient on future import price inflation is not very informative about the true value of this parameter. We note, however, that the estimate of the *sum* of the coefficients on the lead and the lag of import price inflation appears to be increasing in the degree of indexation χ.

In the theoretical model, the coefficients on the lead and the lag of import price

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[15]A similar finding is reported by Mavroeidis (2005) who shows that under weak identification, the GMM estimate of the coefficient on the forward term in the New Keynesian Phillips Curve is roughly invariant to changing the weight on future inflation in the data generating process.

inflation do not sum exactly to unity, except in the case of full indexation ($\chi = 1.00$). It is nevertheless worthwhile to examine whether imposing the restriction $\alpha_1 + \alpha_4 = 1$ improves the finite-sample properties of the GMM estimates. The results reported in table 8 suggest that there are indeed some gains from imposing this restriction in terms of increased accuracy of the estimate of the coefficient on future import price inflation. For example, when $\chi = 0.5$, the median estimate of α_1 is 0.634, compared to 0.587 when the coefficients are unrestricted. Additionally, imposing the restriction increases the precision of the estimates: when $\chi = 0.5$ the median standard error of the estimate of α_1 falls from 0.179 to 0.073.

In the presence of weak instruments, the IV estimator in a linear regression model with one endogenous regressor is biased towards the probability limit of the OLS estimate (see e.g., Stock et al., 2002; Woglom, 2001). To cast more light on the simulation results, table 9 reports the probability limits of the OLS estimates of the coefficients on $\Delta p_{F,t+1}$ and $\Delta p_{F,t-1}$ for different values of the indexation parameter in the data generating process.[16] Three findings are immediately apparent from the table. First, the probability limit of the OLS estimate of α_1 is *increasing* in the degree of indexation. Second, the probability limit of the OLS estimate of the sum of the coefficients on the lead and the lag of import price growth is increasing in the degree of indexation. Third, when the restriction $\alpha_4 = 1 - \alpha_1$ is imposed during estimation, the probability limit of the OLS estimate of the weight on the forward term tends to approximately 0.5 regardless of the weight on future expected import price inflation in the data generating process.

Table 10 reports the results obtained when the data are generated from a model with $\chi = 0.50$, and the estimation is based on the smaller instrument sets $z_{2,t}$ and $z_{3,t}$. Again, the bias is reduced when variables dated $t-3$ and $t-4$ are excluded from the instrument set. When the estimation is based on the instrument set $z_{3,t}$, which only includes variables dated $t-1$, the median estimates are very close to their true values. The rejection frequencies of the t-statistics on the coefficients on the levels terms α_2 and α_3 are only 0.03, however.

4.1.3 Models with local- and producer currency pricing

Table 11 reports the results of the simulation experiments for the LCP-PCP model for different values of the share of PCP exporters $\phi = \{0.25, 0.5, 0.75, 1.00\}$. The sample size in the experiments is $T = 100$, and the driving variables follow a VAR without feedback from import prices.

The key result emerging from the table is that the share of PCP firms is accurately estimated regardless of the true value of the parameter. The estimates of the other coefficients display a similar pattern to those obtained in the pure LCP model: the estimate of

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[16]The (approximate) probability limits are computed from simulations with 500000 observations.

α_1 exhibits a strong negative bias, while the estimates of α_2 and α_3 are biased upwards. The bias in the estimate of α_1 is increasing in the true value of ϕ.

In the final set of experiments, the data generating process is a New Keynesian import price equation with both LCP and PCP price setters ($\phi = 0.25$) and inflation indexation ($\chi = 0.5$). The results reported in table 12 are based on a sample size $T = 100$. For the coefficient on the exchange rate term (α_5) the results are similar to those obtained for the LCP-PCP model without indexation. The other coefficients exhibit somewhat smaller biases than what was reported for the 'hybrid' LCP model with indexation.

4.1.4   Summary of results and reinterpretation of previous empirical evidence

The main results of the GMM experiments can be summarised as follows. First, the GMM estimates exhibit a significant small-sample bias. In particular, the GMM estimate of the coefficient on future import price inflation understates the true value when the degree of indexation is small and overstates the true value when the degree of indexation is high. Second, the accuracy of the GMM estimates increases when the number of lagged instruments is reduced. However, when fewer lagged instruments are used, the coefficient estimates are typically less precise and often statistically insignificant. Finally, the results suggest that the $J-$test has low power to detect that the current values of the driving variables are wrongly included in the instrument set.

What do these results imply for the interpretation of the GMM estimates reported in chapter 2 of this thesis? First, and most importantly, an economically implausible estimate of the coefficient on future import price growth in the purely forward-looking LCP model and statistically insignificant estimates of the coefficients on the levels terms $(s + ulc_F - p_F)_t$ and $(s + p_{COM} - p_F)_t$ could reflect small-sample estimation bias and are not necessarily inconsistent with the data being generated from a New Keynesian import price equation. On the other hand, the simulation evidence suggests that the GMM estimate of the coefficient on lagged inflation is fairly accurate. Thus, small-sample bias does not seem to explain why we get a small positive, or negative, estimate of this coefficient on actual data. Assuming the model is correctly specified, this adds support to the conclusion that the weight on lagged inflation in the New Keynesian import price equation is small. Similarly, the finding that the estimate of the share of PCP firms is accurately estimated supports the conclusion that the import price equation should incorporate both LCP and PCP. Finally, the simulation evidence indicates that we should put more weight on the results obtained using few lagged instruments, and given that the J-test has low power to detect invalid instruments, to focus on the estimates obtained using only lagged instruments.

4.2   Maximum likelihood

Turning next to the small-sample properties of the ML estimator, table 13 summarises the results of Monte Carlo experiments for the purely forward-looking LCP model when $\eta = 0.75$. Two different sample sizes are considered: $T = 100$ and $T = 200$. The key result emerging from the table is that the median estimates are very close to the true parameter values, even when the sample size is small. In addition, the ML estimates are more precise than the GMM estimates: the median standard errors of the GMM estimates (obtained using the instrument set $z_{1,t}$) are about 1.5 times larger than those of the ML estimates. Figure 2 shows the histogram and the fitted normal densities of the ML estimates for $T = 100$. The empirical distributions are asymmetric. In particular, the empirical distribution of the coefficient on expected future inflation (α_1) is left-skewed, whereas the distributions of the coefficients on real unit labour costs and real commodity prices (α_2 and α_3) are right-skewed.

Table 14 reports the results for the LCP model with inflation indexation and the LCP-PCP model. The sample size is $T = 100$. Again, the most striking finding is that the median estimates are close to the true parameter values. The median standard errors are of similar magnitude to what was obtained for GMM using the instrument set $z_{1,t}$.

When the data were generated by the hybrid LCP model, the estimation procedure yielded highly implausible estimates in a non-negligible proportion of the replications. In particular, the procedure occasionally produced large negative estimates of the coefficient on expected future inflation and correspondingly large positive estimates of the coefficient on lagged inflation. This is reflected in the mean and standard deviations of the coefficient estimates in these experiments. The problems were most acute when the data were generated from a model with full indexation ($\chi = 1.00$). For this data generating process, moreover, the ML procedure often required a large number of iterations to converge.

One potential caveat is that the ML estimates are sensitive to the choice of initial values for the parameters. This could reflect that the numerical optimisation routine gets trapped in different local minima, or that the likelihood function is (nearly) flat in some directions.[17] The results reported in table 15 give an idea of how sensitive the Monte Carlo results are to changing the initial conditions. The results are based on 500 replications. In general, the median estimates are close to the true parameters regardless of the choice of initial values. The exception is the LCP model with full indexation: when the initial value of the indexation parameter is set to zero, the median estimates overstate the weight on future inflation and understate the weight on lagged inflation.

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[17]In fact, because the `csminwel` routine makes use of a random number generator in the search process, if the likelihood function has multiple local minima or is flat in some direction, the estimation routine may produce different results depending on the random draws, even if the initial conditions are held fixed.

Another potential caveat is that the results could be sensitive to the exact implementation of the ML procedure. Table 16 reports the results obtained when using the procedures for solving and estimating linear rational expectations models provided by Jeffrey Fuhrer.[18] The rational expectations model is solved using the Anderson-Moore algorithm (Anderson & Moore, 1985), and the likelihood function is maximised using Matlab's sequential quadratic programming algorithm `constr`. The estimation procedure does not impose any restrictions on the variance-covariance matrix of the structural shocks, and the elements of the variance-covariance matrix of the auxiliary VAR are now estimated as part of the ML procedure. The simulation results are based on 5000 replications and the same initial conditions as in the benchmark experiments. As is apparent from the table, the median estimates are close to the true values and the median standard errors are of similar magnitude to what was reported for the Dynare algorithm.

5   MAXIMUM LIKELIHOOD ESTIMATION OF NEW KEYNESIAN IMPORT PRICE EQUATIONS ON UK DATA

The Monte Carlo evidence suggests that when the model is correctly specified, ML performs better than GMM in small samples. This finding is in line with other Monte Carlo studies that compare the performance of the two estimators in the context of single-equation rational expectations models (e.g., Fuhrer et al., 1995; Fuhrer & Rudebusch, 2004; Lindé, 2005). Fuhrer & Rudebusch (2004) argue that the poor small-sample performance of the conventional GMM estimator in rational expectations models can be attributed to the fact that the instruments do not embody the rational expectations restrictions implied by the model, but are constructed from linear projections of the endogenous regressor on the instrument set.

Motivated by this evidence, this section provides ML estimates of New Keynesian import price equations for the UK. The standard argument against the use of full information ML is that it may lead to incorrect inferences when the completing model for the driving variables is misspecified. In an attempt to protect against this risk, I use a data-consistent reduced-form model for the driving variables. Specifically, the completing model is the first-differenced VAR without feedback from import prices documented in table 1. The VAR coefficients are held fixed at their OLS estimates.[19]

All the variables are demeaned prior to estimation. The VAR for the driving variables was estimated with three seasonal dummies. These are not included in the VAR equations used in the ML estimation. Instead, I perform a preliminary regression to remove

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[18]The programs and documentation are available to download from http://www.bos.frb.org/economic/econbios/fuhrer/matlab.htm.

[19]This is also the approach taken by e.g., Fuhrer & Rudebusch (2004) and Roberts (2005) when estimating single-equation rational expectations models using ML.

the effects of the seasonal dummies on the first-differenced driving variables. The ML estimation is based on detrended data for $(s + ulc_F - p_F)_{t-1}$ and $(s + p_{COM} - p_F)_{t-1}$. This is in line with the results of the cointegration analysis in chapter 2, which suggested the presence of a deterministic trend in the cointegrating relation between import prices and marginal costs.

The ML estimates are reported in table 17. The discussion focuses on the estimates reported in the top part of the table, which are obtained by maximising the likelihood function with respect to the structural parameters $\beta, \eta, \delta, \chi, \phi, \sigma_v$.

The first thing to notice is that the estimate of the share of PCP firms in the LCP-PCP models is numerically and statistically significant. The estimate of the standard deviation of the shock to the import price equation σ_v is also smaller in the models that allow for both LCP and PCP. This is consistent with the results in Choudhri et al. (2005). The authors estimate 'new open economy macroeconomics' models on data for non-US G7 countries (including the UK) using an impulse-response matching approach and find that the models that allow for both LCP and PCP outperform the pure LCP model. The point estimate of the share of LCP firms $1 - \phi = 0.55$ is somewhat higher than what is suggested by data on invoicing currency in UK trade: the share of UK imports invoiced in sterling in the years 1999 to 2002 is approximately 40%.[20]

A second finding is that the estimate of the indexation parameter is numerically small and statistically insignificant. This is consistent with the GMM estimates of the coefficient on lagged import price inflation in the New Keynesian import price equation reported in chapter 2 of this thesis. It is also in line with the results reported by Smets & Wouters (2002) for the euro area. On the basis of estimates obtained using an impulse-response matching approach, they conclude that the degree of indexation to past import price inflation is limited.

The remainder of the discussion focuses on the purely forward-looking LCP-PCP model. The point estimate of the discount factor β is 0.96 and the estimate of the weight on commodity prices in marginal costs (δ) is 0.23. This is somewhat smaller than the estimate of 0.36 obtained using cointegration techniques in chapter 2. The point estimate of the price stickiness parameter η is 0.87 and implies an average time between price changes of about seven quarters. This is at the high end of the range of reasonable parameter values, suggesting a high degree of stickiness in import prices. The estimates are, however, in line with the estimates reported by Smets & Wouters (2002) for the euro area. Their estimates point to a value for the price stickiness parameter around 0.85.[21]

To check the sensitivity of the estimates with respect to the initial conditions, I started the ML routine from 100 different initial values drawn from a uniform distribution in the

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[20]These numbers can be found on `http://customs.hmrc.gov.uk/`.

[21]Using micro data on import prices at the docks for the US, Gopinath & Rigobon (2006) find that the trade weighted average price duration in dollars is 12 months.

interval $[0,1]$ for each parameter. For 66% of the draws of the initial values the estimation procedure returned point estimates that were within a range of ± 0.0001 of the estimates reported in table 17.

To make a comparison with the GMM estimates in chapter 2 easier, the table also reports the estimates obtained when the likelihood is maximised with respect to $\left\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \sigma_v^2\right\}$. The estimates are very close to those implied by the estimates of the underlying structural parameters.

The closed form solution of the rational expectations model can be written as a restricted equilibrium-correction model (EqCM) for import prices and VAR equations for the first-differenced driving variables. Following Fuhrer et al. (1995), it is worthwhile to compare the restricted EqCM with an EqCM with data-based dynamics. Since the completing model for the driving variables is a fourth-order VAR, the restricted EqCM has three lags of the driving variables. Estimation of an EqCM for import prices with three lags over the period 1981Q2–2003Q2 yields the following estimates

$$
\begin{aligned}
\widehat{\Delta p_{F,t}} = \ & - \underset{(0.107)}{0.264} \ \Delta p_{F,t-1} + \underset{(0.108)}{0.153} \ \Delta p_{F,t-2} + \underset{(0.098)}{0.347} \ \Delta p_{F,t-3} \\
& + \underset{(0.045)}{0.481} \ \Delta s_t + \underset{(0.072)}{0.258} \ \Delta s_{t-1} + \underset{(0.074)}{0.041} \ \Delta s_{t-2} - \underset{(0.072)}{0.233} \ \Delta s_{t-3} \\
& + \underset{(0.025)}{0.049} \ \Delta p_{COM,t} + \underset{(0.026)}{0.046} \ \Delta p_{COM,t-1} + \underset{(0.026)}{0.033} \ \Delta p_{COM,t-2} \\
& - \underset{(0.027)}{0.004} \ \Delta p_{COM,t-3} + \underset{(0.341)}{0.294} \ \Delta ulc_{F,t} - \underset{(0.410)}{0.427} \ \Delta ulc_{F,t-1} \\
& + \underset{(0.418)}{0.065} \ \Delta ulc_{F,t-2} + \underset{(0.320)}{0.326} \ \Delta ulc_{F,t-3} \\
& + \underset{(0.028)}{0.050} \ (s_{t-1} + ulc_{F,t-1} - p_{F,t-1}) + \underset{(0.016)}{0.024} \ (s_{t-1} + p_{COM,t-1} - p_{F,t-1})
\end{aligned}
$$

<div align="center">

Diagnostics

$\hat{\sigma} = 0.0120 \quad F_{AR(1-4)} = 0.347 \ [0.846] \quad \chi^2_{normality} = 0.812 \ [0.666]$

</div>

The standard error of the regression is 1.20%. We fail to reject the test of zero auto-correlation up to order four ($F_{AR(1-4)}$) and the test for non-normality in the residuals ($\chi^2_{normality}$).

The closed form solution of the estimated purely forward-looking LCP-PCP model

implies the following EqCM for import prices:

$$\widehat{\Delta p_{F,t}} = 0.520\,\Delta s_t \; - \; 0.024\,\Delta s_{t-1} \; - \; 0.003\,\Delta s_{t-2} \; - \; 0.013\,\Delta s_{t-3}$$

$$+ \; 0.045\,\Delta p_{COM,t} \; + \; 0.004\,\Delta p_{COM,t-1} \; + \; 0.011\,\Delta p_{COM,t-2} \; - \; 0.001\,\Delta p_{COM,t-3}$$

$$+ \; 0.240\,\Delta ulc_{F,t} \; + \; 0.035\,\Delta ulc_{F,t-1} \; + \; 0.098\,\Delta ulc_{F,t-2} \; + \; 0.108\,\Delta ulc_{F,t-3}$$

$$+ \; 0.104\,(s + ulc_F - p_F)_{t-1} \; + \; 0.031\,(s + p_{COM} - p_F)_{t-1}$$

<center>Diagnostics</center>

$$\hat\sigma = 0.0145 \qquad F_{AR(1-4)} = 7.531\;[0.000] \qquad \chi^2_{normality} = 0.848\;[0.654]$$

The restricted EqCM differs from the data-based model in that the coefficients on lagged import price growth $\Delta p_{F,t-1}$, $\Delta p_{F,t-2}$ and $\Delta p_{F,t-3}$, are constrained to zero. The standard error of the restricted model is 1.45%, which is somewhat higher than the standard error of the data-based model. Moreover, the autocorrelation test is strongly rejected. Formally, this constitutes a rejection of the rational expectations model and suggests that either the New Keynesian import price equation or the completing model for the driving variables is misspecified. Note, however, that we fail to reject the hypothesis that lagged import price growth can be excluded from the completing VAR model for the driving variables.

It is nevertheless of interest to compare the historical fit of the two models. Figure 3 plots the residuals and the actual and fitted values of import price growth for both models. The restricted model tracks the actual values reasonably well compared with the data-based model. Finally, figure 4 plots the dynamic response of import prices to a permanent 1% increase in each of the driving variables. The dynamic responses of import prices to an exogenous exchange rate change (a measure of the exchange rate pass-through) are similar in the two models: the short-run pass-through is around 0.5, and the pass-through is nearly complete after twenty quarters. The two models also display similar short-run responses to increases in foreign unit labour costs and commodity prices. However, the data-based EqCM implies a smaller response to commodity prices and a larger response to foreign unit labour costs in the medium- and long-run. This reflects that the estimate of δ, which corresponds to the long-run effect of commodity prices in the New Keynesian import price equation, is larger than the estimate of the long-run effect of commodity prices in the data-based model. Consistent with the cointegration analysis in chapter 2, the long-run coefficient on the commodity price index in the data-based EqCM is 0.32.

6 CONCLUDING REMARKS

This paper provides evidence that GMM estimation of New Keynesian import price equations may produce biased estimates in small samples. Small-sample estimation bias could be part of the explanation behind the economically implausible and often statistically insignificant estimates we obtained on UK data.

Fuhrer & Rudebusch (2004) argue that the poor small-sample performance of the GMM in rational expectations models is due to a failure to impose the cross-equation restrictions implied by the rational expectations model. This is consistent with the simulation evidence in this paper which suggests that the estimates obtained using ML, which imposes the rational expectations restrictions, are fairly accurate.

ML estimation of New Keynesian import price equations on UK data produces statistically significant coefficients within the ranges suggested by the theory. The empirical evidence favours a specification that incorporates both LCP and PCP, but no indexation to the previous period's import price inflation. The estimates suggest a high degree of price stickiness in UK import prices. The historical fit of the single-equation model for import prices implied by the rational expectations model is not much inferior to that of an import price equation with data-based dynamics. Moreover, the two models imply similar estimates of exchange rate pass-through.

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Table 1: Estimated VAR models. UK data 1981Q2–2003Q2.

	Model 1			Model 2			
	Δs_t	$\Delta ulc_{F,t}$	$\Delta p_{COM,t}$	Δs_t	$\Delta ulc_{F,t}$	$\Delta p_{COM,t}$	$\Delta p_{F,t}$
Δs_{t-1}	0.169 (0.105)	0.034 (0.016)	0.194 (0.2212)	0.306 (0.182)	−0.016 (0.026)	0.586 (0.375)	0.419 (0.113)
Δs_{t-2}	−0.244 (0.110)	−0.012 (0.017)	−0.091 (0.230)	−0.220 (0.195)	−0.042 (0.028)	−0.122 (0.402)	−0.096 (0.121)
Δs_{t-3}	0.004 (0.110)	−0.009 (0.017)	0.252 (0.231)	−0.139 (0.186)	−0.003 (0.027)	0.263 (0.382)	−0.310 (0.115)
Δs_{t-4}	−0.133 (0.107)	−0.022 (0.016)	−0.002 (0.225)	−0.064 (0.187)	0.008 (0.027)	−0.331 (0.385)	−0.012 (0.116)
$\Delta ulc_{F,t-1}$	0.598 (0.753)	0.796 (0.116)	−2.386 (1.582)	0.135 (0.840)	0.689 (0.121)	−2.44 (1.729)	−0.435 (0.520)
$\Delta ulc_{F,t-2}$	−0.708 (0.963)	0.103 (0.148)	−0.467 (2.023)	−0.757 (1.037)	0.173 (0.149)	−0.366 (2.134)	−0.237 (0.641)
$\Delta ulc_{F,t-3}$	1.242 (0.987)	−0.223 (0.152)	−0.783 (2.074)	1.656 (1.042)	−0.297 (0.150)	−1.129 (2.143)	1.016 (0.644)
$\Delta ulc_{F,t-4}$	−0.043 (0.727)	0.197 (0.112)	2.998 (1.526)	−0.282 (0.768)	0.173 (0.110)	1.87 (1.581)	−0.049 (0.475)
$\Delta p_{COM,t-1}$	0.022 (0.056)	0.000 (0.009)	0.252 (0.117)	0.016 (0.061)	−0.008 (0.009)	0.363 (0.126)	0.063 (0.038)
$\Delta p_{COM,t-2}$	−0.043 (0.056)	0.007 (0.009)	−0.096 (0.118)	−0.021 (0.065)	−0.007 (0.009)	−0.034 (0.134)	0.010 (0.040)
$\Delta p_{COM,t-3}$	0.006 (0.056)	0.013 (0.009)	0.247 (0.119)	−0.013 (0.065)	0.002 (0.009)	0.299 (0.134)	−0.005 (0.040)
$\Delta p_{COM,t-4}$	0.097 (0.057)	0.002 (0.009)	−0.219 (0.120)	0.096 (0.065)	−0.001 (0.009)	−0.173 (0.134)	0.014 (0.040)
$\Delta p_{F,t-1}$	–	–	–	−0.200 (0.292)	0.105 (0.042)	−0.436 (0.601)	−0.313 (0.181)
$\Delta p_{F,t-2}$	–	–	–	0.141 (0.285)	0.033 (0.041)	0.438 (0.587)	0.290 (0.176)
$\Delta p_{F,t-3}$	–	–	–	0.267 (0.275)	−0.036 (0.040)	0.172 (0.566)	0.503 (0.170)
$\Delta p_{F,t-4}$	–	–	–	−0.184 (0.258)	−0.036 (0.037)	0.739 (0.530)	−0.123 (0.159)
$(s+ulc_F-p_F)_{t-1}$	–	–	–	−0.074 (0.064)	−0.008 (0.009)	−0.168 (0.131)	−0.014 (0.039)
$(s+p_{COM}-p_F)_{t-1}$	–	–	–	−0.018 (0.039)	0.007 (0.006)	−0.171 (0.081)	0.020 (0.024)

Notes: The numbers in brackets are the standard errors of the estimates. The results are obtained using PcGive 10.0 (see Hendry & Doornik, 2001).

Table 2: Diagnostics and residual correlation matrices for estimated VARs.

Model 1

Single-equation tests

	Δs_t	$\Delta ulc_{F,t}$	$\Delta pcom_t$
$F_{AR(1-5)}$	0.94 [0.46]	1.65 [0.16]	1.55 [0.19]
$\chi^2_{Normality}$	2.98 [0.23]	1.45 [0.48]	0.91 [0.63]
$F_{ARCH(1-4)}$	0.74 [0.57]	1.41 [0.24]	0.62 [0.65]
F_{Hetero}	0.50 [0.97]	0.95 [0.55]	0.91 [0.59]

Vector tests

$F_{AR(1-5)}$	$\chi^2_{Normality}$	F_{Hetero}
1.33 [0.10]	5.09 [0.53]	0.72 [0.99]

Residual correlation matrix

	Δs_t	$\Delta ulc_{F,t}$	$\Delta pcom_t$
Δs_t	0.029		
$\Delta ulc_{F,t}$	0.013	0.004	
$\Delta pcom_t$	−0.179	−0.247	0.060

Model 2

Single-equation tests

	Δs_t	$\Delta ulc_{F,t}$	$\Delta pcom_t$	Δpf_t
$F_{AR(1-5)}$	0.93 [0.47]	1.85 [0.12]	0.89 [0.49]	1.28 [0.28]
$\chi^2_{Normality}$	2.81 [0.25]	5.09 [0.08]	2.10 [0.35]	0.17 [0.92]
$F_{ARCH(1-4)}$	0.68 [0.61]	0.38 [0.82]	0.59 [0.67]	0.76 [0.55]
F_{Hetero}	0.31 [1.00]	0.78 [0.77]	0.60 [0.93]	0.44 [0.99]

Vector tests

$F_{AR(1-5)}$	$\chi^2_{Normality}$	F_{Hetero}
1.23 [0.13]	14.36 [0.07]	0.57 [1.00]

Residual correlation matrix

	Δs_t	$\Delta ulc_{F,t}$	$\Delta pcom_t$	Δpf_t
Δs_t	0.029			
$\Delta ulc_{F,t}$	0.022	0.004		
$\Delta pcom_t$	−0.202	−0.238	0.059	
Δpf_t	0.719	−0.005	0.000	0.018

Notes: The numbers in square brackets denote p−values. The misspecification tests are the F-transform of the LM test for residual autocorrelation up to order five ($F_{AR(1-5)}$), a test for autoregressive conditional heteroskedasticity up to order four ($F_{ARCH(1-4)}$), a test for non-normality ($\chi^2_{Normality}$), and a test for residual heteroskedasticity (F_{Hetero}). See Hendry & Doornik (2001) for details.

Table 3: Properties of GMM estimators in the purely forward-looking LCP model. Structural parameters: $\beta = 0.99$, $\delta = 0.25$. Instrument set: $z_{1,t}$. Auxiliary model: VAR without feedback from import prices.

DGP: $\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \dfrac{(1-\eta)(1-\beta\eta)}{\eta}\left(s_t + (1-\delta)ulc_{F,t} + \delta pcom_t - p_{F,t}\right) + v_t$

Model: $\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + pcom_t - p_{F,t}) + \omega_t$

$\eta = 0.75$

Parameter	True	T = 100				T = 200				T = 1000			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.001	−0.000	0.028	0.011	−0.000	−0.000	0.009	0.006	−0.000	−0.000	0.001	0.002
α_1	0.990	0.422	0.473	0.373	0.203	0.648	0.683	0.259	0.189	0.921	0.922	0.080	0.108
α_2	0.064	0.142	0.133	0.086	0.054	0.117	0.113	0.055	0.045	0.076	0.076	0.019	0.024
α_3	0.022	0.049	0.046	0.029	0.018	0.040	0.038	0.019	0.015	0.026	0.025	0.006	0.008

$\eta = 0.5$

Parameter	True	T = 100				T = 200				T = 1000			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.000	0.000	0.029	0.011	−0.000	−0.000	0.011	0.006	0.000	−0.000	0.002	0.002
α_1	0.990	0.424	0.455	0.294	0.164	0.655	0.669	0.196	0.154	0.918	0.914	0.083	0.094
α_2	0.379	0.430	0.424	0.147	0.094	0.414	0.411	0.098	0.080	0.388	0.388	0.044	0.045
α_3	0.126	0.145	0.144	0.049	0.032	0.139	0.138	0.033	0.027	0.130	0.130	0.015	0.015

$\eta = 0.875$

Parameter	True	T = 100				T = 200				T = 1000			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.001	−0.000	0.030	0.012	−0.000	−0.000	0.010	0.006	−0.000	−0.000	0.001	0.001
α_1	0.990	0.292	0.341	0.400	0.217	0.456	0.509	0.339	0.218	0.846	0.856	0.114	0.144
α_2	0.014	0.077	0.070	0.056	0.034	0.066	0.059	0.038	0.028	0.029	0.028	0.013	0.016
α_3	0.005	0.026	0.024	0.019	0.012	0.022	0.020	0.013	0.010	0.010	0.009	0.004	0.005

Table 4: Properties of GMM estimators in the purely forward-looking LCP model. $\beta = 0.99$, $\delta = 0.25$, $\eta = 0.75$. Cointegration parameter δ held fixed at its true value. $T = 100$. Instrument set $z_{1,t}$. Auxiliary model: VAR without feedback from import prices.

$$\text{DGP: } \Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta}\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + v_t$$

$$\text{Model: } \Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + \omega_t$$

Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.002	0.003
α_1	0.990	0.443	0.492	0.364	0.199
α_2	0.086	0.183	0.174	0.109	0.069

Table 5: Properties of GMM estimators in the purely forward-looking LCP model. $\beta = 0.99$, $\delta = 0.25$, $\eta = 0.75$. Instrument set $z_{1,t}$. Auxiliary model: VAR with feedback from import prices.

$$\text{DGP: } \Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta}\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + v_t$$

$$\text{Model: } \Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2\left(s_t + ulc_{F,t} - p_{F,t}\right) + \alpha_3\left(s_t + p_{COM,t} - p_{F,t}\right) + \omega_t$$

Parameter	True	T = 100				T = 200				T = 1000			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.002	0.002	0.000	−0.000	0.001	0.001	−0.000	−0.000	0.000	0.001
α_1	0.990	0.421	0.481	0.350	0.181	0.640	0.687	0.239	0.169	0.917	0.917	0.065	0.101
α_2	0.064	0.114	0.110	0.079	0.054	0.099	0.097	0.047	0.043	0.073	0.073	0.019	0.023
α_3	0.022	0.032	0.031	0.021	0.014	0.029	0.028	0.012	0.011	0.023	0.023	0.004	0.006

Table 6: Properties of GMM estimators in the purely forward-looking LCP model. $\beta = 0.99$, $\delta = 0.25$, $\eta = 0.75$, $T = 100$.

$$\text{DGP: } \Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta}\left(s_t + (1-\delta)ulc_{F,t} + \delta pcom_{,t} - p_{F,t}\right) + \nu_t$$

$$\text{Model: } \Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + pcom_{,t} - p_{F,t}) + \omega_t$$

Auxiliary VAR: no feedback from import prices

Parameter	True	$z_{2,t}$				$z_{3,t}$			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.024	0.013	0.000	0.000	0.002	0.016
α_1	0.990	0.539	0.595	0.427	0.306	0.634	0.673	0.364	0.465
α_2	0.064	0.136	0.126	0.091	0.072	0.129	0.124	0.109	0.098
α_3	0.022	0.046	0.043	0.030	0.024	0.044	0.042	0.017	0.033

Auxiliary VAR: with feedback from import prices

Parameter	True	$z_{2,t}$				$z_{3,t}$			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.000	0.000	0.002	0.002	−0.000	0.000	0.002	0.002
α_1	0.990	0.538	0.603	0.389	0.267	0.633	0.673	0.524	0.405
α_2	0.064	0.102	0.099	0.077	0.067	0.095	0.095	0.090	0.080
α_3	0.022	0.030	0.029	0.020	0.017	0.029	0.028	0.022	0.020

Table 7: Properties of GMM estimators in the hybrid LCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75$. *Instrument set:* $z_{1,t}$. $T = 100$.

$$\text{DGP:} \quad \Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi}\Delta p_{F,t-1} + \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)}\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + v_t$$

$$\text{Model:} \quad \Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + p_{COM,t} - p_{F,t}) + \alpha_4\Delta p_{F,t-1} + \omega_t$$

| | | | $\chi = 0.25$ | | | | | | $\chi = 0.50$ | | |
Parameter	True	Mean	Median	Std	Median SE	Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.001	−0.000	0.025	0.016	α_0	0.000	−0.001	−0.000	0.023	0.012
α_1	0.794	0.501	0.556	0.311	0.208	α_1	0.662	0.553	0.587	0.245	0.179
α_2	0.052	0.090	0.079	0.082	0.059	α_2	0.043	0.052	0.045	0.067	0.050
α_3	0.017	0.031	0.027	0.028	0.020	α_3	0.014	0.018	0.015	0.023	0.018
α_4	0.200	0.236	0.243	0.117	0.086	α_4	0.334	0.360	0.364	0.097	0.078

| | | | $\chi = 0.75$ | | | | | | $\chi = 1.00$ | | |
Parameter	True	Mean	Median	Std	Median SE	Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.001	−0.000	0.021	0.012	α_0	0.000	−0.000	0.000	0.019	0.011
α_1	0.568	0.575	0.592	0.193	0.157	α_1	0.498	0.577	0.585	0.159	0.139
α_2	0.037	0.027	0.023	0.051	0.041	α_2	0.032	0.013	0.011	0.038	0.033
α_3	0.012	0.010	0.008	0.018	0.015	α_3	0.011	0.005	0.004	0.014	0.012
α_4	0.430	0.432	0.433	0.092	0.079	α_4	0.503	0.468	0.469	0.093	0.082

Table 8: Properties of GMM estimators in the hybrid LCP model. Restriction $\alpha_4 = 1 - \alpha_1$ imposed during estimation. $\beta = 0.99, \delta = 0.25, \eta = 0.75$. Instrument set: $z_{1,t}$. $T = 100$.

$$\text{DGP:} \quad \Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)} \left(s_t + (1-\delta) ulc_{F,t} + \delta p_{COM,t} - p_{F,t} \right)$$

$$\text{Model:} \quad \Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_3 (s_t + p_{COM,t} - p_{F,t}) + (1-\alpha_1)\Delta p_{F,t-1} + \omega_t$$

Parameter	$\chi = 0.25$					$\chi = 0.50$				
	True	Mean	Median	Std	Median SE	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	0.000	0.018	0.011	0.000	−0.000	0.000	0.019	0.011
α_1	0.794	0.732	0.727	0.100	0.078	0.662	0.636	0.634	0.089	0.073
α_2	0.052	0.042	0.039	0.041	0.038	0.043	0.036	0.034	0.038	0.035
α_3	0.017	0.015	0.014	0.014	0.013	0.014	0.013	0.012	0.013	0.012

Parameter	$\chi = 0.75$					$\chi = 1.00$				
	True	Mean	Median	Std	Median SE	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	0.000	0.019	0.011	0.000	−0.000	0.000	0.018	0.011
α_1	0.568	0.567	0.566	0.086	0.074	0.498	0.521	0.519	0.088	0.076
α_2	0.037	0.030	0.028	0.034	0.031	0.032	0.023	0.022	0.029	0.026
α_3	0.012	0.011	0.010	0.012	0.011	0.011	0.008	0.008	0.011	0.010

Table 9: Probability limits of OLS estimates of the coefficients in the hybrid LCP model. $\beta = 0.99, \eta = 0.75, \delta = 0.25$.

DGP: $\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)} \left(s_t + (1-\delta) u l c_{F,t} + \delta p_{COM,t} - p_{F,t} \right) + v_t$

Model: $\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2 (s_t + u l c_{F,t} - p_{F,t}) + \alpha_3 (s_t + p_{COM,t} - p_{F,t}) + \alpha_4 \Delta p_{F,t-1} + \omega_t$

$\chi = 0$

Parameter	True	Plim OLS	Plim OLS ($\alpha_4 = 1 - \alpha_1$)
α_1	0.990	0.100	0.506
α_4	0.000	0.167	0.494

$\chi = 0.50$

Parameter	True	Plim OLS	Plim OLS ($\alpha_4 = 1 - \alpha_1$)
α_1	0.662	0.386	0.524
α_4	0.334	0.372	0.476

$\chi = 1.00$

Parameter	True	Plim OLS	Plim OLS ($\alpha_4 = 1 - \alpha_1$)
α_1	0.498	0.511	0.530
α_4	0.503	0.452	0.470

$\chi = 0.25$

Parameter	True	Plim OLS	Plim OLS ($\alpha_4 = 1 - \alpha_1$)
α_1	0.794	0.257	0.515
α_4	0.200	0.293	0.485

$\chi = 0.75$

Parameter	True	Plim OLS	Plim OLS ($\alpha_4 = 1 - \alpha_1$)
α_1	0.568	0.468	0.529
α_4	0.430	0.420	0.471

Table 10: Properties of GMM estimators in the hybrid LCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75, \chi = 0.5$. $T = 100$.

DGP: $\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)} \left(s_t + (1-\delta)ulc_{F,t} + \delta pCOM,t - p_{F,t}\right) + v_t$

Model: $\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + pCOM,t - p_{F,t}) + \alpha_4 \Delta p_{F,t-1} + \omega_t$

Parameter	True	$z_{2,t}$				$z_{3,t}$			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.023	0.015	−0.001	0.000	0.029	0.018
α_1	0.662	0.579	0.613	0.331	0.284	0.652	0.666	0.602	0.453
α_2	0.043	0.059	0.051	0.084	0.073	0.049	0.045	0.148	0.108
α_3	0.014	0.020	0.017	0.028	0.025	0.017	0.015	0.049	0.036
α_4	0.334	0.353	0.356	0.099	0.095	0.337	0.342	0.133	0.119

Table 11: Properties of GMM estimators in the LCP-PCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75$. *Instrument set:* $z_{1,t}$. $T = 100$.

$$\text{DGP: } \Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta\beta)(1-\eta)}{\eta}\left(s_t + (1-\delta)ulc_{F,t} + \delta p_{COM,t} - p_{F,t}\right) + \phi(\Delta s_t - \beta E_t \Delta s_{t+1}) + v_t$$

$$\text{Model: } \Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + p_{COM,t} - p_{F,t}) + \alpha_5(\Delta s_t - \alpha_1 \Delta s_{t+1}) + \omega_t$$

$\phi = 0.25$

Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	0.000	0.027	0.012
α_1	0.990	0.381	0.454	0.448	0.239
α_2	0.064	0.146	0.136	0.109	0.069
α_3	0.022	0.050	0.047	0.036	0.023
α_5	0.250	0.275	0.276	0.167	0.108

$\phi = 0.50$

Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	0.000	0.028	0.012
α_1	0.990	0.328	0.399	0.474	0.254
α_2	0.064	0.154	0.141	0.123	0.078
α_3	0.022	0.052	0.048	0.039	0.025
α_5	0.500	0.520	0.521	0.161	0.105

$\phi = 0.75$

Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.028	0.012
α_1	0.990	0.289	0.360	0.487	0.264
α_2	0.064	0.157	0.142	0.132	0.083
α_3	0.022	0.053	0.048	0.041	0.027
α_5	0.750	0.764	0.767	0.158	0.102

$\phi = 1.00$

Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.028	0.012
α_1	0.990	0.270	0.332	0.481	0.263
α_2	0.064	0.153	0.136	0.127	0.082
α_3	0.022	0.051	0.046	0.039	0.026
α_5	1.000	1.008	1.009	0.157	0.101

Table 12: Properties of GMM estimators in the hybrid LCP–PCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75, \chi = 0.5, \phi = 0.25. T = 100$. Instrument set: $\overline{z_{1,t}}$.

DGP: $\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \frac{(1-\eta\beta)(1-\eta)}{\eta(1+\beta\chi)} \left(s_t + (1-\delta)ulc_{F,t} + \delta pcom_t - p_{F,t}\right) + \phi\left(\Delta s_t - \frac{\beta}{1+\beta\chi} E_t \Delta s_{t+1} - \frac{\chi}{1+\beta\chi} \Delta s_{t-1}\right) + v_t$

Model: $\Delta p_{F,t} = \alpha_0 + \alpha_1 \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + pcom_t - p_{F,t}) + \alpha_4 \Delta p_{F,t-1} + \alpha_5(\Delta s_t - \alpha_1 \Delta s_{t+1} - \alpha_4 \Delta s_{t-1}) + \omega_t$

Parameter	True	Mean	Median	Std	Median SE
α_0	0.000	−0.000	−0.000	0.021	0.013
α_1	0.662	0.580	0.620	0.268	0.197
α_2	0.043	0.045	0.038	0.072	0.058
α_3	0.014	0.016	0.013	0.025	0.020
α_4	0.334	0.354	0.361	0.106	0.087
α_5	0.250	0.266	0.264	0.201	0.134

Table 13: Properties of ML estimates in the purely forward-looking LCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75.$

DGP: $\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta} \left(s_t + (1-\delta) ulc_{F,t} + \delta p_{COM,t} - p_{F,t} \right) + v_t$

Model: $\Delta p_{F,t} = \alpha_1 E_t \Delta p_{F,t+1} + \alpha_2 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_3 (s_t + p_{COM,t} - p_{F,t}) + v_t$

Parameter	True	$T = 100$				$T = 200$			
		Mean	Median	Std	Median SE	Mean	Median	Std	Median SE
α_1	0.990	0.954	1.007	0.230	0.123	0.981	1.001	0.111	0.084
α_2	0.064	0.077	0.065	0.053	0.029	0.068	0.064	0.026	0.020
α_3	0.022	0.026	0.022	0.018	0.010	0.023	0.021	0.009	0.007

Table 14: Properties of ML estimates in the hybrid LCP model and the LCP-PCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75, T = 100.$

DGP: $\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \frac{(1-\eta)(1-\beta\eta)}{(1+\beta\chi)\eta} (s_t + (1-\delta)ulc_{F,t} + \delta pcom_{F,t} - p_{F,t}) + v_t$

Model: $\Delta p_{F,t} = \alpha_1 E_t \Delta p_{F,t+1} + \alpha_2(s_t + ulc_{F,t} - p_{F,t}) + \alpha_3(s_t + pcom_{F,t} - p_{F,t}) + \alpha_4 \Delta p_{F,t-1} + v_t$

Parameter		$\chi = 0.50$					$\chi = 1.00$			
	True	Mean	Median	Std	Median SE	True	Mean	Median	Std	Median SE
α_1	0.662	0.484	0.659	0.611	0.156	0.498	−0.146	0.511	6.103	0.178
α_2	0.043	0.094	0.049	0.155	0.036	0.032	0.176	0.034	1.495	0.035
α_3	0.014	0.031	0.016	0.052	0.012	0.011	0.059	0.011	0.501	0.012
α_4	0.334	0.360	0.339	0.170	0.072	0.503	0.850	0.503	5.090	0.076

DGP : $\Delta p_{F,t} = \beta E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta} (s_t + (1-\delta)ulc_{F,t} + \delta pcom_{F,t} - p_{F,t}) + \phi (\Delta s_t - \beta E_t \Delta s_{t+1}) + v_t$

Model: $\Delta p_{F,t} = \alpha_1 E_t \Delta p_{F,t+1} + \alpha_2 (s_t + ulc_{F,t} - p_{F,t}) + \alpha_3 (s_t + pcom_{F,t} - p_{F,t}) + \alpha_4 (\Delta s_t - \alpha_1 E_t \Delta s_{t+1}) + v_t$

Parameter		$\phi = 0.25$					$\phi = 0.75$			
	True	Mean	Median	Std	Median SE	True	Mean	Median	Std	Median SE
α_1	0.990	0.918	1.008	0.325	0.150	0.990	0.833	1.006	0.516	0.242
α_2	0.064	0.090	0.066	0.080	0.038	0.064	0.114	0.065	0.132	0.052
α_3	0.022	0.030	0.022	0.027	0.013	0.022	0.038	0.022	0.044	0.017
α_4	0.250	0.240	0.243	0.076	0.071	0.750	0.749	0.749	0.067	0.064

Table 15: Sensitivity of ML estimates with respect to initial values in the estimation procedure.

LCP

			Experiment 1			Experiment 2		
	Parameter	True	Initial	Mean	Median	Initial	Mean	Median
	α_1	0.990	0.985	0.941	0.987	0.950	0.949	0.993
	α_2	0.064	0.136	0.080	0.067	0.042	0.079	0.065
	α_3	0.022	0.058	0.027	0.022	0.018	0.026	0.022

Hybrid LCP ($\chi = 0.5$)

			Experiment 1			Experiment 2		
	Parameter	True	Initial	Mean	Median	Initial	Mean	Median
	α_1	0.662	0.985	0.530	0.698	0.660	0.487	0.669
	α_2	0.043	0.037	0.089	0.043	0.025	0.094	0.045
	α_3	0.014	0.016	0.030	0.014	0.011	0.031	0.015
	α_4	0.334	0.000	0.338	0.325	0.335	0.360	0.345

Hybrid LCP ($\chi = 1$)

			Experiment 1			Experiment 2		
	Parameter	True	Initial	Mean	Median	Initial	Mean	Median
	α_1	0.498	0.985	0.389	0.670	0.660	-0.166	0.519
	α_2	0.032	0.037	0.093	0.004	0.025	0.183	0.033
	α_3	0.011	0.016	0.034	0.001	0.011	0.064	0.011
	α_4	0.503	0.000	0.539	0.400	0.335	0.848	0.498

LCP-PCP ($\phi = 0.25$)

			Experiment 1			Experiment 2		
	Parameter	True	Initial	Mean	Median	Initial	Mean	Median
	α_1	0.990	0.985	0.891	1.005	0.985	0.890	0.996
	α_2	0.064	0.037	0.095	0.067	0.037	0.095	0.068
	α_3	0.022	0.016	0.032	0.022	0.016	0.032	0.023
	α_4	0.250	0.000	0.246	0.252	1.000	0.247	0.255

LCP-PCP ($\phi = 0.75$)

			Experiment 1			Experiment 2		
	Parameter	True	Initial	Mean	Median	Initial	Mean	Median
	α_1	0.990	0.985	0.813	0.952	0.985	0.837	1.018
	α_2	0.064	0.037	0.115	0.075	0.037	0.110	0.060
	α_3	0.022	0.016	0.038	0.025	0.016	0.037	0.020
	α_4	0.750	0.000	0.753	0.756	1.000	0.755	0.758

Notes: Results based on 500 Monte Carlo replications. The values of the structural parameters in the data generating process are $\beta = 0.99, \eta = 0.75$, and $\delta = 0.25$. In the benchmark experiments discussed in the text, the initial values of the parameters in the estimated model are consistent with the following values of the structural parameters: $\eta = 0.8$, $\delta = 0.3$ and $\beta = 0.985$. For the purely forward-looking LCP model the table reports the effects of changing the initial value of η to 0.65 (experiment 1) and the initial value of β to 0.95 (experiment 2). The initial value of the indexation parameter in the benchmark experiments on the hybrid LCP model with indexation is $\chi = 1$. The table reports the effects of changing the initial value to $\chi = 0$ (experiment 1) and to $\chi = 0.5$ (experiment 2). The experiments are conducted for two different data generating processes: $\chi = 0.5$ and $\chi = 1.0$. In the benchmark LCP-PCP model experiments, the initial value of the share of PCP firms is $\phi = 0.5$. The table reports results for initial values $\phi = 0$ (experiment 1) and $\phi = 1$ (experiment 2). The LCP-PCP experiments are conducted for two data generating processes: $\phi = 0.25$ and $\phi = 0.75$.

Table 16: Properties of ML estimates in simulation experiments using alternative estimation algorithm

LCP

Parameter	True	Mean	Median	Std	Median SE
α_1	0.990	0.929	0.993	0.278	0.141
α_2	0.064	0.086	0.072	0.062	0.038
α_3	0.022	0.029	0.024	0.021	0.013

Hybrid LCP ($\chi = 0.5$)

Parameter	True	Mean	Median	Std	Median SE
α_1	0.662	0.240	0.639	4.987	0.157
α_2	0.043	0.166	0.053	2.303	0.038
α_3	0.014	0.054	0.018	0.734	0.013
α_4	0.334	0.486	0.346	3.412	0.071

Hybrid LCP ($\chi = 1.0$)

Parameter	True	Mean	Median	Std	Median SE
α_1	0.498	−0.235	0.494	2.613	0.173
α_2	0.032	0.189	0.038	0.620	0.036
α_3	0.011	0.063	0.013	0.204	0.012
α_4	0.503	0.881	0.513	1.985	0.096

LCP-PCP ($\phi = 0.25$)

Parameter	True	Mean	Median	Std	Median SE
α_1	0.990	0.931	0.990	0.272	0.132
α_2	0.064	0.090	0.074	0.070	0.038
α_3	0.022	0.030	0.025	0.023	0.013
α_4	0.250	0.233	0.236	0.171	0.159

LCP-PCP ($\phi = 0.75$)

Parameter	True	Mean	Median	Std	Median SE
α_1	0.990	0.956	0.985	0.172	0.111
α_2	0.064	0.091	0.075	0.065	0.040
α_3	0.022	0.030	0.025	0.021	0.013
α_4	0.750	0.745	0.746	0.169	0.160

Table 17: ML estimates of New Keynesian import price equations. UK data 1980Q2–2003Q2. Standard errors in parentheses.

$$\Delta p_{F,t} = \frac{\beta}{1+\beta\chi} E_t \Delta p_{F,t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\beta\chi)} (1-\delta)\left(s_t + ulc_{F,t} - p_{F,t}\right) + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\beta\chi)} \delta(s_t + p_{COM,t} - p_{F,t})$$
$$+ \frac{\chi}{1+\beta\chi} \Delta p_{F,t-1} + \phi\left(\Delta s_t - \frac{\beta}{1+\beta\chi} E_t \Delta s_{t+1} - \frac{\chi}{1+\beta\chi} \Delta s_{t-1}\right) + \nu_t$$

	LCP	Hybrid LCP	LCP-PCP	Hybrid LCP-PCP
β	0.758 (0.252)	0.759 (0.253)	0.936 (0.203)	0.959 (0.169)
η	0.742 (0.033)	0.742 (0.034)	0.865 (0.032)	0.869 (0.032)
δ	0.208 (0.035)	0.208 (0.035)	0.231 (0.056)	0.231 (0.056)
χ	–	0.018 (0.112)	–	−0.081 (0.109)
ϕ	–	–	0.439 (0.055)	0.446 (0.053)
σ_ν	0.023 (0.002)	0.022 (0.003)	0.016 (0.001)	0.017 (0.002)

$$\Delta p_{F,t} = \alpha_1 E_t \Delta p_{F,t+1} + \alpha_2\left(s_t + ulc_{F,t} - p_{F,t}\right) + \alpha_3(s_t + p_{COM,t} - p_{F,t})$$
$$+ \alpha_4 \Delta p_{F,t-1} + \phi\left(\Delta s_t - \alpha_1 E_t \Delta s_{t+1} - \alpha_4 \Delta s_{t-1}\right) + \nu_t$$

	LCP	Hybrid LCP	LCP-PCP	Hybrid LCP-PCP
α_1	0.758 (0.252)	0.759 (0.253)	0.937 (0.202)	1.039 (0.246)
α_2	0.121 (0.059)	0.119 (0.060)	0.023 (0.025)	0.021 (0.023)
α_3	0.032 (0.016)	0.031 (0.016)	0.007 (0.007)	0.006 (0.006)
α_4	–	0.018 (0.109)	–	−0.087 (0.128)
α_5	–	–	0.439 (0.055)	0.442 (0.054)
σ_ν	0.023 (0.002)	0.022 (0.003)	0.016 (0.001)	0.017 (0.002)
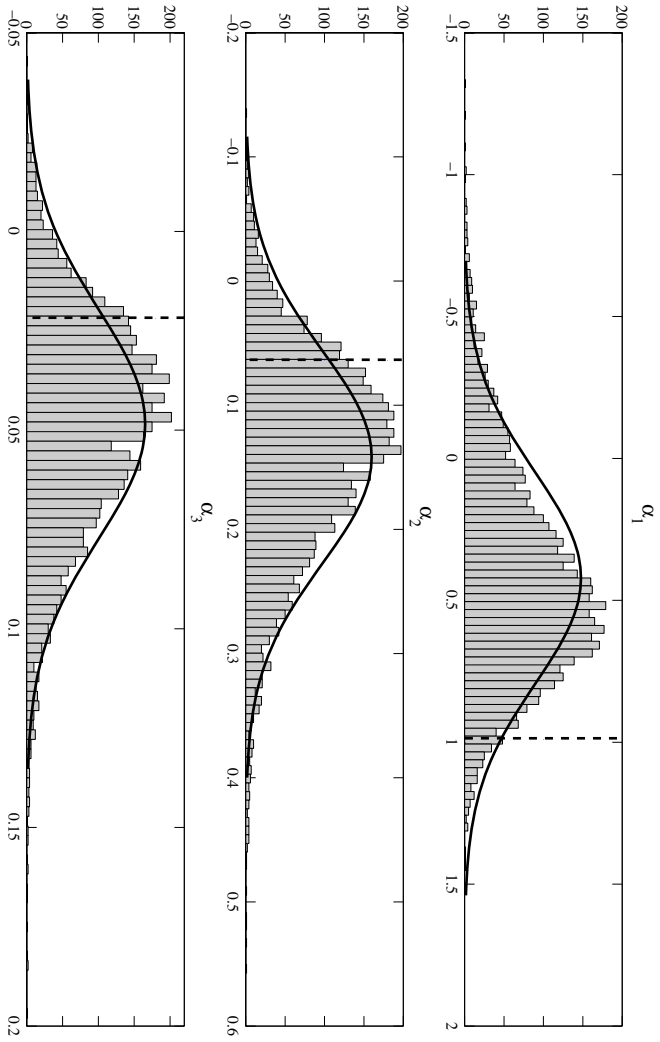
Figure 1: Distribution of GMM estimates in the purely forward-looking LCP model. $\beta = 0.99$, $\delta = 0.25$, $\eta = 0.75$. $T = 100$. Auxiliary model: VAR without feedback from import prices. Instrument set $z_{1,t}$
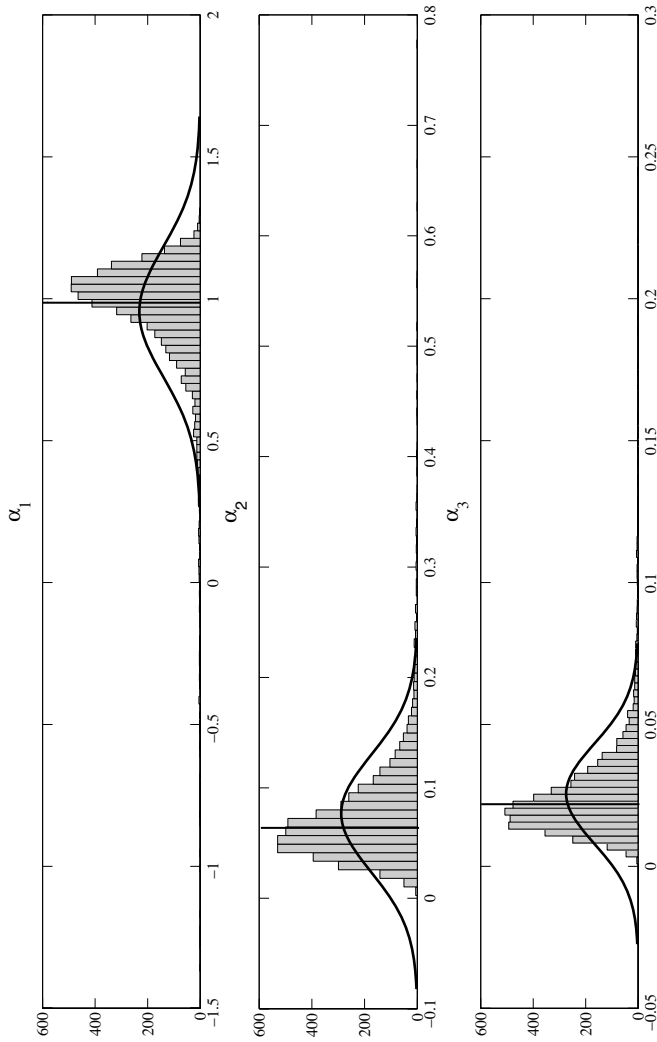
Figure 2: Distribution of ML estimates in the purely forward-looking LCP model. $\beta = 0.99, \delta = 0.25, \eta = 0.75. T = 100.$
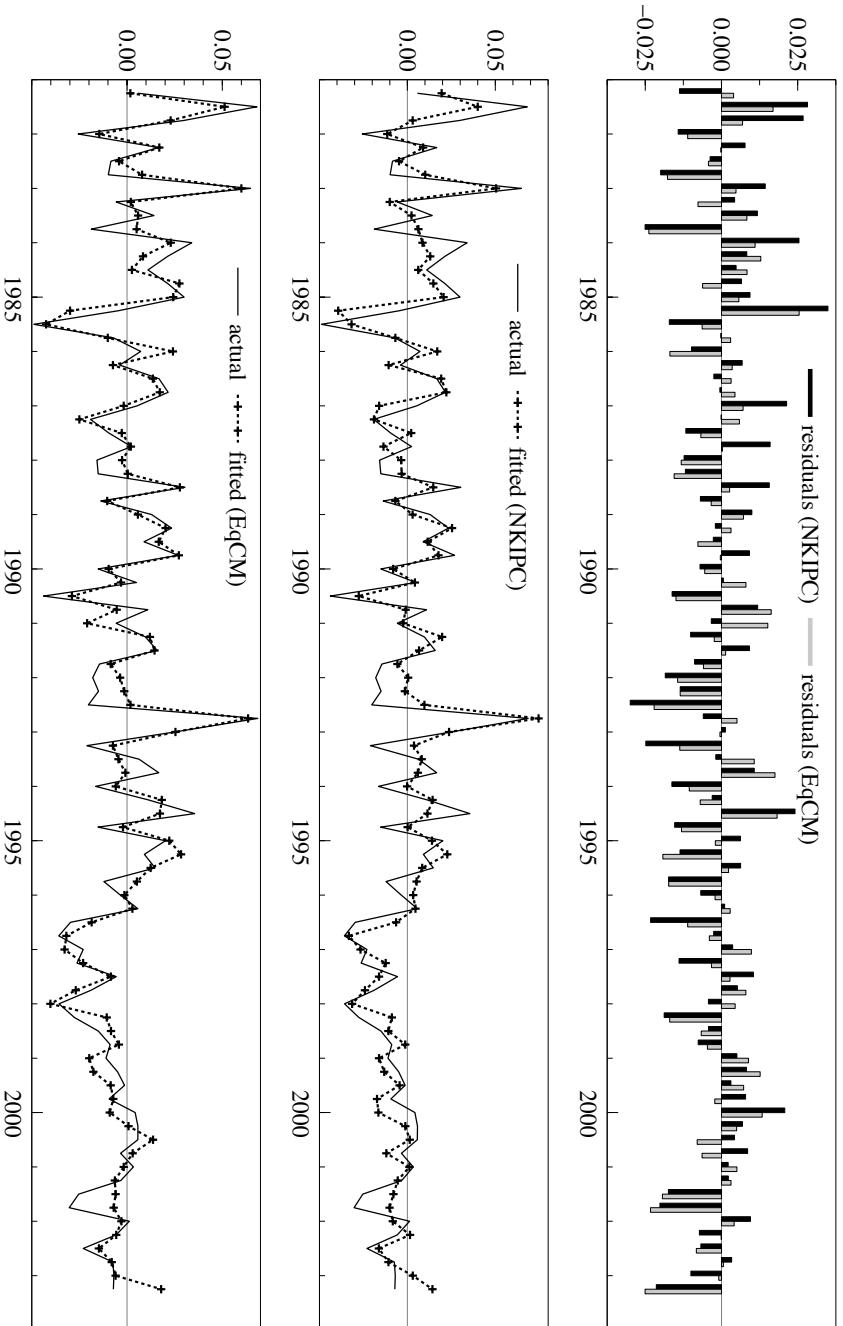
Figure 3: Actual and fitted values from restricted (NKIPC) and unrestricted (EqCM) conditional single-equation models for UK import prices 1981Q2–2003Q2.
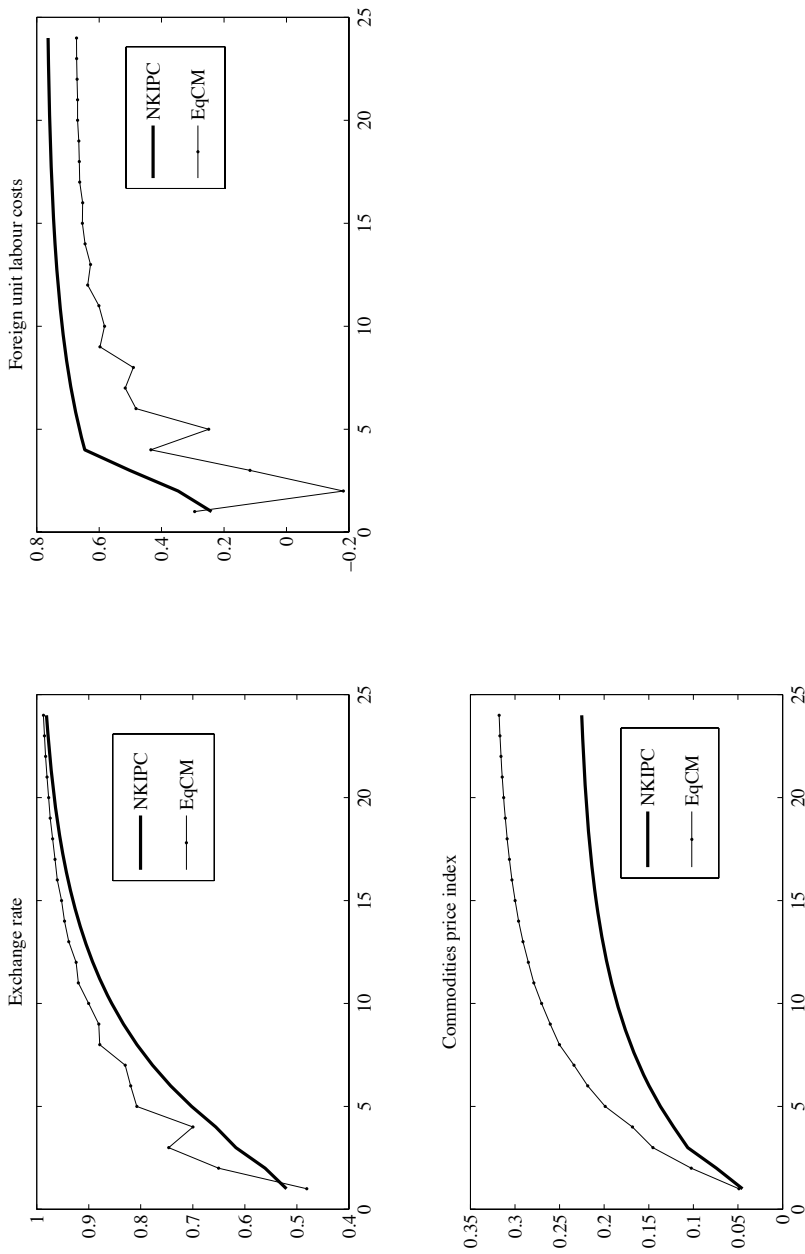
Figure 4: Interim-multipliers in restricted (NKIPC) and unrestricted (EqCM) conditional single-equation models for UK import prices.

CHAPTER 4

UNIT ROOTS AND EXCHANGE RATE PASS-THROUGH TO UK PRICES

1   INTRODUCTION

Since the publication of Sims (1980), structural vector autoregressions (VARs) have become one of the most widely used tools in applied macroeconometrics. Recently, structural VARs have also become a popular method to estimate the degree of exchange rate pass-through.[1] One motivation for using the structural VAR approach is that it takes explicit account of the endogeneity of the exchange rate and permits the estimation of pass-through to a set of prices, such as import prices, producer prices and consumer prices, simultaneously. Another motivation is that a structural VAR can be a useful tool to estimate dynamic stochastic general equilibrium (DSGE) models (see e.g., Rotemberg & Woodford, 1997; Christiano et al., 2005). The basic idea is to estimate the parameters in the DSGE model by minimising a measure of the distance between the impulse responses from the DSGE model and those obtained from a data-consistent VAR, relying only on a minimal set of identifying restrictions. Choudhri et al. (2005) and Faruqee (2006) estimate 'new open economy macroeconomics' (NOEM) models with incomplete exchange rate pass-through by matching the impulse responses of a set of prices to an exogenous exchange rate shock.

   This paper provides structural VAR evidence on the degree of exchange rate pass-through to UK prices. The price indices included in the VAR are import prices, export prices, producer prices and consumer prices. The comovement between prices and the exchange rate in a VAR depends on which shock causes the variables to move. In this sense the VAR does not provide a unique measure of the exchange rate pass-through. The structural VAR literature has resolved this ambiguity by defining exchange rate pass-through as the impulse responses of prices to a particular shock, namely an exogenous exchange rate shock. The same convention is observed in this paper. Also in line with common practice in the pass-through literature, I identify the exchange rate shock by imposing a recursive ordering on the variables in the VAR. Recursive identification schemes have been criticised for being arbitrary and for lacking justification from economic theory. Hence, it would be a worthwhile exercise to examine the robustness of the pass-through estimates to alternative, non-recursive identification schemes.[2] However, my paper focuses instead on two other issues that have received relatively little attention in the existing literature, namely small-sample estimation bias and the sensitivity of the estimates to different ways of dealing with the apparent non-stationarity in the data. The paper thus adds to and complements the previous studies by McCarthy (2000) and Choudhri et al. (2005) who provide structural VAR evidence on exchange rate pass-through for several countries, including the UK.

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[1]See e.g., McCarthy (2000), Hahn (2003), Mihailov (2003), Choudhri et al. (2005), and Faruqee (2006).

[2]Hahn (2003) considers an alternative identification scheme based on a mixture of restrictions on the long-run impulse responses and restrictions on the impact matrix of the shocks.

The model is estimated on quarterly data for the period 1980Q1–2003Q2. Univariate and multivariate unit root tests suggest that the levels of UK prices and the exchange rate are well approximated by unit root processes over the sample period. The question is how to deal with the non-stationarity in the variables. The OLS estimates of the auto-regressive coefficients are consistent even if some or all the variables in the VAR have unit roots (see Sims et al., 1990). Moreover, estimation of the VAR in levels ensures that information about the long-run relations between the variables is not lost. One possible approach is therefore to ignore the non-stationarity and estimate a VAR in levels. However, if there are unit roots (or near unit roots) in the VAR, the estimated long-horizon impulse responses from an unrestricted VAR are unreliable (Phillips, 1998). Additionally, when the variables are highly persistent, the small-sample bias in the estimated autoregressive coefficients can be substantial. This suggests that we should obtain a stationary representation of the VAR prior to computing the impulse responses.

There are two common approaches to obtaining a stationary representation of the VAR. The first approach is to difference the non-stationary variables. This is by far the most common approach in the pass-through literature (see e.g., McCarthy, 2000; Hahn, 2003; Choudhri et al., 2005; Faruqee, 2006).[3] Differencing the variables is likely to improve the small-sample performance of the estimates if the true model is a VAR in first differences, however, if the variables are cointegrated, estimating a model in first differences involves a loss of information. In fact, if the data are generated by a stationary (or cointegrated) process, first-differencing induces a unit root in the moving average (MA) representation of the transformed process, in which case the latter cannot be approximated by a finite-order VAR. This is known as the problem of 'overdifferencing' (see e.g., Plosser & Schwert, 1977).

The second approach is to test for cointegration and impose cointegration restrictions. The imposition of valid cointegration restrictions increases the efficiency of the estimates, but this gain in efficiency must be weighed against the bias that may result if invalid restrictions are imposed (Hamilton, 1994, chap. 20). Moreover, if the purpose of the exercise is to estimate a DSGE model using an impulse response matching approach, one could argue that the cointegration restrictions imposed on the VAR should be consistent with the cointegration restrictions implied by the DSGE model. If the latter are not supported by the data, one strategy is to go back to the drawing board and re-specify the DSGE model. However, recognising that, particularly if the sample size is small, the power of the cointegration test is low and the results from the cointegration analysis are often not clear-cut, an alternative approach is to include the cointegration vectors implied by the DSGE model in the VAR.

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[3]However, the decision to difference the variables is often made after testing for cointegration and finding little evidence of cointegration among the variables.

The cointegration tests suggest that there is one, or possibly two, cointegrating relations among the variables in the UK data. The cointegration restrictions implied by many open-economy DSGE models, namely that relative prices are stationary, are strongly rejected by the data.

I proceed by computing the impulse responses of prices to an exchange rate shock from three different specifications of the VAR: a VAR in levels, a VAR in first differences and a vector equilibrium-correction model (VEqCM) that imposes stationarity of relative prices. The purpose of the exercise is to identify features of the pass-through process that are robust to alternative assumptions about the time-series properties of the data. This could potentially be useful for deciding on the number of impulse response horizons to use when constructing an impulse response matching estimator for open-economy DSGE models. To take account of small-sample bias in the estimated impulse responses, the confidence bands for the impulse response estimator are computed using the bias-corrected bootstrap procedure proposed by Kilian (1998).

The estimates from the first-differenced VAR are in line with the estimates in the existing literature. First, exchange rate pass-through is incomplete, even in the long run. Second, the size and speed of pass-through decline along the distribution chain: import prices respond stronger and faster to exchange rate shocks than producer and consumer prices. Finally, consumer prices are largely unresponsive to exchange rate shocks. The VEqCM imposes the same degree of long-run pass-through to all prices. Compared to the first-differenced model, the VEqCM implies a lower degree of pass-through to export prices and import prices in the medium- and long run and higher pass-through to producer prices, consumer prices and unit labour costs. The medium- and long-run pass-through estimates obtained from the levels VAR are substantially higher than for the other specifications: for some horizons the pass-through (measured as the response of the price level divided by the exchange rate response) exceeds one. The results thus indicate that the estimates of pass-through are highly sensitive to the assumptions made about the time-series properties of the data, even at relatively short horizons.

In order to cast more light on the results, the last part of the paper considers two sets of simulation experiments. In the first set of experiments, the data generating process and the model coincide. The purpose of these experiments is to gain insight into the small-sample properties of the impulse response estimator and the accuracy of confidence intervals constructed using the bias-adjusted bootstrap procedure. When the data generating process and the model are a first-differenced VAR, there is essentially no bias in the impulse responses. However, if the data generating process and the model are a VAR in levels, the impulse responses display a downward bias. For both specifications the coverage rates of the confidence intervals are lower than the nominal level at short horizons, but close to the nominal level at longer horizons.

The second set of experiments addresses the following questions: what would an econometrician find on average if she estimated a VAR in levels, but the data were generated by a first-differenced VAR? And conversely; what would the econometrician find if she estimated a VAR in first-differences when the data generating process was a VAR in levels? These experiments also shed light on a slightly different issue, namely whether any of the two data generating processes can account for the pass-through estimates obtained on actual data. Christiano et al. (2003) proposed a method for selecting between the levels and first difference specification of the VAR based on an encompassing criterion. A model is said to encompass the other if it is able to predict the results using the opposing model. If only one of the two specifications can simultaneously account for the pass-through estimates obtained for both specifications on actual data, then this specification is to be preferred. However, the encompassing test does not offer an unambiguous answer as to which is the most plausible model of UK prices and exchange rates; the VAR in levels or the first-differenced VAR.

The structure of the paper is as follows. Section 2 gives a brief overview of the structural VAR methodology. Section 3 presents the outcome of unit root tests and tests for cointegration and provides estimates of the exchange rate pass-through from different VAR specifications. Section 4 reports the results of the simulation experiments. Section 5 concludes.

2   THE STRUCTURAL VAR METHODOLOGY

Let X_t be a $p \times 1$ vector of variables observed at time t. The unrestricted k-th order VAR for X_t (omitting any deterministic terms) is

$$A(L)X_t = \varepsilon_t, \tag{1}$$

where $A(L) = I - \sum_{i=1}^{k} A_i L^i$ is a $p \times p$ matrix polynomial in the lag operator L ($L^j X_t \equiv X_{t-j}$), $A_1, A_2, \ldots, A_k$ are $p \times p$ matrices of autoregressive coefficients, and ε_t is a $p \times 1$ vector of innovations. The innovations are assumed to be identically and independently distributed, $\varepsilon_t \sim IID(0, \Omega)$ with Ω positive definite. The initial values $X_{-k+1}, \ldots, X_0$ are assumed fixed. If all the roots of the characteristic polynomial $\left| I - \sum_{i=1}^{k} A_i z^i \right| = 0$ are outside the unit circle, the process X_t is covariance stationary. In this case $A(L)$ is invertible, and X_t has an MA representation

$$X_t = A(L)^{-1} \varepsilon_t = C(L) \varepsilon_t, \tag{2}$$

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is a convergent matrix polynomial in the lag operator, and $C_0 = I$. The $\{j, i)$ element in C_s identifies the response of $X_{j,t+s}$ to a one-unit increase in $\varepsilon_{i,t}$,

$\partial X_{j,t+s}/\partial \varepsilon_{i,t}$. For stationary processes, the responses die out as the horizon increases, that is,

$$\lim_{s\to\infty}\frac{\partial X_{j,t+s}}{\partial \varepsilon_{i,t}}=0 \tag{3}$$

However, the persistence in macroeconomic time series is often found to be well described by a unit root process. The VAR in (1) can alternatively be parameterised as

$$A^*(L)\Delta X_t = -A(1)X_{t-1}+\varepsilon_t, \tag{4}$$

where $A^*(L)=I-\sum_{j=1}^{k-1}A_i^*L^i$, and $A_i^*=-\sum_{j=i+1}^{k}A_j$. If X_t is a unit root process, that is, if at least one of the roots of $\left|I-\sum_{i=1}^{k}A_iz^i\right|=0$ is on the unit circle, $A(1)$ will have reduced rank $r<p$.[4] If $r=0$, then $A(1)=0$, and the model is a VAR in ΔX_t. In this case, all shocks will have permanent effects on X_t, and the impulse responses will not die out as the horizon increases. In the intermediate case $0<r<p$, there exists r stationary relations between the variables in X_t. Specifically,

$$-A(1)=\alpha\beta', \tag{5}$$

where α and β are $p\times r$ matrices with rank r. The vectors in β are the so-called cointegration vectors, and α is a matrix of adjustment coefficients. The corresponding MA representation for X_t is (see Johansen, 1995, p. 49)

$$X_t = C\sum_{i=1}^{t}\varepsilon_i+C^*(L)\varepsilon_t+G, \tag{6}$$

where C is a matrix of rank $p-r$ which represents the long-run effects of the shocks, $C^*(L)=\sum_{i=0}^{\infty}C_i^*L^i$ is a convergent matrix polynomial in the lag operator, and G depends on initial values. If the variables in X_t are not cointegrated, C has full rank p. If X_t is stationary, $C=0$. In this case, transforming the model into a model in first differences will induce a unit root in the MA representation of the transformed process. The unit root in the MA representation implies that, in this case, the first-differenced process cannot be approximated by a finite-order VAR.[5]

---

[4]Following Johansen (1995, p. 14), all the roots are assumed to be on or outside the unit circle.

[5]Plosser & Schwert (1977) provide the following simple example. Assume that x_t is stationary around a linear trend

$$x_t = \alpha+\beta t+\varepsilon_t,$$

where ε_t is a mean zero i.i.d random variable. The first-differenced process then follows a non-invertible MA(1) process,

$$\Delta x_t = \beta+\varepsilon_t-\varepsilon_{t-1},$$

and does not have an autoregressive representation. See Hamilton (1994, chap. 19) for an example with cointegrated processes.

In general, the elements in ε_t will be correlated. This makes it difficult to interpret the innovations as 'structural' shocks. However, the VAR in (1) can be interpreted as the reduced form of a *structural* VAR with orthogonal disturbances u_t,

$$B(L)X_t = u_t, \tag{7}$$

where $B(L) = \sum_{i=0}^{k} B_i L^i$, and $u_t \sim IID(0, \Sigma)$ where Σ is diagonal. Normalising the diagonal elements of Σ to one (i.e., assuming $\Sigma = I$) is without loss of information. If B_0 is non-singular, the relationship between the parameters of the structural VAR and the reduced form VAR is

$$A(L) = B_0^{-1} B(L) \text{ and } \Omega = B_0^{-1} \left(B_0^{-1} \right), \tag{8}$$

and the reduced form innovations ε_t are related to the structural disturbances u_t by $\varepsilon_t = B_0^{-1} u_t$. Expressed in terms of the orthogonal shocks, the MA representation of X_t is

$$X_t = C(L) B_0^{-1} u_t \tag{9}$$

Absent further restrictions, knowledge of the reduced form parameters $A_1, A_2, \ldots, A_k$ and Ω is not sufficient to recover the structural form parameters $B_0, B_1, \ldots, B_k$. In general, an infinite number of structural models is consistent with the same reduced form, and this gives rise to an identification problem. The necessary order condition for (local) identification is that a minimum of $p \times (p-1)/2$ restrictions are imposed on the parameters of the structural form model. For the rank condition for identification to be satisfied, these restrictions must be linearly independent.

In the structural VAR literature on exchange rate pass-through, identification is typically achieved by imposing zero restrictions on the matrix of contemporaneous responses, that is, by setting $p \times (p-1)/2$ of the parameters in B_0 to zero. A common assumption is that B_0 is lower triangular. In this case, the parameters in B_0 can be recovered from the Choleski decomposition of Ω, that is, by setting B_0 equal to Γ^{-1}, where Γ is the unique lower triangular matrix satisfying $\Omega = \Gamma\Gamma'$.

The assumption that B_0 is lower triangular imposes a recursive structure on the variables in X_t. Letting γ_{ij} denote the $\{i, j\}$ element of Γ, the relationship between the reduced form innovations and the structural shocks can be written as

$$
\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{pt} \end{bmatrix} = \begin{bmatrix} \gamma_{11} & 0 & 0 & 0 \\ \gamma_{21} & \gamma_{22} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ \gamma_{p1} & \gamma_{p2} & \cdots & \gamma_{pp} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{pt} \end{bmatrix}. \tag{10}
$$

The first variable in the ordering is contemporaneously affected only by the shock to the first equation, the second variable is affected by the shocks to the first and second equations and so on. The last variable in the ordering is contemporaneously affected by all the shocks in the system. Unless the reduced form innovations are uncorrelated, the impulse response functions will not be invariant to the ordering of the variables in the VAR.

If interest is only in the response to a single shock, an exchange rate shock say, identification can be achieved by assuming that B_0 (and hence B_0^{-1}) is *block* triangular (see e.g., Christiano et al., 1998). Letting s_t denote the exchange rate, and partitioning X_t into three blocks

$$
\underset{(1\times p)}{X_t'} = \left[\begin{array}{ccc} \underset{(1\times p_1)}{X_{1,t}} & \underset{(1\times 1)}{s_t} & \underset{(1\times p_2)}{X_{2,t}} \end{array} \right],
\tag{11}
$$

the block triangular B_0 can be written

$$
\underset{(p\times p)}{B_0^{-1}} = \left[\begin{array}{ccc} \underset{(p_1\times p_1)}{\beta_{11}} & \underset{(p_1\times 1)}{0} & \underset{(p_1\times p_2)}{0} \\ \underset{(1\times p_1)}{\beta_{21}} & \underset{(1\times 1)}{\beta_{22}} & \underset{(1\times p_2)}{0} \\ \underset{(p_2\times p_1)}{\beta_{31}} & \underset{(p_2\times 1)}{\beta_{32}} & \underset{(p_2\times p_2)}{\beta_{33}} \end{array} \right],
\tag{12}
$$

where $p = p_1 + p_2 + 1$. The block recursive structure implies that the exchange rate s_t responds contemporaneously to shocks to the variables in $X_{1,t}$, but not to shocks to the variables in $X_{2,t}$. Moreover, only the variables in $X_{2,t}$, respond within period to a shock to s_t. The impulse responses of a shock to s_t can be shown to be invariant to the ordering of variables within $X_{1,t}$ and $X_{2,t}$ (but not to the ordering of variables across groups).

3   ESTIMATES OF EXCHANGE RATE PASS-THROUGH TO UK PRICES

The VARs used to estimate exchange rate pass-through typically include a nominal exchange rate, one or several price indices (typically, import prices, producer prices and consumer prices) and sometimes additional variables such as oil prices, a measure of the output gap, wages and interest rates. The variables included in the baseline VAR in this paper are: UK import prices of manufactures (P_t^m), export prices of manufactures (P_t^x), producer prices of manufactures (P_t^y), consumer prices (P_t^c), nominal unit labour costs (ULC_t), and the nominal effective exchange rate (S_t). This is the same set of variables considered by Faruqee (2006), except that I have replaced the nominal wage rate by nominal unit labour costs. The data are quarterly, seasonally adjusted series covering the period 1980Q1 to 2003Q2. Details on the variable definitions and the sources of the data are provided in appendix A. In what follows, lower case letters denote variables in

natural logs.

Figures 1 and 2 plot the log-levels and the first difference of the logs of the variables in the VAR. The nominal price series all exhibit a high degree of persistence, consistent with the series containing a unit root. If the series were integrated of order one, the first differences of the series would be stationary. However, looking at the plots of the first differences, it is evident that there has been a decline in average inflation over the period. This has led some authors to suggest that UK prices are appropriately modeled as being integrated of order two, $I(2)$ (e.g., Bowdler & Nielsen, 2006). The fall in inflation in the 1990s should be viewed in light of the introduction of inflation targeting in 1992 and the delegation of the interest rate decision to the Monetary Policy Committee in 1997.

Figure 3 plots the level of prices relative to consumer prices, $p_t^m - p_t^c$, $p_t^x - p_t^c$ and $p_t^y - p_t^c$ and real unit labour costs $ulc_t - p_t^c$. As is evident from the graph, the relative prices display a distinct negative trend over the sample period. A partial explanation for the downward drift in relative prices could be faster technological growth in tradable goods than in the rest of the economy (see Gagnon, 2003). There is a less marked negative trend in real unit labour costs, however, the series displays large and persistent swings over the sample period. It is also apparent from the figure that import prices have fallen relative to export prices and relative to domestic producer prices.

The sharp exchange rate depreciation in the fourth quarter of 1992 followed UK's exit from the European Exchange Rate Mechanism (ERM) in September of that year. Four years later, the UK experienced a sharp exchange rate appreciation. These large swings in the exchange rate are reflected in import prices and export prices, however, the effects on consumer prices appear to have been small.[6] The spike in consumer price inflation in the second quarter of 1990 coincides with the introduction of the community charge in April of that year.

3.1   Time-series properties

The first step in the empirical analysis is to determine the order of integration of the variables using formal univariate and multivariate unit root tests.

3.1.1   Univariate unit root tests

Table 1 reports the results of two different unit root tests on the dataseries in the VAR: the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979; Said & Dickey, 1984) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al., 1992).[7] The null hypothesis in the ADF test is that the variable has a unit root, while the null

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[6]This has been noted by several authors. See e.g., Cunningham & Haldane (2000) for an event study of the 1992 depreciation and the 1996–1997 appreciation.

[7]The results are obtained using EViews version 5.

hypothesis in the KPSS test is that the variable is (trend-) stationary. The selection of lag-order for the ADF test is based on the Akaike Information Criterion (AIC). The numbers in brackets denote the number of lags chosen by the AIC in each case. The maximum number of lagged differenced terms is set to four for the levels of the variables and three for the first-differenced variables. The critical values for the ADF test are taken from MacKinnon (1996). The KPSS tests were run using a Bartlett kernel. The number in brackets is the bandwidth selected by the Newey & West (1994) method. Single asterisks (*) and double asterisks (**) denote statistical significance at the 5% level and the 1% level, respectively. For the levels of the variables and for relative prices I report results for two deterministic specifications: a model with a constant term and a model with a constant and a linear trend. For the first-differenced series the test regressions include a constant term.

Both tests indicate that the levels of nominal prices and unit labour costs are non-stationary unit root processes. For the level of the exchange rate, the KPSS test does not reject the null hypothesis that the nominal exchange rate is stationary. At the same time, however, the ADF test does not reject the hypothesis that the nominal exchange rate contains a unit root. Regarding the first-differenced series, both tests suggest that the first-differences of unit labour costs and the nominal exchange rate are stationary, whereas the tests indicate that the first-difference of producer prices contains a unit root. For import prices, export prices and consumer prices the tests give conflicting results; the ADF test rejects the null hypothesis that the series contain a unit root, whereas the KPSS test rejects the null hypothesis that the series are stationary. Both tests suggest that relative prices contain a unit root, however, the KPSS test rejects the null of stationarity for real unit labour costs. According to both tests, the first-differences of relative prices are stationary.

Overall, bearing in mind that unit root tests have low power to distinguish between a unit root process and a stationary, but highly persistent process, the tests indicate that the exchange rate and unit labour costs are at most integrated of order one, $I(1)$. Additionally, the tests suggest that there is a unit root in relative prices and real unit labour costs. The evidence is mixed regarding whether there is a unit root in the first difference of the nominal price series.

3.1.2 Multivariate cointegration analysis

The cointegration analysis is conducted within the maximum likelihood framework of Johansen (1988).[8] The first step in the cointegration analysis is to estimate an unrestricted VAR in $X_t' = \{s_t, p_t^m, p_t^x, p_t^y, p_t^c, ulc_t\}$. The effective sample is 1981Q2-2003Q2. The deterministic specification allows for linear trends in all directions, including the coin-

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[8]See chapter 2 of this thesis for a more detailed description of the Johansen framework.

tegration relations, but excludes the possibility of a quadratic trend in the levels of the data. This is achieved by including a constant and a linear drift term in the VAR and by restricting the linear drift term to lie inside the cointegration space. The VAR also includes impulse dummies for the second quarter of 1990 and the fourth quarter of 1992 to control for, respectively, the introduction of the community charge and sterling's exit from the ERM.

Table 2 reports the values of the Akaike (AIC), Schwarz (SC), and Hannan-Quinn (HQ) information criteria for different lag orders as well as the p-values from likelihood ratio (LR) tests for successive lag deletions (see Lütkepohl, 1991, for details). The LR test is implemented using the small-sample correction suggested in Sims (1980) and using a 5% significance level for the individual tests. The maximum lag-order is set to five. The LR-tests, the AIC and the HQ all point to a lag-length of two as being appropriate, whereas the SC is minimised for a first-order VAR. In the subsequent analysis the lag-length is set to two. Table 3 reports misspecification tests for the system and for the six equations individually. Except for some evidence of autocorrelation in the residuals in the equation for p_t^y and non-normality in the residuals in the equation for s_t, the VAR appears statistically well-specified.[9]

Table 4 reports the trace test statistics with the corresponding p-values, as well as the moduli of the largest eigenvalues of the companion matrix associated with the VAR for different values of the cointegration rank.[10,11] The asymptotic critical values and the p-values are approximated using the gamma-distribution (see Doornik, 1998). The test statistics in the third column are the small sample Bartlett-corrected trace statistics (Johansen, 2002). The effect of the small-sample correction is to reduce the size of the test-statistic. The VAR is clearly non-stationary; in the unrestricted VAR (corresponding to cointegration rank equal to six) there is one root very close to the unit circle, and two more roots exceeding 0.9. Using a 5% significance level, the trace test suggests the existence of two cointegration relations. The Bartlett-corrected test-statistics indicate a cointegration rank of one.

Due to the presence of two impulse-dummies in the system, the approximated distribution used to compute the critical values for the trace test is not strictly valid. CATS includes a procedure for simulating the asymptotic distribution of the trace-test statistic.[12] The p-values in table 5 are based on the simulated critical values. As is evident from the table, the simulated critical values are very similar to the approximated ones used above and do not lead to a different conclusion about cointegration rank.

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[9]The autocorrelation in the residuals in the equation for p_t^y is not removed by increasing the lag-length.

[10]The eigenvalues are the inverse of the roots of the characteristic polynomial $\left| I - \sum_{i=1}^{k} A_i z^i \right| = 0$

[11]The results are obtained using the program CATS in RATS version 2. See Dennis (2006).

[12]The simulation is based on $N = 2500$ replications and a length $T = 400$ for the random walk processes. See Dennis (2006) for details.

Additional information about cointegration rank is provided by the eigenvalues of the companion matrix. If a non-stationary relation is wrongly included in the model, then the largest unrestricted eigenvalue will be close to one (see Juselius, 2006, chap. 8). With cointegration rank three, the largest unrestricted eigenvalue is 0.9. Imposing cointegration rank two, the largest unrestricted eigenvalue is 0.8. This root is not removed by lowering the cointegration rank. This could be an indication that there are $I(2)$ trends in the data; if the process X_t is $I(2)$ then unit roots will remain in the system even after the correct cointegration rank has been imposed on $-A(1) = \alpha\beta'$ (Johansen, 1995, p. 53).[13] However, the maintained assumption in the following is that the variables are at most integrated of order one.

Overall, the evidence points to the existence of one, or possibly two, cointegrating relations among the variables. It is difficult to give an economic interpretation of the cointegration relations, however. Table 6 reports the LR statistics for various restrictions on the cointegration parameters under the assumption that the cointegration rank is one. All the tests allow for a restricted trend in the cointegration relation. Consistent with the findings from the univariate tests, the hypothesis of stationarity of the series in levels are strongly rejected, as are the tests of stationarity of the relative prices $p_t^m - p_t^c$, $p_t^x - p_t^c$, $p_t^y - p_t^c$ and $ulc_t - p_t^c$. Stationarity of the terms of trade ($p_t^m - p_t^x$), the producer real wage ($ulc_t - p_t^y$) and the price of exports relative to the domestic producer price ($p_t^x - p_t^y$) is also strongly rejected. Finally, the LR tests reject the hypotheses that there exists a long-run homogenous relation between either consumer-, producer- or export prices and a linear combination of unit labour costs and import prices.[14]

An alternative to the data-based procedure for determining cointegration rank is to start with a hypothesis about cointegration rank from economic theory and test this against the unrestricted model (see Johansen, 1995, p. 98). A theoretical scenario consistent with many open-economy macroeconomic models, including many recent DSGE models (e.g., the DSGE model considered in chapter 5 of this thesis), is that nominal prices and the nominal exchange rate are $I(1)$, but that relative prices and real unit labour costs are stationary. According to this scenario, there should be four cointegrating relations among the variables in the VAR. Note that, under this hypothesis, the cointegrated VAR can be written as a VAR in the first-differences of exchange rates and consumer prices and four relative prices, that is, $\widetilde{X}_t' = \left\{ \Delta p_t^c, \Delta s_t, p_t^m - p_t^c, p_t^x - p_t^c, p_t^y - p_t^c, ulc_t - p_t^c \right\}$.[15] As can be seen from table 4, the trace test does not reject the hypothesis of at least two unit roots in the $I(1)$ model, corresponding to cointegration rank four. However, when

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[13]In fact, formal statistical procedures for determining cointegration ranks in the $I(2)$ model suggest the existence of two stationary relations and one, or possibly two, $I(2)$ trends in the system. Appendix B offers a discussion of the cointegrated $I(2)$ model and presents the results of the $I(2)$ cointegration tests.

[14]Imposing cointegration rank two, all the hypotheses are still rejected based on non-corrected test-statistics.

[15]For the two models to be identical, the coefficients on the second lags of Δs_t and Δp_t^c should be restricted to zero and there should be no trend in the equations for Δs_t and Δp_t^c in the VAR in $\widetilde{X}_t$.

cointegration rank four is imposed, a root close to the unit circle (0.91) remains in the model. Moreover, conditional on $r = 4$, the LR test-statistic for the joint hypothesis that $p_t^m - p_t^c$, $p_t^x - p_t^c$, $p_t^y - p_t^c$ and $ulc_t - p_t^c$ are (trend) stationary is $\chi^2(8) = 58.85$ which is statistically significant at the 1% level. Subject to the caveat that the LR tests of restrictions on the cointegration space tend to be oversized in small samples (see e.g., Gredenhoff & Jacobson, 2001), the finding that relative prices are non-stationary is consistent with the outcome of the univariate unit root tests and is supported by the visual inspection of the variables.

The results of the cointegration analysis can be summarised as follows. First, the levels process is clearly non-stationary. Second, there is evidence of cointegration, indicating that the model in first-differences is misspecified and could produce biased estimates of the exchange rate pass-through. However, a typical theoretical open-economy model would suggest fewer common trends and hence, more cointegration relations among the variables. Finally, the hypotheses that relative prices are stationary, jointly or individually, are strongly rejected. The next subsection asks the question whether the estimates of exchange rate pass-through are sensitive to the assumptions made about unit roots.

3.2   Impulse response analysis

I first consider the estimates of exchange rate pass-through from a first-order VAR estimated in first differences.[16] The exchange rate shock is identified by imposing a recursive structure on the variables in the VAR. Given that interest is only in identifying a single shock, there are six different recursive orderings to consider. Table 7 reports the correlation structure of the residuals in the first-differenced VAR. The most striking finding is that the residuals are highly correlated. The high correlation between the residuals implies that the ordering of the variables will matter for the impulse responses. In particular, noting that the correlation coefficients between the residuals in the exchange rate equation and the equations for import prices and export prices are, respectively, 0.63 and 0.29, it will matter whether the exchange rate is placed before or after the traded goods prices in the VAR.

When the exchange rate is ordered prior to the price indices, unexpected movements in prices do not have an impact effect on exchange rates. Choudhri et al. (2005) present a

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[16]The first-differenced VAR(1) is a restricted sub-model of the unrestricted VAR(2) in levels. Starting from a maximum lag length of four for the first-differenced VAR, the LR-test for sequential lag deletions suggests a lag-order of four, whereas the information criteria (AIC, SC and HQ) suggest a lag-order of one. Monte Carlo evidence reported in Ivanov & Kilian (2005) indicates that underestimation of the true lag order is beneficial in very small samples because the bias induced by choosing a low lag order is more than offset by a reduction in variance. If the primary purpose of the analysis is to construct accurate impulse responses, the authors recommend using the SC for sample sizes up to 120 quarters and the HQ for larger sample sizes. However, Ivanov & Kilian (2005) do not explore the case where the VAR is overdifferenced, in which case the trade-offs between bias and variance are likely to be different.

small open economy DSGE model that is consistent with the notion that exchange rates do not move in the impact period of a shock to prices in goods and labour markets. The argument is that, due to time lags in the publication of official statistics, participants in the foreign exchange market observe prices in goods and labour markets with at least a one period lag. In the absence of such information delays, however, we would expect the exchange rate to respond contemporaneously to all shocks that are relevant for its determination, including prices in goods and labour markets that signal future monetary policy.[17] According to this argument, exchange rates should be placed last in the recursive ordering, or alternatively, we should use an identification scheme that allows for contemporaneous simultaneity between exchange rates and prices.

Table 8 reports the results of pairwise Granger-causality tests for the exchange rate and each of the price indices. The numbers in square brackets are p-values. The hypotheses that the exchange rate does not Granger-cause traded goods prices are strongly rejected. At the same time there is little evidence that import prices and export prices Granger-cause the exchange rate. This gives informal support to an identification scheme where the exchange rate is placed prior to traded goods prices. For producer prices, the causation seems to run from prices to exchange rates, but for consumer prices and unit labour costs, the picture is less clear. Below I therefore report results for two identification schemes; one where the exchange rate is placed first in the ordering, and another where the exchange rate is placed after producer prices, consumer prices and unit labour costs, but before the traded goods prices.

Figure 4 plots the impulse responses of the levels of import prices, export prices, producer prices, consumer prices and unit labour costs to a one standard deviation shock to the exchange rate. The exchange rate shock is identified by placing the exchange rate first in a recursive ordering of the variables. The bottom panel of the figure shows the normalised impulse responses, that is, the impulse responses of the price levels divided by the exchange rate response. This normalisation facilitates a comparison with single-equation estimates of pass-through defined as the dynamic responses of prices to a one per cent permanent exchange rate change.

The 90% confidence intervals are computed using the bias-adjusted bootstrap procedure proposed by Kilian (1998). This procedure explicitly accounts for the bias and the skewness in the small-sample distribution of the impulse response functions.[18] The

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[17]See e.g., the discussion about efficient market variables and structural VARs in Sarno & Thornton (2004).

[18]The Monte Carlo evidence in Kilian (1998) and Kilian & Chang (2000) indicates that the bias-corrected bootstrap intervals are as accurate in finite samples as intervals based on the Monte Carlo integration method of Sims & Zha (1999) and more accurate than intervals based on the asymptotic normal approximation (see Lütkepohl, 1990) and the standard bootstrap procedure originally suggested by Runkle (1987). The bootstrap procedure is asymptotically valid in stationary VARs, but not in non-stationary VARs estimated in levels. Kilian (1998) shows, however, that the bias-corrected bootstrap procedure performs well compared to alternative methods even in the latter case.

bootstrap samples are generated by first drawing with replacement from the estimated residuals and then constructing a new dataset using the bootstrapped residuals and the OLS estimates of the autoregressive coefficients. The bootstrap samples are initialised using the actual observations. The bias-corrected bootstrap procedure is implemented by first performing a preliminary bootstrap to obtain an estimate of the mean bias in the VAR coefficients. A stationarity correction is applied if the bias-corrected estimates imply that the VAR becomes non-stationary or explosive. In the second stage, the bias-corrected estimates are used to generate new bootstrap replications. As a short-cut, the first-stage bias estimate is used to correct for bias in the second stage of the bootstrap procedure.[19] Finally, the bias-corrected estimates are used to compute the empirical distribution of the impulse responses. The 90% confidence interval is obtained by reading off the 5th and 95th percentiles of the ordered responses at each horizon. Following Kilian (1998), the first-stage bias estimation is based on 1000 replications, and the construction of the confidence intervals is based on 2000 bootstrap replications.[20]

As is evident from the figure, the responses of traded goods prices to the exchange rate shock are numerically and statistically significant at all horizons. The response of the producer price index is smaller, whereas the responses of consumer prices and unit labour costs are close to zero. We note that the confidence intervals are wide and asymmetric.

According to the normalised impulse responses, the pass-through to import prices is 0.40 within the first quarter, increases to 0.58 within the first year and stabilises at 0.64 after about two years. The long-run pass-through is significantly different from one. The pass-through to export prices is somewhat smaller at all horizons: the normalised response to the exchange rate shock is 0.14 within the first quarter and reaches 0.37 after a year. The estimate of long-run pass-through to export prices is 0.43. The pass-through to producer prices is both smaller and more gradual; the pass-through is close to zero in the first quarter, but eventually reaches 0.15 after three years. The pass-through to consumer prices and unit labour costs is close to zero at all horizons and is statistically insignificant. These findings are in line with those reported in other structural VAR studies of pass-through. In particular, the pass-through estimates are similar to those reported by Faruqee (2006). Faruqee estimates a first-differenced VAR on monthly UK data for the period 1990–2002 and identifies the exchange rate shock by placing the exchange rate first in a recursive ordering of the variables. He finds that the pass-through to import prices is 0.28 in the first month after the shock, increasing to 0.57 after one year. For export prices the pass-through is 0.16 in the first month, increasing to 0.46 after one year. The similarity of the estimates adds confidence that the results obtained

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[19]This short-cut was suggested by Kilian (1998). Alternatively, the mean bias in the coefficient estimates can be estimated in a separate bootstrap loop nested inside each of the second-stage bootstrap loops. This increases the number of bootstrap replications from 1000+2000 to $1000 + 2000 \times 1000$.

[20]I have scripted the bias-corrected bootstrap procedure in Matlab 7.04.

in this paper are not specific to the particular estimation period used or the frequency of the data.[21]

Figure 5 plots the impulse responses to an exchange rate shock when the exchange rate is placed after producer prices, consumer prices and unit labour costs, but before the traded goods prices in the recursive ordering. Under this identification scheme the contemporaneous effect of the exchange rate shock on the first three variables is restricted to be zero. Focusing on the normalised responses we see that the overall pattern of pass-through estimates remains intact, however, the estimated pass-through is somewhat smaller. The short-run pass-through to import prices is now 0.36 and the first-quarter response of export prices is 0.11. The long-run pass-through to import prices is 0.55, whereas the pass-through to export prices stabilises at 0.33 in the long-run. The pass-through to producer prices reaches 0.06 after about three years. The pass-through to consumer prices and unit labour costs is close to zero at all horizons.

The next step is to examine whether the impulse responses are sensitive to the assumptions made about the time-series properties of the data. In what follows, the exchange rate shock is identified by placing the exchange rate first in the recursive ordering. Figure 6 plots the impulse responses of prices to an exchange rate shock when the VAR is a second-order VAR in levels. The first thing to note is that, because all the roots of the companion matrix are smaller than one, all the variables eventually revert back to their original levels. This means that the long-run pass-through to all prices is complete, in the sense that the long-run responses of prices and exchange rates are the same (i.e., zero). As before, import prices and export prices display a significant response to the exchange rate shock, whereas the effects on consumer prices and unit labour costs are negligible. For horizons up to three or four quarters the pass-through estimates (as measured by the normalised impulse responses) are of similar magnitude to what was obtained for the first-differenced VAR. For longer horizons, however, the responses are much larger and even exceed one for a long period. This reflects that the exchange rate is reverting faster to its original level than the remaining variables.

Finally, figure 7 plots the responses obtained when the estimated model is a VEqCM, or more precisely, a VAR in $\widetilde{X}_t' = \left\{ \Delta p_t^c, \Delta s_t, p_t^m - p_t^c, p_t^x - p_t^c, p_t^y - p_t^c, ulc_t - p_t^c \right\}$. We notice that the confidence levels are much wider than in the previous specifications. The exchange rate shock is permanent, however, the mean-reverting component is stronger in the VEqCM than in the first-differenced VAR, where the exchange rate jumps almost directly to its new long-run level. Again there is a significant response of traded goods prices to the exchange rate shock, whereas the remaining variables display smaller and

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[21]As a robustness check I also computed impulse responses from a first-differenced VAR which excluded the impulse dummy for the fourth quarter of 1992. The exclusion of the dummy caused a slight increase in the estimates of exchange rate pass-through to all the price indices (e.g., the long-run pass-through to import prices is increased to 0.66).

statistically insignificant responses. The normalised responses are similar to those obtained for the other specifications for horizons up to two or three quarters. Compared to the first-differenced model, the VEqCM implies a lower degree of pass-through to export prices and import prices in the medium- and long run and higher pass-through to producer prices, consumer prices and unit labour costs. This reflects that, by construction, the VEqCM imposes the same degree of long-run pass-through to all prices. Specifically, the estimate of long-run pass-through is 0.21.

In summary, the pass-through estimates are highly sensitive to the time-series specification of the VAR, except at very short horizons. The results suggest that, if there is uncertainty about the time-series properties of the data, one should be careful to interpret medium to long-run responses of prices to an exchange rate shock as 'stylised facts' that the theoretical models should reproduce. One practical implication is that one should focus on impulse horizons at relatively short horizons when constructing an impulse response matching estimator based on the VAR responses.

4   SIMULATION EVIDENCE

The preceding section showed that the estimates of exchange rate pass-through were highly sensitive to the assumptions made about unit roots. The question is which of the specifications is most plausible. The trace test for cointegration rejects the first-differenced specification in favour of the unrestricted VAR in levels. This suggests that the first-differenced VAR in misspecified: if the VAR is stationary, the first-differenced VAR is over-differenced and, strictly speaking, a finite-order VAR representation does not exist. By contrast, the levels specification is not misspecified if there are unit roots in the VAR; the levels specification nests the first-differenced specification (and any other representation with cointegration rank $r < p$). However, there is also the issue of small-sample estimation bias.

In order to cast more light on the results, this section reports the results from two sets of simulation experiments. In the first set of experiments the data generating process and the model coincide. The purpose of these experiments is to gain insight into the small-sample properties of the impulse response estimator and the accuracy of confidence intervals constructed using the bias-adjusted bootstrap procedure. The second set of experiments addresses the following questions: what would an econometrician find on average if she estimated a VAR in levels, but the data were generated by a first-differenced VAR? And conversely; what would the econometrician find if she estimated a VAR in first-differences when the data generating process was a VAR in levels?

In each simulation experiment I generate 5000 dataseries of length 1100. The first 1000 observations are discarded to limit the effect of the initial conditions. The errors are drawn from a multivariate normal distribution. In each artificial dataset I estimate a

VAR and compute the impulse responses to an orthogonalised exchange rate innovation, placing the exchange rate first in the recursive ordering. The properties of the distribution of the simulated impulse responses are summarised by the pointwise mean and a 90% pointwise confidence interval of the simulated responses.

The data generating processes (and the models) in the first set of experiments are the estimated second-order VAR in levels and the first-order VAR in first-differences from section 3. Figures 8 and 9 summarise the results.[22] The lines with circles correspond to the 'true' impulse responses, the solid line is the pointwise median of the simulated responses and the grey area covers a 90% probability interval for the simulated responses. The latter is obtained by first ordering the impulse responses from smallest to the largest and then reading off the 5th and the 95th percentiles at each horizon. Finally, the lines with points indicate the average across artificial datasets of the 90% confidence intervals computed using the bias-adjusted bootstrap procedure. In the simulation experiments the first-stage bias estimation in the bootstrap procedure is based on 500 replications and the construction of the confidence intervals is based on 1000 bootstrap replications. If the bootstrap confidence intervals were accurate, we would expect them on average to coincide with the probability interval of the simulated responses (see Christiano et al., 2006). The bottom panel in the figures displays the coverage rates for the bootstrap confidence interval (i.e., the fraction of artificial datasets in which the bootstrap interval contains the true impulse response function). Ideally, the coverage rates should be 90%.

For the first-differenced specification there is virtually no bias in the estimated impulse responses: the median responses are very close to the true responses. At short horizons, the coverage rates of the confidence intervals are below 90%, but the average bootstrap interval coincides closely with the probability interval of the simulated responses. At longer horizons the coverage rate is close to 90%. However, the average bootstrap interval is wider than the simulated probability interval at longer horizons, suggesting that, on average, an econometrician using the bias-adjusted bootstrap procedure for constructing confidence intervals would overestimate the degree of uncertainty surrounding the responses somewhat. Overall, however, the bias-corrected confidence intervals have good properties in the first-differenced VAR. It should be stressed that the results are conditional on several maintained assumptions, including that the disturbances are multivariate normal and that the lag-order is finite and known.

When the data generating process and the estimated model are a VAR in levels, there is evidence of downward bias in the impulse responses. However, the true responses lie inside the probability interval of the simulated responses at all horizons. As was the case for the first-differenced VAR, the average bootstrap intervals are wider than the simulated

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[22]The deterministic terms are not included in the data generating processes. The estimated models include a constant term, but no trend or impulse dummies. The lag order is assumed to be known.

probability interval for long-horizon responses, and the coverage rates of the confidence intervals are below 90% at shorter horizons. In conclusion, when there are roots near unity and the VAR is estimated in levels, small-sample estimation bias becomes an issue and the performance of the bias-corrected bootstrap interval deteriorates somewhat.

The next set of experiments examines what happens if the data generating process and the model do not coincide. In the first experiment the data are generated by a VAR in levels, but the estimated model is a first-differenced VAR. In the second experiment the data generating process is a first-differenced VAR, but the estimated model is a VAR in levels. In both experiments the lag-order is determined endogenously for each dataset using the sequential LR test. The LR test is implemented using the small-sample correction suggested in Sims (1980) and a 5% significance level for the individual tests. The minimum and maximum lag-orders are set to one and five, respectively.[23] In addition to providing information on the relative size of the biases caused by overdifferencing, or by the failure to impose a unit root in the non-stationary VAR, the experiments shed light on a slightly different issue, namely whether any of the two data generating processes can account for the pass-through estimates obtained on actual data. Christiano et al. (2003) propose a method for selecting between the levels and first-difference specification of the VAR based on an encompassing criterion. A model is said to encompass the other if it is able to predict the results using the opposing model. If only one of the two specifications can simultaneously account for the pass-through estimates obtained for both specifications on actual data, then this specification is to be preferred.[24]

Figure 10 reports the outcome of the experiment where the data are generated by a levels VAR, but the econometrician estimates a VAR in first differences. The distribution of lag-orders chosen by the LR test is in this case: 0.02 ($k = 1$), 0.24 ($k = 2$), 0.31 ($k = 3$), 0.25 ($k = 4$), 0.19 ($k = 5$). The lines with circles are the true responses, and the solid lines reproduce the estimated responses from the first-differenced VAR in figure 4. The lines with crosses represent the median simulated responses, and the grey area is the 90% probability interval of the simulated responses. As can be seen from the figure, the impulse responses of exchange rates, import prices and export prices exhibit a significant upward bias. Except for the short-horizon responses, the true impulse responses lie outside of the 90% interval of the simulated responses. The producer price responses are reasonably accurate, whereas the responses of consumer prices and unit labour costs display a downward bias. The estimates of exchange rate pass-through are thus biased downward. Are the estimated responses from the first-differenced VAR consistent with the data being generated by the levels specification? The figure shows that the estimated responses from the first-differenced VAR lie above the median simulated response for

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[23]I also conducted experiments using the AIC as the lag-order selection criterion. The distributions of the impulse responses were very similar to those obtained using the LR-test.

[24]See also Christiano & Ljungqvist (1988) and the discussion in Hamilton (1994, chap. 19).

all variables at all horizons. However, the estimated responses all lie (albeit just barely) inside the 90% interval of the simulated responses.

Figure 11 plots the results for the experiment where the data generating process is non-stationary, but the econometrician estimates a VAR in levels. In this case, the solid lines represent the responses generated by estimating a VAR in levels on UK data (see figure 6). The distribution of the lag-orders selected by the LR test is: 0.00 ($k = 1$), 0.72 ($k = 2$), 0.08 ($k = 3$), 0.10 ($k = 4$), 0.10 ($k = 5$). As is evident from the figure, the failure to impose a unit root in estimation causes a downward bias in the impulse responses. The bias is most evident in the responses of exchange rates, import prices and export prices. However, the median impulse response functions emerging from the levels VAR are close to the impulse response functions generated when estimating a VAR in levels on actual data. This finding suggests that the estimates obtained when estimating a VAR in levels on UK could be consistent with the data being generated by a first-differenced VAR. Viewed in isolation, this adds support to the first-differenced specification. However, this reasoning does not take into account the finding reported above, namely that estimating a VAR in levels yields biased estimates when the true model is a VAR in levels. Overall, the encompassing criterion does not provide us with a clear answer as to which is the most plausible specification in this case.

5   CONCLUDING REMARKS

The main theme of this paper is that structural VAR estimates of exchange rate pass-through are sensitive to the treatment of the apparent non-stationarity in the data. This is illustrated by comparing the impulse responses of UK prices to an orthogonalised exchange rate shock in three different VAR specifications: a first-differenced VAR, a VAR in levels and a VEqCM that imposed stationarity of relative prices.

Simulation evidence indicates that, when the data generating process is stationary, the estimates obtained from a VAR estimated in first differences exhibit a strong upward bias. When the VAR is non-stationary, but the econometrician estimates a VAR in levels, the opposite holds: the estimated responses are biased upwards.

As to whether which specification is the most plausible, the trace test for cointegration suggests the existence of one, or possibly two, cointegration relations between the variables. However, the estimated cointegration relation is not easily interpretable in terms of economic theory. Another finding is that the transformation of the VAR in nominal variables into a model in relative prices and inflation rates does not eliminate the unit roots from the process. Stationarity of relative prices is a key implication of many open-economy DSGE models. One obvious possibility is that relative prices are non-stationary, and hence that the theoretical models are misspecified. Another possibility is that, because of small-sample estimation bias, we would not recover the cointegration

relations implied by the DSGE model even if it were the correct data generating process. This possibility is examined in chapter 5 of this thesis.

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A   VARIABLE DEFINITIONS AND SOURCES

- P^m: Import price of manufactured goods, local currency (source: OECD International Trade and Competitiveness Indicators).[25]

- S: Nominal effective exchange rate (source: OECD Economic Outlook [Q.GBR.EXCHEB]).

- ULC: Unit labour costs (source: OECD Economic Outlook [Q.GBR.ULC]).

- P^c: RPIX, retail price index excl. mortgage interest payments (source: UK National Statistics [CHMK]/Bank of England).[26]

- P^x: Export price of manufactured goods, local currency (source: OECD International Trade and Competitiveness Indicators).

- P^y: Producer price index all manufacturing excl. duty (source: UK National Statistics [PVNQ]).

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[25]All nominal variables are converted to a common baseyear 2000=100.

[26]As no official seasonally adjusted RPIX exists, this series was seasonally adjusted using the X12 method as implemented in EViews.

B   I(2) COINTEGRATION ANALYSIS

In recent years a number of authors have analysed price formation in open economies within a framework that allows nominal prices to be integrated up to order two (e.g., Banerjee et al., 2001; Kongsted, 2003; Bowdler & Nielsen, 2006). This appendix provides a brief discussion of the cointegrated $I(2)$ model and reports the outcome of tests of cointegration ranks in the $I(2)$ model for UK prices.

The VAR model with p endogenous variables and k lags can be written as

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \cdots + A_k X_{t-k} + \mu_0 + \mu_1 t + \varepsilon_t, \qquad t = 1, \dots, T \qquad \text{(B1)}$$

where ε_t is identically and independently distributed as $N(0, \Omega)$ with Ω positive definite, and the initial values $X_{-k+1}, \dots, X_0$ are fixed. A parameterisation of the VAR in (B1) which is convenient for analysing $I(1)$ series is

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \phi D_t + \mu_0 + \mu_1 t + \varepsilon_t, \qquad \text{(B2)}$$

where $\Pi = -A(1) = \sum_{i=1}^{k} A_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{k} A_j$. An alternative parameterisation, useful for $I(2)$ cointegration analysis, is

$$\Delta^2 X_t = \Pi X_{t-1} - \Gamma \Delta X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} + \mu_0 + \mu_1 t + \varepsilon_t, \qquad \text{(B3)}$$

where $\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i$ and $\Psi_i = -\sum_{j=i+1}^{k-1} \Gamma_i$ for $i = 1, \dots, k-2$. The unrestricted VAR model is denoted $H(p)$.

Both the $I(1)$ and $I(2)$ models are restricted sub-models of the VAR in (B1). The $I(1)$ model with r stationary relations, denoted $H(r)$, is defined by the reduced rank condition (see Johansen, 1995, p. 71).

$$\Pi = \alpha \beta', \qquad \text{(B4)}$$

where α and β are $p \times r$ matrices of rank $r < p$. The number of unit roots in the $I(1)$ model is $p - r$. The $I(2)$ model, denoted $H(r,s)$, is defined by the two reduced rank conditions (see Johansen, 1995, p. 133)

$$\Pi = \alpha \beta' \quad \text{and} \quad \alpha'_\perp \Gamma \beta_\perp = \xi \eta', \qquad \text{(B5)}$$

where ξ and η are $(p-r) \times s$ matrices of rank $s < p - r$. The $p \times (p-r)$ matrices $\alpha_\perp$ and $\beta_\perp$ are the orthogonal complements to α and β, respectively, that is, $\alpha' \alpha_\perp = \beta' \beta_\perp = 0$. The number of unit roots in the cointegrated $I(2)$ model is $2(p-r) - s$. Hence, determining the rank of Π is not sufficient to determine the number of unit roots in the system. If

X_t is an $I(2)$ process then unit roots will remain in the system even after the correct rank of Π has been imposed. The $I(2)$ model thus allows a richer cointegration structure than the $I(1)$ model. In particular, even if the levels of the variables do not cointegrate to stationarity, a linear combination of the levels could cointegrate with the first differences of the process. For example, in their analysis of Australian inflation, Banerjee et al. (2001) find that nominal prices and costs can be characterised as being integrated of order two, and that the mark-up of price on labour and import costs cointegrates with the rate of inflation.

Table 9 reports the outcome of the likelihood ratio (LR) test for cointegration ranks in the $I(2)$ model for UK data.[27] The estimates are based on a second-order VAR with impulse dummies for the second quarter of 1990 and the fourth quarter of 1992. The sample period is 1981Q2–2003Q2. Following Rahbek et al. (1999), I exclude the possibility of cubic and quadratic trends in the data, but allow for linear trends in all directions, including the cointegration relations. The LR test-statistic is

$$Q(r,s) = -2\log Q(H(r,s)|H(p)) = -T \log \left| \widetilde{\Omega}^{-1}\widehat{\Omega} \right|, \qquad \text{(B6)}$$

where $\widetilde{\Omega}$ and $\widehat{\Omega}$ are, respectively, the variance-covariance matrices estimated under $H(r,s)$ and $H(p)$ (see Nielsen & Rahbek, 2003). The test-statistic has a non-standard distribution under the null hypothesis. The number of stationary relations is r, the number of $I(1)$ trends is s and the number of $I(2)$ trends equals $p - s - r$. The LR test is based on the maximum likelihood procedure for estimation of the parameters in the $I(2)$ model outlined in Johansen (1997). The p-values reported in brackets below the test-statistics are approximated as in Doornik (1998).[28] Notice that the test-statistics in the last column correspond to the trace-statistics for cointegration rank in the $I(1)$ model (see table 4 in the main text).

Estimates of the cointegration ranks are obtained by starting from the most restrictive hypothesis $Q(0,0)$, and if the model is rejected, proceeding from the left-to-right and from the top-to-bottom in the table until the first insignificant test statistic is reached. The estimate of cointegration ranks in the UK data obtained using the sequential procedure are $r = 2$ and $s = 2$, corresponding to a model with two stationary relations and two $I(2)$ trends. In summary, we cannot reject the hypothesis that the process $X_t = \{p_t^m, p_t^x, p_t^y, p_t^c, ulc_t, s_t\}$ is $I(2)$. This is consistent with the $I(2)$ analysis of UK inflation in Bowdler & Nielsen (2006).

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[27]The results are obtained using CATS version 2. See Dennis (2006).
[28]The asymptotic critical values are computed for a model without dummies.

Table 1: Univariate unit root tests 1980Q1–2003Q2.

	ADF t-statistic		KPSS LM-statistic	
	constant+trend	constant	constant+trend	constant
s_t	−2.01 [2]	−1.68 [2]	0.25** [7]	0.33 [7]
p_t^m	−1.26 [0]	–	0.28** [7]	–
p_t^x	−1.07 [0]	–	0.29** [7]	–
p_t^y	−1.03 [4]	–	0.30** [7]	–
p_t^c	−1.37 [2]	–	0.31** [7]	–
ulc_t	−1.30 [4]	–	0.25** [7]	–
Δs_t	–	−7.12** [1]	–	0.12 [3]
Δp_t^m	–	−8.57** [0]	–	0.76** [4]
Δp_t^x	–	−5.07** [1]	–	0.91** [5]
Δp_t^y	–	−2.33 [2]	–	1.09** [6]
Δp_t^c	–	−3.33* [1]	–	0.80** [7]
Δulc_t	–	−3.53** [3]	–	0.43 [6]
$p_t^m - p_t^c$	−1.78 [1]	0.51 [0]	0.18* [7]	1.19** [7]
$p_t^x - p_t^c$	−1.59 [2]	0.75 [0]	0.23** [7]	1.10** [7]
$p_t^y - p_t^c$	−1.48 [1]	0.78 [1]	0.19* [7]	1.21** [7]
$ulc_t - p_t^c$	−2.20 [1]	−2.83 [2]	0.09 [7]	0.14 [7]
$\Delta(p_t^m - p_t^c)$	–	−8.24** [0]	–	0.16 [2]
$\Delta(p_t^x - p_t^c)$	–	−5.25** [1]	–	0.22 [4]
$\Delta(p_t^y - p_t^c)$	–	−5.42** [0]	–	0.18 [5]
$\Delta(ulc_t - p_t^c)$	–	−7.66** [0]	–	0.15 [4]

Table 2: Lag-length determination. UK VAR 1981Q2-2003Q2.

Lags	LR	AIC	SC	HQ
5	45.623	−40.288	−34.583	−37.988
4	43.625	−40.267	−35.569	−38.374
3	39.893	−40.361	−36.670	−38.873
2	93.245*	−40.574*	−37.890	−39.492*
1	1061.277	−40.106	−38.428*	−39.430

Note: The lag-order selection criteria are the Akaike (AIC), Schwarz (SC) and the Hannan-Quinn (HQ) information criteria and the LR test-statistic for sequential lag deletions (LR).

Table 3: Misspecification tests VAR(2).

Variable	$F_{AR(1-5)}$	$F_{ARCH(1-5)}$	$\chi^2_{NORMALITY}$	F_{HETERO}
s_t	0.45 [0.81]	0.19 [0.94]	14.36 [0.00]	0.86 [0.65]
p_t^m	0.98 [0.44]	0.62 [0.65]	1.98 [0.37]	1.41 [0.15]
p_t^x	0.68 [0.64]	0.65 [0.63]	1.03 [0.60]	0.92 [0.58]
p_t^y	3.27 [0.01]	1.67 [0.17]	1.56 [0.46]	0.72 [0.82]
p_t^c	0.54 [0.74]	0.64 [0.63]	1.34 [0.51]	0.74 [0.79]
ulc_t	1.94 [0.10]	1.15 [0.34]	3.56 [0.17]	0.60 [0.92]
Vector tests	1.22 [0.08]	–	22.29 [0.03]	0.78 [1.00]

Note: The misspecification tests are the LM tests for residual autocorrelation ($F_{AR(1-5)}$) and autoregressive conditional heteroskedasticity ($F_{ARCH(1-5)}$) up to order 5, a test for normality of the residuals ($\chi^2_{NORMALITY}$), and a test for residual heteroskedasticity (F_{HETERO}). See Hendry & Doornik (2001) for details. The numbers in brackets are the corresponding *p*-values.

Table 4: I(1) cointegration analysis. p-values in brackets.

Rank	Unit roots	Trace test	Trace test (Bartlett corrected)	Eigenvalues of companion matrix					
0	6	166.77 [0.00]	134.09 [0.00]	1.00	1.00	1.00	1.00	1.00	0.81
1	5	101.02 [0.00]	81.29 [0.15]	1.00	1.00	1.00	1.00	0.80	0.58
2	4	56.42 [0.18]	44.48 [0.67]	1.00	1.00	1.00	1.00	0.69	0.68
3	3	28.26 [0.61]	20.08 [0.95]	1.00	1.00	1.00	0.91	0.68	0.59
4	2	13.62 [0.69]	9.97 [0.92]	1.00	1.00	0.91	0.76	0.59	0.46
5	1	3.75 [0.77]	3.12 [0.85]	1.00	0.91	0.90	0.76	0.72	0.48
6	0	—	—	0.99	0.92	0.92	0.77	0.73	0.49

Table 5: I(1) cointegration analysis. Results based on simulating the asymptotic distribution for the trace test statistic. p-values in brackets.

Rank	Unit roots	Trace test	Trace test (Bartlett corrected)
0	6	166.77 [0.00]	134.09 [0.00]
1	5	101.02 [0.00]	81.29 [0.12]
2	4	56.42 [0.14]	44.48 [0.61]
3	3	28.26 [0.58]	20.08 [0.95]
4	2	13.62 [0.67]	9.97 [0.91]
5	1	3.75 [0.76]	3.12 [0.84]

Table 6: LR tests of restrictions on the cointegration parameters when r = 1. Asymptotic p-values in brackets.

Null hypothesis	Test-statistic	Test-statistic (Bartlett corrected)
$s_t \sim I(0)$	$\chi^2(5) = 47.18\ [0.000]$	$\chi^2(5) = 32.48\ [0.000]$
$p_t^m \sim I(0)$	$\chi^2(5) = 46.57\ [0.000]$	$\chi^2(5) = 32.06\ [0.000]$
$p_t^x \sim I(0)$	$\chi^2(5) = 48.68\ [0.000]$	$\chi^2(5) = 33.51\ [0.000]$
$p_t^y \sim I(0)$	$\chi^2(5) = 48.36\ [0.000]$	$\chi^2(5) = 32.29\ [0.000]$
$p_t^c \sim I(0)$	$\chi^2(5) = 50.59\ [0.000]$	$\chi^2(5) = 34.83\ [0.000]$
$ulc_t \sim I(0)$	$\chi^2(5) = 51.27\ [0.000]$	$\chi^2(5) = 35.29\ [0.000]$
$p_t^m - p_t^c \sim I(0)$	$\chi^2(5) = 43.48\ [0.000]$	$\chi^2(5) = 29.93\ [0.000]$
$p_t^x - p_t^c \sim I(0)$	$\chi^2(5) = 47.18\ [0.000]$	$\chi^2(5) = 32.48\ [0.000]$
$p_t^y - p_t^c \sim I(0)$	$\chi^2(5) = 41.03\ [0.000]$	$\chi^2(5) = 28.24\ [0.000]$
$ulc_t - p_t^c \sim I(0)$	$\chi^2(5) = 32.65\ [0.000]$	$\chi^2(5) = 22.48\ [0.000]$
$p_t^m - p_t^x \sim I(0)$	$\chi^2(5) = 51.62\ [0.000]$	$\chi^2(5) = 35.54\ [0.000]$
$p_t^x - p_t^y \sim I(0)$	$\chi^2(5) = 49.35\ [0.000]$	$\chi^2(5) = 33.98\ [0.000]$
$ulc_t - p_t^y \sim I(0)$	$\chi^2(5) = 33.50\ [0.000]$	$\chi^2(5) = 23.06\ [0.000]$
$p_t^c - \gamma ulc_t - (1-\gamma)p_t^m \sim I(0)$	$\chi^2(4) = 32.22\ [0.000]$	$\chi^2(4) = 21.86\ [0.000]$
$p_t^y - \gamma ulc_t - (1-\gamma)p_t^m \sim I(0)$	$\chi^2(4) = 31.67\ [0.000]$	$\chi^2(4) = 21.49\ [0.000]$
$p_t^x - \gamma ulc_t - (1-\gamma)p_t^m \sim I(0)$	$\chi^2(4) = 36.67\ [0.000]$	$\chi^2(4) = 24.87\ [0.000]$

Table 7: Correlation of reduced form VAR residuals.

	Δs_t	Δp_t^m	Δp_t^x	Δp_t^y	Δp_t^c	Δulc_t
Δs_t	1.000					
Δp_t^m	0.625	1.000				
Δp_t^x	0.290	0.438	1.000			
Δp_t^y	0.173	0.385	0.311	1.000		
Δp_t^c	0.116	0.341	0.263	0.463	1.000	
Δulc_t	0.028	0.062	0.072	0.110	0.074	1.000

Table 8: Pairwise Granger-causality tests.

	Exchange rate $\nRightarrow$ price index	Price index $\nRightarrow$ exchange rate
Δp_t^m	5.912 [0.004]	0.945 [0.393]
Δp_t^x	10.011 [0.000]	0.149 [0.862]
Δp_t^y	1.134 [0.327]	3.190 [0.046]
Δp_t^c	0.941 [0.394]	0.691 [0.504]
Δulc_t	0.426 [0.655]	0.569 [0.568]

Table 9: Maximum likelihood inference on cointegration ranks UK data

$p-r$	r			$Q(r,s)$				$Q(r)$
6	0	531.69 [0.000]	420.28 [0.000]	323.11 [0.000]	257.35 [0.000]	208.38 [0.000]	178.66 [0.000]	166.77 [0.000]
5	1		312.944 [0.000]	237.41 [0.000]	185.00 [0.000]	135.09 [0.006]	113.52 [0.009]	101.03 [0.004]
4	2			177.89 [0.000]	120.71 [0.022]	81.14 [0.308]	67.23 [0.228]	56.42 [0.181]
3	3				72.71 [0.455]	49.44 [0.692]	37.01 [0.646]	28.26 [0.611]
2	4					29.46 [0.823]	20.25 [0.730]	13.62 [0.692]
1	5						10.06 [0.651]	3.75 [0.774]
$p-r-s$		6	5	4	3	2	1	0
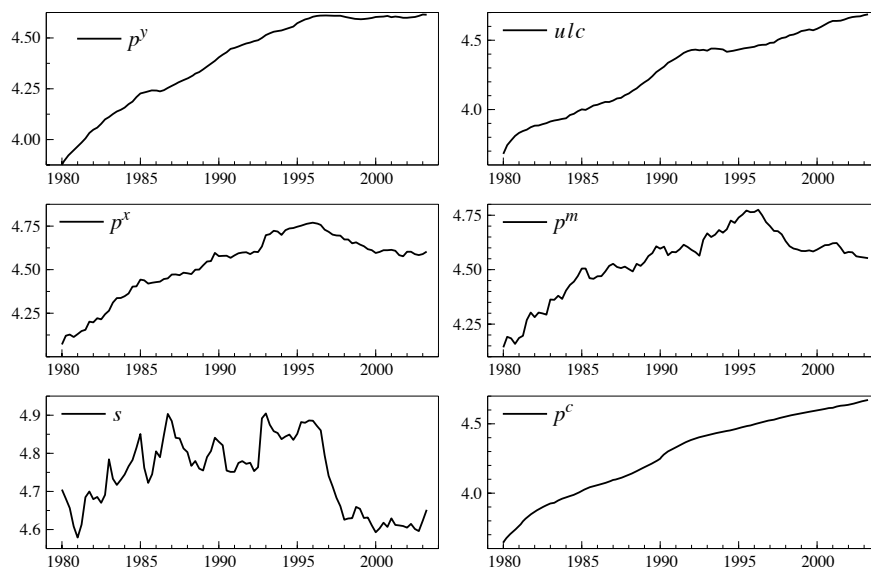
Figure 1: UK data series 1980Q1–2003Q2. Log-levels.
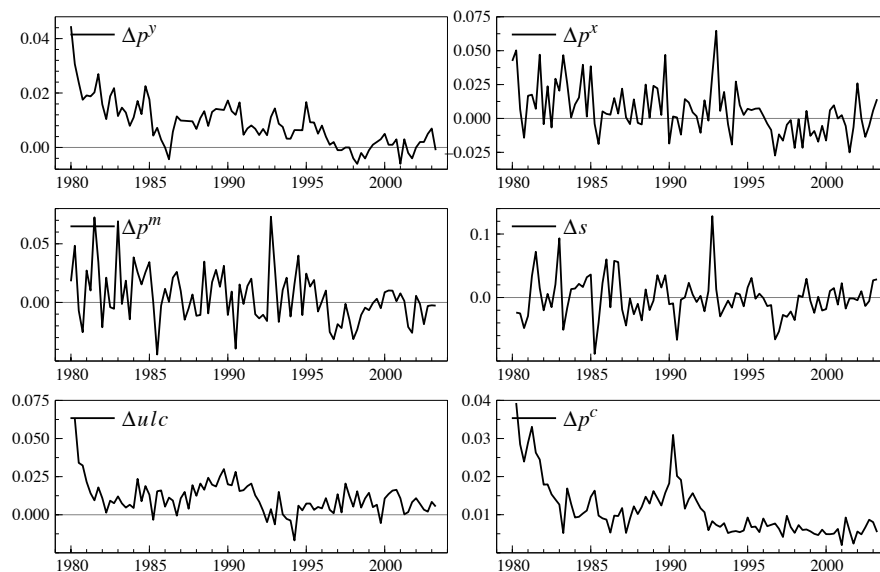
Figure 2: UK data series 1980Q1–2003Q2. First differences.
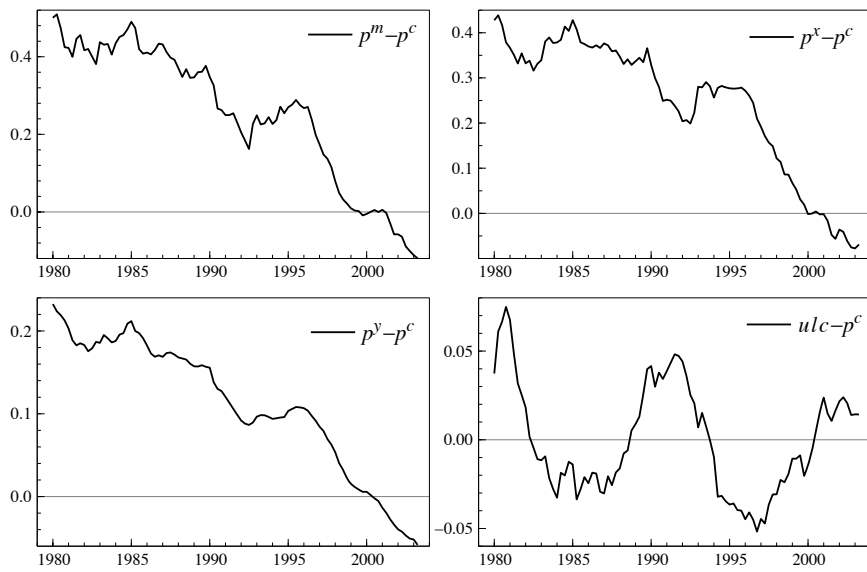
Figure 3: UK data series 1980Q1–2003Q2. Relative prices.
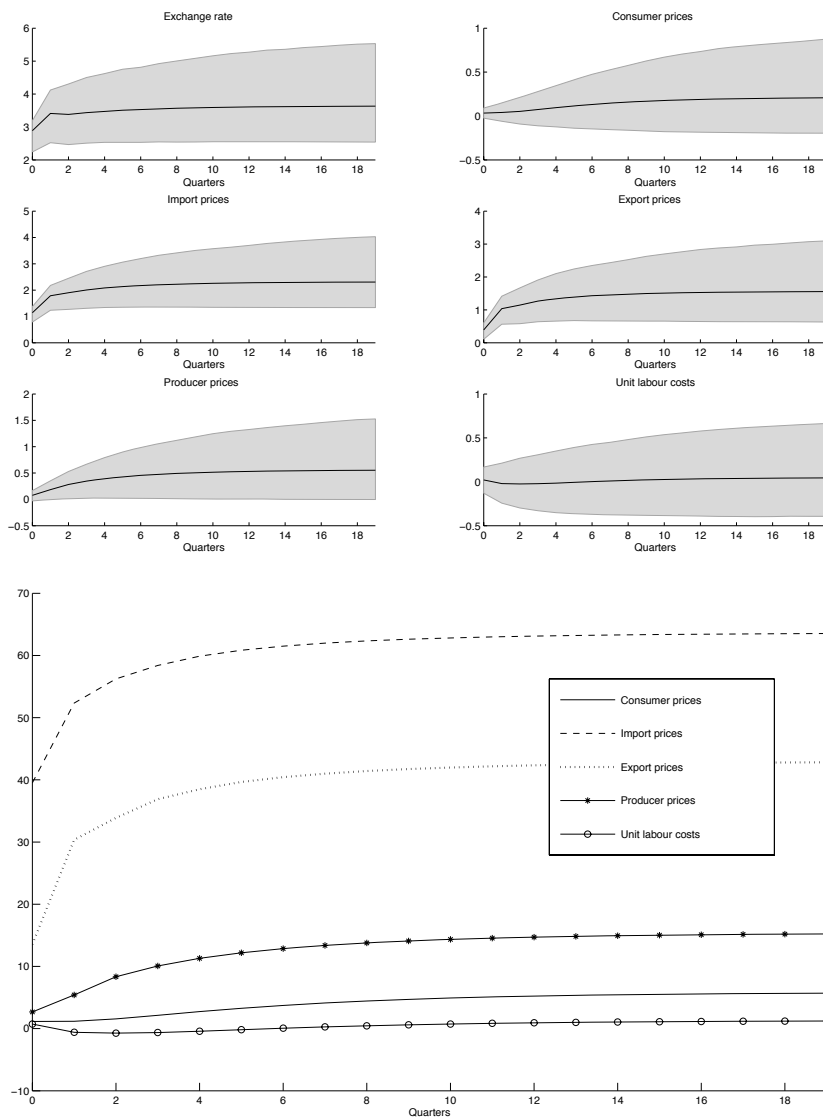
Figure 4: Impulse responses to orthogonalised exchange rate shock. UK data. Model: First-differenced VAR. Exchange rate first in recursive ordering.
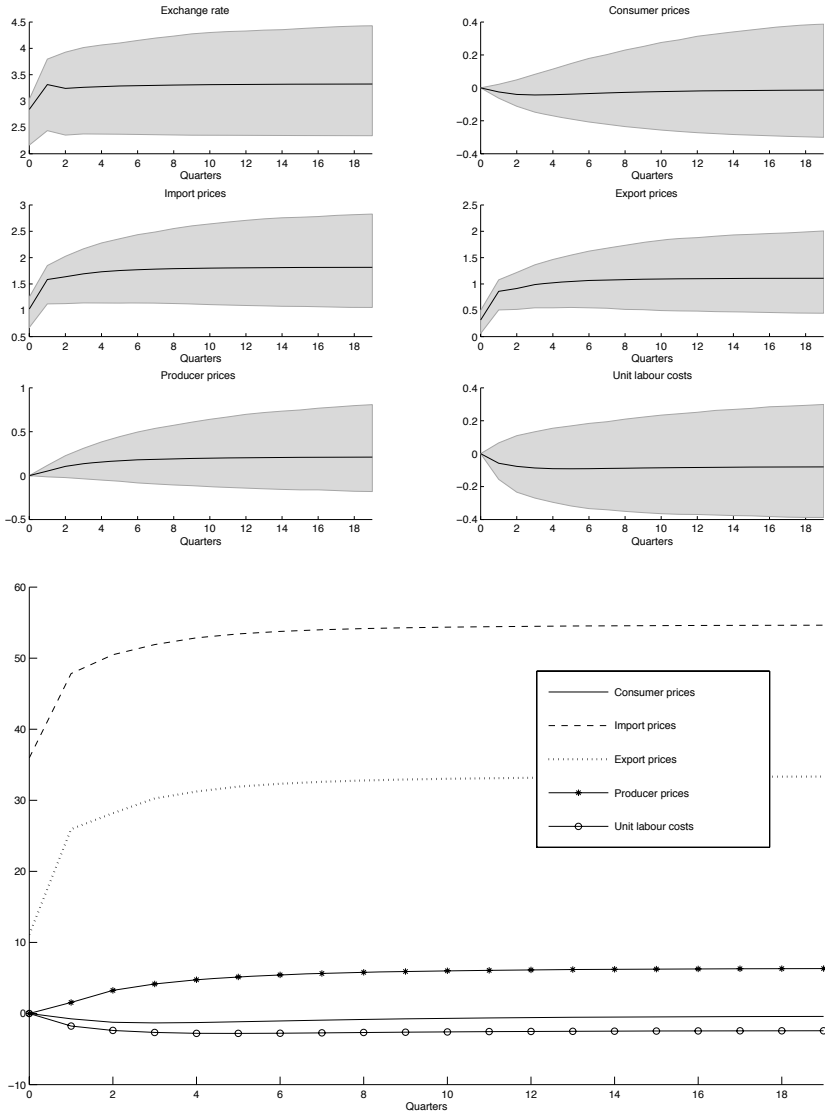
Figure 5: Impulse responses to orthogonalised exchange rate shock. UK data. Model: First-differenced VAR. Recursive ordering: $\Delta p_t^y, \Delta ulc_t, \Delta p_t^c, \Delta s_t, \Delta p_t^m, \Delta p_t^x$
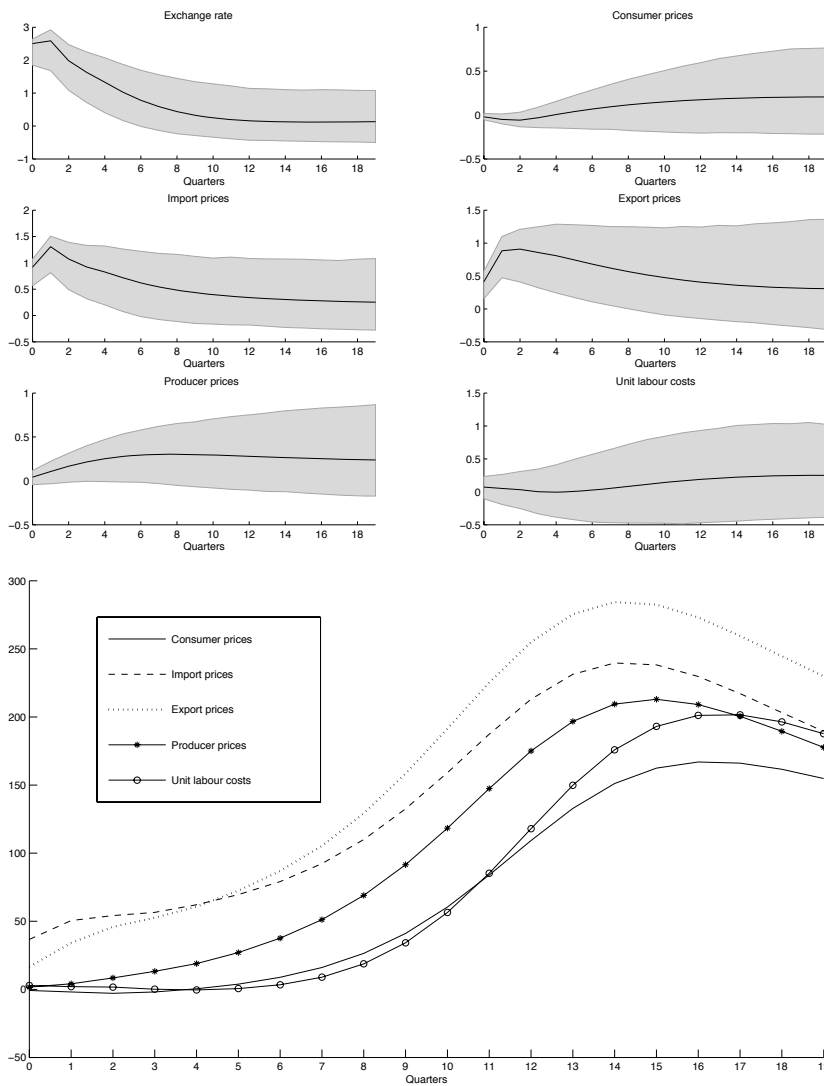
Figure 6: Impulse responses to orthogonalised exchange rate shock. UK data. Model: VAR in levels.
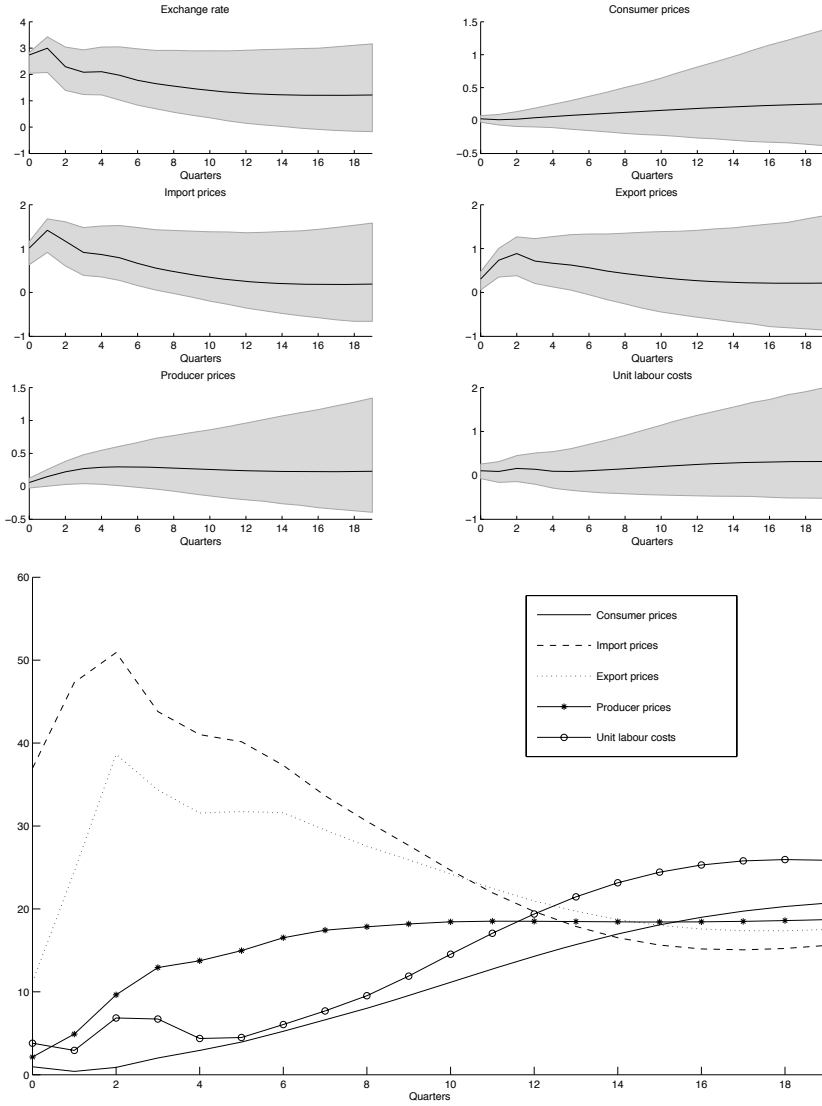
Figure 7: Impulse responses to orthogonalised exchange rate shock. UK data. Model: VEqCM.
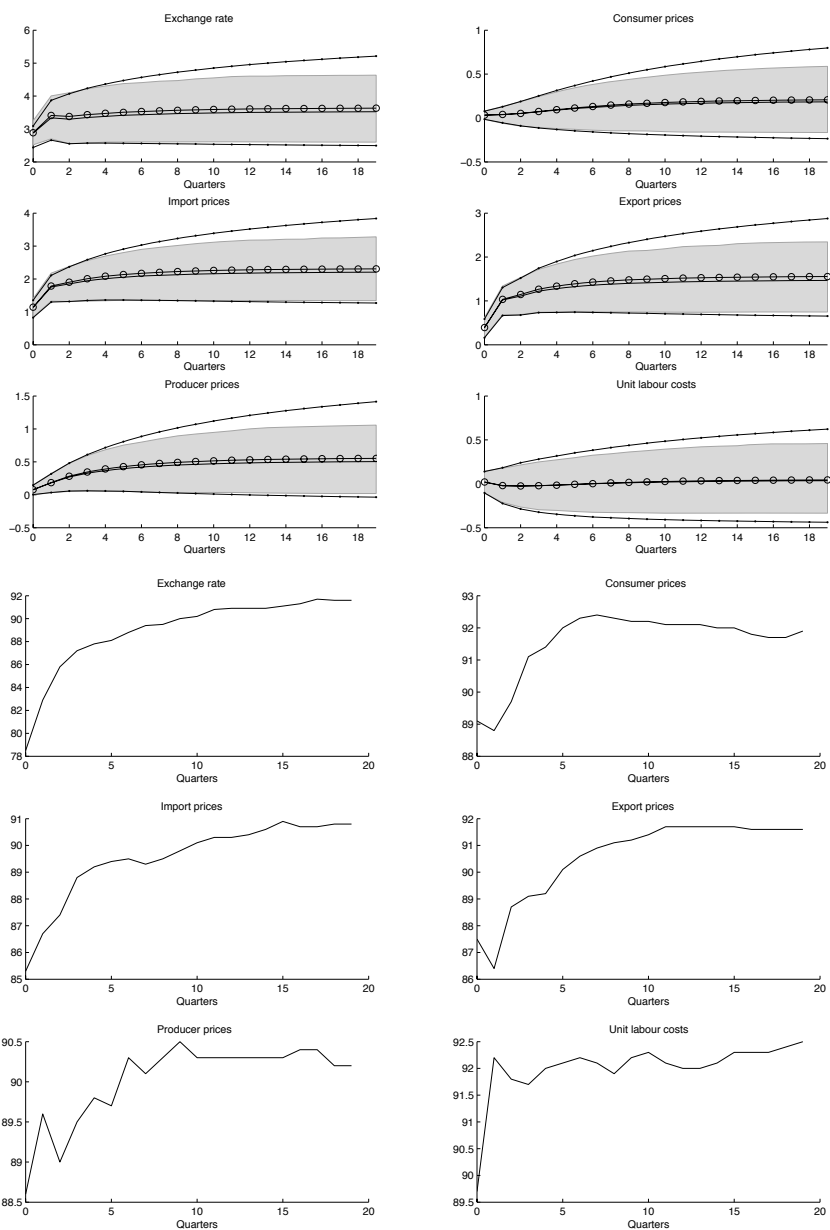
Figure 8: Top panel: Impulse responses to orthogonalised exchange rate shock. DGP: First-differenced VAR. Model: First-differenced VAR. Line with circles: True responses. Solid line: Median simulated responses. Grey area: 90% probability interval of simulated responses. Line with points: Average 90% bias-corrected confidence interval. Lower panel: Coverage rates for bias-corrected bootstrap confidence interval.
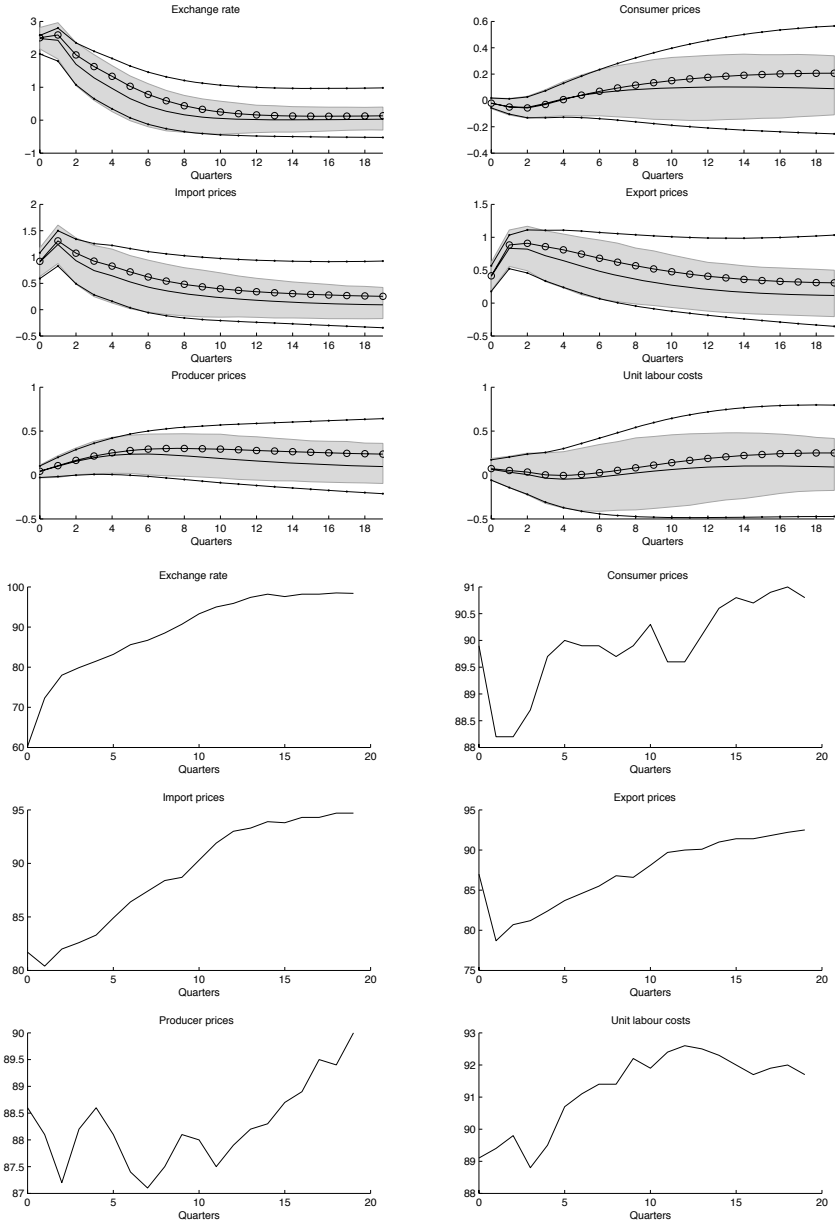
Figure 9: Top panel: Impulse responses to orthogonalised exchange rate shock. DGP: First-differenced VAR. Model: First-differenced VAR. Line with circles: True responses. Solid line: Median simulated responses. Grey area: 90% probability interval of simulated responses. Lines with points: Average 90% bias-corrected confidence interval. Lower panel: Coverage rates for bias-corrected bootstrap confidence interval.
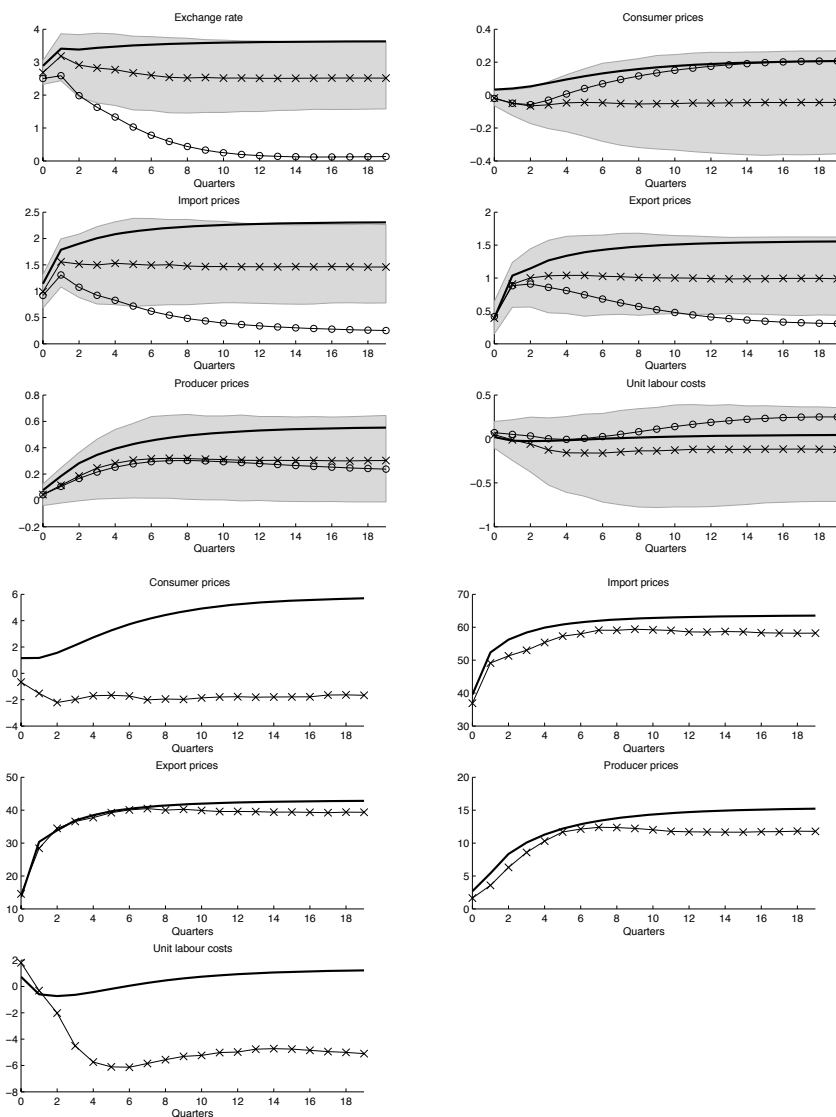
Figure 10: Impulse responses to orthogonalised exchange rate shock. DGP: VAR in levels VAR. Model: First-differenced VAR. Lower panel: Normalised responses. Line with circles: True responses. Line with crosses: Median simulated responses. Grey area: 90% probability interval of simulated responses. Solid line: Estimated responses from first-differenced VAR on UK data.
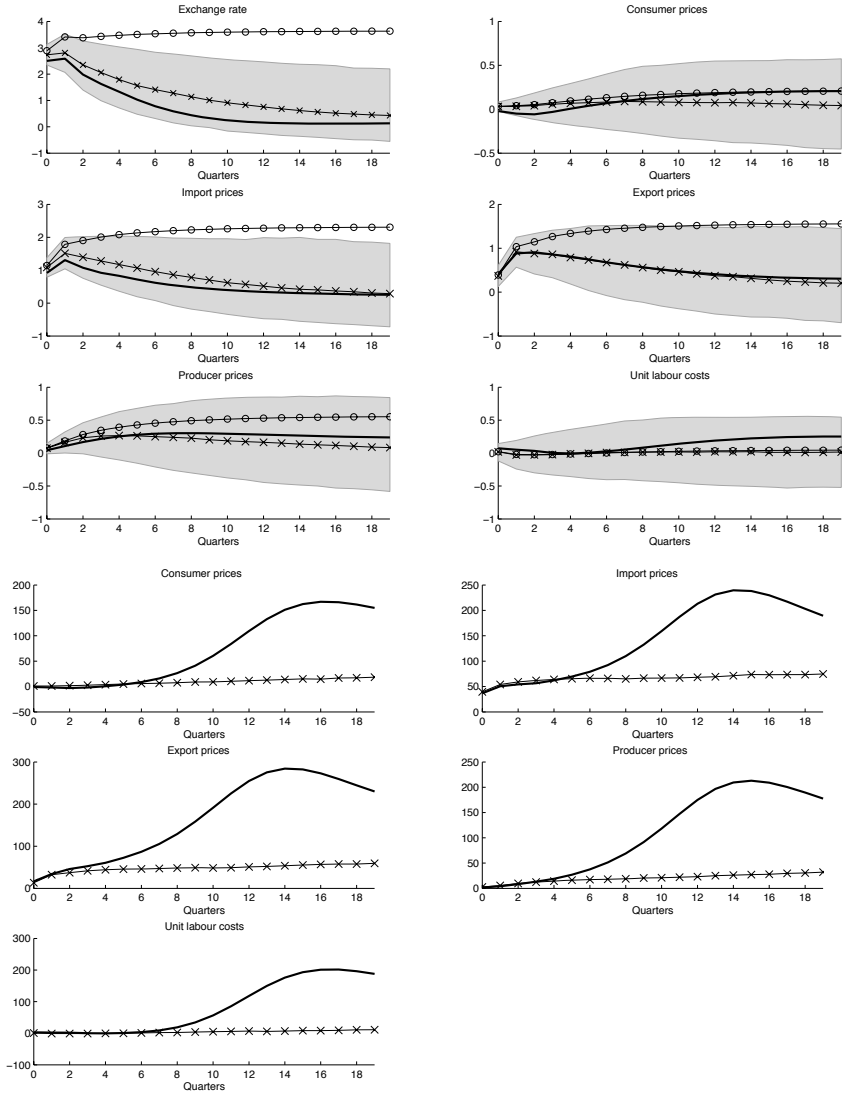
Figure 11: Impulse responses to orthogonalised exchange rate shock. DGP: First-differenced VAR. Model: VAR in levels. Lower panel: Normalised responses. Line with circles: True responses. Line with crosses: Median simulated responses. Grey area: 90% probability interval of simulated responses. Solid line: Estimated responses from first-differenced VAR on UK data.

ASSESSING THE STRUCTURAL VAR APPROACH TO EXCHANGE RATE PASS-THROUGH

1 INTRODUCTION

A common approach to evaluate dynamic stochastic general equilibrium (DSGE) models is to compare impulse response functions from the DSGE model and impulse responses obtained from identified vector autoregressions (VARs). The VAR responses, which rely only on a minimum set of theoretical restrictions, are interpreted as 'stylised facts' that empirically relevant DSGE models should reproduce. Prominent examples are Rotemberg & Woodford (1997) and Christiano et al. (2005) who estimate the parameters of DSGE models by minimising a measure of the distance between the impulse responses to a monetary policy shock generated by an identified VAR and the responses to the monetary policy shock in the DSGE model. Choudhri et al. (2005) and Faruqee (2006) employ the same strategy to estimate 'new open economy macroeconomics' (NOEM) models with incomplete exchange rate pass-through, defining exchange rate pass-through as the impulse responses of a set of prices (import prices, export prices, producer prices, consumer prices) to a shock to the uncovered interest rate parity (UIP) condition.

Recently, several papers have examined the reliability of the structural VAR approach using Monte Carlo simulations. The basic idea in this literature is to generate artificial data from a DSGE model, construct impulse responses from a VAR estimated on the artificial data and ask whether the VAR recovers the DSGE model's responses. A maintained assumption is that the identification scheme used to identify the structural shocks in the VAR is consistent with the theoretical model. Chari et al. (2005), Erceg et al. (2005) and Christiano et al. (2006) assess the ability of a structural VAR to recover the impulse responses to a technology shock in a real business cycle (RBC) model. Their conclusions are not unanimous. Chari et al. (2005) conclude that a very large number of lags is needed for the VAR to well approximate their log-linearised RBC model. Erceg et al. (2005) find that, while the VAR responses have the same sign and shape as the true responses, quantitatively, the bias in the estimated responses could be considerable. Christiano et al. (2006) reach a more optimistic conclusion. They find that the VAR does a good job in recovering the responses from the RBC model, particularly if the technology shock is identified using short-run restrictions. Kapetanios et al. (2005) estimate a five variable VAR on data generated from a small open economy model and derive the impulse responses to shocks to productivity, monetary policy, foreign demand, fiscal policy and the risk premium. Their results suggest that the ability of the VAR to reproduce the theoretical shock responses varies across shocks. In particular, a high lag-order is required for the VAR to recover the responses to a risk premium shock and a domestic fiscal shock.

My paper extends this literature to assess the reliability of the structural VAR approach to estimating exchange rate pass-through. The motivating question is: are im-

pulse responses of prices to a UIP shock a useful tool to evaluate and estimate DSGE models with incomplete exchange rate pass-through? To address this question I generate a large number of artificial datasets from a small open economy DSGE model, estimate a VAR on the artificial data and compare the responses of prices to a UIP shock in the VAR and the DSGE model. The DSGE model that serves as the data generating process incorporates many of the mechanisms for generating imperfect pass-through that have been proposed in the NOEM literature, including local currency price stickiness and distribution costs. In addition, the model incorporates mechanisms such as habit formation in consumption and structural inflation persistence that have been found to improve the empirical fit of monetary DSGE models.

The specification of the DSGE model implies that the nominal exchange rate and nominal prices are non-stationary unit root processes, but that relative prices and the real exchange rate are stationary. Given that exchange rate pass-through is usually defined in terms of the levels of prices and the nominal exchange rate, a conjecture is that the magnitude of the bias in the estimated VAR responses will depend on whether the correct cointegration rank has been imposed during estimation. To test this conjecture I compare the performance of two different VAR specifications: a pure first-differenced VAR and a VAR that includes the cointegration relations implied by the DSGE model. The first-differenced specification is by far the most common in the structural VAR literature on exchange rate pass-through.[1] As a second exercise, I investigate whether an econometrician would be able to infer the true cointegration rank and identify the cointegration relations using the maximum likelihood framework of Johansen (1988). My findings can be summarised as follows. The estimates of exchange rate pass-through obtained from a VAR estimated in first differences are biased downwards. This is true even when the VAR is specified with a large number of lags. The bias is attributable to the fact that the finite-order VAR in first differences is not a good approximation to the infinite order VAR implied by the DSGE model. By contrast, a low order vector equilibrium-correction model (VEqCM) that includes the cointegration relations implied by the DSGE model does a good job recovering the theoretical impulse responses. However, the results from the cointegration analysis raise doubts about whether, in practice, an econometrician would be able to infer the cointegration properties implied by the DSGE model.

The paper is organised as follows. Section 2 lays out the DSGE model that serves as the data generating process in the Monte Carlo exercise. Section 3 discusses the mapping from the DSGE model to a VAR, and the results of the simulation experiments are presented in section 4. Section 5 concludes the paper.

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[1] See e.g., McCarthy (2000), Hahn (2003), Choudhri et al. (2005), and Faruqee (2006).

2 THE MODEL ECONOMY

This section presents the small open economy DSGE model that is used as the data generating process in the simulation experiments.

2.1 Firms

The production structure is the same as that considered by Choudhri et al. (2005). The home economy produces two goods: a non-tradable final consumption good and a tradable intermediate good. Firms in both sectors use domestic labour and a basket of domestic and imported intermediate goods as inputs. The assumption that imports do not enter directly in the consumption basket of households is consistent with the notion that all goods in the consumer price index contain a significant non-traded component. It follows that the direct effect of import prices on consumer prices will be muted, and this acts to limit the degree of exchange rate pass-through to consumer prices. The assumption that imported goods are used as inputs in the production of domestic goods implies a direct link between import prices and the production costs of domestic firms. The latter is potentially an important transmission channel for exchange rate changes in a small open economy (see e.g., McCallum & Nelson, 2000).

2.1.1 Final goods firms

Technology and factor demand There is a continuum of firms indexed by $c \in [0,1]$ that produces differentiated non-tradable final consumption goods. The market for final goods is characterised by monopolistic competition. The consumption good is produced using the following Cobb-Douglas technology

$$C_t(c) = Q_t(c)^{\gamma_c} H_t^c(c)^{1-\gamma_c}, \tag{1}$$

where $C_t(c)$ is the output of final good variety c at time t, $Q_t(c)$ and $H_t^c(c)$ are, respectively, the amounts of intermediate goods and labour used in the production of final good c and $\gamma_c \in [0,1]$. The aggregate labour index H_t is a constant elasticity of substitution (CES) aggregate of differentiated labour inputs indexed by $j \in [0,1]$

$$H_t \equiv \left[\int_0^1 H_t(j)^{\frac{\theta^h-1}{\theta^h}} dj \right]^{\frac{\theta^h}{\theta^h-1}}, \tag{2}$$

where $\theta^h > 1$ is the elasticity of substitution between labour types. Q_t is a composite of imported and domestically produced intermediate goods

$$Q_t \equiv \left[\alpha^{\frac{1}{v}} \left(Q_t^d \right)^{\frac{v-1}{v}} + (1-\alpha)^{\frac{1}{v}} \left(Q_t^m \right)^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}}, \tag{3}$$

where $\alpha \in [0,1]$ and $v > 0$ denotes the elasticity of substitution between domestic and imported goods. Q_t^d and Q_t^m are quantity indices of differentiated domestic and foreign intermediate goods indexed by $i \in [0,1]$ and $m \in [0,1]$, respectively:[2]

$$Q_t^d \equiv \left[\int_0^1 Y_t^{dq}(i)^{\frac{\theta_t^y - 1}{\theta_t^y}} \, di \right]^{\frac{\theta_t^y}{\theta_t^y - 1}} \tag{4}$$

$$Q_t^m \equiv \left[\int_0^1 Y_t^{mq}(m)^{\frac{\theta_t^m - 1}{\theta_t^m}} \, dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}, \tag{5}$$

where $Y_t^{dq}(i)$ and $Y_t^{mq}(m)$ denote the quantities of individual domestic and imported intermediate goods, respectively, used in the production of domestic final goods. The elasticities of substitution between varieties of domestic and imported intermediate goods in the domestic market are $\theta_t^y > 1$ and $\theta_t^m > 1$, respectively. Following e.g., Smets & Wouters (2003) and Adolfson et al. (2005), the substitution elasticities are assumed to be time-varying.

Final goods firms take the prices of intermediate goods and labour inputs as given. Cost minimisation implies that the demands for individual varieties of intermediate goods are

$$Y_t^{dq}(i) = \left(\frac{P_t^y(i)}{P_t^y} \right)^{-\theta_t^y} Q_t^d \tag{6}$$

$$Y_t^{mq}(m) = \left(\frac{P_t^m(m)}{P_t^m} \right)^{-\theta_t^m} Q_t^m. \tag{7}$$

The CES preference specification thus implies that the elasticities of substitution are equal to the elasticities of demand for individual goods. The price indices P_t^y and P_t^m are

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[2]See table 1 for a schematic overview of the price and quantity indices in the DSGE model.

defined as

$$P_t^y \equiv \left[\int_0^1 P_t^y(i)^{1-\theta_t^y} di \right]^{\frac{1}{1-\theta_t^y}} \tag{8}$$

$$P_t^m \equiv \left[\int_0^1 P_t^m(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}. \tag{9}$$

The price index for the composite intermediate good is

$$P_t^q \equiv \left[\alpha \left(P_t^y \right)^{1-\nu} + (1-\alpha) \left(P_t^m \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}. \tag{10}$$

The demand for labour input j is

$$H_t^c(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta^h} H_t^c, \tag{11}$$

where $W_t(j)$ is the nominal wage paid to labour input j, and W_t is the aggregate wage index defined as

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\theta^h} dj \right]^{\frac{1}{1-\theta^h}}. \tag{12}$$

Aggregating over firms and using the fact that all firms are identical, the cost minimising choices of H_t^c and Q_t are characterised by

$$W_t = \xi_t^c (1-\gamma_c) \frac{C_t}{H_t^c} \tag{13}$$

$$P_t^q = \xi_t^c \gamma_c \frac{C_t}{Q_t}, \tag{14}$$

where ξ_t^c denotes the nominal marginal costs of final goods firms. The marginal costs can be expressed as

$$\xi_t^c = \frac{W_t^{1-\gamma_c} (P_t^q)^{\gamma_c}}{(1-\gamma_c)^{1-\gamma_c} \gamma_c^{\gamma_c}}. \tag{15}$$

Finally, the final goods firms' demands for the composite imported and domestic intermediate goods are

$$Q_t^d = \alpha \left(\frac{P_t^y}{P_t^q} \right)^{-\nu} Q_t \tag{16}$$

$$Q_t^m = (1-\alpha) \left(\frac{P_t^m}{P_t^q} \right)^{-\nu} Q_t \tag{17}$$

Price setting The aggregate consumption index is defined as

$$C_t \equiv \left[\int_0^1 C_t(c)^{\frac{\theta_t^c - 1}{\theta_t^c}} \, dc \right]^{\frac{\theta_t^c}{\theta_t^c - 1}}, \tag{18}$$

where $\theta_t^c > 1$ is the time-varying elasticity of substitution between individual consumption goods. The corresponding ideal price index is

$$P_t^c \equiv \left[\int_0^1 P_t^c(c)^{1 - \theta_t^c} \, dc \right]^{\frac{1}{1 - \theta_t^c}}, \tag{19}$$

and the demand for a single variety of the consumption good is

$$C_t(c) = \left(\frac{P_t^c(c)}{P_t^c} \right)^{-\theta_t^c} C_t. \tag{20}$$

Nominal price stickiness is modelled using the quadratic adjustment cost framework of Rotemberg (1982).[3] Following e.g., Price (1992), Ireland (2001) and Laxton & Pesenti (2003), I assume that there are costs associated with changing the inflation rate relative to past observed inflation. Specifically, adjustment costs are given by:

$$\Upsilon_{t+l}^c(c) \equiv \frac{\phi_c}{2} \left(\frac{P_{t+l}^c(c)/P_{t+l-1}^c(c)}{P_{t+l-1}^c/P_{t+l-2}^c} - 1 \right)^2, \tag{21}$$

where $\phi_c >$ is an adjustment cost parameter.

Since all firms in the economy are owned by households, future profits are valued according to the households' stochastic discount factor $D_{t,t+l}$ (to be defined below). Firms set prices to maximise the expected discounted value of future profits subject to adjustment costs, that is, they maximise

$$E_t \left[\sum_{l=0}^\infty D_{t,t+l} \left(P_{t+l}^c(c) - \xi_{t+l}^c \right) \left(\frac{P_{t+l}^c(c)}{P_{t+l}^c} \right)^{-\theta_{t+l}^c} C_{t+l} \left(1 - \Upsilon_{t+l}^c(c) \right) \right] \tag{22}$$

---

[3]The list of NOEM papers which model price stickiness by assuming quadratic costs of price adjustment includes Bergin (2006), Corsetti et al. (2005), Laxton & Pesenti (2003), and Hunt & Rebucci (2005).

subject to (21). In a symmetric equilibrium, $P_t^c(c) = P_t^c$, and the optimal price satisfies

$$
\begin{aligned}
0 \quad = \quad & -(P_t^c - \xi_t^c)\,\phi_c\left(\frac{\pi_t^c}{\pi_{t-1}^c} - 1\right)\frac{\pi_t^c}{\pi_{t-1}^c} \\
& + ((1 - \theta_t^c)P_t^c + \theta_t^c\xi_t^c)\left(1 - \frac{\phi_c}{2}\left(\frac{\pi_t^c}{\pi_{t-1}^c} - 1\right)^2\right) \\
& + E_t\left[\; D_{t,t+1}\left(P_{t+1}^c - \xi_{t+1}^c\right)\frac{C_{t+1}}{C_t}\phi_c\left(\frac{\pi_{t+1}^c}{\pi_t^c} - 1\right)\frac{\pi_{t+1}^c}{\pi_t^c}\;\right],
\end{aligned}
\tag{23}
$$

where π_t^c is the gross inflation rate, $\pi_t^c \equiv P_t^c/P_{t-1}^c$. The price-setting rule is forward-looking and balances the costs of deviating from the optimal (frictionless) price and the costs associated with changing the inflation rate. The log-linearised inflation equation implied by this model (see equation A90 in appendix A.3) can be written as a forward-looking equation in the first difference of inflation. It is observationally equivalent to the inflation equation implied by the Calvo (1983) model when firms index non-optimised prices perfectly to last period's aggregate inflation rate (see e.g., Christiano et al., 2005).

If prices were flexible (i.e., $\phi_c = 0$), firms would set prices according to the familiar mark-up rule:

$$
P_t^c = \frac{\theta_t^c}{\theta_t^c - 1}\xi_t^c.
\tag{24}
$$

2.1.2 Intermediate goods firms

Technology and factor demand   There is a continuum of intermediate goods firms indexed by $i \in [0,1]$ operating in a monopolistically competitive market. The intermediate goods are produced with the following technology

$$
Y_t(i) = Z_t(i)^{\gamma_y} H_t^y(i)^{1-\gamma_y},
\tag{25}
$$

where $Y_t(i)$ denotes the output of intermediate good i, $\gamma_y \in [0,1]$ and $Z_t(i)$ and $H_t^y(i)$ are, respectively, units of the composite intermediate good and the composite labour index used in the production of variety i of the domestic intermediate good. The composite intermediate good is defined as

$$
Z_t \equiv \left[\alpha^{\frac{1}{v}}\left(Z_t^d\right)^{\frac{v-1}{v}} + (1-\alpha)^{\frac{1}{v}}\left(Z_t^m\right)^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}},
\tag{26}
$$

where Z_t^d and Z_t^m are quantity indices of differentiated domestic and foreign intermediate goods, that is,

$$Z_t^d \equiv \left[\int_0^1 Y_t^{dz}(i)^{\frac{\theta_t^y - 1}{\theta_t^y}} \, di \right]^{\frac{\theta_t^y}{\theta_t^y - 1}} \tag{27}$$

$$Z_t^m \equiv \left[\int_0^1 Y_t^{mz}(m)^{\frac{\theta_t^m - 1}{\theta_t^m}} \, dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}, \tag{28}$$

where $Y_t^{dz}(i)$ and $Y_t^{mz}(m)$ denote the quantities of individual domestic and imported intermediate goods, respectively, used in the production of domestic intermediate goods. The price index for the composite intermediate good, P_t^z, is defined as

$$P_t^z \equiv \left[\alpha \left(P_t^y \right)^{1-\nu} + (1 - \alpha) \left(P_t^m \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}. \tag{29}$$

Firms take the prices of the composite intermediate good and labour inputs as given. Cost minimisation with respect to H_t^y and Z_t implies (again using the fact that all intermediate goods firms are identical)

$$W_t = \xi_t^y (1 - \gamma_y) \frac{Y_t}{H_t^y} \tag{30}$$

$$P_t^z = \xi_t^y \gamma_y \frac{Y_t}{Z_t}, \tag{31}$$

where ξ_t^y denotes nominal marginal costs

$$\xi_t^y = \frac{W_t^{1-\gamma_y} (P_t^z)^{\gamma_y}}{(1 - \gamma_y)^{1-\gamma_y} \gamma_y^{\gamma_y}}. \tag{32}$$

Demands for domestic and imported intermediate goods from domestic intermediate goods firms are

$$Z_t^d = \alpha \left(\frac{P_t^y}{P_t^z} \right)^{-\nu} Z_t \tag{33}$$

$$Z_t^m = (1 - \alpha) \left(\frac{P_t^m}{P_t^z} \right)^{-\nu} Z_t \tag{34}$$

Price setting   As pointed out by Obstfeld & Rogoff (2000), there are more possibilities for modelling nominal rigidities in an open-economy setting than in a closed-economy setting. One issue is whether international goods markets should be characterised as being integrated or segmented. Another issue is that, with nominal price stickiness, the

choice of price-setting currency will matter. In the following it is assumed that international goods markets are segmented, due to for example, transportation costs or formal or informal trade barriers. Intermediate goods firms thus have the option to set different prices in the domestic and foreign markets.

Domestic market   The demand facing firm i in the domestic market is

$$Y_t^d(i) = \left(\frac{P_t^y(i)}{P_t^y} \right)^{-\theta_t^y} Y_t^d, \tag{35}$$

where $Y_t^d = Q_t^d + Z_t^d$ is the total demand for domestic intermediate goods from domestic firms. Firm i's price setting problem in the domestic market is

$$\max_{P_t^y(i)} E_t \left[\sum_{l=0}^{\infty} D_{t,t+l} \left(P_{t+l}^y(i) - \xi_{t+l}^y \right) \left(\frac{P_{t+l}^y(i)}{P_{t+l}^y} \right)^{-\theta_{t+l}^y} Y_{t+l}^d \left(1 - \Upsilon_{t+l}^y(i) \right) \right], \tag{36}$$

where the form of adjustment costs $\Upsilon_{t+l}^y(i)$ is

$$\Upsilon_{t+l}^y(i) \equiv \frac{\phi_y}{2} \left(\frac{P_{t+l}^y(i)/P_{t+l-1}^y(i)}{P_{t+l-1}^y/P_{t+l-2}^y} - 1 \right)^2. \tag{37}$$

In equilibrium, $P_t^y(i) = P_t^y$, and the optimal price satisfies

$$
\begin{aligned}
0 = & - \left(P_t^y - \xi_t^y \right) \phi_y \left(\frac{\pi_t^y}{\pi_{t-1}^y} - 1 \right) \frac{\pi_t^y}{\pi_{t-1}^y} \\
& + \left((1 - \theta_t^y) P_t^y + \theta_t^y \xi_t^y \right) \left(1 - \frac{\phi_y}{2} \left(\frac{\pi_t^y}{\pi_{t-1}^y} - 1 \right)^2 \right) \\
& + E_t \left[D_{t,t+1} \left(P_{t+1}^y - \xi_{t+1}^y \right) \frac{Y_{t+1}^d}{Y_t^d} \phi_y \left(\frac{\pi_{t+1}^y}{\pi_t^y} - 1 \right) \frac{\pi_{t+1}^y}{\pi_t^y} \right],
\end{aligned}
\tag{38}
$$

where $\pi_t^y \equiv P_t^y/P_{t-1}^y$ is the gross inflation rate.

Foreign market   In the Obstfeld & Rogoff (1995) Redux model, international goods markets are integrated, and the law of one price holds continuously. Moreover, because prices are set in the currency of the producer (so-called producer currency pricing, PCP), exchange rate pass-through to import prices is immediate and complete. Betts & Devereux (1996) extended the Redux model to allow for market segmentation and to allow a share of prices to be sticky in the currency of the buyer (so-called local currency pricing, LCP). Local currency pricing implies that import prices will respond only gradually to

exchange rate changes, a feature consistent with the findings of a large empirical lite-rature on exchange rate pass-through.[4] In this paper, following Choudhri et al. (2005) and Bergin (2006), I assume that a proportion ϖ of domestic intermediate goods firms engages in PCP, and a proportion $1 - \varpi$ engages in LCP. Both PCP and LCP firms have the option to price discriminate between foreign and domestic markets.[5]

Corsetti & Dedola (2005) extended the basic NOEM framework to allow for distri-bution costs. In their model, the distribution of traded goods requires the input of local, non-traded goods and services. Here, following Choudhri et al. (2005), I assume that the distribution of one unit of the domestic traded good to foreign firms requires the in-put of δ_f units of foreign labour. The distribution sector is perfectly competitive. Let $\overline{P}_t^{xp}(i)$ and $\overline{P}_t^{xl}(i)$ be the ('wholesale') prices set by a representative PCP firm and LCP firm, respectively.[6] The Leontief production technology and the zero profit condition in the distribution sector imply that the ('retail') prices paid by foreign firms for a type i domestic good, $P_t^{xp}(i)$ and $P_t^{xl}(i)$, satisfy

$$\frac{P_t^{xp}(i)}{S_t} = \frac{\overline{P}_t^{xp}(i)}{S_t} + \delta_f W_t^f \tag{39}$$

$$P_t^{xl}(i) = \overline{P}_t^{xl}(i) + \delta_f W_t^f, \tag{40}$$

where S_t is the nominal exchange rate and W_t^f is the foreign wage level. The existence of a distribution sector thus implies that there will be a wedge between the wholesale and the retail price of imports in the foreign economy.

The aggregate export price index (in domestic currency) is

$$P_t^x \equiv \left[\varpi \left(P_t^{xp} \right)^{1-\theta_t^x} + (1-\varpi)(S_t P_t^{xl})^{1-\theta_t^x} \right]^{\frac{1}{1-\theta_t^x}}, \tag{41}$$

where P_t^{xp} and P_t^{xl} are the export price indices obtained by aggregating over PCP firms and LCP firms, respectively, that is

$$P_t^{xp} \equiv \left[\frac{1}{\varpi} \int_0^\varpi P_t^{xp}(i)^{1-\theta_t^x} di \right]^{\frac{1}{1-\theta_t^x}} \tag{42}$$

$$P_t^{xl} \equiv \left[\frac{1}{1-\varpi} \int_\varpi^1 P_t^{xl}(i)^{1-\theta_t^x} di \right]^{\frac{1}{1-\theta_t^x}}, \tag{43}$$

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[4]See Campa & Goldberg (2005) for a recent study.

[5]In this paper ϖ is treated as an exogenous parameter. Several recent papers have examined the optimal choice of invoicing currency in the context of NOEM models (e.g., Devereux et al., 2004; Bacchetta & van Wincoop, 2005; Goldberg & Tille, 2005). The choice is found to depend on several factors, including the exporting firm's market share in the foreign market, the degree of substitutability between foreign and domestic goods and relative monetary stability.

[6]The wholesale export prices correspond to the export prices 'at the docks'.

and $\theta_t^x > 1$ denotes the elasticity of substitution between domestic intermediate goods in the foreign economy. The corresponding quantity indices are

$$Y_t^x \equiv \left[(\varpi)^{\frac{1}{\theta_t^x}} \left(Y_t^{xp} \right)^{\frac{\theta_t^x - 1}{\theta_t^x}} + (1 - \varpi)^{\frac{1}{\theta_t^x}} \left(Y_t^{xl} \right)^{\frac{\theta_t^x - 1}{\theta_t^x}} \right]^{\frac{\theta_t^x}{\theta_t^x - 1}} \tag{44}$$

and

$$Y_t^{xp} \equiv \left[\left(\frac{1}{\varpi} \right)^{\frac{1}{\theta_t^x}} \int_0^{\varpi} Y_t^{xp}(x)^{\frac{\theta_t^x - 1}{\theta_t^x}} di \right]^{\frac{\theta_t^x}{\theta_t^x - 1}} \tag{45}$$

$$Y_t^{xl} \equiv \left[\left(\frac{1}{1 - \varpi} \right)^{\frac{1}{\theta_t^x}} \int_{\varpi}^1 Y_t^{xl}(x)^{\frac{\theta_t^x - 1}{\theta_t^x}} di \right]^{\frac{\theta_t^x}{\theta_t^x - 1}} \tag{46}$$

A representative PCP firm sets $\overline{P}_t^{xp}(i)$ to maximise

$$E_t \left[\sum_{l=0}^{\infty} D_{t,t+l} \left(\overline{P}_{t+l}^{xp}(i) - \xi_{t+l}^y \right) Y_{t+l}^{xp}(i) \left(1 - \Upsilon_{t+l}^{xp}(i) \right) \right] \tag{47}$$

subject to demand[7]

$$Y_{t+l}^{xp}(i) = \left(\frac{\overline{P}_{t+l}^{xp}(i)/S_{t+l} + \delta_f W_{t+l}^f}{P_{t+l}^x / S_{t+l}} \right)^{-\theta_{t+l}^x} Y_{t+l}^x, \tag{48}$$

and adjustment costs

$$\Upsilon_{t+l}^{xp}(i) \equiv \frac{\phi_x}{2} \left(\frac{\overline{P}_{t+l}^{xp}(i)/\overline{P}_{t+l-1}^{xp}(i)}{\overline{P}_{t+l-1}^{xp}/\overline{P}_{t+l-2}^{xp}} - 1 \right)^2. \tag{49}$$

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[7]This can be derived from

$$Y_{t+l}^{xp}(i) = \frac{1}{\varpi} \left(\frac{P_{t+l}^{xp}(i)}{P_{t+l}^{xp}} \right)^{-\theta_{t+l}^x} Y_{t+l}^{xp} = \left(\frac{P_{t+l}^{xp}(i)}{P_{t+l}^x} \right)^{-\theta_{t+l}^x} Y_{t+l}^x$$

In equilibrium, $\overline{P}_t^{xp}(i) = \overline{P}_t^{xp}$, and the optimal price satisfies

$$
\begin{aligned}
0 \quad = \quad & \left(\overline{P}_t^{xp} - \theta_t^x \left(\overline{P}_t^{xp} - \xi_{it}^y \right) \frac{\overline{P}_t^{xp}}{P_t^{xp}} \right) \left(1 - \frac{\phi_x}{2} \left(\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} - 1 \right)^2 \right) \\
& - \left(\overline{P}_t^{xp} - \xi_{it}^y \right) \phi_x \left(\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} - 1 \right) \frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} \\
& + E_t \left[D_{t,t+1} \left(P_{t+1}^{xp} - \xi_{t+1}^y \right) \frac{Y_{t+1}^{xp}}{Y_t^{xp}} \phi_x \left(\frac{\overline{\pi}_{t+1}^{xp}}{\overline{\pi}_t^{xp}} - 1 \right) \frac{\overline{\pi}_{t+1}^{xp}}{\overline{\pi}_t^{xp}}, \right]
\end{aligned}
\tag{50}
$$

where $\overline{\pi}_t^{xp} \equiv \overline{P}_t^{xp} / \overline{P}_{t-1}^{xp}$.

The wedge between prices at the wholesale and retail levels implies that the price elasticity of demand as perceived by the exporter will be a function of the exchange rate. To see this, note that in the absence of costs of price adjustment (i.e., if $\phi_x = 0$), the optimal export price is

$$
\overline{P}_t^{xp} = \frac{\theta_t^x}{\theta_t^x - 1} \xi_{it}^y + \frac{\delta_f}{\theta_t^x - 1} S_t W_t^f .
\tag{51}
$$

In the absence of distribution costs ($\delta_f = 0$), the export price in domestic currency is independent of the exchange rate, and the price-setting rule collapses to the standard mark-up rule. Moreover, if the elasticities of demand are the same across countries (i.e., $\theta_t^x = \theta_t^y$), the firm sets identical prices to the home and foreign markets. The existence of distribution costs thus creates a motive for price discrimination between markets. Moreover, distribution costs cause the optimal mark-up to vary positively with the level of the exchange rate. This can be seen more clearly by rewriting (51) as

$$
\overline{P}_t^{xp} = \frac{\theta_t^x}{\theta_t^x - 1} \xi_{it}^y \left(1 + \frac{\delta_f}{\theta_t^x} \frac{S_t W_t^f}{\xi_{it}^y} \right) .
\tag{52}
$$

In the face of an exchange rate depreciation, the exporter will find it optimal to absorb part of the exchange rate movement in her mark-up. From the point of view of the importing country, exchange rate pass-through to import prices at the docks is incomplete, even in the absence of nominal rigidities.

A representative LCP firm sets $\overline{P}_t^{xl}(i)$ to maximise

$$
\max_{\overline{P}_t^{xl}(i)} E_t \left[\sum_{l=0}^{\infty} D_{t,t+l} \left(S_{t+l} \overline{P}_{t+l}^{xl}(i) - \xi_{t+l}^y \right) Y_{t+l}^{xl}(i) \left(1 - \Upsilon_{t+l}^{xl}(i) \right) \right]
\tag{53}
$$

subject to demand,[8]

$$Y_{t+l}^{xl}(i) = \left(\frac{\overline{P}_{t+l}^{xl}(i) + \delta_f W_{t+l}^f}{P_{t+l}^x/S_{t+l}} \right)^{-\theta_{t+l}^x} Y_{t+l}^x, \tag{54}$$

and adjustment costs

$$\Upsilon_{t+l}^{xl}(i) \equiv \frac{\phi_x}{2} \left(\frac{\overline{P}_{t+l}^{xl}(i)/\overline{P}_{t+l-1}^{xl}(i)}{\overline{P}_{t+l-1}^{xl}/\overline{P}_{t+l-2}^{xl}} - 1 \right)^2. \tag{55}$$

The degree of price stickiness as measured by the parameter ϕ_x is thus assumed to be the same for PCP and LCP firms.[9] In equilibrium $\overline{P}_t^{xl}(i) = \overline{P}_t^{xl}$, and the first-order condition can be written

$$\begin{aligned}
0 =& \left(S_t \overline{P}_t^{xl} - \theta_t^x \left(S_t \overline{P}_t^{xl} - \xi_t^y \right) \frac{\overline{P}_t^{xl}}{P_t^{xl}} \right) \left(1 - \frac{\phi_x}{2} \left(\frac{\overline{\pi}_t^{xl}}{\overline{\pi}_{t-1}^{xl}} - 1 \right)^2 \right) \\
& - \left(S_t \overline{P}_t^{xl} - \xi_t^y \right) \phi_x \left(\frac{\overline{\pi}_t^{xl}}{\overline{\pi}_{t-1}^{xl}} - 1 \right) \frac{\overline{\pi}_t^{xl}}{\overline{\pi}_{t-1}^{xl}} \\
& + E_t \left[D_{t,t+1} \left(S_{t+1} P_{t+1}^{xl} - \xi_{t+1}^y \right) \frac{Y_{t+1}^{xl}}{Y_t^{xl}} \phi_x \left(\frac{\overline{\pi}_{t+1}^{xl}}{\overline{\pi}_t^{xl}} - 1 \right) \frac{\overline{\pi}_{t+1}^{xl}}{\overline{\pi}_t^{xl}} \right],
\end{aligned} \tag{56}$$

where $\overline{\pi}_t^{xl} \equiv \overline{P}_t^{xl}/\overline{P}_{t-1}^{xl}$. In the absence of adjustment costs (i.e., if $\phi_x = 0$), the optimal price is

$$\overline{P}_t^{xl} = \frac{\theta_t^x}{\theta_t^x - 1} \frac{\xi_t^y}{S_t} + \frac{\delta_f}{\theta_t^x - 1} W_t^f. \tag{57}$$

Thus, when prices are flexible, LCP and PCP firms set the same price. The choice of price-setting currency only matters in a situation where nominal prices are sticky.

Finally, aggregate export demand is assumed to be given by

$$Y_t^x = \alpha_f \left(\frac{P_t^x/S_t}{P_t^f} \right)^{-\nu_f} Y_t^f, \tag{58}$$

where α_f is (approximately) the share of home goods and ν_f the elasticity of substitution

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[8]This follows from

$$Y_{t+l}^{xl}(i) = \frac{1}{1-\varpi} \left(\frac{P_{t+l}^{xl}(i)}{P_{t+l}^{xl}} \right)^{-\theta_{t+l}^x} Y_{t+l}^{xl} = \left(\frac{P_{t+l}^{xl}(i)}{P_{t+l}^x/S_{t+l}} \right)^{-\theta_{t+l}^x} Y_{t+l}^x$$

[9]This assumption has some empirical support. Using micro data for traded goods prices at the docks for the US, Gopinath & Rigobon (2006) find that the stickiness of prices invoiced in foreign currencies in terms of foreign currency is similar to the stickiness of prices invoiced in dollars in terms of dollars.

between home and foreign goods in the composite index of intermediate goods in the foreign economy, P_t^f is the foreign price level, and Y_t^f denotes aggregate demand for domestic intermediate goods in the foreign economy. Foreign output, prices and wages are assumed to exogenous to the small open economy.

2.1.3 Foreign firms

Foreign intermediate goods firms are treated symmetrically with domestic intermediate goods firms. A subset ϖ_f of firms engages in PCP, and a subset $1 - \varpi_f$ engages in LCP. The aggregate import quantity index is

$$Y_t^m \equiv \left[(\varpi_f)^{\frac{1}{\theta_t^m}} \left(Y_t^{mp} \right)^{\frac{\theta_t^m - 1}{\theta_t^m}} + (1 - \varpi_f)^{\frac{1}{\theta_t^m}} \left(Y_t^{ml} \right)^{\frac{\theta_t^m - 1}{\theta_t^m}} \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}, \tag{59}$$

where Y_t^{mp} and Y_t^{ml} are, respectively, the production indices of PCP firms and LCP firms, defined as

$$Y_t^{mp} \equiv \left[\left(\frac{1}{\varpi_f} \right)^{\frac{1}{\theta_t^m}} \int_0^{\varpi_f} Y_t^{mp}(m)^{\frac{\theta_t^m - 1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}} \tag{60}$$

$$Y_t^{ml} \equiv \left[\left(\frac{1}{1 - \varpi_f} \right)^{\frac{1}{\theta_t^m}} \int_{\varpi_f}^1 Y_t^{ml}(m)^{\frac{\theta_t^m - 1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}. \tag{61}$$

The distribution of one unit of the imported good to domestic firms requires the input of δ units of domestic labour. The zero profit condition in the distribution sector implies that the prices paid by domestic firms for a type m imported good, $P_t^{mp}(m)$ and $P_t^{ml}(m)$, will be

$$S_t P_t^{mp}(m) = S_t \overline{P}_t^{mp}(m) + \delta W_t \tag{62}$$

$$P_t^{ml}(m) = \overline{P}_t^{ml}(m) + \delta W_t. \tag{63}$$

The aggregate import price index (in the importing country's currency) is

$$P_t^m \equiv \left[\varpi_f \left(S_t P_t^{mp} \right)^{1 - \theta_t^m} + (1 - \varpi_f)(P_t^{ml})^{1 - \theta_t^m} \right]^{\frac{1}{1 - \theta_t^m}}, \tag{64}$$

where P_t^{mp} and P_t^{ml} are the price indices obtained by aggregating over PCP firms and LCP firms, respectively, that is

$$P_t^{mp} \equiv \left[\frac{1}{\varpi_f} \int_0^{\varpi_f} P_t^{mp}(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}} \tag{65}$$

$$P_t^{ml} \equiv \left[\frac{1}{1-\varpi_f} \int_{\varpi_f}^1 P_t^{ml}(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}. \tag{66}$$

Let $D_{t,t+l}^f$ denote the stochastic discount factor of foreign households and let ξ_t^f denote the marginal costs of foreign intermediate goods firms. A representative foreign LCP firm sets $\overline{P}_t^{ml}(m)$ to maximise

$$E_t \left[\sum_{l=0}^{\infty} D_{t,t+l}^f \left(\frac{\overline{P}_{t+l}^{ml}(m)}{S_{t+l}} - \xi_{t+l}^f \right) Y_{t+l}^{ml}(m) \left(1 - \Upsilon_{t+l}^{ml}(m) \right) \right] \tag{67}$$

subject to demand[10]

$$Y_{t+l}^{ml}(m) = \left(\frac{\overline{P}_{t+l}^{ml}(m) + \delta W_{t+l}}{P_{t+l}^m} \right)^{-\theta_{t+l}^m} Y_{t+l}^m, \tag{68}$$

where $Y_t^m = Q_t^m + Z_t^m$ is the aggregate demand for imported intermediate goods from domestic firms. The specification of adjustment costs is

$$\Upsilon_{t+l}^{ml}(m) \equiv \frac{\phi_m}{2} \left(\frac{\overline{P}_{t+l}^{ml}(m)/\overline{P}_{t+l-1}^{ml}(m)}{\overline{P}_{t+l-1}^{ml}/\overline{P}_{t+l-2}^{ml}} - 1 \right)^2. \tag{69}$$

In equilibrium, $\overline{P}_t^{ml} = \overline{P}_t^{ml}(m)$, and the optimal price satisfies

$$
\begin{aligned}
0 = & \left(\frac{\overline{P}_t^{ml}}{S_t} - \theta_t^m \left(\frac{\overline{P}_t^{ml}}{S_t} - \xi_t^f \right) \frac{\overline{P}_t^{ml}}{P_t^{ml}} \right) \left(1 - \frac{\phi_m}{2} \left(\frac{\overline{\pi}_t^{ml}}{\overline{\pi}_{t-1}^{ml}} - 1 \right)^2 \right) \\
& - \left(\frac{\overline{P}_t^{ml}}{S_t} - \xi_t^f \right) \phi_m \left(\frac{\overline{\pi}_t^{ml}}{\overline{\pi}_{t-1}^{ml}} - 1 \right) \frac{\overline{\pi}_t^{ml}}{\overline{\pi}_{t-1}^{ml}} \\
& + E_t \left[D_{t,t+1}^f \left(\frac{\overline{P}_{t+1}^{ml}}{S_{t+1}} - \xi_{t+1}^f \right) \frac{Y_{t+1}^{ml}}{Y_t^{ml}} \phi_m \left(\frac{\overline{\pi}_{t+1}^{ml}}{\overline{\pi}_t^{ml}} - 1 \right) \frac{\overline{\pi}_{t+1}^{ml}}{\overline{\pi}_t^{ml}} \right],
\end{aligned}
\tag{70}
$$

---

[10]This follows from

$$Y_{t+l}^{ml}(m) = \frac{1}{1-\varpi_f} \left(\frac{P_{t+l}^{ml}(m)}{P_{t+l}^{ml}} \right)^{-\theta_{t+l}^m} Y_{t+l}^{ml} = \left(\frac{P_{t+l}^{ml}(m)}{P_{t+l}^m} \right)^{-\theta_{t+l}^m} Y_{t+l}^m$$

where $\bar{\pi}_t^{ml} \equiv \overline{P}_t^{ml}/\overline{P}_{t-1}^{ml}$. If $\phi_m = 0$, the first-order condition simplifies to

$$\overline{P}_t^{ml} = \frac{\theta_t^m}{\theta_t^m - 1} S_t \xi_t^f + \frac{\delta}{\theta_t^m - 1} W_t \tag{71}$$

The foreign firm's optimal mark-up is a function of the exchange rate. Conditional on domestic wages and the marginal costs of foreign exporters, the exchange rate pass-through to domestic currency import prices at the wholesale level is incomplete, even if prices are perfectly flexible.

Finally, a representative foreign PCP firm sets $\overline{P}_t^{mp}(m)$ to maximise

$$E_t \left[\sum_{l=0}^{\infty} D_{t,t+l}^f \left(\overline{P}_{t+l}^{mp}(m) - \xi_{t+l}^f \right) Y_{t+l}^{mp}(m) \left(1 - \Upsilon_{t+l}^{mp}(m) \right) \right] \tag{72}$$

subject to demand[11]

$$Y_{t+l}^{mp}(m) = \left(\frac{S_{t+l} \overline{P}_{t+l}^{mp}(m) + \delta W_{t+l}}{P_{t+l}^m} \right)^{-\theta_{t+l}^m} Y_{t+l}^m \tag{73}$$

and adjustment costs

$$\Upsilon_{t+l}^{mp}(m) \equiv \frac{\phi_m}{2} \left(\frac{\overline{P}_{t+l}^{mp}(m)/\overline{P}_{t+l-1}^{mp}(m)}{\overline{P}_{t+l-1}^{mp}/\overline{P}_{t+l-2}^{mp}} - 1 \right)^2. \tag{74}$$

Imposing $\overline{P}_t^{mp} = \overline{P}_t^{mp}(m)$, the first-order condition can be written

$$\begin{aligned}
0 = & \left(\overline{P}_t^{mp} - \theta_t^m \left(\overline{P}_t^{mp} - \xi_t^f \right) \frac{\overline{P}_t^{mp}}{P_t^m} \right) \left(1 - \frac{\phi_m}{2} \left(\frac{\bar{\pi}_t^{mp}}{\bar{\pi}_{t-1}^{mp}} - 1 \right)^2 \right) \\
& - \left(\overline{P}_t^{mp} - \xi_t^f \right) \phi_m \left(\frac{\bar{\pi}_t^{mp}}{\bar{\pi}_{t-1}^{mp}} - 1 \right) \frac{\bar{\pi}_t^{mp}}{\bar{\pi}_{t-1}^{mp}} \\
& + E_t \left[D_{t,t+1}^f \left(\overline{P}_{t+1}^{mp} - \xi_{t+1}^f \right) \frac{Y_{t+1}^{mp}}{Y_t^{mp}} \phi_m \left(\frac{\bar{\pi}_{t+1}^{mp}}{\bar{\pi}_t^{mp}} - 1 \right) \frac{\bar{\pi}_{t+1}^{mp}}{\bar{\pi}_t^{mp}} \right],
\end{aligned} \tag{75}$$

where $\bar{\pi}_t^{mp} \equiv \overline{P}_t^{mp}/\overline{P}_{t-1}^{mp}$.

---

[11] The demand function is derived from

$$Y_{t+l}^{mp}(m) = \frac{1}{\varpi_f} \left(\frac{P_{t+l}^{mp}(m)}{P_{t+l}^{mp}} \right)^{-\theta_{t+l}^m} Y_{t+l}^{mp} = \left(\frac{S_{t+l} P_{t+l}^{mp}(m)}{P_{t+l}^m} \right)^{-\theta_{t+l}^m} Y_{t+l}^m,$$

2.2 Households

The economy is inhabited by a continuum of symmetric, infinitely lived households indexed by $j \in [0,1]$ that derive utility from leisure and consumption of the final good. Households get income from selling labour services, from holding one-period domestic and foreign bonds, and they receive the real profits from domestic firms. The adjustment costs incurred by domestic firms are also rebated to households. Each household is a monopoly supplier of a differentiated labour service and sets the wage rate subject to labour demand

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta^h} H_t, \tag{76}$$

and quadratic costs of wage adjustment. The adjustment costs are measured in terms of the total wage bill. The specification of adjustment costs follows Laxton & Pesenti (2003) and is given by:

$$\Upsilon_t^w(j) \equiv \frac{\phi_w}{2} \left(\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1 \right)^2. \tag{77}$$

The return on the foreign bond is given by $\kappa_t R_t^f$, where R_t^f is the gross nominal interest rate on foreign bonds and κ_t is a premium on foreign bond holdings. The premium is assumed to be a function of the economy's real net foreign asset position

$$\kappa_t = \exp\left(-\psi \frac{S_t B_t^f}{P_t^c} + u_t \right), \tag{78}$$

where B_t^f is the aggregate holding of nominal foreign bonds in the economy, and u_t is a time-varying 'risk premium' shock.[12] The risk premium shock is assumed to follow a first-order autoregressive process

$$\ln u_t = \rho_u \ln u_{t-1} + \varepsilon_{u,t} \tag{79}$$

where $0 \le \rho_u < 1$, and $\varepsilon_{u,t}$ is a white noise process. The specification of the risk premium implies that if the domestic economy is a net borrower ($B_t^f < 0$), it has to pay a premium on the foreign interest rate. This assumption ensures that net foreign assets are stationary.[13]

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[12] As discussed by Bergin (2006), the mean-zero disturbance term u_t can be interpreted as a proxy for a time-varying risk premium omitted by linearisation, or as capturing the stochastic bias in exchange rate expectations in a noise trader model.

[13] See Schmitt-Grohe & Uribe (2003) for a discussion of alternative ways to ensure stationary net foreign assets in a small open economy. In the standard small open economy model with incomplete international asset markets, equilibrium dynamics have a random walk component. That is, transitory shocks have permanent effects on wealth and consumption.

Household j's period $t+l$ budget constraint is

$$P_{t+l}^c C_{t+l}(j) + \frac{B_{t+l}(j)}{R_{t+l}} + \frac{S_{t+l}B_{t+l}^f(j)}{\kappa_{t+l}R_{t+l}^f}$$ (80)

$$= (1 - \Upsilon_{t+l}^w(j))W_{t+l}(j)H_{t+l}(j) + B_{t+l-1}(j) + S_{t+l}B_{t+l-1}^f(j) + \Pi_{t+l},$$

where R_{t+l} is the (gross) nominal interest rate on domestic bonds, $B_{t+l}(j)$ and $B_{t+l}^f(j)$ are household j's holdings of nominal domestic and foreign bonds, and the variable Π_{t+l} includes all profits accruing to domestic households and the nominal adjustment costs that are rebated to households.

A representative household chooses a sequence $\left\{C_{t+l}(j), B_{t+l}(j), B_{t+l}^f(j), W_{t+l}(j)\right\}_{l=0}^{\infty}$ to maximise

$$E_t \sum_{l=0}^{\infty} \beta^l \left(\ln \left(\frac{C_{t+l}(j) - \zeta C_{t+l-1}}{1 - \zeta} \right) - \eta \frac{H_{t+l}^{1+\chi}}{1+\chi} \right)$$

subject to the budget constraint (80). The parameter $\zeta \in [0,1)$ reflects the assumption of (external) habit formation in consumption, and $\chi \in (0,\infty)$ is the inverse of the Frisch elasticity of labour supply (i.e., the elasticity of labour supply with respect to real wages for a constant marginal utility of wealth). The parameter $\eta > 0$ is a scale parameter and $\beta \in (0,1]$ is the subjective discount factor. The stochastic discount factor $D_{t,t+l}$ is defined as

$$D_{t,t+l} = \beta^l \frac{C_t - \zeta C_{t-1}}{C_{t+l} - \zeta C_{t+l-1}} \frac{P_t^c}{P_{t+l}^c}$$ (82)

Making use of the fact that all households are identical, the first-order conditions with respect to consumption and bond holdings can be combined to give

$$\frac{1}{R_t} = E_t D_{t,t+1}$$ (83)

$$\frac{1}{\kappa_t R_t^f} = E_t \left[D_{t,t+1} \frac{S_{t+1}}{S_t} \right].$$ (84)

The first equation is the consumption Euler equation reflecting the households' desire to smooth consumption over time. With habit formation, consumption dated $t-1$ enters the Euler equation. The second equation is the UIP condition which characterises the optimal portfolio allocation of foreign and domestic bonds. The first-order condition

with respect to wages can be written

$$
\begin{aligned}
0 \;=\; & \frac{P_t^c \left(C_t - \zeta C_{t-1}\right)}{1 - \zeta} \eta \frac{\theta^h H_t^{\chi}}{W_t} \\
& -(\theta^h - 1)\left(1 - \frac{\phi_w}{2}\left(\frac{\pi_t^w}{\pi_{t-1}^w} - 1\right)^2\right) \\
& -\phi_w \left(\frac{\pi_t^w}{\pi_{t-1}^w} - 1\right)\frac{\pi_t^w}{\pi_{t-1}^w} \\
& +E_t \left[\; D_{t,t+1}\pi_{t+1}^w \frac{H_{t+1}}{H_t}\phi_w\left(\frac{\pi_{t+1}^w}{\pi_t^w} - 1\right)\frac{\pi_{t+1}^w}{\pi_t^w}\;\right],
\end{aligned}
\tag{85}
$$

where $\pi_t^w \equiv W_t/W_{t-1}$. In the absence of adjustment costs ($\phi_w = 0$), the optimal real wage is a mark-up over the marginal rate of substitution between leisure and consumption

$$
\frac{W_t}{P_t^c} = \frac{\theta^h}{\theta^h - 1}\eta H_t^{\chi}\frac{\left(C_t - \zeta C_{t-1}\right)}{1 - \zeta}.
\tag{86}
$$

2.3   Monetary authorities

The central bank sets short-term interest rates according to the following simple feedback rule

$$
R_t = \rho_R R_{t-1} + (1 - \rho_R)\left(R + \rho_\pi \left(\pi_t^c - \pi^c\right)\right),
\tag{87}
$$

where R is the steady-state level of the nominal interest rate, π^c is the inflation target and $\rho_\pi > 0$. The parameter $0 < \rho_R < 1$ measures the degree of interest rate smoothing.

2.4   Market clearing

The market clearing conditions for the domestic labour market and the intermediate goods market are

$$
H_t = H_t^y + H_t^c + H_t^m
\tag{88}
$$

$$
Y_t = Y_t^d + Y_t^x,
\tag{89}
$$

where $H_t^m = \delta Y_t^m$. Only foreign bonds are assumed to be traded internationally, hence the domestic bond is in zero net supply at the domestic level (i.e., $B_t = 0$). Net foreign assets evolve according to

$$
\frac{S_t B_t^f}{\kappa_t R_t^f} = S_t B_{t-1}^f + \overline{P}_t^x Y_t^x - \overline{P}_t^m Y_t^m
\tag{90}
$$

2.5 Mark-up shocks

The model derived above only has one shock; the risk premium or UIP shock. If the purpose is to estimate the DSGE model by matching impulse responses to a UIP shock, there is no need to introduce additional shocks. In fact, one of the advantages of the impulse response matching approach is that it allows the researcher to leave most of the exogenous shocks unspecified. However, if the dimension of the VAR is greater than the number of shocks, a VAR fitted to data generated from the DSGE model will have a singular variance-covariance matrix. This is the stochastic singularity problem discussed by e.g., Ingram et al. (1994). One strategy for dealing with this problem is to add shocks until the number of shocks is at least as great as the number of variables in the VAR. This is the approach taken in this paper. More precisely, I introduce four mark-up shocks. The elasticities of substitution between varieties of goods are characterised by the following processes

$$\ln \theta_t^c = (1 - \rho_c) \ln \theta^c + \rho_c \ln \theta_{t-1}^c + \varepsilon_{c,t} \tag{91}$$

$$\ln \theta_t^y = (1 - \rho_y) \ln \theta^y + \rho_y \ln \theta_{t-1}^y + \varepsilon_{y,t} \tag{92}$$

$$\ln \theta_t^x = (1 - \rho_x) \ln \theta^x + \rho_x \ln \theta_{t-1}^x + \varepsilon_{x,t} \tag{93}$$

$$\ln \theta_t^m = (1 - \rho_m) \ln \theta^m + \rho_m \ln \theta_{t-1}^m + \varepsilon_{m,t} \tag{94}$$

where $0 \leq \rho_i < 1$ and the $\varepsilon_{i,t}$ are independent white noise processes, $i = \{c, y, x, m\}$. Variables without time-subscripts denote steady-state values. The motivation for adding this particular set of shocks is that the mark-up shocks have a direct effect on the price-setting equations in the structural model and hence, on the variables included in the VAR. This turned out to be important to avoid a (near) singular variance-covariance matrix. However, I do not attach a strong structural interpretation to the mark-up shocks. An alternative would be to add serially correlated errors to the observation equations in the state space representation. Such 'measurement errors' could be interpreted as capturing the effects of structural shocks that are omitted from the model or other forms of misspecification of the DSGE model.

2.6 Calibration

In the calibration one period is taken to be one quarter. The calibration is guided by the following principles: first, the parameters should be within the range suggested by the literature and second, the model should loosely match the standard deviations and first-order autocorrelations of UK prices and exchange rates over the period 1980–2003.

Table 2 lists the values of the parameters in the baseline calibration of the model. The subjective discount factor is set to $1.03^{-0.25}$ to yield a steady-state annualised real interest

rate of 3%. The habit persistence parameter (ζ) is set to 0.85, which is close to the value chosen by Kapetanios et al. (2005) for the UK. There appears to be little consensus in the literature about the appropriate value for the inverse of the Frisch elasticity of labour demand (χ). Choudhri et al. (2005) choose an initial value of 0.5 for this parameter, but later allow it to vary between zero and infinity. In the baseline calibration in this paper, the inverse Frisch elasticity is set to 3, which is the same value used in Hunt & Rebucci (2005) in a version of the IMF's Global Economy Model. The weight on leisure in the utility function (η) is chosen to yield a steady-state level of labour supply equal to unity ($H = 1$).

Based on the data for revenue shares of intermediate goods reported in Choudhri et al. (2005), the Cobb-Douglas shares of intermediate goods in the production functions for final goods and intermediate goods (γ_c, γ_y) are set to 0.42 and 0.77, respectively. The share of domestic intermediate goods in the aggregate intermediate good (α) is set to 0.85, and the elasticity of substitution between domestic and foreign intermediate goods (ν) is 1.5. The range considered by the literature for the latter is quite large. Groen & Matsumoto (2004) use the value 1.5 in their calibrated model of the UK economy. The distribution cost parameters (δ, δ_f) are set to 0.4, slightly higher than the 0.3 used by Hunt & Rebucci (2005).

The steady-state values of the elasticities of substitution between varieties of goods sold in the domestic market (i.e., θ^c, θ^y and θ^m) are set to 6. This implies a steady-state mark-up of 20% for final goods and domestic intermediate goods. Again, these numbers are comparable to what has been used in models of the UK economy. Benigno & Thoenissen (2003) assume that the substitution elasticity between traded goods is 6.5, and Kapetanios et al. (2005) set the elasticity of substitution between varieties of domestic goods sold in the domestic market to 5. The elasticity of substitution between types of labour services is also set to 6, in line with the values in Hunt & Rebucci (2005) and Benigno & Thoenissen (2003). Finally, the elasticity of substitution between varieties of domestic goods sold in foreign markets is set to 15. This is based on the argument in Kapetanios et al. (2005) that domestic firms face more competitive demand conditions in foreign markets.

The annual domestic inflation target is 2%. The parameters in the monetary policy rule are taken from Kapetanios et al. (2005). The weight on interest rate smoothing in the monetary policy rule (ρ_R) is 0.65, and the weight on inflation (ρ_π) is 1.8.

The adjustment cost parameters associated with changing the rates of change in prices and wages ($\phi_c, \phi_y, \phi_m, \phi_x, \phi_w$) are set to 400.

The share of PCP firms in the foreign economy (ω_f) is set to 0.4, while the share of PCP firms in exports (ω) is 0.6. Data on invoicing currency in UK trade from the years 1999 to 2002 show that the share of UK imports and exports that are invoiced in

sterling is around 40% and 50% respectively.[14] To get short-run pass-through to import prices more in line with the empirical estimates I had to use a somewhat higher value for the share of LCP firms in the foreign economy than what is suggested by the data on invoicing currency. Admittedly, this is not entirely satisfying.

The steady-state levels of foreign output y_f and real wages w_f are normalised to unity. The implicit inflation target in the foreign economy (π_f) is identical to the domestic inflation target. This implies that the rate of exchange rate depreciation is zero in the steady-state. Moreover, assuming that domestic and foreign households have the same subjective discount rates, the steady-state interest rates will be the same. This is consistent with a zero risk premium ($\kappa = 1$) and zero net foreign assets ($B_f = 0$) in the steady-state. The elasticity of substitution between foreign and domestic goods in the foreign economy (ν_f) is set to 1.5, the same as in the domestic economy.

The sensitivity of the risk premium to net foreign assets is set to 0.02. During the calibration process I found that setting this parameter too low caused the model to become non-invertible (see section 3). The parameters in the processes for the risk premium and the demand elasticities were chosen to make the standard deviation and autocorrelation of the inflation rates and exchange rate depreciation roughly match those in the data. Table 3 reports the standard deviations and the first-order autocorrelations in the model and in the UK data 1980Q1–2003Q4.

2.7   Model solution and properties

To solve the model, I first compute a first-order approximation (in logs) of the equilibrium conditions around a non-stochastic steady state. Several solution algorithms are available for linearised rational expectations models (e.g., Blanchard & Kahn, 1980; Anderson & Moore, 1985; Klein, 2000; Sims, 2002). Depending on the eigenvalues of the system there are three possibilities: there are no stable rational expectations solutions, there exists a unique stable solution, or there are multiple stable solutions. According to Blanchard & Kahn (1980, prop. 1), there exists a unique stable solution if the number of eigenvalues outside the unit circle equals the number of non-predetermined ('forward-looking') variables. In this paper, the log-linearised model is solved using the procedures implemented in Dynare, which is a collection of Matlab routines for solving rational expectations models (see Juillard, 2005). The non-linear equilibrium conditions, the steady-state equations and the equations in the log-linearised model are listed in appendix A.

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[14]These numbers can be found on `http://customs.hmrc.gov.uk/`

2.8   Is the model empirically relevant?

As a check on the calibration I examined whether the DSGE model is empirically relevant in the following sense: the estimation of a VAR on artificial data generated from the DSGE model should yield similar estimates of exchange rate pass-through to those obtained when estimating a VAR on actual UK data when using the same sample size, the same set of variables and the same identification scheme.

The estimated fourth-order VAR includes the following variables: UK import prices of manufactures ($\overline{P}_t^m$), export prices of manufactures ($\overline{P}_t^x$), producer prices of manufactures (P_t^y), consumer prices (P_t^c), and a nominal effective exchange rate (S_t).[15] An increase in the exchange rate S_t corresponds to a depreciation of sterling. The data are quarterly, covering the period 1980Q1–2003Q4, and all the price series are seasonally adjusted and measured in domestic currency. Variable definitions and sources are provided in appendix D.

In line with common practice in the literature, the variables are differenced prior to estimation. Also in line with common practice, the exchange rate shock is identified by placing the exchange rate first in a recursive ordering of the variables. Under this identification scheme, exchange rate shocks have a contemporaneous effect on the price indices, but shocks to the price equations affect the exchange rate with at least a one-period lag. This assumption could be justified by the existence of time lags in the publication of official statistics (see Choudhri et al., 2005). Note that, if interest is only in the exchange rate shock, the ordering of the variables placed before or after the exchange rate is irrelevant.

Figure 1 plots the accumulated impulse responses of import prices, export prices, producer prices and consumer prices to a one standard deviation shock to the exchange rate. The responses are normalised by the accumulated response of the exchange rate. The normalised impulse responses can be interpreted as a measure of exchange rate pass-through.[16] Exchange rate pass-through to import prices is 39% within the first quarter, increasing to 55% within one year and to about 70% in the longer run. The immediate response of export prices is somewhat lower; pass-through is 16% after one quarter, 47% after one year and increasing to 60% in the long run. The response of producer prices is smaller and more gradual; pass-through is 15% within one year and increases to 27% after five years. The pass-through to consumer prices is close to zero at all horizons. The long-run pass-through is approximately 7%. These estimates are broadly in line with the estimates reported for the UK in other structural VAR studies such as McCarthy (2000) and Faruqee (2006).

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[15]This is the same set of variables as considered by Faruqee (2006), with the exception that he also includes wages in the VAR. I have confirmed that the pass-through estimates reported in this section are robust to the inclusion of wages in the model.

[16]The normalisation facilitates a comparison with single-equation estimates of pass-through defined as the dynamic responses of prices to a one per cent permanent exchange rate change.

As a next step, I conducted the following simulation experiment: using the log-linearised solution to the DSGE model as the data generating process, I simulated 5000 synthetic datasets of length $T = 100$ for $y_t' = \{\Delta \ln S_t, \Delta \ln \overline{P}_t^m, \Delta \ln \overline{P}_t^x, \Delta \ln P_t^y, \Delta \ln P_t^c\}$. For each synthetic dataset I estimated a VAR(4) and computed the impulse responses to an exchange rate shock using the same recursive identification scheme as above.[17] Figure 2 plots the pointwise mean of the normalised responses to an exchange rate shock. Exchange rate pass-through to import prices is 45% in the first quarter and stabilises at 75% after about 12 quarters. The pass-through to export prices is lower; 32% in the first quarter and close to 40% in the long-run. The short-run pass-through to producer and consumer prices is close to zero. After twenty periods the pass-through is 25% and 10%, respectively. Evidently, the overall pattern of the pass-through estimates is broadly similar to the estimates obtained using actual UK data.

3   MAPPING FROM THE DSGE MODEL TO A VAR

Adopting the notation in Fernández-Villaverde et al. (2005), the log-linear transition equations describing the model solution can be expressed in state space form as

$$
\begin{aligned}
x_{t+1} &= Ax_t + Bw_t \\
y_t &= Cx_t + Dw_t,
\end{aligned}
\tag{95}
$$

where w_t is an $m \times 1$ vector of structural shocks satisfying $E[w_t] = 0, E[w_t w_t'] = I$ and $E[w_t w_{t-j}] = 0$ for $j \neq 0$, x_t is an $n \times 1$ vector of state variables, and y_t is a $k \times 1$ vector of variables observed by the econometrician. The eigenvalues of A are all strictly less than one in modulus, hence the model is stationary. In what follows I will focus on the case where D is square (i.e., $m = k$) and D^{-1} exists. The impulse responses from the structural shocks w_t to y_t are given by the moving average (MA) representation

$$
y_t = d(L)w_t = \sum_{j=0}^{\infty} d_j L^j w_t,
\tag{96}
$$

where L is the lag operator ($L^j y_t \equiv y_{t-j}$), $d_0 = D$ and $d_j = CA^{j-1}B$ for $j \geq 1$.

---

[17]This identification scheme is not consistent with the DSGE model presented above. However, the point of this exercise is to show that, if I use a similar sample size and the same identification scheme, I get results that are not too dissimilar from what was found using actual UK data. In the Monte Carlo experiments in section 4 I use an identification scheme that is compatible with the DSGE model.

3.1   Invertibility

An infinite order VAR is defined by

$$y_t = \sum_{j=1}^{\infty} A_j y_{t-j} + G v_t, \qquad (97)$$

where $E[v_t] = 0$, $E[v_t v_t'] = I$, $E[v_t v_{t-j}'] = 0$ for $j \neq 0$. The orthogonalisation of the VAR innovations implicit in G is void of economic content and does not impose any restrictions on the model. The covariance matrix of the VAR innovations $u_t = G v_t$ is $E[G v_t v_t' G'] = GG' = \Sigma_u$. The MA representation of (97) is

$$y_t = c(L) v_t \qquad (98)$$

where $c(L) = \sum_{j=0}^{\infty} c_j L^j = (I - \sum_{j=1}^{\infty} A_j L^j)^{-1} G$.

A potential source of discrepancies between the VAR impulse responses and the responses from the log-linearised solution to the DSGE model is that the MA representation (96) is non-invertible. By construction, the MA representation associated with the infinite order VAR (98) is fundamental in the sense that the innovations v_t can be expressed as a linear combination of current and past observations of y_t. However, there exists an infinite number of other, non-fundamental, MA representations that are observationally equivalent to (98), but which cannot be recovered from the infinite order VAR. These MA representations are non-invertible, meaning that they cannot be inverted to yield an infinite order VAR. In general, we cannot rule out the possibility that a DSGE model has a non-invertible MA representation for a given set of observables.[18] That is, we cannot rule out the possibility that some of the roots of the characteristic equation associated with (96) are inside the unit circle. If this is the case, the impulse responses derived from an infinite order VAR will be misleading, as the structural shocks cannot be recovered from the innovations to the VAR. Whether the MA components of a model are invertible or non-invertible will in general depend on which variables are included in the VAR.

Fernández-Villaverde et al. (2005) show that when D is square and D^{-1} exists, a necessary and sufficient condition for invertibility is that the eigenvalues of $A - BD^{-1}C$ are strictly less than one in modulus. If this condition is satisfied, y_t has an infinite order VAR representation given by

$$y_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1} BD^{-1} y_{t-j} + D w_t. \qquad (99)$$

---

[18] Lippi & Reichlin (1994) and Fernández-Villaverde et al. (2005) provide examples of economic models with non-invertible MA components.

The rate at which the autoregressive coefficients converge to zero is determined by the largest eigenvalue of $A - BD^{-1}C$. If this eigenvalue is close to unity, a low order VAR is likely to be a poor approximation to the infinite order VAR. Two special cases are worth noting. First, as can be seen from (95), if all the variables in x_t are observed by the econometrician (implying that $A = C$ and $B = D$), the process for y_t will be a VAR(1). Second, if all the endogenous state variables are observable and included in y_t, and the exogenous state variables follow a VAR(1), then y_t has a VAR(2) representation (see e.g., Kapetanios et al., 2005; Ravenna, 2005).

If one or more of the eigenvalues of $A - BD^{-1}C$ are exactly equal to one in modulus, y_t does not have a VAR representation; the autoregressive coefficients do not converge to zero as the number of lags tends to infinity. Fernández-Villaverde et al. (2005) refer to this as a 'benign borderline case'. Often, roots on the unit circle indicate that the variables in the VAR have been overdifferenced (see Watson, 1994).

3.2 Identification

If the model is invertible, the impulse responses from the infinite order VAR (97) with $Gv_t = Dw_t$ correspond to the impulse responses to the structural shocks in the DSGE model (96). In practice, however, D is unknown, and the econometrician is faced with an identification problem. A prerequisite for estimating DSGE models by matching impulse responses, is that the identification restrictions imposed on the VAR are compatible with the theoretical model. As discussed above, the pass-through literature has typically achieved exact identification by setting $G = \Gamma_{tr}$, where Γ_{tr} is the lower triangular Choleski factor of the estimated variance-covariance matrix of the VAR residuals, $\widehat{\Sigma}_u$. However, this identification scheme is not consistent with the DSGE model set out in section 2. Hence, $G = \Gamma_{tr}$ will yield biased estimates of the model's impulse responses.[19]

In the simulation experiments in this paper I employ an identification scheme suggested by Del Negro & Schorfheide (2004). Using a QR decomposition of D, the impact responses of y_t to the structural shocks w_t can be expressed as

$$\left(\frac{\partial y_t}{\partial w_t} \right)_{DSGE} = D = \Gamma_{tr}^* \Omega^*, \tag{100}$$

where Γ_{tr}^* is lower triangular and Ω^* satisfies $(\Omega^*)'\Omega^* = I$. The VAR is identified by setting $G = \Gamma_{tr}\Omega^*$. With this identification scheme, the impact responses computed from the VAR will differ from D only to the extent that Γ_{tr} differs from Γ_{tr}^* (that is, only to the extent that the estimated variance-covariance matrix $\widehat{\Sigma}_u$ differs from DD'). Thus, in the

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[19]Canova & Pina (2005) show that when the DSGE model does not imply a recursive ordering of the variables, the VAR responses to a monetary policy shock identified with a recursive identification scheme can be very misleading.

absence of misspecification of the VAR, the identification scheme succeeds in recovering the true impact responses.

4   SIMULATION EXPERIMENTS

This section presents the results of the simulation experiments. I consider two different VARs: a VAR in first differences of nominal prices and the exchange rate, and a VAR in relative prices and the first difference of consumer prices. The latter is equivalent to a VEqCM that includes the cointegration relations implied by the DSGE model as regressors. As a second exercise, I examine whether an econometrician who uses standard techniques for determining cointegration rank and for testing restrictions on the cointegration relations will be able to infer the cointegration properties of the DSGE model.

4.1   Monte Carlo design

I generate $M = 5000$ datasets of lengths $T = 1100$ and $T = 1200$ using the state space representation of the log-linearised DSGE model as the data generating process.[20] Each sample is initialised using the steady-state values of the variables. To limit the influence of the initial conditions, I discard the first 1000 observations in each replication and leave $T = 100$ and $T = 200$ observations for estimation of the VAR. The simulations are performed in Matlab, and the built-in function randn.m is used to generate the pseudo-random normal errors. I use the same random numbers in all experiments. This is achieved by fixing the seed for the random number generator.

For each dataset I estimate a VAR and compute the accumulated responses of prices to a UIP shock. The UIP shock is identified using the Del Negro & Schorfheide (2004) identification scheme discussed in the previous section.

The selection of lag-order is an important preliminary step in VAR analyses. I conduct experiments for four different methods of lag-order selection: the Akaike information criterion (AIC), the Hannan-Quinn criterion (HQ), the Schwarz criterion (SC), and the sequential likelihood-ratio test (LR) (see Lütkepohl, 1991 for a discussion). The LR test is implemented using the small-sample correction suggested in Sims (1980) and using a 5% significance level for the individual tests. I also report results for a fixed lag-length ($L = 2$ and $L = 4$ for the VAR in first differences, $L = 3$ and $L = 5$ for the VEqCM and the VAR in levels).

Lütkepohl (1990) shows that, as long as the lag-order goes to infinity with the sample size, the orthogonalised impulse response functions computed from a finite order VAR estimated by OLS are consistent and asymptotically normal, even if the true order of the

---

[20]To examine the sensitivity of the results to the number of Monte Carlo replications I conducted preliminary experiments using $M = \{1000, 2000, \ldots, 10000\}$ and found that the pointwise mean and standard deviations of the impulse responses obtained with $M = 5000$ and $M = 10000$ are essentially indistinguishable.

process is infinite. In this sense, any discrepancies between the impulse responses from the VAR and the log-linearised DSGE model can be attributed to a small-sample bias. It is nevertheless instructive to decompose the overall difference between the DSGE model's impulse responses and the VAR impulse responses into (i) bias arising from approximating an infinite order VAR with a finite order VAR, and (ii) small-sample estimation bias for a given lag-order. The first source of bias, which Chari et al. (2005) label the 'specification error', is given by the difference between the DSGE model's responses and those obtained from the population version of the finite order VAR for a given lag-order. The coefficients in the population version of a finite order VAR can be interpreted as the probability limits of the OLS estimators or, what the OLS estimates would converge to if the number of observations went to infinity while keeping the lag-order fixed (Christiano et al., 2006). Fernández-Villaverde et al. (2005) provide formulas for these coefficients as functions of the matrices A, B, C and D in the state space representation (95). Hence, the magnitude of the specification error can be assessed without resorting to simulation exercises.[21] For a given lag-order, the bias arising from the specification error persists even in large samples. Regarding the small-sample estimation bias; VAR impulse responses are non-linear functions of the autoregressive coefficients and the covariance matrix of the VAR residuals. It is well known that OLS estimates of the autoregressive coefficients in VARs are biased downward in small samples.

4.2   VAR in first differences

The first model I consider is a VAR in first differences of nominal prices and the exchange rate:

$$\Delta y_t = A_1 \Delta y_{t-1} + A_2 \Delta y_{t-2} + \ldots + A_p \Delta y_{t-p} + \varepsilon_t \qquad (101)$$

where

$$\Delta y_t' = \{\Delta \ln \overline{P}_t^m, \Delta \ln \overline{P}_t^x, \Delta \ln P_t^y, \Delta \ln P_t^c, \Delta \ln S_t\}.$$

With this vector of observables, the matrix $A - BD^{-1}C$ has four roots equal to one, while the remaining roots are all smaller than one in modulus. This implies that, technically, the model does not have a VAR representation. The reason why a VAR representation fails to exist in this case, is that the variables included in the VAR are overdifferenced.

Table 4 reports the distribution of the lag-orders chosen by the different lag-order selection criteria for sample sizes $T = 100$ and $T = 200$. The maximum lag-length is set to five. As expected, the SC is the most conservative and selects the lowest average lag-order. For sample size $T = 100$ the SC chooses a lag-length of one in 71.5% of the replications. By contrast, the AIC and the HQ select a lag-order of two in approximately

---

[21]I am grateful to Jesús Fernández-Villaverde for sharing the Matlab program `ssvar.m` which calculates the coefficients of the population version VAR.

90% of the replications. With a sample size of $T = 200$, the average lag-order increases for all the criteria: the SC picks a lag-order of two in 98% of the datasets, the AIC and the HQ select a lag-order of two in 89% and 100% of the replications respectively. For both sample sizes, the LR test selects a somewhat higher lag-order than the information criteria.

Figure 3 plots the outcome of the simulation experiment with $T = 100$ and a fixed lag-length $L = 2$. The solid lines represent the pointwise mean of the accumulated impulse responses, and the shaded areas correspond to the pointwise mean plus/minus 1.96 times the pointwise standard deviations. The lines with points correspond to a 95% interval for the pointwise responses, calculated by reading off the 2.5 and 97.5 percentiles of the ordered responses at each horizon. Finally, the lines with circles depict the impulse responses from the DSGE model. Figure 4 plots the accumulated responses normalised on the exchange rate response.

Looking at the normalised responses, we see that the VAR estimates of exchange rate pass-through are biased downwards. Whereas in the DSGE model the exchange rate pass-through is nearly complete after twenty quarters, the mean of the VAR estimates of long-run pass-through is 72% for import prices, 35% for export prices, 15% for consumer prices and 20% for producer prices.[22] From the bottom panel of figure 3 it is evident that the downward bias to some extent reflects that the exchange rate behaves almost like a random walk in the VAR, whereas there is significant (but not complete) reversion in the exchange rate towards the original level following a UIP shock in the DSGE model. The bias in the nominal exchange rate response is transmitted to import prices. By contrast, the estimated VAR responses of consumer and producer prices are smaller than the true responses. This suggests that the downward bias in the VAR estimates of pass-through to these prices would remain even if the VAR had accurately captured the exchange rate response. Figures 5 and 6 plot the outcome of an experiment with $L = 2$ and $T = 200$. The biases in the impulse responses remain in the larger sample, the main effect of adding observations is to lower the standard deviations of the simulated responses.

Figures 7 and 8 decompose the overall bias into small-sample bias and bias arising from approximating an infinite order VAR with a VAR(2). The latter is measured as the difference between the true impulse responses (lines with circles) and the responses from the population version of a VAR(2) (solid lines). It is evident that the dominant source of bias is the specification error. For a given lag-order, this bias persists in large samples. The small-sample bias is measured as the difference between the responses from the population VAR(2) and the mean responses from the Monte Carlo experiments for $T = 100$ (dotted lines) and $T = 200$ (lines with points). The impulse responses of the exchange

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[22]Extending the horizon beyond twenty quarters, the exchange rate pass-through to all prices is 100% in the DSGE model.

rate and import prices are biased downward in small samples. For these variables, the small-sample bias and the specification error bias are of opposite signs. Hence, the effect of adding more observations is to increase the overall bias in the impulse responses. For consumer and producer prices, the opposite is true. For these variables the small-sample bias reinforces the downward bias induced by the specification error.

Next, I examine how many lags are needed for the VAR to be able to recover the true impulse responses. Figures 9 and 10 show the impulse responses from the DSGE model (circled lines), together with the responses from the population version of the VAR for lag-orders $L = \{2, 4, 10, 20\}$. As expected, increasing the number of lags reduces the biases. However, even with as many as twenty lags, the VAR does not accurately capture the responses of prices to a UIP shock.

As we have seen, standard lag-order selection criteria do not detect the need for longer lags. This raises the question whether the misspecification of the lag order is picked up by standard misspecification tests. Table 5 reports the rejection frequencies at the 5% significance level for single-equation and vector tests for residual autocorrelation up to order four. The test is the $F-$approximation to the Lagrange Multiplier (LM) test for autocorrelation described in Doornik (1996). In the small sample ($T = 100$), the tests reject only slightly more often than expected when using a 5% significance level when $L = 4$ or the lag-order is determined by the sequential LR tests. However, when a conservative criterion such as the SC is used, the autocorrelation tests are rejected in a large number of the replications. The autocorrelation tests thus help detect misspecification when the lag-order is very low. When the sample size is increased to $T = 200$, the rejection frequencies increase for all the criteria and for both the fixed lag-lengths.

Erceg et al. (2005) suggest measuring the bias in the impulse responses by the average absolute per cent difference between the mean response and the theoretical response for each variable, that is

$$bias_i^H = \frac{1}{H} \sum_{j=1}^{H} \left| \frac{r_{i,j}^{VAR} - r_{i,j}^{DSGE}}{r_{i,j}^{DSGE}} \right|, \tag{102}$$

where $r_{i,j}^{DSGE}$ and $r_{i,j}^{VAR}$ are the DSGE model's responses and the mean across datasets of the VAR responses of variable i to a UIP shock at horizon j respectively. Tables 6 and 7 report the biases for $H = 10$ and $H = 20$ for different lag-order criteria and sample sizes $T = 100$ and $T = 200$. The results confirm that adding observations increases the bias in the responses of exchange rates and import prices, but reduces the biases in the estimated responses of consumer prices and producer prices. At both horizons and for both sample sizes the average bias is minimised for $L = 4$. The average bias is largest when the lag-order is chosen to minimise the SC.

As a final point, note that a reduction in bias from estimating a higher order VAR may come at the cost of higher variance. Using VARs estimated by leading practitioners as

data generating processes, Ivanov & Kilian (2005) find that underestimation of the true lag-order is beneficial in very small samples because the bias induced by choosing a low lag-order is more than offset by a reduction in variance. If the primary purpose of the VAR analysis is to construct accurate impulse responses, the authors recommend using the SC for sample sizes up to 120 quarters and the HQ for larger sample sizes. However, Ivanov & Kilian (2005) do not explore the case where the data generating process is an infinite order VAR, in which case the trade-offs between bias and variance are likely to be different.

4.3 VEqCM

The fact that the monetary policy rule is specified in terms of inflation and not the price level induces a common stochastic trend in the nominal variables in the log-linearised DSGE model.[23] Hence, while nominal prices and the exchange rate contain a unit root, the real exchange rate and relative prices are stationary. Estimating a VAR in first differences implies a loss of information, and in this sense it is not surprising that a VAR that omits the cointegration relations does a poor job in recovering the responses of the levels of prices and the exchange rate. Here I examine whether I obtain a better approximation of the DSGE model by estimating a VEqCM that includes the cointegration relations implied by the theoretical model. That is, I consider the system

$$\Delta y_t = \alpha \beta' y_{t-1} + A_1^* \Delta y_{t-1} + A_2^* \Delta y_{t-2} + \ldots + A_p^* \Delta y_{t-p} + \varepsilon_t \tag{103}$$

with

$$\beta' y_{t-1} = \left\{ \begin{array}{c} \ln \overline{P}_{t-1}^m - \ln P_{t-1}^c \\ \ln \overline{P}_{t-1}^x - \ln P_{t-1}^c \\ \ln P_{t-1}^y - \ln P_{t-1}^c \\ \ln S_{t-1} + \ln P_{t-1}^f - \ln P_{t-1}^c \end{array} \right\}$$

Estimating (103) is (almost) the same as estimating a VAR in the real exchange rate, relative prices and consumer price inflation[24], that is,

$$y_t^\dagger = A_1^\dagger y_{t-1}^\dagger + A_2^\dagger y_{t-2}^\dagger + \ldots + A_{p+1}^\dagger y_{t-(p+1)}^\dagger + \varepsilon_t^\dagger \tag{104}$$

where

$$(y_t^\dagger)' = \left\{ \Delta \ln P_t^c, \ln \left(\overline{P}_t^m / P_t^c \right), \ln \left(\overline{P}_t^x / P_t^c \right), \ln \left(P_t^y / P_t^c \right), \ln \left(S_t P_t^f / P_t^c \right) \right\}$$

---

[23]The foreign price level is stationary around a deterministic trend.

[24]The only difference is that an extra lag of $\ln P_t^c$ is included in the latter from the inclusion of $\Delta \ln P_{t-(p+1)}^c \equiv \ln P_{t-(p+1)}^c - \ln P_{t-(p+2)}^c$. The mapping between the VEqCM and the VAR in levels is described in appendix B.

When the observation vector is $y_t^\dagger$, all the roots of the matrix $A - BD^{-1}C$ are smaller than one in modulus. Hence, the model is invertible, and $y_t^\dagger$ has a VAR representation. Including the cointegration relations thus removes the unit roots in the MA components that appear in the VARMA representation for the first differences. This is a common finding in the literature (see e.g., Del Negro et al., 2005).

Table 8 reports the distribution of the lag-orders chosen by different selection criteria for $T = 100$ and $T = 200$. The maximum lag-length is six. The SC selects a lag-length of two for both sample sizes. On average, the AIC chooses a higher lag-order: when the sample size is $T = 100$ the AIC chooses $L = 2$ in 47.4% of the datasets and $L = 3$ in 46.2% of the datasets. Figures 11 and 12 plot the outcome of the simulation experiment with $T = 100$. and a fixed lag-length $L = 3$. The VAR approximation to the DSGE model is good even with a moderate number of lags. This is confirmed in figures 13 and 14 which plot the responses computed from the population version of the VAR for lag-orders $L = \{2, 3, 20\}$. There is some bias in the impulse responses for $L = 2$, but for $L = 3$ the estimated responses are close to the true responses. Note that this holds even if the VAR does not include all the state-variables in the DSGE model (e.g., the VAR does not include consumption or the nominal interest rate).

Figures 15 and 16 plot the impulse responses from the population version of the VEqCM(3) together with the true responses and the mean responses from a VEqCM(3) estimated on sample sizes $T = 100$ and $T = 200$. In this case, the small-sample estimation bias is the dominant source of bias in the responses. For all prices except import prices, the estimate of exchange rate pass-through is biased upwards, implying that for a given lag-order, adding observations does not reduce the bias. This is confirmed in tables 9 and 10 which report the average biases over the first ten and twenty quarters respectively, for different lag-order criteria and sample sizes $T = 100$ and $T = 200$. Notice that, in contrast to what was the case for the first-differenced model, the average bias is minimised when the lag-order is based on a conservative lag-order criterion such as the SC.

To summarise, provided the cointegration relations implied by the model are included as additional regressors, the state space representation of the log-linearised DSGE models can be approximated with a low-order VAR. This raises the question of whether, in practice, the econometrician would be able to infer the cointegration rank and identify the cointegration relations using standard techniques.

4.4   Cointegration analysis

This section asks the question: will an econometrician armed with standard techniques be able to infer the correct cointegration rank and identify the cointegration relations implied by the theory?

The experiment is constructed as follows. I generate 5000 artificial datasets of lengths $T = 100$ and $T = 200$ from the DSGE model. Dataseries for the levels of the variables are obtained by cumulating the series for the first differences.[25] For a given synthetic dataset I estimate an unrestricted VAR in levels of the variables and determine the cointegration rank using the sequential testing procedure based on the trace test-statistic proposed by Johansen (1988).[26] Next, I test the restrictions on the cointegration space implied by the DSGE model using the standard LR test for known cointegration vectors (see Johansen, 1995, chap. 7).

The VAR is fitted with an unrestricted constant term and a restricted drift term. The specification of the deterministic terms is consistent with the data generating process. To see this, note that the monetary policy rule and the positive inflation target imply that nominal prices will have both a deterministic trend and a stochastic trend. Both trends are cancelled in the cointegrating relations, implying that relative prices are stationary around a constant mean. That is, $\ln \overline{P}_t^m - \ln P_t^c \sim I(0), \ln \overline{P}_t^x - \ln P_t^c \sim I(0)$, and $\ln P_t^y - \ln P_t^c \sim I(0)$. Since the inflation target in the foreign economy is assumed to be the same as the domestic inflation target, the process for the foreign price level contains the same deterministic trend as the domestic price level, and there is no linear trend in the nominal exchange rate. However, since the foreign price level is not included in the VAR, the fourth cointegration relation will be stationary around a deterministic trend. That is, $\ln S_t - \ln P_t^c + 0.005t \sim I(0)$.

Table 11 reports the distribution of lag-orders chosen by the different selection criteria when the maximum lag-length is set to six. The average lag-order selected is two or three for both sample sizes, with SC being the most conservative criterion.

The trace test is derived under the assumption that the errors are serially uncorrelated and normally distributed with mean zero. Good practice dictates that these assumptions be checked before testing for cointegration. Table 12 reports the rejection frequencies across 5000 datasets for the single-equation and vector tests for non-normality in the residuals described in Doornik & Hansen (1994). The rejection frequencies are close to the nominal 5% level for both sample sizes and across different lag-order criteria. Table 13 reports the rejection frequencies for tests of no autocorrelation up to order five in the residuals. For sample size $T = 100$ and lag-length $L = 3$, the rejection frequencies for the single-equation tests are around 10%. The vector test rejects the null hypothesis in 23% of the datasets. Similar rejection frequencies are obtained when the lag-length is determined using the AIC or sequential LR tests. However, when a conservative criterion like the SC or HQ is used, the rejection frequencies are much higher. When the lag-order is

---

[25]The initial values of the (log) levels of the variables are set to zero. Since the levels series are unit root processes and thus have infinite memory, dropping observations at the beginning of the sample does not eliminate the dependence on the initial values.

[26]See chapter 2 of this thesis for details on the cointegration tests.

chosen to minimise the SC, the vector test rejects the null of no autocorrelation in 59.1% of the datasets. For all criteria except the SC, the rejection frequencies are lower in the larger sample $T = 200$. Below I report the outcome of the cointegration tests for all the lag-order selection criteria. In practice, however, researchers often supplement the information criteria with tests for residual autocorrelation, and when there is a contradiction, overrule the lag-order selected by the former. This suggests that less weight should be placed on the results of the cointegration analysis obtained when the lag-order is selected using the SC or the HQ criterion.

Table 14 shows the frequencies of preferred cointegration rank for different sample sizes and for different methods of lag-order selection. The non-standard 5% critical values for the trace test are taken from MacKinnon et al. (1999). The numbers in parentheses correspond to the frequencies of preferred rank when the test statistic is adjusted using the small-sample correction suggested by Reinsel & Ahn (1988). When $T = 100$ and $L = 3$, the correct cointegration rank is selected in only 2.7% of the datasets. In 17.5% of the datasets the trace test suggests that the rank is zero, in which case a model in first differences is appropriate. Using the small-sample adjusted test statistics, the trace test chooses the correct rank in only 0.3% of the datasets. In 61.2% of the datasets the trace test would lead us to conclude that the variables are not cointegrated. The results are more encouraging when a sample size of $T = 200$ is used. However, for $L = 3$ the trace test still picks the true cointegration rank in only 31% of the replications.

When the lag-order is endogenous, the correct rank is chosen most frequently when the lag-order is determined using the SC. For $T = 100$ the correct rank is chosen in 20% of the datasets. With a sample size of $T = 200$ the corresponding number is 53%. For the purpose of choosing the correct cointegration rank, a low lag-order appears to be beneficial.

As a second exercise, I examine how often the restrictions on the cointegration vector implied by the DSGE model are rejected when using the standard LR test for known cointegration vectors. Table 15 reports the rejection frequencies for the individual and joint tests of the following hypotheses: $\ln \overline{P}_t^m - \ln P_t^c \sim I(0)$, $\ln \overline{P}_t^x - \ln P_t^c \sim I(0)$, $\ln P_t^y - \ln P_t^c \sim I(0)$, and $\ln S_t - \ln P_t^c + 0.005t \sim I(0)$. The tests are conditional on the maintained hypothesis that the cointegration rank is 4 ($r = 4$). For $T = 100$ and $L = 3$, the rejection frequencies for the individual hypotheses are 20% when using a nominal test size of 5%. The rejection frequency for the joint hypothesis is 88%. These results raise doubts about whether, in practice, the econometrician will be able to identify the cointegration relations implied by the DSGE model.

Again it is instructive to see whether the results are driven by the specification error or by small-sample estimation bias. In particular, it is of interest to see whether the frequent rejections of the autocorrelation tests are due to the omission of MA terms or due

to the fact that the autocorrelation tests are oversized in small samples. To address this issue I redo the above Monte Carlo experiments, this time using the population version of a VEqCM(5) and a VEqCM(3) as the data generating processes. Table 16 reports the distribution of chosen lag-lengths and table 17 reports the outcome of the trace test in this case. The results are similar to the results obtained when the log-linearised solution to the DSGE model is used as the data generating process. This finding suggests that the poor performance of the sequential testing procedure is not due to approximating an infinite order VAR with a low order VAR, but is due to small-sample problems.[27] Interestingly, the same seems to hold for the autocorrelation test. When the data generating process is a VEqCM(3) and the estimated model is a VAR(3) in levels of the data, the rejection frequencies of the autocorrelation tests are 10% for the single-equation tests and 23% for the vector test (see table 18). This suggests that the autocorrelation test is oversized in small samples. This is consistent with the Monte Carlo evidence presented in Brüggemann et al. (2004). Table 19 illustrates a well known result in the literature (see e.g. Gredenhoff & Jacobson, 2001), namely that the LR tests for restrictions on the cointegration space are oversized in small samples.

Most of the existing literature that has examined the finite-sample performance of cointegration tests assumes that the data generating process is a (reduced form) VAR(MA), in which case it is possible to study the effects of marginal changes in the reduced form parameters (e.g., in the elements in the adjustment coefficients to the cointegration relations).[28] Here, the underlying data generating process is a restricted VAR where the parameters are explicit functions of the parameters in the DSGE model. Changing a structural parameter in the DSGE model will typically affect all the dynamic coefficients in the population VAR as well as the variance-covariance matrix of the error terms. A natural extension of the analysis in this paper would be to identify which feature(s) of the DSGE model is responsible for the low power of the cointegration test.

5   CONCLUDING REMARKS

This paper has examined the ability of a structural VAR to recover the dynamic responses of a set of prices to a risk premium shock. The main results can be summarised as follows. The estimates of exchange rate pass-through obtained from a first-differenced VAR are systematically biased downwards. The bias in the estimated responses can largely be attributed to the fact that a low order first-differenced VAR is not a good approximation to the VARMA model implied by the DSGE model. Moreover, small-sample estimation bias sometimes acts to offset the bias arising from the approximation error. When the

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[27]See appendix C for some additional simulation evidence on the power of the cointegration test.

[28]One notable exception is Söderlind & Vredin (1996) who examine the properties of cointegration tests when the data are generated by a monetary equilibrium business cycle model.

cointegration relations implied by the DSGE model are included in the VAR, even a VAR with a modest number of lags is able to recover the true impulse responses. However, an econometrician using standard tests for cointegration rank and for testing restrictions on the cointegration space would in general not be able to infer the correct rank or identify the true cointegration relations. Interesting extensions of the analysis in this paper are therefore to examine the properties of structural VAR estimates of exchange rate pass-through when the model is estimated in levels, or when the cointegration rank is chosen on the basis of data-based cointegration tests.

A EQUILIBRIUM CONDITIONS DSGE MODEL

A.1 Non-linear model

Defining

$$
p_t^q = \frac{P_t^q}{P_t^c}, p_t^{\tilde{z}} = \frac{P_t^z}{P_t^c}, p_t^y = \frac{P_t^y}{P_t^c}, p_t^m = \frac{P_t^m}{P_t^c}, p_t^{ml} = \frac{P_t^{ml}}{P_t^c}, p_t^{mp} = \frac{S_t P_t^{mp}}{P_t^c}, \bar{p}_t^m = \frac{\bar{P}_t^m}{P_t^c},
$$

$$
\bar{p}_t^{ml} = \frac{\bar{P}_t^{ml}}{P_t^c}, \bar{p}_t^{mp} = \frac{S_t \bar{P}_t^{mp}}{P_t^c}, p_t^{xl} = \frac{S_t P_t^{xl}}{P_t^c}, p_t^{xp} = \frac{P_t^{xp}}{P_t^c}, p_t^x = \frac{P_t^x}{P_t^c}, \bar{p}_t^x = \frac{\bar{P}_t^x}{P_t^c}, \bar{p}_t^{xl} = \frac{S_t \bar{P}_t^{xl}}{P_t^c},
$$

$$
\bar{p}_t^{xp} = \frac{\bar{P}_t^{xp}}{P_t^c}, w_t = \frac{W_t}{P_t^c}, s_t = \frac{S_t P_t^f}{P_t^c}, \vartheta_t^c = \frac{\xi_t^c}{P_t^c}, \vartheta_t^y = \frac{\xi_t^y}{P_t^c}, b_t^f = \frac{B_t^f}{P_t^f}, \vartheta_t^f = \frac{\xi_t^f}{P_t^f}, \Delta S_t = \frac{S_t}{S_{t-1}}
$$

the model's equilibrium conditions can be written

$$
C_t = Q_t^{\gamma_c} (H_t^c)^{1-\gamma_c} \tag{A1}
$$

$$
Q_t^d = \alpha \left(\frac{p_t^y}{p_t^q} \right)^{-\nu} Q_t \tag{A2}
$$

$$
Q_t^m = (1-\alpha) \left(\frac{p_t^m}{p_t^q} \right)^{-\nu} Q_t \tag{A3}
$$

$$
w_t = \vartheta_t^c (1-\gamma_c) \frac{C_t}{H_t^c} \tag{A4}
$$

$$
p_t^q = \vartheta_t^c \gamma_c \frac{C_t}{Q_t} \tag{A5}
$$

$$
p_t^q = \left[\alpha \left(p_t^y \right)^{1-\nu} + (1-\alpha) \left(p_t^m \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{A6}
$$

$$
0 = -(1-\vartheta_t^c) \phi_c \left(\frac{\pi_t^c}{\pi_{t-1}^c} - 1 \right) \frac{\pi_t^c}{\pi_{t-1}^c} + ((1-\theta_t^c) + \theta_t^c \vartheta_t^c) \left(1 - \frac{\phi_c}{2} \left(\frac{\pi_t^c}{\pi_{t-1}^c} - 1 \right)^2 \right) \tag{A7}
$$

$$
+ E_t \left[D_{t,t+1} \pi_{t+1}^c \left(1 - \vartheta_{t+1}^c \right) \frac{C_{t+1}}{C_t} \phi_c \left(\frac{\pi_{t+1}^c}{\pi_t^c} - 1 \right) \frac{\pi_{t+1}^c}{\pi_t^c} \right]
$$

$$
Y_t = Z_t^{\gamma_y} (H_t^y)^{1-\gamma_y} \tag{A8}
$$

$$
Z_t^d = \alpha \left(\frac{p_t^y}{p_t^{\tilde{z}}} \right)^{-\nu} Z_t \tag{A9}
$$

$$
Z_t^m = (1-\alpha) \left(\frac{p_t^m}{p_t^{\tilde{z}}} \right)^{-\nu} Z_t \tag{A10}
$$

$$
w_t = \vartheta_t^y (1-\gamma_y) \frac{Y_t}{H_t^y} \tag{A11}
$$

$$
p_t^{\tilde{z}} = \vartheta_t^y \gamma_y \frac{Y_t}{Z_t} \tag{A12}
$$

$$
p_t^{\tilde{z}} = p_t^q \tag{A13}
$$

$$Y_t^d = Q_t^d + Z_t^d \tag{A14}$$

$$Y_t^m = Q_t^m + Z_t^m \tag{A15}$$

$$Y_t = Y_t^d + Y_t^x \tag{A16}$$

$$
\begin{aligned}
0 = {} & -\left(p_t^y - \vartheta_t^y\right)\phi_y\left(\frac{\pi_t^y}{\pi_{t-1}^y} - 1\right)\frac{\pi_t^y}{\pi_{t-1}^y} \\[2mm]
& + \left((1-\theta_t^y)p_t^y + \theta_t^y\vartheta_t^y\right)\left(1 - \frac{\phi_y}{2}\left(\frac{\pi_t^y}{\pi_{t-1}^y} - 1\right)^2\right) \\[2mm]
& + E_t\left[D_{t,t+1}\pi_{t+1}^c\left(p_{t+1}^y - \vartheta_{t+1}^y\right)\frac{Y_{t+1}^d}{Y_t^d}\phi_y\left(\frac{\pi_{t+1}^y}{\pi_t^y} - 1\right)\frac{\pi_{t+1}^y}{\pi_t^y}\right]
\end{aligned}
\tag{A17}
$$

$$\overline{p}_t^x = \left[\varpi\left(\overline{p}_t^{xp}\right)^{1-\theta_t^x} + (1-\varpi)(\overline{p}_t^{xl})^{1-\theta_t^x}\right]^{\frac{1}{1-\theta_t^x}} \tag{A18}$$

$$p_t^{xp} = \overline{p}_t^{xp} + \delta_f s_t w_t^f \tag{A19}$$

$$p_t^{xl} = \overline{p}_t^{xl} + \delta_f s_t w_t^f \tag{A20}$$

$$p_t^x = \left[\varpi\left(p_t^{xp}\right)^{1-\theta_t^x} + (1-\varpi)(p_t^{xl})^{1-\theta_t^x}\right]^{\frac{1}{1-\theta_t^x}} \tag{A21}$$

$$
\begin{aligned}
0 = {} & \left(\overline{p}_t^{xp} - \theta_t^x\left(\overline{p}_t^{xp} - \vartheta_t^y\right)\frac{\overline{p}_t^{xp}}{p_t^{xp}}\right)\left(1 - \frac{\phi_x}{2}\left(\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} - 1\right)^2\right) \\[2mm]
& - \left(\overline{p}_t^{xp} - \vartheta_t^y\right)\phi_x\left(\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} - 1\right)\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} \\[2mm]
& + E_t\left[D_{t,t+1}\pi_{t+1}^c\left(\overline{p}_{t+1}^{xp} - \vartheta_{t+1}^y\right)\frac{Y_{t+1}^{xp}}{Y_t^{xp}}\phi_x\left(\frac{\overline{\pi}_{t+1}^{xp}}{\overline{\pi}_t^{xp}} - 1\right)\frac{\overline{\pi}_{t+1}^{xp}}{\overline{\pi}_t^{xp}}\right]
\end{aligned}
\tag{A22}
$$

$$
\begin{aligned}
0 = {} & \left(\overline{p}_t^{xl} - \theta_t^x\left(\overline{p}_t^{xl} - \vartheta_t^y\right)\frac{\overline{p}_t^{xl}}{p_t^{xl}}\right)\left(1 - \frac{\phi_x}{2}\left(\frac{\overline{\pi}_t^{xl}}{\overline{\pi}_{t-1}^{xl}} - 1\right)^2\right) \\[2mm]
& - \left(\overline{p}_t^{xl} - \vartheta_t^y\right)\phi_x\left(\frac{\overline{\pi}_t^{xl}}{\overline{\pi}_{t-1}^{xl}} - 1\right)\frac{\overline{\pi}_t^{xl}}{\overline{\pi}_{t-1}^{xl}} \\[2mm]
& + E_t\left[D_{t,t+1}\pi_{t+1}^c\left(\overline{p}_{t+1}^{xl} - \vartheta_{t+1}^y\right)\frac{Y_{t+1}^{xl}}{Y_t^{xl}}\phi_x\left(\frac{\overline{\pi}_{t+1}^{xl}}{\overline{\pi}_t^{xl}} - 1\right)\frac{\overline{\pi}_{t+1}^{xl}}{\overline{\pi}_t^{xl}}\right]
\end{aligned}
\tag{A23}
$$

$$Y_t^{xl} = \left(\frac{p_t^{xl}}{p_t^x}\right)^{-\theta_t^x} Y_t^x \tag{A24}$$

$$Y_t^{xp} = \left(\frac{p_t^{xp}}{p_t^x}\right)^{-\theta_t^x} Y_t^x \tag{A25}$$

$$Y_t^x = \alpha_f \left(\frac{p_t^x}{s_t} \right)^{-\nu_f} Y_t^f \tag{A26}$$

$$H_t^m = \delta Y_t^m \tag{A27}$$

$$\overline{p}_t^m = \left[\varpi_f \left(\overline{p}_t^{mp} \right)^{1-\theta_t^m} + (1-\varpi_f)(\overline{p}_t^{ml})^{1-\theta_t^m} \right]^{\frac{1}{1-\theta_t^m}} \tag{A28}$$

$$p_t^{ml} = \overline{p}_t^{ml} + \delta w_t \tag{A29}$$

$$p_t^{mp} = \overline{p}_t^{mp} + \delta w_t \tag{A30}$$

$$p_t^m = \left[\varpi_f \left(p_t^{mp} \right)^{1-\theta_t^m} + (1-\varpi_f)(p_t^{ml})^{1-\theta_t^m} \right]^{\frac{1}{1-\theta_t^m}} \tag{A31}$$

$$
\begin{aligned}
0 \quad = \quad & \left(\overline{p}_t^{ml} - \theta_t^m \left(\overline{p}_t^{ml} - s_t \vartheta_t^f \right) \frac{\overline{p}_t^{ml}}{p_t^{ml}} \right) \left(1 - \frac{\phi_m}{2} \left(\frac{\overline{\pi}_t^{ml}}{\overline{\pi}_{t-1}^{ml}} - 1 \right)^2 \right) \\[2ex]
& - \left(\overline{p}_t^{ml} - s_t \vartheta_t^f \right) \phi_m \left(\frac{\overline{\pi}_t^{ml}}{\overline{\pi}_{t-1}^{ml}} - 1 \right) \frac{\overline{\pi}_t^{ml}}{\overline{\pi}_{t-1}^{ml}} \\[2ex]
& + E_t \left[D_{t,t+1}^f \frac{\pi_{t+1}^c}{\Delta S_{t+1}} \left(\overline{p}_{t+1}^{ml} - s_{t+1}\vartheta_{t+1}^f \right) \frac{Y_{t+1}^{ml}}{Y_t^{ml}} \phi_m \left(\frac{\overline{\pi}_{t+1}^{ml}}{\overline{\pi}_t^{ml}} - 1 \right) \frac{\overline{\pi}_{t+1}^{ml}}{\overline{\pi}_t^{ml}} \right]
\end{aligned} \tag{A32}
$$

$$
\begin{aligned}
0 \quad = \quad & \left(\overline{p}_t^{mp} - \theta_t^m \left(\overline{p}_t^{mp} - s_t \vartheta_t^f \right) \frac{\overline{p}_t^{mp}}{p_t^{mp}} \right) \left(1 - \frac{\phi_m}{2} \left(\frac{\overline{\pi}_t^{mp}}{\overline{\pi}_{t-1}^{mp}} - 1 \right)^2 \right) \\[2ex]
& - \left(\overline{p}_t^{mp} - s_t \vartheta_t^f \right) \phi_m \left(\frac{\overline{\pi}_t^{mp}}{\overline{\pi}_{t-1}^{mp}} - 1 \right) \frac{\overline{\pi}_t^{mp}}{\overline{\pi}_{t-1}^{mp}} \\[2ex]
& + E_t \left[D_{t,t+1}^f \frac{\pi_{t+1}^c}{\Delta S_{t+1}} \left(\overline{p}_{t+1}^{mp} - s_{t+1}\vartheta_{t+1}^f \right) \frac{Y_{t+1}^{mp}}{Y_t^{mp}} \phi_m \left(\frac{\overline{\pi}_{t+1}^{mp}}{\overline{\pi}_t^{mp}} - 1 \right) \frac{\overline{\pi}_{t+1}^{mp}}{\overline{\pi}_t^{mp}} \right]
\end{aligned} \tag{A33}
$$

$$Y_t^{ml} = \left(\frac{p_t^{ml}}{p_t^m} \right)^{-\theta_t^m} Y_t^m \tag{A34}$$

$$Y_t^{mp} = \left(\frac{p_t^{mp}}{p_t^m} \right)^{-\theta_t^m} Y_t^m \tag{A35}$$

$$\kappa_t = \exp \left(-\psi s_t b_t^f + u_t \right) \tag{A36}$$

$$D_{t,t+1} = \beta \frac{C_t - \zeta C_{t-1}}{C_{t+1} - \zeta C_t} \frac{1}{\pi_{t+1}^c} \tag{A37}$$

$$\frac{1}{R_t} \quad = \quad E_t D_{t,t+1} \tag{A38}$$

$$\frac{1}{\kappa_t R_t^f} \quad = \quad E_t \left[D_{t,t+1} \Delta S_{t+1} \right] \tag{A39}$$

$$0 = \frac{(C_t - \zeta C_{t-1})}{1 - \zeta} \frac{\theta^h \eta H_t^\chi}{w_t} - (\theta^h - 1) \left(1 - \frac{\phi_w}{2} \left(\frac{\pi_t^w}{\pi_{t-1}^w} - 1\right)^2\right) \tag{A40}$$

$$-\phi_w \left(\frac{\pi_t^w}{\pi_{t-1}^w} - 1\right) \frac{\pi_t^w}{\pi_{t-1}^w} + E_t \left[D_{t,t+1} \pi_{t+1}^w \frac{H_{t+1}}{H_t} \phi_w \left(\frac{\pi_{t+1}^w}{\pi_t^w} - 1\right) \frac{\pi_{t+1}^w}{\pi_t^w}\right]$$

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(R + \rho_\pi (\pi_t^c - \pi^c)) \tag{A41}$$

$$H_t = H_t^c + H_t^y + H_t^m \tag{A42}$$

$$\frac{s_t b_t^f}{\kappa_t R_t^f} = \frac{s_t b_{t-1}^f}{\pi_t^f} + \bar{p}_t^x Y_t^x - \bar{p}_t^m Y_t^m \tag{A43}$$

$$\ln \theta_t^c = (1 - \rho_c) \ln \theta^c + \rho_c \ln \theta_{t-1}^c + \varepsilon_{c,t} \tag{A44}$$

$$\ln \theta_t^y = (1 - \rho_y) \ln \theta^y + \rho_y \ln \theta_{t-1}^y + \varepsilon_{y,t} \tag{A45}$$

$$\ln \theta_t^x = (1 - \rho_x) \ln \theta^x + \rho_x \ln \theta_{t-1}^x + \varepsilon_{x,t} \tag{A46}$$

$$\ln \theta_t^m = (1 - \rho_m) \ln \theta^m + \rho_m \ln \theta_{t-1}^m + \varepsilon_{m,t} \tag{A47}$$

$$\ln u_t = \rho_u \ln u_{t-1} + \varepsilon_{u,t} \tag{A48}$$

$$\pi_t^y = \frac{p_t^y}{p_{t-1}^y} \pi_t^c \tag{A49}$$

$$\bar{\pi}_t^m = \frac{\bar{p}_t^m}{\bar{p}_{t-1}^m} \pi_t^c \tag{A50}$$

$$\bar{\pi}_t^{ml} = \frac{\bar{p}_t^{ml}}{\bar{p}_{t-1}^{ml}} \pi_t^c \tag{A51}$$

$$\bar{\pi}_t^{mp} = \frac{\bar{p}_t^{mp}}{\bar{p}_{t-1}^{mp}} \frac{\pi_t^c}{\Delta S_t} \tag{A52}$$

$$\pi_t^m = \frac{p_t^m}{p_{t-1}^m} \pi_t^c \tag{A53}$$

$$\Delta S_t = \frac{s_t}{s_{t-1}} \frac{\pi_t^c}{\pi_t^f} \tag{A54}$$

$$\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t^c \tag{A55}$$

$$\bar{\pi}_t^x = \frac{\bar{p}_t^x}{\bar{p}_{t-1}^x} \pi_t^c \tag{A56}$$

$$\bar{\pi}_t^{xl} = \frac{\bar{p}_t^{xl}}{\bar{p}_{t-1}^{xl}} \frac{\pi_t^c}{\Delta S_t} \tag{A57}$$

$$\bar{\pi}_t^{xp} = \frac{\bar{p}_t^{xp}}{\bar{p}_{t-1}^{xp}} \pi_t^c \tag{A58}$$

A.2   Steady-state model

Assuming that $\pi^c = \pi^f$ it follows that exchange rate depreciation is zero in the steady state ($\Delta S = 1$) and that $R = \kappa R^f$. If the discount factor is the same in both countries, we have $R^f = \pi^f/\beta = R$, which implies $\kappa = 1$ and zero net foreign assets in the steady-state ($b^f = 0$). Moreover, in the steady-state, $\overline{p}^{ml} = \overline{p}^{mp} = \overline{p}^m$, $p^{ml} = p^{mp} = p^m$ and $p^{xl} = p^{xp} = p^x$. Hence $Y^{ml} = Y^{mp} = Y^m$ and $Y^{xl} = Y^{xp} = Y^x$. The steady-state levels of foreign demand and foreign wages are normalised to unity ($y^f = w^f = 1$). The following system of equations defines the steady-state levels of $C, Q, Q^d, Q^m, Y, Y^d, Y^x, Y^m, Z, Z^d, Z^m, H, H^c, H^y, H^m, s, p^y, p^q, p^z, \overline{p}^m, p^m, p^x,$ $\overline{p}^x, w, \vartheta^c$ and ϑ^y.

$$C = Q^{\gamma_c}(H^c)^{1-\gamma_c} \tag{A59}$$

$$Q^d = \alpha\left(\frac{p^y}{p^q}\right)^{-\nu}Q \tag{A60}$$

$$Q^m = (1-\alpha)\left(\frac{p^m}{p^q}\right)^{-\nu}Q \tag{A61}$$

$$w = \vartheta^c(1-\gamma_c)\frac{C}{H^c} \tag{A62}$$

$$p^q = \vartheta^c\gamma_c\frac{C}{Q} \tag{A63}$$

$$p^q = \left[\alpha(p^y)^{1-\nu} + (1-\alpha)(p^m)^{1-\nu}\right]^{\frac{1}{1-\nu}} \tag{A64}$$

$$1 = \frac{\theta^c}{\theta^c-1}\vartheta^c$$

$$Y = Z^{\gamma_y}(H^y)^{1-\gamma_y} \tag{A65}$$

$$Z^d = \alpha\left(\frac{p^y}{p^z}\right)^{-\nu}Z \tag{A66}$$

$$Z^m = (1-\alpha)\left(\frac{p^m}{p^z}\right)^{-\nu}Z \tag{A67}$$

$$w = \vartheta^y(1-\gamma_y)\frac{Y}{H^y} \tag{A68}$$

$$p^z = \vartheta^y\gamma_y\frac{Y}{Z} \tag{A69}$$

$$p^z = \left[\alpha(p^y)^{1-\nu} + (1-\alpha)(p^m)^{1-\nu}\right]^{\frac{1}{1-\nu}} \tag{A70}$$

$$Y^d = Q^d + Z^d \tag{A71}$$

$$Y^m = Q^m + Z^m \tag{A72}$$

$$Y = Y^d + Y^x \tag{A73}$$

$$p^y = \frac{\theta^y}{\theta^y-1}\vartheta^y \tag{A74}$$

$$Y^x = \alpha_f\left(\frac{p^x}{s}\right)^{-\nu_f} \tag{A75}$$

$$\overline{p}^x = \frac{\theta^x}{\theta^x - 1}\vartheta^y + \frac{\delta_f}{\theta_x - 1}s \tag{A76}$$

$$p^x = \overline{p}^x + \delta_f s \tag{A77}$$

$$H^m = \delta Y^m \tag{A78}$$

$$p^m = \overline{p}^m + \delta w \tag{A79}$$

$$\overline{p}^m = \frac{\theta^m}{\theta^m - 1}s\vartheta^f + \frac{\delta}{\theta^m - 1}w \tag{A80}$$

$$w = \frac{\theta^h}{\theta^h - 1}\eta H^\chi C \tag{A81}$$

$$H = H^c + H^y + H^m \tag{A82}$$

$$\overline{p}^m Y^m = \overline{p}^x Y^x \tag{A83}$$

A.3   Log-linearised model

Letting variables with a hat denote percentage deviations from the deterministic steady state (i.e., $\widehat{X}_t = \ln X_t - \ln X$), the log-linearised equilibrium conditions can be written

$$\widehat{C}_t = \gamma_c \widehat{Q}_t + (1 - \gamma_c)\widehat{H}_t^c \tag{A84}$$

$$\widehat{Q}_t^d = -\nu\left(\widehat{p}_t^y - \widehat{p}_t^q\right) + \widehat{Q}_t \tag{A85}$$

$$\widehat{Q}_t^m = -\nu\left(\widehat{p}_t^m - \widehat{p}_t^q\right) + \widehat{Q}_t \tag{A86}$$

$$\widehat{w}_t = \widehat{\vartheta}_t^c + \widehat{C}_t - \widehat{H}_t^c \tag{A87}$$

$$\widehat{p}_t^q = \widehat{\vartheta}_t^c + \widehat{C}_t - \widehat{Q}_t \tag{A88}$$

$$\widehat{p}_t^q = \alpha\left(\frac{p^y}{p^q}\right)^{1-\nu}\widehat{p}_t^y + (1-\alpha)\left(\frac{p^m}{p^q}\right)^{1-\nu}\widehat{p}_t^m \tag{A89}$$

$$\widehat{\pi}_t^c = \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^c + \frac{1}{1+\beta}\widehat{\pi}_{t-1}^c + \frac{\theta^c(\theta^c-1)}{(1+\beta)\phi_c}\widehat{\vartheta}_t^c - \frac{\theta^c}{(1+\beta)\phi_c}\widehat{\theta}_t^c \tag{A90}$$

$$\widehat{Y}_t = \gamma_y\widehat{Z}_t + (1-\gamma_y)\widehat{H}_t^y \tag{A91}$$

$$\widehat{Z}_t^d = -\nu\left(\widehat{p}_t^y - \widehat{p}_t^{\tilde{z}}\right) + \widehat{Z}_t \tag{A92}$$

$$\widehat{Z}_t^m = -\nu\left(\widehat{p}_t^m - \widehat{p}_t^{\tilde{z}}\right) + \widehat{Z}_t \tag{A93}$$

$$\widehat{w}_t = \widehat{\vartheta}_t^y + \widehat{Y}_t - \widehat{H}_t^y \tag{A94}$$

$$\widehat{p}_t^{\tilde{z}} = \widehat{\vartheta}_t^y + \widehat{Y}_t - \widehat{Z}_t \tag{A95}$$

$$\widehat{p}_t^{\tilde{z}} = \alpha\left(\frac{p^y}{p^z}\right)^{1-\nu}\widehat{p}_t^y + (1-\alpha)\left(\frac{p^m}{p^z}\right)^{1-\nu}\widehat{p}_t^m \tag{A96}$$

$$\widehat{Y}_t^d = \left(\frac{Q^d}{Y^d}\right)\widehat{Q}_t^d + \left(\frac{Z^d}{Y^d}\right)\widehat{Z}_t^d \tag{A97}$$

$$\widehat{Y}_t^m = \left(\frac{Q^m}{Y^m}\right)\widehat{Q}_t^m + \left(\frac{Z^m}{Y^m}\right)\widehat{Z}_t^m \tag{A98}$$

$$\widehat{Y}_t = \left(\frac{Y^d}{Y}\right)\widehat{Y}_t^d + \left(\frac{Y^x}{Y}\right)\widehat{Y}_t^x \tag{A99}$$

$$\widehat{\pi}_t^y = \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^y + \frac{1}{1+\beta}\widehat{\pi}_{t-1}^y + \frac{\theta^y(\theta^y-1)}{(1+\beta)\phi_y}\left(\widehat{\vartheta}_t^y - \widehat{p}_t^y\right) - \frac{\theta^y}{(1+\beta)\phi_y}\widehat{\theta}_t^y \tag{A100}$$

$$\widehat{\overline{p}}_t^x = \varpi\widehat{\overline{p}}_t^{xp} + (1-\varpi)\widehat{\overline{p}}_t^{xl} \tag{A101}$$

$$\widehat{p}_t^x = \left(\frac{\overline{p}^x}{p^x}\right)\widehat{\overline{p}}_t^x + \delta_f\left(\frac{sw^f}{p^x}\right)\left(\widehat{w}_t^f + \widehat{s}_t\right) \tag{A102}$$

$$\widehat{\overline{\pi}}_t^{xl} = \frac{\beta}{1+\beta}E_t\widehat{\overline{\pi}}_{t+1}^{xl} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{xl} - \frac{\theta^x-1}{(1+\beta)\phi_x}\frac{\overline{p}^x}{\overline{p}^x-\vartheta^y}\frac{\overline{p}^x}{p^x}\left(\widehat{\overline{p}}_t^{xl} - \left(\widehat{\overline{p}}_t^x\right)^{opt}\right) \tag{A103}$$

$$\widehat{\overline{\pi}}_t^{xp} = \frac{\beta}{1+\beta}E_t\widehat{\overline{\pi}}_{t+1}^{xp} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{xp} - \frac{\theta^x-1}{(1+\beta)\phi_x}\frac{\overline{p}^x}{\overline{p}^x-\vartheta^y}\frac{\overline{p}^x}{p^x}\left(\widehat{\overline{p}}_t^{xp} - \left(\widehat{\overline{p}}_t^x\right)^{opt}\right) \tag{A104}$$

$$\overline{p}^x\left(\widehat{\overline{p}}_t^x\right)^{opt} = \frac{\theta^x}{\theta^x-1}\vartheta^y\left(\widehat{\vartheta}_t^y - \frac{1}{\theta^x-1}\widehat{\theta}_t^x\right) + \frac{\delta_f}{\theta^x-1}sw^f\left(\widehat{s}_t + \widehat{w}_t^f - \frac{\theta^x}{\theta^x-1}\widehat{\theta}_t^x\right) \tag{A105}$$

$$\widehat{Y}_t^x = -\nu_f\left(\widehat{p}_t^x - \widehat{s}_t\right) + \widehat{Y}_t^f \tag{A106}$$

$$\widehat{H}_t^m = \widehat{Y}_t^m \tag{A107}$$

$$\widehat{p}_t^m = \left(\frac{\overline{p}^m}{p^m}\right)\widehat{\overline{p}}_t^m + \delta\left(\frac{w}{p^m}\right)\widehat{w}_t \tag{A108}$$

$$\widehat{\overline{p}}_t^m = \varpi_f\widehat{\overline{p}}_t^{mp} + (1-\varpi_f)\widehat{\overline{p}}_t^{ml} \tag{A109}$$

$$\widehat{\overline{\pi}}_t^{ml} = \frac{\beta}{1+\beta}E_t\widehat{\overline{\pi}}_{t+1}^{ml} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{ml} - \frac{\theta^m-1}{(1+\beta)\phi_m}\frac{\overline{p}^m}{\overline{p}^m-s\vartheta^f}\frac{\overline{p}^m}{p^m}\left(\widehat{\overline{p}}_t^{ml} - \left(\widehat{\overline{p}}_t^m\right)^{opt}\right) \tag{A110}$$

$$\widehat{\overline{\pi}}_t^{mp} = \frac{\beta}{1+\beta}E_t\widehat{\overline{\pi}}_{t+1}^{mp} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{mp} - \frac{\theta^m-1}{(1+\beta)\phi_m}\frac{\overline{p}^m}{\overline{p}^m-s\vartheta^f}\frac{\overline{p}^m}{p^m}\left(\widehat{\overline{p}}_t^{mp} - \left(\widehat{\overline{p}}_t^m\right)^{opt}\right) \tag{A111}$$

$$\overline{p}^m\left(\widehat{\overline{p}}_t^m\right)^{opt} = \frac{\theta^m}{\theta^m-1}s\vartheta^f\left(\widehat{s}_t + \widehat{\vartheta}_t^f - \frac{1}{\theta^m-1}\widehat{\theta}_t^m\right) + \frac{\delta}{\theta^m-1}w\left(\widehat{w}_t - \frac{\theta^m}{\theta^m-1}\widehat{\theta}_t^m\right) \tag{A112}$$

$$\widehat{C}_t = \frac{1}{1+\zeta}E_t\widehat{C}_{t+1} + \frac{\zeta}{1+\zeta}\widehat{C}_{t-1} - \frac{1-\zeta}{1+\zeta}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}^c\right) \tag{A113}$$

$$\widehat{R}_t = \widehat{R}_t^f + E_t\widehat{\Delta S}_{t+1} + \widehat{\kappa}_t \tag{A114}$$

$$\widehat{\pi}_t^w = \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^w + \frac{1}{1+\beta}\widehat{\pi}_{t-1}^w - \frac{\theta^h-1}{(1+\beta)\phi_w}\left(\widehat{w}_t - \chi\widehat{H}_t - \frac{\widehat{C}_t - \zeta\widehat{C}_{t-1}}{1-\zeta}\right) \tag{A115}$$

$$\widehat{\kappa}_t = -\psi s\widehat{b}_t^f + \widehat{u}_t \tag{A116}$$

$$\widehat{R}_t = \rho_R\widehat{R}_{t-1} + \frac{(1-\rho_R)\rho_\pi\pi_c}{R}\widehat{\pi}_t^c \tag{A117}$$

$$\widehat{H}_t = \left(\frac{H^c}{H}\right)\widehat{H}_t^c + \left(\frac{H^y}{H}\right)\widehat{H}_t^y + \left(\frac{H^m}{H}\right)\widehat{H}_t^m \tag{A118}$$

$$\frac{s\beta\widehat{b}_t^f}{\pi^f} = \frac{s\widehat{b}_{t-1}^f}{\pi^f} + \overline{p}^xY^x\left(\widehat{\overline{p}}_t^x + \widehat{Y}_t^x\right) - \overline{p}^mY^m\left(\widehat{\overline{p}}_t^m + \widehat{Y}_t^m\right) \tag{A119}$$

$$\widehat{\theta}_t^c \quad = \quad \rho_c \widehat{\theta}_{t-1}^c + \varepsilon_{c,t} \tag{A120}$$

$$\widehat{\theta}_t^y \quad = \quad \rho_y \widehat{\theta}_{t-1}^y + \varepsilon_{y,t} \tag{A121}$$

$$\widehat{\theta}_t^x \quad = \quad \rho_x \widehat{\theta}_{t-1}^x + \varepsilon_{x,t} \tag{A122}$$

$$\widehat{\theta}_t^m \quad = \quad \rho_m \widehat{\theta}_{t-1}^m + \varepsilon_{m,t} \tag{A123}$$

$$\widehat{u}_t = \rho_u \widehat{u}_{t-1} + \widehat{\varepsilon}_{u,t} \tag{A124}$$

$$\widehat{\pi}_t^y = \widehat{p}_t^y - \widehat{p}_{t-1}^y + \widehat{\pi}_t^c \tag{A125}$$

$$\widehat{\overline{\pi}}_t^m = \widehat{\overline{p}}_t^m - \widehat{\overline{p}}_{t-1}^m + \widehat{\pi}_t^c \tag{A126}$$

$$\widehat{\pi}_t^m = \widehat{p}_t^m - \widehat{p}_{t-1}^m + \widehat{\pi}_t^c \tag{A127}$$

$$\widehat{\overline{\pi}}_t^{mp} = \widehat{\overline{p}}_t^{mp} - \widehat{\overline{p}}_{t-1}^{mp} + \widehat{\pi}_t^c - \widehat{\Delta S}_t \tag{A128}$$

$$\widehat{\overline{\pi}}_t^{ml} = \widehat{\overline{p}}_t^{ml} - \widehat{\overline{p}}_{t-1}^{ml} + \widehat{\pi}_t^c \tag{A129}$$

$$\widehat{\Delta S}_t = \widehat{s}_t - \widehat{s}_{t-1} + \widehat{\pi}_t^c - \widehat{\pi}_t^f \tag{A130}$$

$$\widehat{\pi}_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \widehat{\pi}_t^c \tag{A131}$$

$$\widehat{\overline{\pi}}_t^x = \widehat{\overline{p}}_t^x - \widehat{\overline{p}}_{t-1}^x + \widehat{\pi}_t^c \tag{A132}$$

$$\widehat{\overline{\pi}}_t^{xl} = \widehat{\overline{p}}_t^{xl} - \widehat{\overline{p}}_{t-1}^{xl} + \widehat{\pi}_t^c - \widehat{\Delta S}_t \tag{A133}$$

$$\widehat{\overline{\pi}}_t^{xp} = \widehat{\overline{p}}_t^{xp} - \widehat{\overline{p}}_{t-1}^{xp} + \widehat{\pi}_t^c \tag{A134}$$

B   MAPPING FROM THE VAR IN RELATIVE PRICES TO A VEQCM

The solution procedure for the rational expectations model requires that the model is cast in stationary form. Dynare provides log-linear decision rules for relative prices, the real exchange rate and consumer price inflation. The population VAR(5) for these variables can be expressed as[29]

$$
\begin{bmatrix}
\ln \overline{P}_t^m - \ln P_t^c \\
\ln \overline{P}_t^x - \ln P_t^c \\
\ln P_t^y - \ln P_t^c \\
\ln S_t + \ln P_t^f - \ln P_t^c \\
\Delta \ln P_t^c
\end{bmatrix}
= \mu + \sum_{s=1}^{5} A_s
\begin{bmatrix}
\ln \overline{P}_{t-s}^m - \ln P_{t-s}^c \\
\ln \overline{P}_{t-s}^x - \ln P_{t-s}^c \\
\ln P_{t-s}^y - \ln P_{t-s}^c \\
\ln S_{t-s} + \ln P_{t-s}^f - \ln P_{t-s}^c \\
\Delta \ln P_{t-s}^c
\end{bmatrix}
+ u_t
$$

or, alternatively, as

$$
\begin{bmatrix} \beta' x_t \\ v' \Delta x_t \end{bmatrix} = \sum_{s=1}^{5} A_s \begin{bmatrix} \beta' x_{t-s} \\ v' \Delta x_{t-s} \end{bmatrix} + u_t
$$

where

$$
\beta' = \begin{bmatrix}
1 & 0 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 \\
0 & 0 & -1 & 0 & 1
\end{bmatrix}
$$

$$
v' = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}
$$

$$
x_t' = \begin{bmatrix} \ln \overline{P}_t^m & \ln \overline{P}_t^x & \ln P_t^c & \ln P_t^y & \ln S_t + \ln P_t^f \end{bmatrix}
$$

To derive the VEqCM representation

$$
\Delta x_t = \mu_x + \alpha \beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-3} + \Gamma_4 \Delta x_{t-4} + e_t,
$$

I write the VAR in relative prices as (see Kongsted & Nielsen, 2004)

$$
\begin{bmatrix} \beta' x_t \\ v' \Delta x_t \end{bmatrix} = \begin{bmatrix} 0 \\ v' \mu_x \end{bmatrix} + \begin{bmatrix} \beta' \Delta x_t + \beta' x_{t-1} \\ v' \alpha \beta' x_{t-1} + v' \Gamma_1 \Delta x_{t-1} + v' \Gamma_2 \Delta x_{t-2} + v' \Gamma_3 \Delta x_{t-3} + v' \Gamma_4 \Delta x_{t-4} + v' e_t \end{bmatrix},
$$

and substitute in for Δx_t in equation for $\beta' x_t$

$$
\begin{bmatrix} \beta' x_t \\ v' \Delta x_t \end{bmatrix} = \begin{bmatrix} \beta' \mu_x \\ v' \mu_x \end{bmatrix} + \begin{bmatrix} \beta'(\alpha \beta' + I) x_{t-1} + \beta' \Gamma_1 \Delta x_{t-1} + \beta' \Gamma_2 \Delta x_{t-2} + \beta' \Gamma_3 \Delta x_{t-3} + \beta' \Gamma_4 \Delta x_{t-4} + \beta' e_t \\ v' \alpha \beta' x_{t-1} + v' \Gamma_1 \Delta x_{t-1} + v' \Gamma_2 \Delta x_{t-2} + v' \Gamma_3 \Delta x_{t-3} + v' \Gamma_4 \Delta x_{t-4} + v' e_t \end{bmatrix}.
$$

---

[29]The population version VAR is derived from the log-linear solution to the DSGE model using the Matlab program `ssvar.m`.

Now, noting that

$$\Delta x_t = v_\perp (\beta' v_\perp)^{-1} \beta' \Delta x_t + \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_t,$$

where $\beta_\perp$ and $v_\perp$ are defined as

$$\beta_\perp = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}'$$

$$v_\perp = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

I substitute in for $\Delta x_{t-1}, \Delta x_{t-2}, \Delta x_{t-3}, \Delta x_{t-4}$ and obtain

$$
\begin{aligned}
\beta' x_t =\ & \beta' \mu_x + \left(\beta' \alpha + I\right) \beta' x_{t-1} \\
& + \beta' \Gamma_1 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-1} - \beta' \Gamma_1 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-2} + \beta' \Gamma_1 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-1} \\
& + \beta' \Gamma_2 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-2} - \beta' \Gamma_2 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-3} + \beta' \Gamma_2 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-2} \\
& + \beta' \Gamma_3 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-3} - \beta' \Gamma_3 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-4} + \beta' \Gamma_3 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-3} \\
& + \beta' \Gamma_4 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-4} - \beta' \Gamma_4 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-5} + \beta' \Gamma_4 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-4} \\
& + \beta' e_t
\end{aligned}
$$

$$
\begin{aligned}
v' \Delta x_t =\ & v' \mu_x + v' \alpha \beta' x_{t-1} \\
& + v' \Gamma_1 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-1} - v' \Gamma_1 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-2} + v' \Gamma_1 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-1} \\
& + v' \Gamma_2 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-2} - v' \Gamma_2 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-3} + v' \Gamma_2 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-2} \\
& + v' \Gamma_3 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-3} - v' \Gamma_3 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-4} + v' \Gamma_3 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-3} \\
& + v' \Gamma_4 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-4} - v' \Gamma_4 v_\perp (\beta' v_\perp)^{-1} \beta' x_{t-5} + v' \Gamma_4 \beta_\perp (v' \beta_\perp)^{-1} v' \Delta x_{t-4} \\
& + v' e_t
\end{aligned}
$$

In matrix notation

$$
\begin{bmatrix} \beta'x_t \\ v'\Delta x_t \end{bmatrix} = \begin{bmatrix} (\beta'\alpha+I)+\beta'\Gamma_1 v_\perp(\beta'v_\perp)^{-1} & \beta'\Gamma_1\beta_\perp(v'\beta_\perp)^{-1} \\ v'\alpha+v'\Gamma_1 v_\perp(\beta'v_\perp)^{-1} & v'\Gamma_1\beta_\perp(v'\beta_\perp)^{-1} \end{bmatrix} \begin{bmatrix} \beta'x_{t-1} \\ v'\Delta x_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} -\beta'\Gamma_1 v_\perp(\beta'v_\perp)^{-1}+\beta'\Gamma_2 v_\perp(\beta'v_\perp)^{-1} & \beta'\Gamma_2\beta_\perp(v'\beta_\perp)^{-1} \\ -v'\Gamma_1 v_\perp(\beta'v_\perp)^{-1}+v'\Gamma_2 v_\perp(\beta'v_\perp)^{-1} & v'\Gamma_2\beta_\perp(v'\beta_\perp)^{-1} \end{bmatrix} \begin{bmatrix} \beta'x_{t-2} \\ v'\Delta x_{t-2} \end{bmatrix}
$$

$$
+ \begin{bmatrix} -\beta'\Gamma_2 v_\perp(\beta'v_\perp)^{-1}+\beta'\Gamma_3 v_\perp(\beta'v_\perp)^{-1} & \beta'\Gamma_3\beta_\perp(v'\beta_\perp)^{-1} \\ -v'\Gamma_2 v_\perp(\beta'v_\perp)^{-1}+v'\Gamma_3 v_\perp(\beta'v_\perp)^{-1} & v'\Gamma_3\beta_\perp(v'\beta_\perp)^{-1} \end{bmatrix} \begin{bmatrix} \beta'x_{t-3} \\ v'\Delta x_{t-3} \end{bmatrix}
$$

$$
+ \begin{bmatrix} -\beta'\Gamma_3 v_\perp(\beta'v_\perp)^{-1}+\beta'\Gamma_4 v_\perp(\beta'v_\perp)^{-1} & \beta'\Gamma_4\beta_\perp(v'\beta_\perp)^{-1} \\ -v'\Gamma_3 v_\perp(\beta'v_\perp)^{-1}+v'\Gamma_4 v_\perp(\beta'v_\perp)^{-1} & v'\Gamma_4\beta_\perp(v'\beta_\perp)^{-1} \end{bmatrix} \begin{bmatrix} \beta'x_{t-4} \\ v'\Delta x_{t-4} \end{bmatrix}
$$

$$
+ \begin{bmatrix} -\beta'\Gamma_4 v_\perp(\beta'v_\perp)^{-1} & 0 \\ -v'\Gamma_4 v_\perp(\beta'v_\perp)^{-1} & 0 \end{bmatrix} \begin{bmatrix} \beta'x_{t-5} \\ v'\Delta x_{t-5} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \beta'e_t \\ v'e_t \end{bmatrix} + \begin{bmatrix} \beta'\mu_x \\ v'\mu_x \end{bmatrix}
$$

More generally, the relationship between the parameters of the VEqCM and the VAR(p) in relative prices, the real exchange rate and consumer price is

$$
A_1 = \begin{bmatrix} (\beta'\alpha+I)+\beta'\Gamma_1 v_\perp(\beta'v_\perp)^{-1} & \beta'\Gamma_1\beta_\perp(v'\beta_\perp)^{-1} \\ v'\alpha+v'\Gamma_1 v_\perp(\beta'v_\perp)^{-1} & v'\Gamma_1\beta_\perp(v'\beta_\perp)^{-1} \end{bmatrix}
$$

$$
A_j = \begin{bmatrix} -\beta'\Gamma_{j-1} v_\perp(\beta'v_\perp)^{-1}+\beta'\Gamma_j v_\perp(\beta'v_\perp)^{-1} & \beta'\Gamma_j\beta_\perp(v'\beta_\perp)^{-1} \\ -v'\Gamma_{j-1} v_\perp(\beta'v_\perp)^{-1}+v'\Gamma_j v_\perp(\beta'v_\perp)^{-1} & v'\Gamma_j\beta_\perp(v'\beta_\perp)^{-1} \end{bmatrix}, \quad j=2,\ldots,p-1
$$

$$
A_p = \begin{bmatrix} -\beta'\Gamma_{p-1} v_\perp(\beta'v_\perp)^{-1} & 0 \\ -v'\Gamma_{p-1} v_\perp(\beta'v_\perp)^{-1} & 0 \end{bmatrix}
$$

$$
\mu = \begin{bmatrix} \beta'\mu_x \\ v'\mu_x \end{bmatrix}
$$

Having obtained estimates of $A_1,\ldots,A_p$ I can now solve for $\alpha,\Gamma_1,\ldots,\Gamma_{p-1}$. The relationship between the variance-covariance matrices in the two representations is

$$
\begin{aligned}
\Omega &= E\left[u_t u_t'\right] \\
&= E\left\{ \begin{bmatrix} \beta'e_t \\ v'e_t \end{bmatrix} \begin{bmatrix} \beta'e_t \\ v'e_t \end{bmatrix}^T \right\} \\
&= \begin{bmatrix} \beta'E\left[e_t e_t'\right]\beta & \beta E\left[e_t e_t'\right]v' \\ v'E\left[e_t e_t'\right]\beta' & v'E\left[e_t e_t'\right]v' \end{bmatrix} \\
&= \begin{bmatrix} \beta' \\ v' \end{bmatrix} E[e_t e_t'] \begin{bmatrix} \beta' \\ v' \end{bmatrix}^T
\end{aligned}
$$

C   THE POWER OF THE COINTEGRATION TEST

This appendix reports additional Monte Carlo evidence on the cointegration test. The data generating process is a population VAR(5) in $x_t' = \{\ln \overline{P}_t^m, \ln \overline{P}_t^x, \ln P_t^c, \ln P_t^y, \ln S_t + \ln P_t^f\}$ derived from the log-linearised solution to the DSGE model using the mapping described in appendix B. The data generating process is I(1), and the cointegration rank is four. The results are based on 5000 Monte Carlo replications in PcNaive version 2.01 (Doornik & Hendry, 2001). The estimated model is a VAR(5) in levels of the variables with an unrestricted constant.

Figure 17 plots the recursively computed rejection frequencies for the trace test for sample sizes from $T = 100$ to $T = 1000$. It is evident from the graph that the power of the trace test is low, even in fairly large samples. As shown by for example, Eitrheim (1992), the power of the trace test depends both on the sample size and properties of the data generating process. In particular, the power depends on the size of the cointegration eigenvalues; power is increasing in the size of the eigenvalues with non-zero asymptotic limits. Figure 18 plots the recursively computed cointegration eigenvalues. The smallest eigenvalue converges to zero, while the four remaining eigenvalues converge to non-zero, but numerically small numbers. Specifically, the eigenvalues converge to[30]

$$\begin{bmatrix} 0.060 & 0.054 & 0.044 & 0.024 & 0.000 \end{bmatrix}$$

Thus, the calibration of the DSGE model implies that the cointegration eigenvalues will be small. The effect is that the power of the trace test is very low in small samples. The size of the cointegration eigenvalues is related to the size of the adjustment coefficients (α). The matrix of adjustment coefficients in the population VAR is

$$\begin{bmatrix} -0.0194 & -0.0014 & 0.0079 & -0.0420 \\ 0.0025 & -0.0167 & 0.0067 & -0.0374 \\ 0.0029 & -0.0010 & 0.0047 & 0.0011 \\ 0.0056 & -0.0008 & -0.0171 & 0.0008 \\ -0.0021 & -0.0027 & 0.0154 & -0.1270 \end{bmatrix}$$

The adjustment coefficients are numerically small, except for the coefficient to the real exchange rate in the exchange rate equation (-0.1270). Eitrheim (1992) refers to the case with 'small' values in a column of the matrix of adjustment coefficients as a case of 'near cointegration'. Consistent with the findings in this paper, he shows that when the cointegration relations enter in the process with very small adjustment coefficients, they are difficult to detect.

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[30]Computed from a VAR estimated on a sample size $T = 500000$.

As a final exercise I examined the power of augmented Dickey-Fuller (ADF) tests (Dickey & Fuller, 1979; Said & Dickey, 1984) for a unit root in relative prices and the real exchange rate. Figure 19 shows the recursively computed rejection frequencies at the 5% level. The ADF regression includes a constant term and two lags (in first differences). The results are based on 5000 Monte Carlo simulations in PcNaive for sample sizes from $T = 25$ to $T = 300$. With a sample size $T = 100$, the rejection frequencies range from 35% for relative export prices to 70% for relative import prices.

D   VARIABLE DEFINITIONS AND SOURCES

- $\overline{P}^m$: Import price of manufactured goods, local currency (source: OECD International Trade and Competitiveness Indicators).[31]

- S: Nominal effective exchange rate (source: OECD Economic Outlook [Q.GBR.EXCHEB]).

- P^c: RPIX, retail price index excl. mortgage interest payments (source: UK National Statistics [CHMK]/Bank of England).[32]

- $\overline{P}^x$: Export price of manufactured goods, local currency (source: OECD International Trade and Competitiveness Indicators).

- P^y: Producer price index all manufacturing excl. duty (source: UK National Statistics [PVNQ]).

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[31] All nominal variables are converted to a common baseyear 2000=100.

[32] As no official seasonally adjusted RPIX exists this series was seasonally adjusted using the X12 method as implemented in EViews.

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Table 1: *The price and quantity indices in the DSGE model*

Quantity index	Price index	Demand functions
$Q_t = \left[\alpha^{\frac{1}{\nu}} (Q_t^d)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} (Q_t^m)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$	$P_t^q = \left[\alpha (P_t^y)^{1-\nu} + (1-\alpha)(P_t^{pm})^{1-\nu} \right]^{\frac{1}{1-\nu}}$	$Q_t^d = \alpha \left(\frac{P_t^y}{P_t^q}\right)^{-\nu} Q_t, \quad Q_t^m = (1-\alpha)\left(\frac{P_t^{pm}}{P_t^q}\right)^{-\nu} Q_t$
$Q_t^d = \left[\int_0^1 Y_t^{dq}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di \right]^{\frac{\theta_t^y}{\theta_t^y-1}}$	$P_t^y = \left[\int_0^1 P_t^y(i)^{1-\theta_t^y} di \right]^{\frac{1}{1-\theta_t^y}}$	$Y_t^{dq}(i) = \left(\frac{P_t^y(i)}{P_t^y}\right)^{-\theta_t^y} Q_t^d$
$Q_t^m = \left[\int_0^1 Y_t^{mq}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^m = \left[\int_0^1 P_t^m(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mq}(m) = \left(\frac{P_t^m(m)}{P_t^m}\right)^{-\theta_t^m} Q_t^m$
$Z_t = \left[\alpha^{\frac{1}{\nu}} (Z_t^d)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} (Z_t^m)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$	$P_t^z = \left[\alpha (P_t^y)^{1-\nu} + (1-\alpha)(P_t^m)^{1-\nu} \right]^{\frac{1}{1-\nu}}$	$Z_t^d = \alpha \left(\frac{P_t^y}{P_t^z}\right)^{-\nu} Z_t, \quad Z_t^m = (1-\alpha)\left(\frac{P_t^m}{P_t^z}\right)^{-\nu} Z_t$
$Z_t^d = \left[\int_0^1 Y_t^{dz}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di \right]^{\frac{\theta_t^y}{\theta_t^y-1}}$	$P_t^y = \left[\int_0^1 P_t^y(i)^{1-\theta_t^y} di \right]^{\frac{1}{1-\theta_t^y}}$	$Y_t^{dz}(i) = \left(\frac{P_t^y(i)}{P_t^y}\right)^{-\theta_t^y} Z_t^d$
$Z_t^m = \left[\int_0^1 Y_t^{mz}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^m = \left[\int_0^1 P_t^m(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mz}(m) = \left(\frac{P_t^m(m)}{P_t^m}\right)^{-\theta_t^m} Z_t^m$
$Y_t^x = \left[\varpi^{\frac{1}{\theta_t^x}} (Y_t^{xp})^{\frac{\theta_t^x-1}{\theta_t^x}} + (1-\varpi)^{\frac{1}{\theta_t^x}} (Y_t^{xl})^{\frac{\theta_t^x-1}{\theta_t^x}} \right]^{\frac{\theta_t^x}{\theta_t^x-1}}$	$P_t^x = \left[\varpi (P_t^{xp})^{1-\theta_t^x} + (1-\varpi)(S_t P_t^{xl})^{1-\theta_t^x} \right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xp} = \varpi \left(\frac{P_t^{xp}}{P_t^x}\right)^{-\theta_t^x} Y_t^x, \quad Y_t^{xl} = (1-\varpi)\left(\frac{S_t P_t^{xl}}{P_t^x}\right)^{-\theta_t^x} Y_t^x$
$Y_t^{xp} = \left[\left(\frac{1}{\varpi}\right)^{\frac{1}{\theta_t^x}} \int_0^\varpi Y_t^{xp}(i)^{\frac{\theta_t^x-1}{\theta_t^x}} di \right]^{\frac{\theta_t^x}{\theta_t^x-1}}$	$P_t^{xp} = \left[\frac{1}{\varpi} \int_0^\varpi P_t^{xp}(i)^{1-\theta_t^x} di \right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xp}(i) = \frac{1}{\varpi} \left(\frac{P_t^{xp}(i)}{P_t^{xp}}\right)^{-\theta_t^x} Y_t^{xp}$
$Y_t^{xl} = \left[\left(\frac{1}{1-\varpi}\right)^{\frac{1}{\theta_t^x}} \int_\varpi^1 Y_t^{xl}(i)^{\frac{\theta_t^x-1}{\theta_t^x}} di \right]^{\frac{\theta_t^x}{\theta_t^x-1}}$	$P_t^{xl} = \left[\frac{1}{1-\varpi} \int_\varpi^1 P_t^{xl}(i)^{1-\theta_t^x} di \right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xl}(i) = \frac{1}{1-\varpi} \left(\frac{P_t^{xl}(i)}{P_t^{xl}}\right)^{-\theta_t^x} Y_t^{xl}$

Quantity index	Price index	Demand functions

$$Y_t^m = \left[(\varpi_f)^{\frac{1}{\theta_t^m}} (Y_t^{mp})^{\frac{\theta_t^m-1}{\theta_t^m}} + (1-\varpi_f)^{\frac{1}{\theta_t^m}} (Y_t^{ml})^{\frac{\theta_t^m-1}{\theta_t^m}} \right]^{\frac{\theta_t^m}{\theta_t^m-1}}$$

$$P_t^m = \left[\varpi_f (S_t P_t^{mp})^{1-\theta_t^m} + (1-\varpi_f)(P_t^{ml})^{1-\theta_t^m} \right]^{\frac{1}{1-\theta_t^m}}$$

$$Y_t^{mp} = \varpi_f \left(\frac{S_t P_t^{mp}}{P_t^m} \right)^{-\theta_t^m} Y_t^m ,$$
$$Y_t^{ml} = (1-\varpi_f) \left(\frac{P_t^{ml}}{P_t^m} \right)^{-\theta_t^m} Y_t^m$$

$$Y_t^{mp} = \left[\left(\frac{1}{\varpi_f}\right)^{\frac{1}{\theta_t^m}} \int_0^{\varpi_f} Y_t^{mp}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m-1}}$$

$$P_t^{mp} = \left[\frac{1}{\varpi_f} \int_0^{\varpi_f} P_t^{mp}(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$$

$$Y_t^{mp}(m) = \frac{1}{\varpi_f} \left(\frac{P_t^{mp}(m)}{P_t^{mp}} \right)^{-\theta_t^m} Y_t^{mp}$$

$$Y_t^{ml} = \left[\left(\frac{1}{1-\varpi_f}\right)^{\frac{1}{\theta_t^m}} \int_{\varpi_f}^1 Y_t^{ml}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m-1}}$$

$$P_t^{ml} = \left[\frac{1}{1-\varpi_f} \int_{\varpi_f}^1 P_t^{ml}(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$$

$$Y_t^{ml}(m) = \frac{1}{1-\varpi_f} \left(\frac{P_t^{ml}(m)}{P_t^{ml}} \right)^{-\theta_t^m} Y_t^{ml}$$

$$C_t = \left[\int_0^1 C_t(c)^{\frac{\theta_t^c-1}{\theta_t^c}} dc \right]^{\frac{\theta_t^c}{\theta_t^c-1}}$$

$$P_t^c = \left[\int_0^1 P_t^c(c)^{1-\theta_t^c} dc \right]^{\frac{1}{1-\theta_t^c}}$$

$$C_t(c) = \left(\frac{P_t^c(c)}{P_t^c} \right)^{-\theta_t^c} C_t$$

$$H_t = \left[\int_0^1 H_t(j)^{\frac{\theta^h-1}{\theta^h}} dj \right]^{\frac{\theta^h}{\theta^h-1}}$$

$$W_t = \left[\int_0^1 W_t(j)^{1-\theta^h} dj \right]^{\frac{1}{1-\theta^h}}$$

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta^h} H_t$$

Table 2: Baseline calibration

Parameter	Value
Share of intermediate goods production of final goods γ_c	0.42
Share of intermediate goods production of intermediate goods γ_y	0.77
Elasticity of substitution varieties of domestic intermediate goods domestic market θ^y	6
Elasticity of substitution varieties of domestic intermediate goods foreign market θ^x	15
Elasticity of substitution varieties of domestic final goods θ^c	6
Elasticity of substitution varieties of imported intermediate goods θ^m	6
Elasticity of substitution differentiated labour services θ^h	6
Share of domestic intermediate goods production of domestic goods α	0.85
Share of domestic intermediate goods production of goods in foreign economy α_f	0.064
Elasticity of substitution domestic and foreign goods domestic economy ν	1.5
Elasticity of substitution between domestic and foreign goods foreign economy ν_f	1.5
Habit persistence parameter ζ	0.85
Inverse of Frisch elasticity of labour supply χ	3
Weight on labour in utility function η	0.570
Discount factor β	$1.03^{-0.25}$
Inflation target π^c	1.005
Units of labour required to distribute one unit of imported intermediate good δ	0.4
Units of labour required to distribute one unit of imported intermediate good foreign economy δ_f	0.4
Adjustment cost parameter domestic final goods prices ϕ_c	400
Adjustment cost parameter domestic intermediate goods prices ϕ_y	400
Adjustment cost parameter export prices ϕ_x	400
Adjustment cost parameter import prices ϕ_m	400
Adjustment cost parameter wages ϕ_w	400
Proportion of PCP firms domestic economy $\overline{\omega}$	0.6
Proportion of PCP firms foreign economy $\overline{\omega}_f$	0.4
Sensitivity of premium on foreign bond holdings w.r.t. net foreign assets ψ	0.02
Coefficient on lagged interest rates in interest rate rule ρ_R	0.65
Coefficient on inflation in interest rate rule ρ_π	1.8
AR coefficient in process for θ_t^y, ρ_y	0.3
AR coefficient in process for θ_t^x, ρ_x	0.75
AR coefficient in process for θ_t^c, ρ_c	0.5
AR coefficient in process for θ_t^m, ρ_m	0.5
AR coefficient in process for risk premium shock ρ_u	0.9
Standard deviation shock to θ_t^y, $\varepsilon_{y,t}$	0.2
Standard deviation shock to θ_t^x, $\varepsilon_{x,t}$	0.35
Standard deviation shock to θ_t^c, $\varepsilon_{c,t}$	0.2
Standard deviation shock to θ_t^m, $\varepsilon_{m,t}$	0.35
Standard deviation risk premium shock $\varepsilon_{u,t}$	0.005
Foreign inflation target π_f	1.005

Table 3: Second order moments: DSGE model and UK data 1980Q1–2003Q4.

Standard deviation	Data	Model
$\Delta \ln S_t$	0.031	0.035
$\Delta \ln \overline{P}_t^m$	0.017	0.019
$\Delta \ln \overline{P}_t^x$	0.013	0.018
$\Delta \ln P_t^y$	0.004	0.006
$\Delta \ln P_t^c$	0.003	0.006

First-order autocorrelation	Data	Model
$\Delta \ln S_t$	0.21	-0.07
$\Delta \ln \overline{P}_t^m$	0.36	0.29
$\Delta \ln \overline{P}_t^x$	0.29	0.30
$\Delta \ln P_t^y$	0.76	0.86
$\Delta \ln P_t^c$	0.79	0.81

Table 4: Distribution of chosen lag-length for different lag-order selection criteria. VAR in first differences. In per cent.

	$T = 100$				
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
LR	0.00	67.34	11.86	11.50	9.30
AIC	0.16	90.56	6.72	1.78	0.78
HQ	8.82	91.16	0.02	0.00	0.00
SC	71.52	28.48	0.00	0.00	0.00

	$T = 200$				
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
LR	0.00	43.08	19.34	20.28	17.30
AIC	0.00	89.42	8.32	1.92	0.34
HQ	0.00	100.00	0.00	0.00	0.00
SC	2.44	97.56	0.00	0.00	0.00

Table 5: Rejection frequencies for single-equation and vector tests for residual autocorrelation up to order four. First-differenced VAR. 5% significance level.

	$\Delta \ln \overline{P}_t^m$	$\Delta \ln \overline{P}_t^x$	$\Delta \ln P_t^c$	$\Delta \ln P_t^y$	$\Delta \ln S_t$	Vector test
			$T = 100$			
LR	4.9	7.1	7.4	7.4	6.4	6.7
AIC	5.7	8.3	8.5	8.1	7.4	12.4
HQ	6.4	11.2	15.1	11.2	8.4	22.5
SC	11.3	37.1	66.4	39.5	11.0	75.9
$L = 2$	6.2	9.3	9.4	8.7	8.2	16.4
$L = 4$	5.6	6.5	6.7	8.8	5.9	10.7
			$T = 200$			
LR						
AIC	6.4	13.8	14.3	15.9	9.6	36.7
HQ	7.3	15.6	16.6	17.3	11.2	42.8
SC	7.5	17.0	18.4	18.8	11.1	44.1
$L = 2$	7.3	15.6	16.6	17.3	11.2	42.8
$L = 4$	6.8	9.9	10.1	15.8	7.4	23.6

Table 6: Absolute value of per cent difference between pointwise mean of estimated accumulated responses and DSGE model's responses over first ten quarters.

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
			$T = 100$		
LR	5.6	8.8	25.6	24.1	30.6
AIC	6.0	10.7	29.7	24.8	34.6
HQ	6.6	11.2	30.4	25.1	36.8
SC	11.0	11.9	33.3	24.8	42.9
$L = 2$	6.2	11.5	30.3	25.2	36.2
$L = 4$	6.0	5.4	25.5	22.2	21.7
			$T = 200$		
LR	8.9	10.2	24.3	20.9	34.8
AIC	10.3	13.4	27.1	23.5	42.4
HQ	10.5	14.1	27.7	24.1	43.8
SC	10.7	14.0	27.7	24.1	43.9
$L = 2$	10.5	14.1	27.7	24.1	43.8
$L = 4$	8.0	8.4	22.0	18.7	29.4

Table 7: *Absolute value of per cent difference between pointwise mean of estimated responses and DSGE model's responses over first twenty quarters.*

$T = 100$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	39.4	6.6	38.7	29.4	81.1
AIC	42.6	7.7	40.0	30.5	88.6
HQ	44.5	8.0	40.2	30.5	92.2
SC	54.3	9.1	38.6	26.7	100.6
$L = 2$	43.6	8.1	40.5	31.0	91.3
$L = 4$	33.0	5.1	35.4	26.9	63.8

$T = 200$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	45.9	7.3	34.7	25.4	88.9
AIC	51.0	9.5	38.1	28.5	101.4
HQ	51.7	9.9	38.7	29.1	103.6
SC	52.1	9.9	38.6	29.0	103.8
$L = 2$	51.7	9.9	38.7	29.1	103.6
$L = 4$	42.5	6.3	33.2	23.0	80.0

Table 8: *Distribution of chosen lag-length for different lag-order selection criteria. VEqCM. In per cent.*

$T = 100$

	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0	27.1	49.9	7.3	7.1	8.5
AIC	0.0	47.4	46.2	3.7	1.4	1.2
HQ	0.0	95.3	4.7	0.0	0.0	0.0
SC	0.0	100.0	0.0	0.0	0.0	0.0

$T = 200$

	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0	0.4	80.7	7.0	5.5	6.4
AIC	0.0	3.4	95.0	1.6	0.1	0.0
HQ	0.0	62.1	37.9	0.0	0.0	0.0
SC	0.0	99.7	0.3	0.0	0.0	0.0

Table 9: Absolute value of per cent difference between pointwise mean of estimated accumulated responses from VEqCM and DSGE model's responses over first ten quarters.

$T = 100$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	21.7	26.3	27.7	22.0	25.0
AIC	19.2	24.2	25.2	19.3	22.7
HQ	15.9	22.4	22.4	16.2	19.9
SC	15.5	22.2	22.0	15.9	19.6
$L = 3$	20.7	24.9	26.2	20.6	23.8
$L = 5$	27.9	31.9	33.2	27.8	30.8

$T = 200$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	7.7	12.6	16.1	11.4	8.7
AIC	7.0	12.0	15.5	10.8	8.1
HQ	4.8	12.3	14.3	9.1	7.0
SC	3.0	12.7	13.4	7.8	6.1
$L = 3$	7.0	11.9	15.5	10.8	8.1
$L = 5$	9.8	14.5	17.6	13.3	10.8

Table 10: Absolute value of per cent difference between pointwise mean of estimated accumulated responses from VEqCM and DSGE model's responses over first twenty quarters.

$T = 100$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	20.0	25.6	27.3	23.8	20.2
AIC	17.7	22.8	24.5	21.0	17.8
HQ	15.3	18.9	20.9	17.4	14.9
SC	15.1	18.5	20.5	17.0	14.7
$L = 3$	18.4	24.6	25.9	22.5	19.3
$L = 5$	26.7	32.5	33.9	30.5	27.4

$T = 200$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	8.4	10.7	14.2	10.4	9.2
AIC	8.3	9.8	13.2	9.5	9.1
HQ	6.2	7.3	11.1	7.3	9.3
SC	4.9	7.5	9.6	5.7	9.7
$L = 3$	8.4	9.8	13.2	9.5	9.1
$L = 5$	8.5	13.5	17.0	13.3	9.3

Table 11: Distribution of chosen lag-length for different lag-order selection criteria. 5% significance level in individual LR tests. Variables in (log) levels. In per cent.

	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$
			$T=100$			
LR	0.0	13.8	55.4	8.9	9.4	12.4
AIC	0.0	27.7	59.3	6.3	3.1	3.6
HQ	0.0	86.5	13.4	0.0	0.0	0.0
SC	0.0	99.9	0.1	0.0	0.0	0.0

	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$
			$T=200$			
LR	0.0	0.0	78.1	8.2	6.4	7.2
AIC	0.0	0.1	97.6	2.2	0.1	0.0
HQ	0.0	14.2	85.8	0.0	0.0	0.0
SC	0.0	90.5	9.5	0.0	0.0	0.0

Table 12: Rejection frequencies for single-equation and vector tests for non-normality. 5% significance level.

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
			$T=100$			
LR	5.0	5.1	4.8	4.8	5.0	5.2
AIC	5.2	5.0	4.9	4.8	5.2	5.2
HQ	4.7	4.9	5.5	4.8	4.9	5.4
SC	4.8	5.0	5.1	4.8	5.1	5.4
$L=3$	5.1	4.9	4.9	4.6	4.6	5.0
$L=5$	5.4	4.8	5.1	5.5	5.4	5.2

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
			$T=200$			
LR	5.2	4.9	5.3	4.9	5.3	5.6
AIC	5.0	4.9	5.4	5.1	5.2	5.5
HQ	5.4	4.9	5.5	5.0	5.3	5.5
SC	5.0	5.1	4.9	5.4	5.3	5.7
$L=3$	5.0	5.0	5.5	5.2	5.2	5.5
$L=5$	5.1	4.9	5.1	5.3	5.4	6.0

Table 13: *Rejection frequencies for single-equation and vector tests for residual autocorrelation up to order 5. 5% significance level.*

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
			$T = 100$			
LR	10.2	9.5	13.0	11.6	9.4	19.5
AIC	9.5	9.0	16.6	11.7	8.9	22.5
HQ	10.5	14.5	46.5	21.0	8.8	50.0
SC	11.5	16.9	55.5	24.1	9.1	59.1
$L = 3$	11.0	9.6	10.5	11.4	10.3	23.2
$L = 5$	12.8	12.0	11.8	13.7	12.4	32.6

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
			$T = 200$			
LR	5.9	4.8	7.1	6.2	5.5	5.5
AIC	7.1	5.6	8.4	7.1	6.6	9.7
HQ	7.4	8.3	20.5	12.3	6.7	22.1
SC	11.1	33.7	89.5	53.2	7.6	89.6
$L = 3$	7.4	5.8	8.6	7.3	6.8	10.7
$L = 5$	6.1	5.3	7.1	6.9	6.1	8.6

Table 14: *Frequencies of chosen cointegration rank using Johansen's trace test. Numbers in parentheses denote the preferred rank when using a small-sample correction to the trace test.*

	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
			$T = 100$			
LR	10.7 (51.7)	31.4 (28.6)	33.6 (11.8)	17.6 (6.1)	5.5 (1.5)	1.2 (0.4)
AIC	10.8 (42.2)	28.4 (26.1)	30.2 (16.9)	21.1 (11.3)	8.2 (3.2)	1.3 (0.5)
HQ	2.5 (8.1)	8.4 (14.5)	28.6 (36.5)	39.7 (30.9)	17.6 (8.7)	3.3 (1.3)
SC	0.0 (0.6)	3.8 (12.2)	29.0 (41.2)	43.8 (34.8)	19.8 (9.8)	3.6 (1.5)
$L = 3$	17.5 (61.2)	41.3 (29.7)	28.0 (7.2)	10.0 (1.6)	2.7 (0.3)	0.5 (0.0)
$L = 5$	13.7 (84.4)	40.1 (13.9)	32.2 (1.7)	10.5 (0.2)	2.5 (0.0)	0.5 (0.0)

	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
			$T = 200$			
LR	0.3 (2.1)	3.9 (11.0)	19.0 (29.5)	43.5 (38.4)	28.1 (16.3)	5.2 (2.6)
AIC	0.1 (0.5)	2.1 (6.9)	16.2 (28.7)	45.5 (41.8)	30.5 (19.1)	5.7 (3.0)
HQ	0.0 (0.5)	1.9 (6.3)	14.4 (25.4)	42.4 (39.4)	33.6 (23.4)	7.6 (5.0)
SC	0.0 (0.1)	0.3 (1.0)	2.4 (4.2)	25.6 (31.0)	53.2 (48.7)	18.4 (15.1)
$L = 3$	0.1 (0.5)	2.0 (6.8)	16.1 (28.5)	45.5 (42.0)	30.6 (19.2)	5.7 (3.0)
$L = 5$	1.5 (12.3)	11.9 (32.9)	32.0 (32.7)	37.2 (16.9)	14.7 (4.3)	2.6 (0.1)

Table 15: Rejection frequencies for LR tests of restrictions on cointegration space conditional on r = 4. 5% significance level. Numbers in parentheses are rejection frequencies based only on the datasets in which the correct cointegration rank is chosen.

$T = 100$

	$\ln(\overline{P}_t^m/P_t^c) \sim I(0)$	$\ln(\overline{P}_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	27.9	24.8	24.4	26.5	81.6 (94.6)
AIC	29.1	24.2	23.9	27.3	79.2 (91.7)
HQ	38.8	29.4	28.4	35.8	83.3 (92.1)
SC	41.2	30.3	29.6	38.1	84.2 (91.0)
L = 3	22.4	20.2	19.7	21.1	73.4 (81.5)
L = 5	26.8	25.9	25.7	26.3	88.1 (94.4)

$T = 200$

	$\ln(\overline{P}_t^m/P_t^c) \sim I(0)$	$\ln(\overline{P}_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	19.4	13.5	15.4	15.4	40.6 (45.7)
AIC	18.2	13.4	14.7	14.5	37.2 (43.2)
HQ	20.0	15.1	16.4	16.6	40.3 (49.0)
SC	29.3	25.5	25.0	26.0	60.5 (64.5)
L = 3	18.2	13.2	14.5	14.3	36.7 (42.8)
L = 5	22.0	14.3	16.7	17.9	48.0 (57.9)

Table 16: Distribution of chosen lag-length for different lag-order selection criteria. Data generated from VEqCM(5) [VEqCM(3)]. 5% significance level in individual LR tests. Variables in (log) levels. In per cent.

	$T = 100$					
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0 [0.0]	14.1 [36.3]	54.4 [32.8]	9.0 [8.6]	10.7 [10.0]	11.8 [12.3]
AIC	0.0 [0.0]	27.5 [61.8]	59.5 [29.5]	6.6 [3.9]	3.0 [2.0]	3.3 [2.8]
HQ	0.0 [0.0]	87.8 [98.4]	12.2 [1.5]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
SC	0.0 [0.0]	99.9 [100.0]	0.1 [0.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]

	$T = 200$					
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0 [0.0]	0.0 [6.9]	79.2 [75.3]	7.1 [5.8]	7.0 [5.3]	6.7 [6.7]
AIC	0.0 [0.0]	0.1 [24.5]	97.7 [74.3]	2.1 [1.1]	0.1 [0.0]	0.0 [0.0]
HQ	0.0 [0.0]	16.1 [93.0]	83.9 [7.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
SC	0.0 [0.0]	92.0 [100.0]	8.0 [0.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]

Table 17: Frequencies of chosen cointegration rank using Johansen's trace test. 5% significance level. Data generated from VEqCM(5) [VEqCM(3)].

	$T = 100$					
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
LR	11.5 [0.6]	33.2 [14.5]	31.6 [39.7]	17.5 [33.6]	5.3 [9.9]	0.8 [1.6]
AIC	11.5 [0.4]	30.6 [10.7]	28.6 [38.0]	20.5 [36.1]	7.6 [12.8]	1.2 [2.0]
HQ	2.1 [0.3]	8.7 [6.8]	29.0 [35.0]	39.8 [40.2]	18.1 [15.4]	2.5 [2.3]
SC	0.0 [0.3]	3.5 [6.7]	29.6 [34.9]	44.0 [40.3]	19.9 [15.5]	2.9 [2.3]
$L = 3$	18.9 [2.5]	42.1 [23.1]	27.2 [41.9]	9.5 [25.7]	1.9 [5.9]	0.3 [0.1]
$L = 5$	14.6 [5.2]	41.6 [30.1]	31.8 [40.3]	9.7 [19.4]	2.0 [4.4]	0.2 [0.6]

	$T = 200$					
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
LR	0.2 [0.0]	5.1 [0.3]	25.1 [6.8]	44.8 [52.3]	21.8 [37.1]	3.0 [3.5]
AIC	0.0 [0.0]	3.1 [0.0]	22.6 [3.3]	46.6 [48.5]	24.5 [43.5]	3.2 [4.7]
HQ	0.0 [0.0]	2.9 [0.0]	20.0 [1.4]	42.8 [37.2]	29.9 [54.5]	4.3 [6.9]
SC	0.0 [0.0]	0.3 [0.0]	2.5 [0.3]	27.3 [27.6]	57.5 [62.8]	12.4 [9.2]
$L = 3$	0.0 [0.0]	3.1 [0.0]	22.5 [4.1]	46.7 [54.0]	24.5 [38.6]	3.1 [3.3]
$L = 5$	1.5 [0.0]	16.2 [1.7]	37.1 [22.6]	33.2 [52.0]	10.2 [21.5]	1.7 [2.3]

Table 18: Rejection frequencies for single-equation and vector tests for residual autocorrelation up to order five. Data generated by VEqCM(5) [VEqCM(3)]. 5% significance level.

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
			$T = 100$			
LR	10.1 [9.2]	9.8 [9.5]	13.1 [25.6]	11.6 [12.3]	9.7 [9.7]	18.5 [18.6]
AIC	9.3 [8.1]	8.9 [8.3]	16.5 [39.0]	12.5 [12.8]	8.6 [8.2]	21.7 [22.8]
HQ	9.7 [9.1]	14.4 [9.1]	48.2 [66.2]	22.3 [14.5]	7.8 [8.5]	50.2 [38.3]
SC	10.2 [9.1]	16.6 [9.1]	56.9 [67.6]	25.5 [14.4]	8.1 [8.4]	58.4 [39.1]
$L = 3$	10.5 [11.0]	8.7 [9.5]	9.8 [10.0]	11.3 [13.3]	10.1 [10.9]	23.3 [23.6]
$L = 5$	12.3 [13.0]	12.1 [13.2]	12.4 [13.2]	12.7 [14.4]	12.0 [12.5]	33.0 [32.8]
			$T = 200$			
LR	5.4 [4.9]	4.7 [4.8]	6.7 [11.0]	6.3 [6.2]	5.3 [5.1]	5.1 [4.6]
AIC	5.9 [5.2]	4.8 [4.9]	8.5 [27.7]	7.1 [6.6]	5.9 [5.6]	8.8 [11.7]
HQ	6.3 [5.8]	8.5 [5.4]	22.6 [92.2]	13.2 [7.9]	5.8 [7.1]	22.1 [51.2]
SC	10.9 [5.6]	36.3 [5.6]	90.9 [98.9]	55.8 [8.3]	6.8 [7.2]	90.2 [58.2]
$L = 3$	6.1 [5.7]	5.0 [5.7]	8.7 [6.1]	7.2 [6.8]	6.1 [5.9]	9.7 [7.9]
$L = 5$	5.5 [6.0]	5.7 [5.6]	5.1 [5.4]	6.2 [6.6]	6.0 [6.1]	7.3 [7.4]

Table 19: Rejection frequencies for LR tests of restrictions on cointegration space conditional on r = 4. 5% significance level. Data generated by VEqCM(5) [VEqCM(3)].

T = 100

	$\ln(\bar{P}_t^m/P_t^c) \sim I(0)$	$\ln(\bar{P}_t^v/P_t^c) \sim I(0)$	$\ln(P_t^v/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	29.0 [34.4]	23.3 [22.3]	23.2 [25.3]	26.9 [35.5]	81.3 [74.6]
AIC	29.7 [33.7]	23.0 [20.5]	22.4 [23.8]	27.4 [8.2]	79.1 [69.7]
HQ	38.9 [33.8]	28.0 [18.9]	27.0 [23.5]	35.9 [35.5]	83.2 [67.1]
SC	17.1 [33.9]	30.0 [18.9]	8.4 [23.4]	26.5 [35.6]	58.4 [66.8]
L = 3	23.1 [29.3]	18.7 [18.3]	18.7 [20.1]	21.9 [29.9]	73.7 [65.9]
L = 5	28.1 [34.9]	24.9 [25.2]	25.1 [26.1]	27.6 [35.9]	87.8 [85.1]

T = 200

	$\ln(\bar{P}_t^m/P_t^c) \sim I(0)$	$\ln(\bar{P}_t^v/P_t^c) \sim I(0)$	$\ln(P_t^v/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	14.5 [16.4]	12.2 [11.8]	12.6 [12.6]	14.2 [16.8]	35.1 [28.6]
AIC	14.3 [18.0]	11.7 [12.6]	12.3 [14.2]	13.8 [18.0]	31.8 [28.1]
HQ	17.5 [24.0]	14.0 [16.2]	14.6 [20.9]	16.4 [24.3]	36.1 [34.0]
SC	31.4 [24.5]	25.8 [16.5]	26.4 [21.8]	27.7 [24.9]	60.7 [34.6]
L = 3	14.4 [15.3]	11.5 [10.9]	12.2 [11.4]	13.7 [15.3]	31.4 [25.2]
L = 5	15.4 [17.5]	12.4 [13.0]	13.3 [13.3]	15.0 [18.1]	40.9 [35.3]
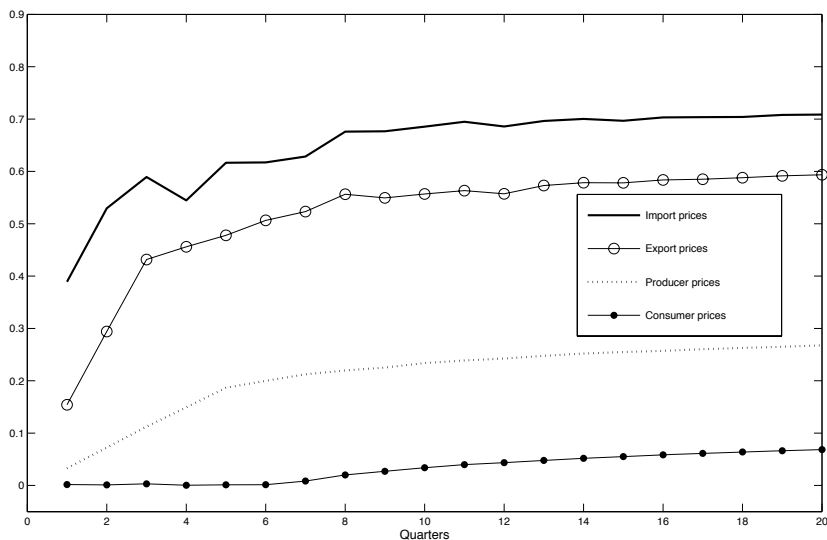
Figure 1: Normalised impulse responses to exchange rate shock. UK data. First-differenced VAR(4). Exchange rate first in recursive ordering.
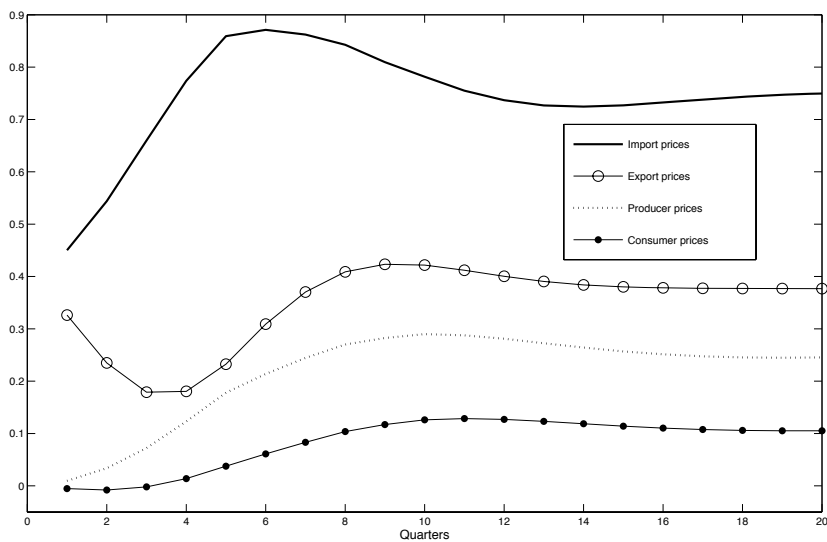
Figure 2: Normalised responses to exchange rate shock. Mean of 5000 datasets from DSGE model. $T = 100$. First-differenced VAR(4). Exchange rate first in recursive ordering.
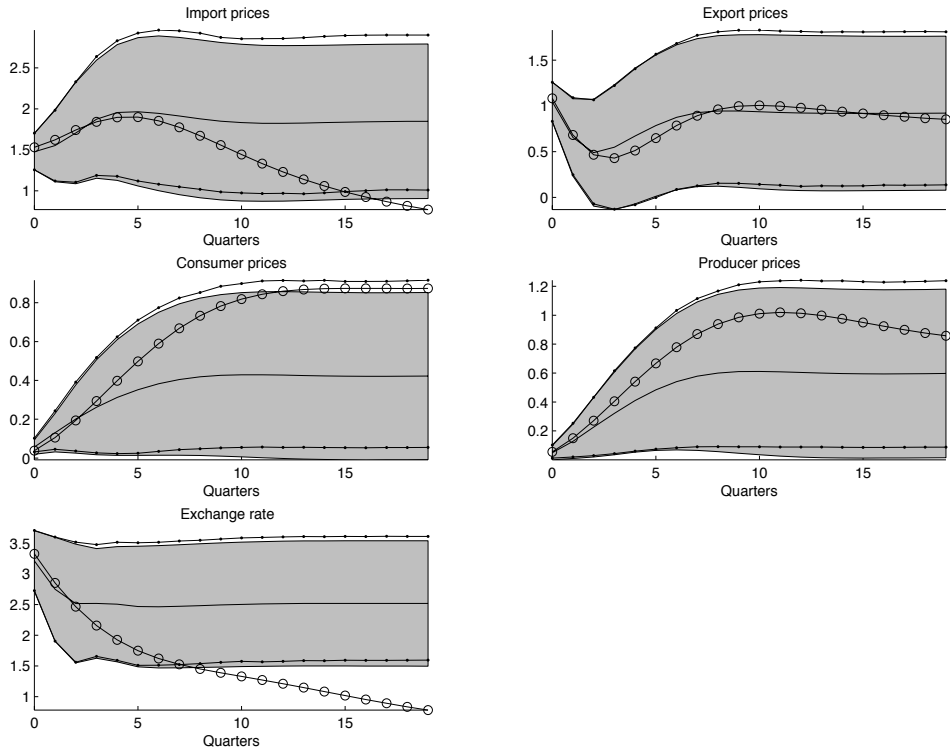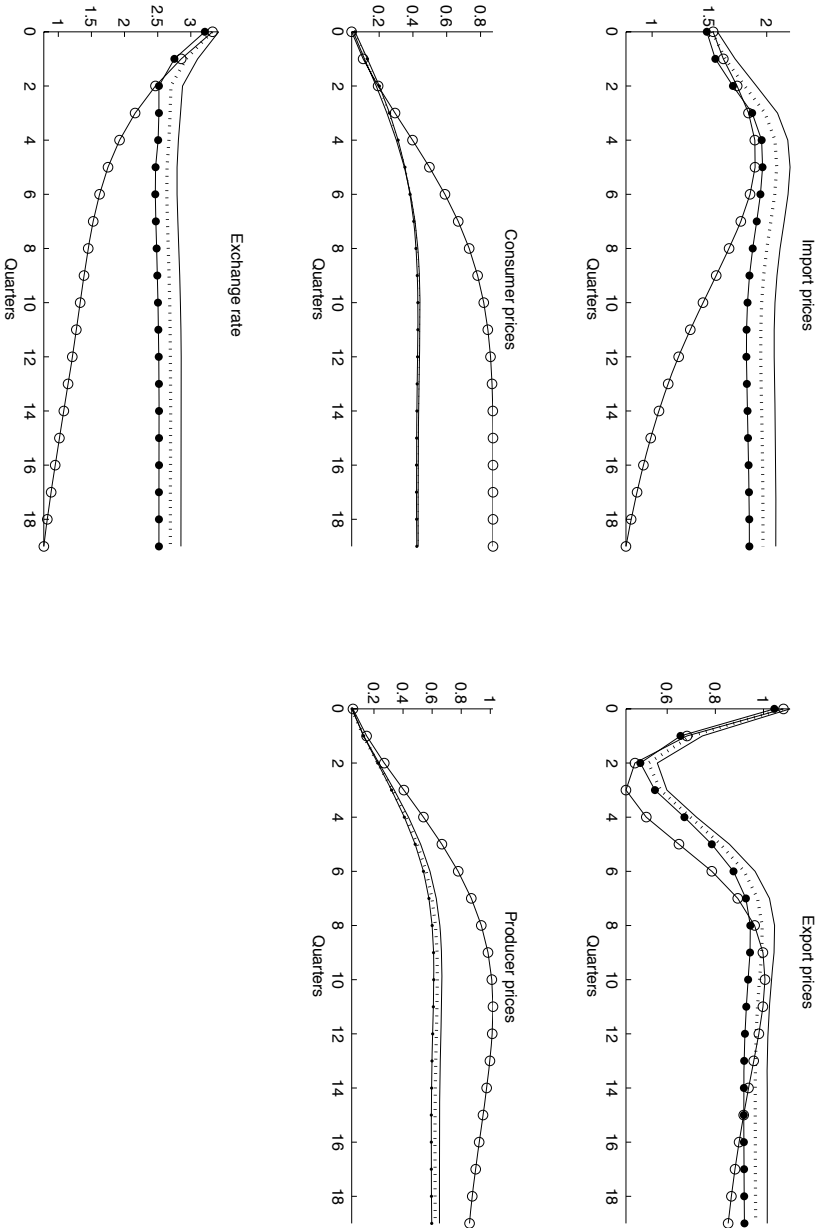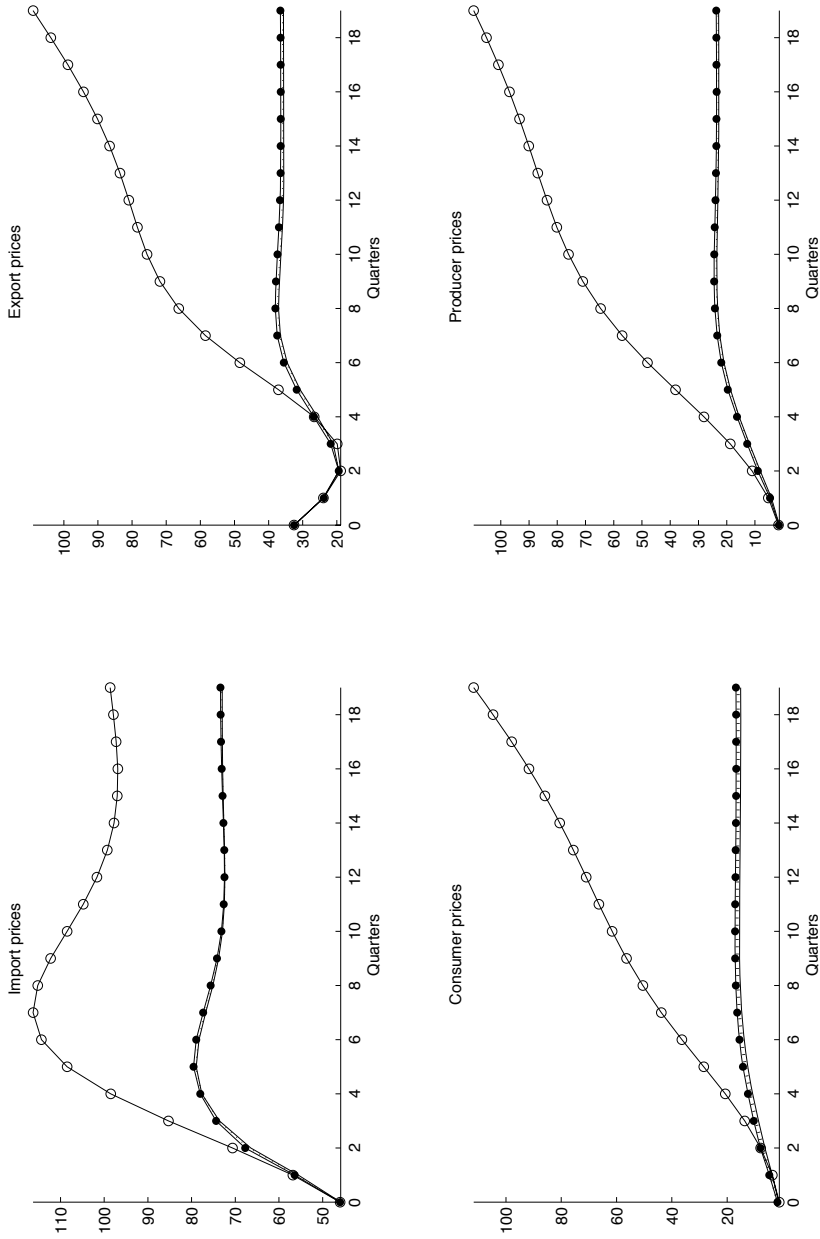
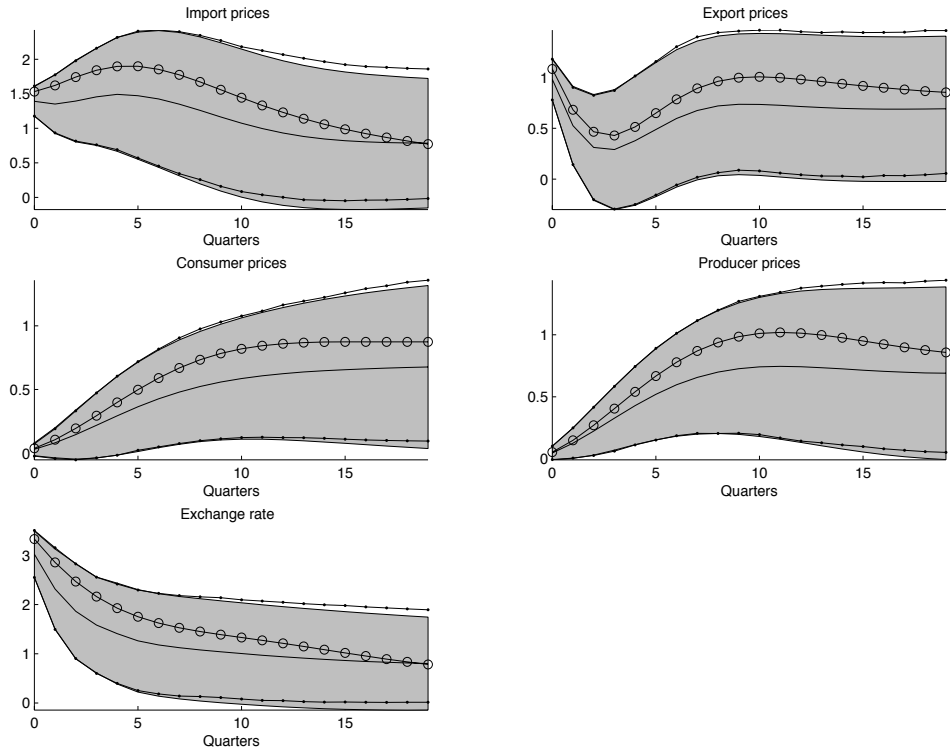*Figure 3: Responses to a one standard deviation UIP shock. In per cent. T = 100, L = 2. Line with circles: DSGE model responses. Solid line: Pointwise mean of simulated responses. Shaded area: pointwise mean plus/minus* 1.96 *times the standard deviations of the simulated responses. Line with points: 95% percentile interval for simulated responses.*
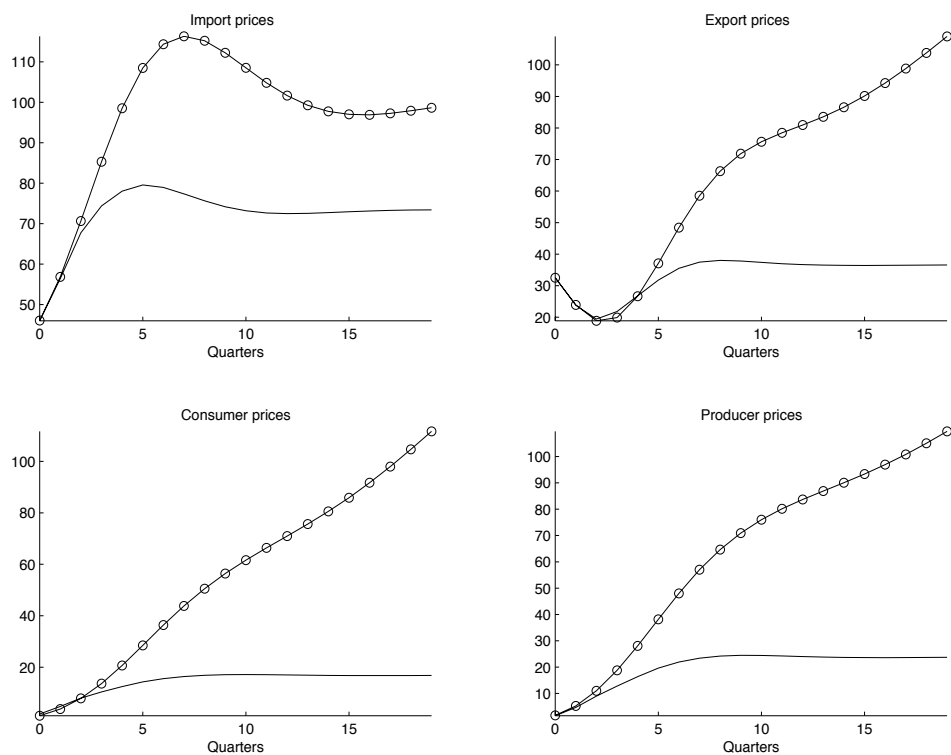
*Figure 4: Responses to UIP shock normalised on exchange rate response. In per cent. T =
100, L = 2. Line with circles: DSGE model responses. Solid line: pointwise mean of simulated
responses.*
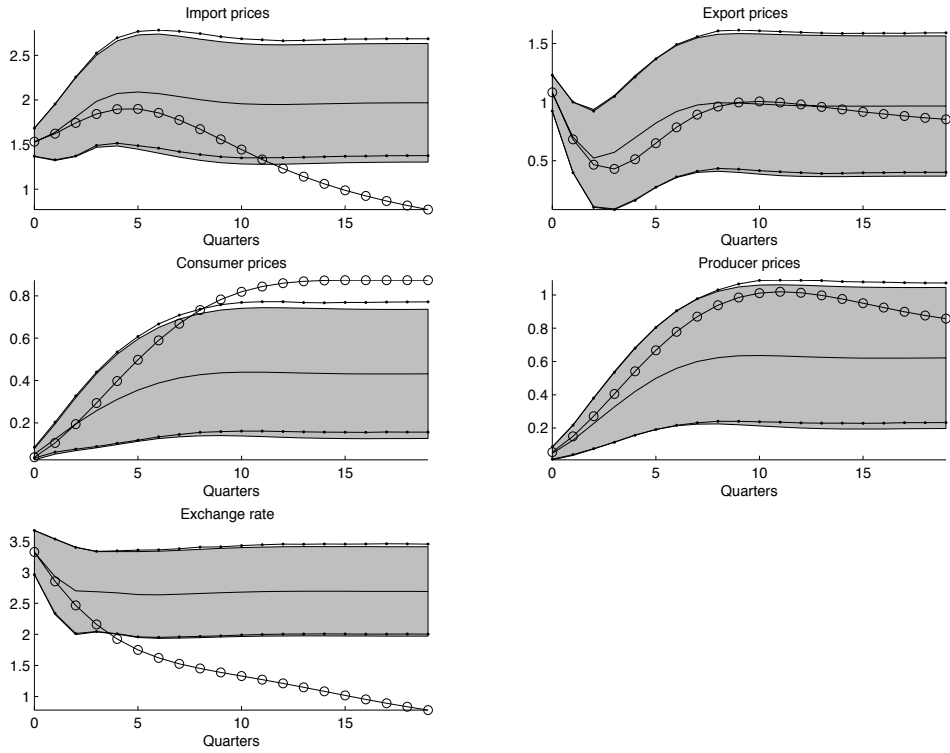
Figure 5: Accumulated responses to one standard deviation UIP shock. In per cent. $T = 200$, $L = 2$. Line with circles: DSGE model responses. Solid line: Pointwise mean of simulated responses. Shaded area: pointwise mean plus/minus 1.96 times the standard deviations of the simulated responses. Line with points: 95% percentile interval for simulated responses.
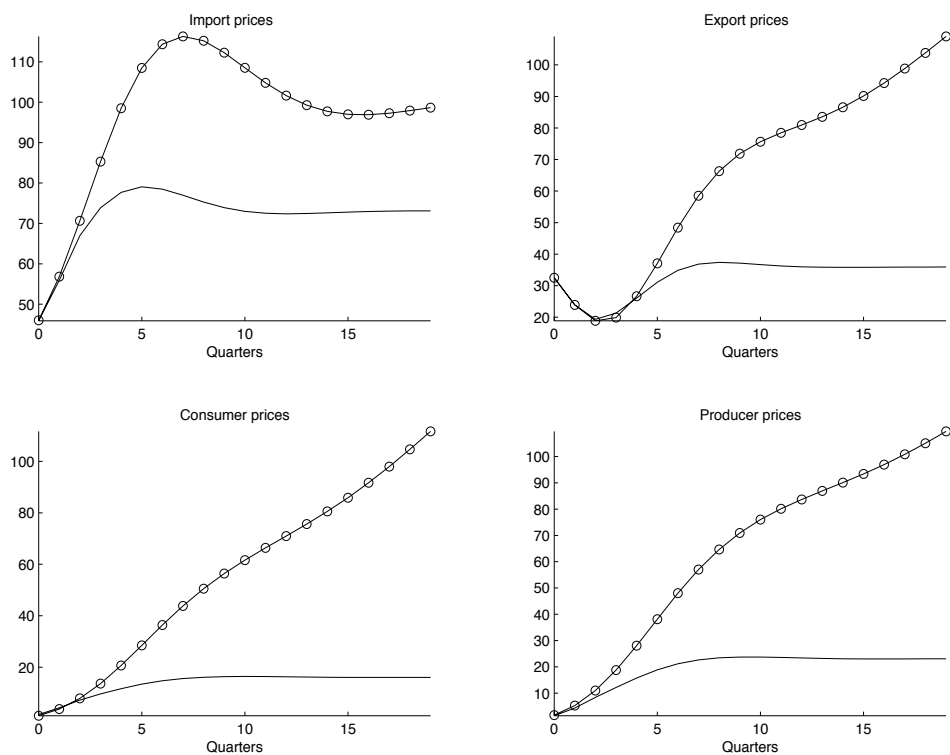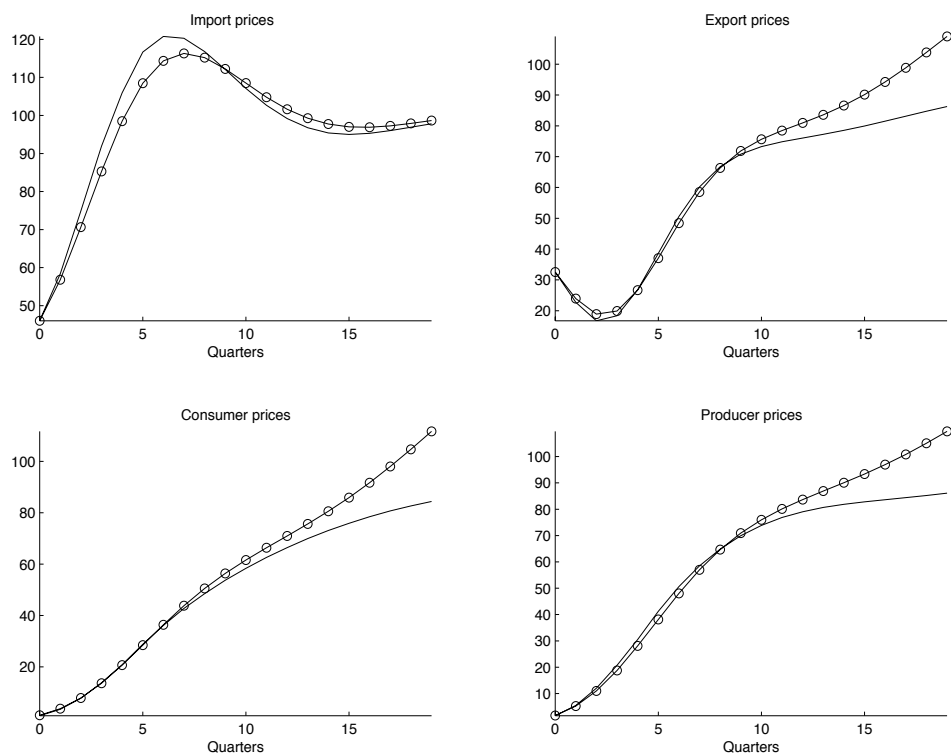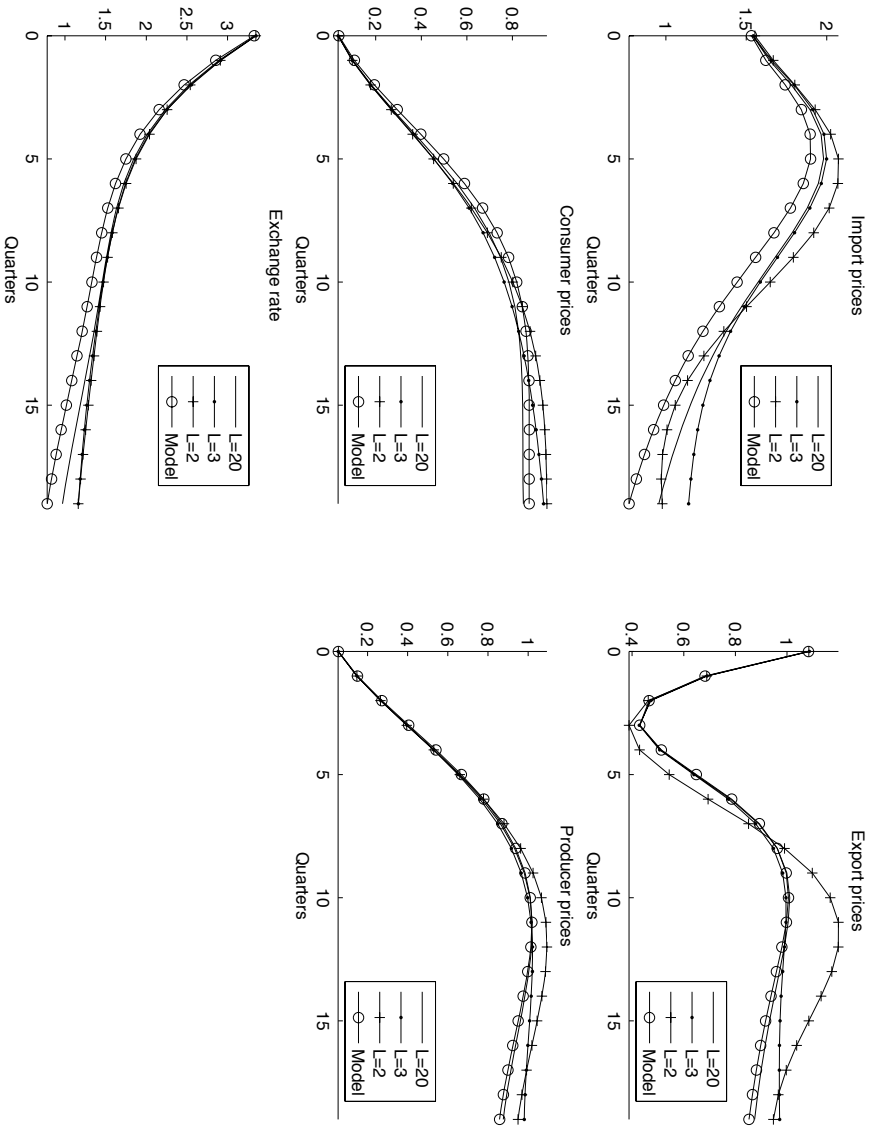
*Figure 6: Responses to UIP shock normalised on exchange rate response. In per cent. T =
200, L = 2. Line with circles: DSGE model responses. Solid line: pointwise mean of simulated
responses.*

Figure 7: Accumulated responses to one standard deviation UIP shock. In per cent. L = 2. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when T = 200. Dashed line: Responses from VAR when T = 100.

Figure 8: Normalised responses to one standard deviation UIP shock. In per cent. L = 2. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when T = 200. Dashed line: Responses from VAR when T = 100.
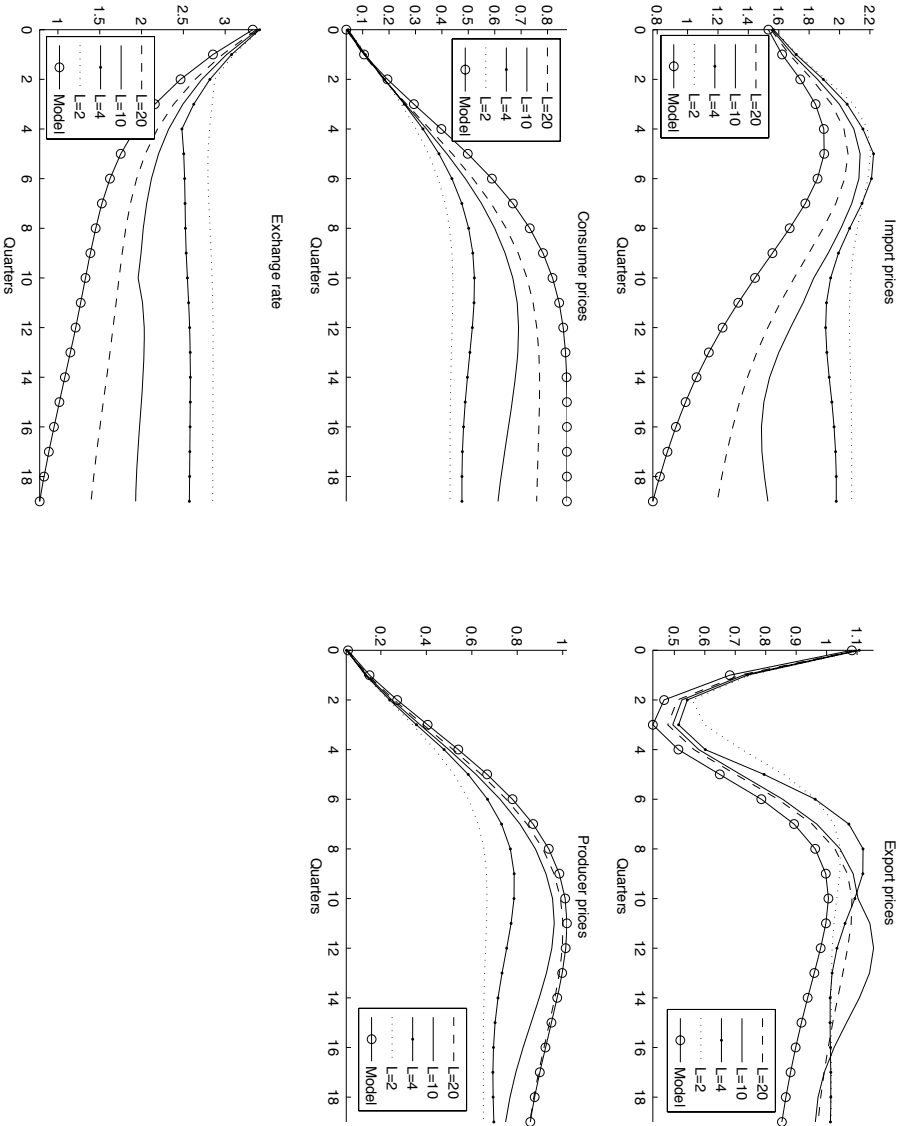
Figure 9: Accumulated responses to one standard deviation UIP shock in population version of VAR for different lag-orders. In per cent.
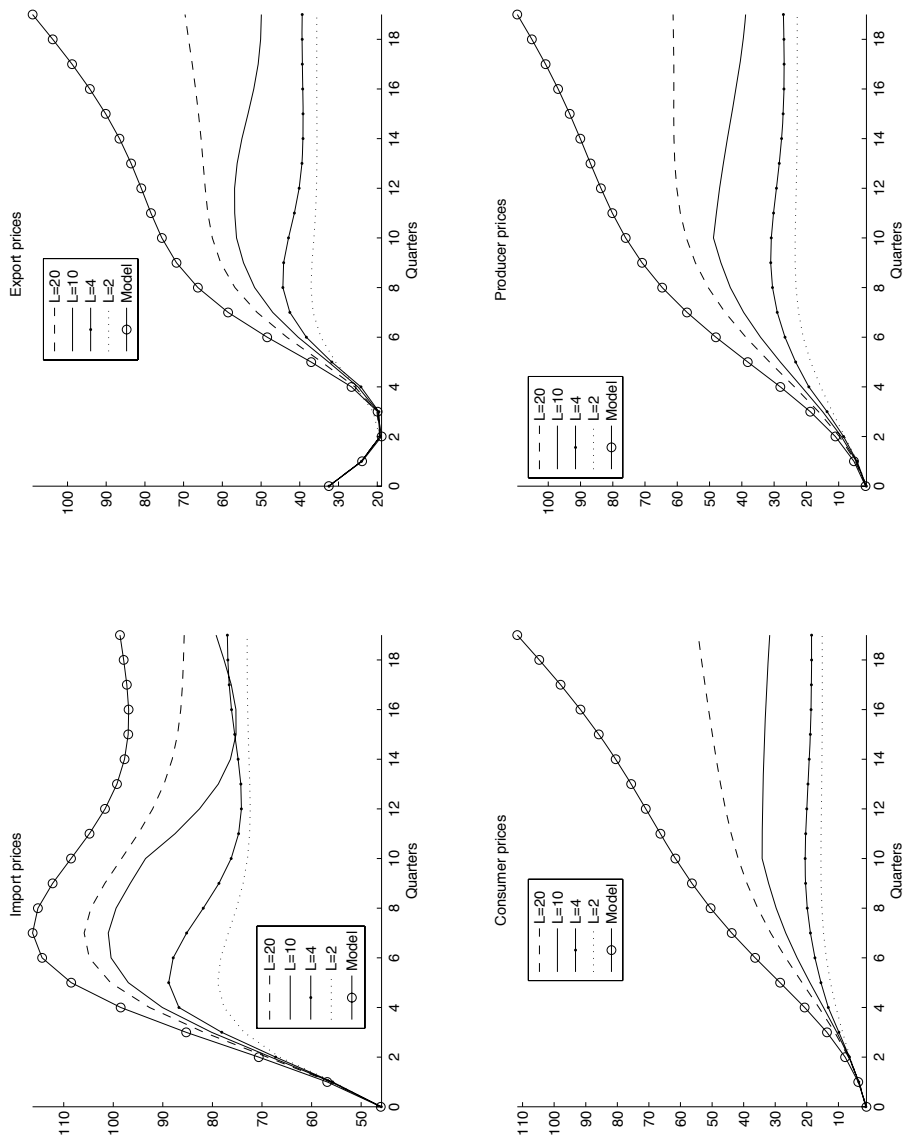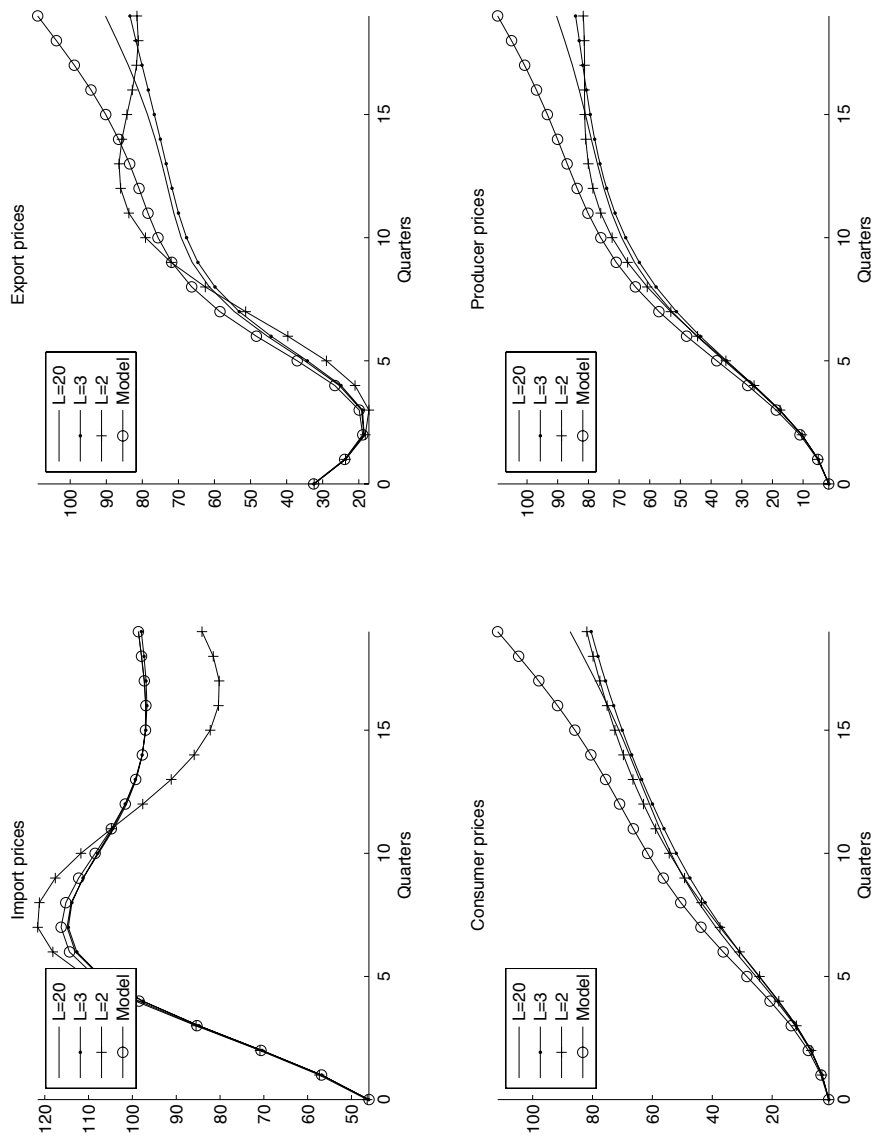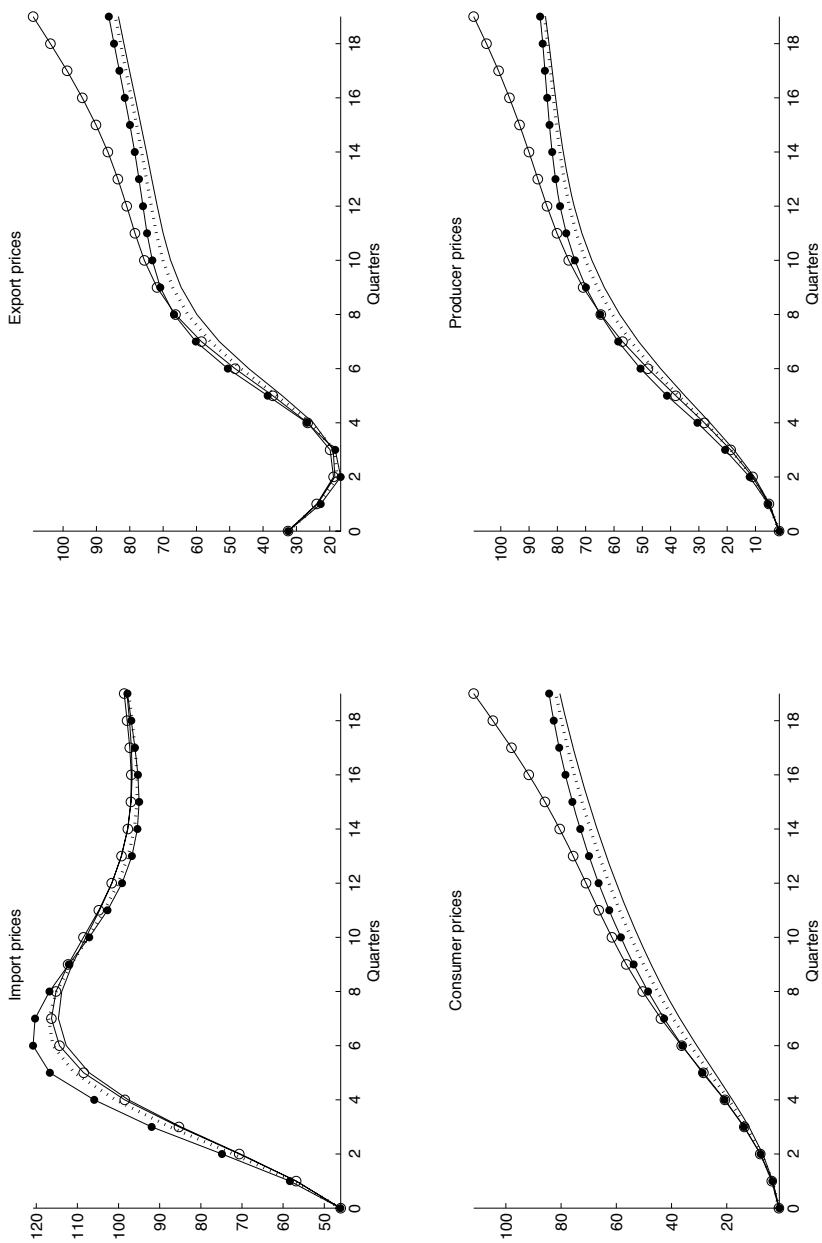
Figure 10: Normalised impulse responses to one standard deviation UIP shock in population version of VAR for different lag-orders. In per cent.

Figure 11: Accumulated responses to one standard deviation UIP shock. In per cent. VEqCM.
$T = 100, L = 3$. *Line with circles: DSGE model responses. Solid line: Pointwise mean of simulated responses. Shaded area: pointwise mean plus/minus* 1.96 *times the standard deviations of the simulated responses. Line with points: 95% percentile interval for simulated responses.*

Figure 12: Normalised responses to one standard deviation UIP shock. In per cent. VEqCM.
$T = 100, L = 3$. *Line with circles: DSGE model responses. Solid line: pointwise mean of simulated responses.*

Figure 13: Accumulated responses to one standard deviation UIP shock from population version of VEqCM for different lag-orders. In per cent.

Figure 14: Normalised responses to one standard deviation UIP shock from population version of VEqCM for different lag-orders. In per cent.
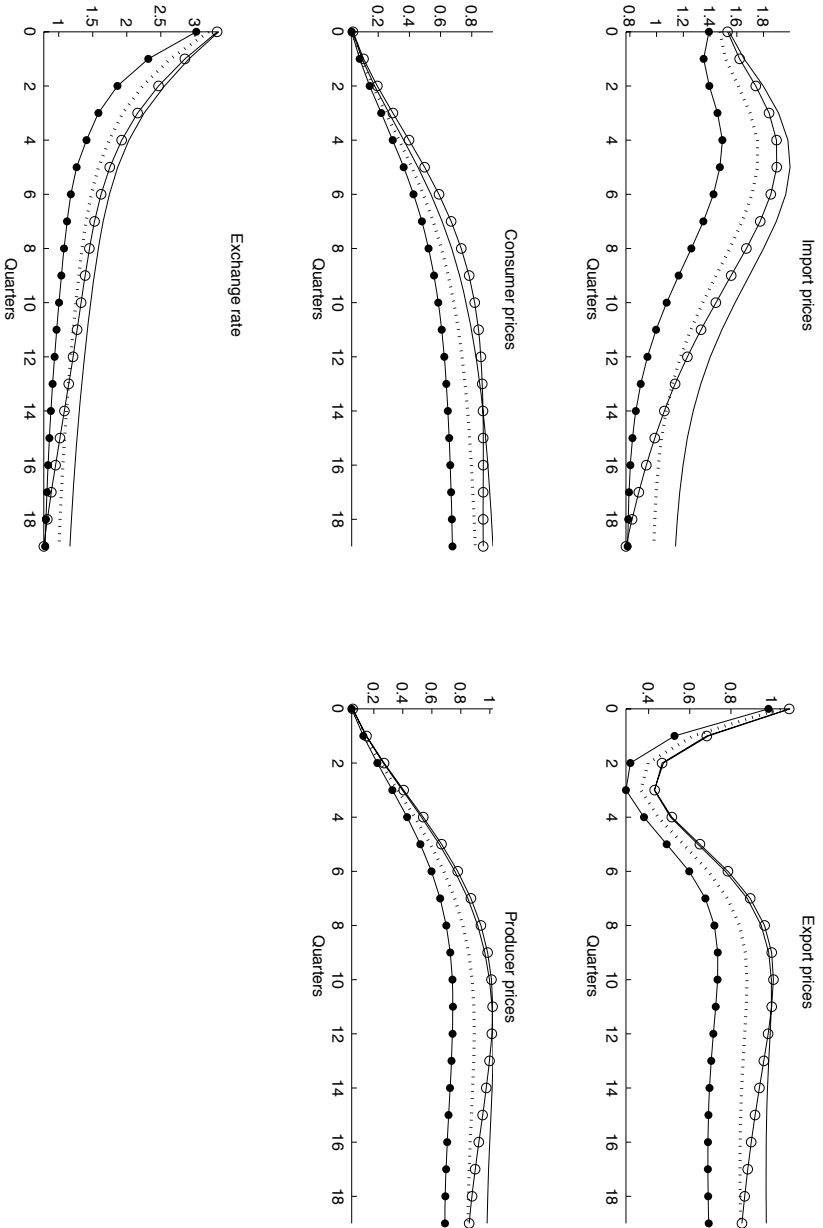
Figure 15: Accumulated responses to one standard deviation UIP shock from VEqCM. In per cent. L = 3. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when T = 200. Dashed line: Responses from VAR when T = 100.
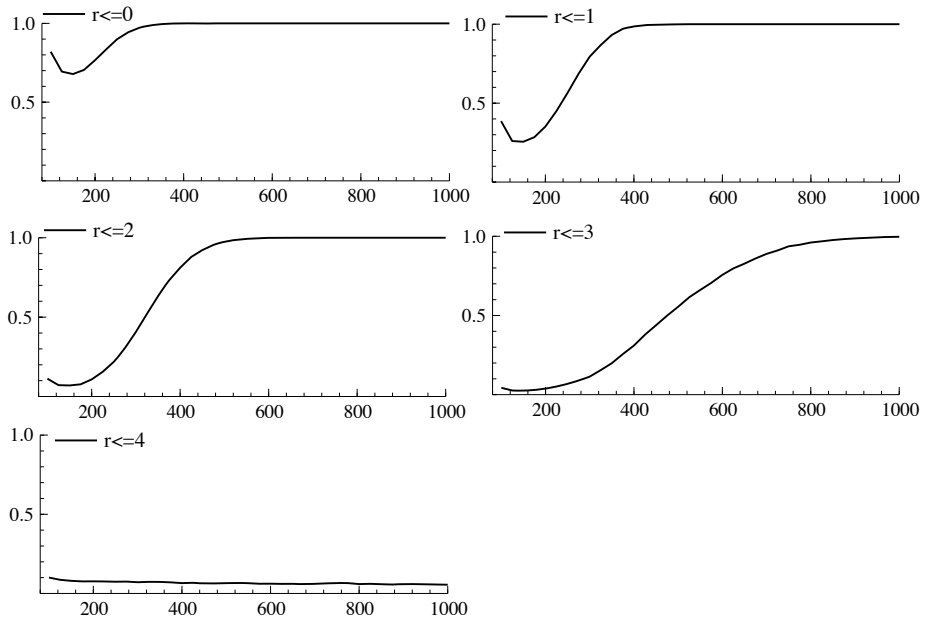
Figure 16: Normalised responses to one standard deviation UIP shock from VEqCM. In per cent. L = 3. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when T = 200. Dashed line: Responses from VAR when T = 100.

Figure 17: Trace test for cointegration rank r. Rejection frequencies based on 5000 Monte Carlo replications. 5% significance level.
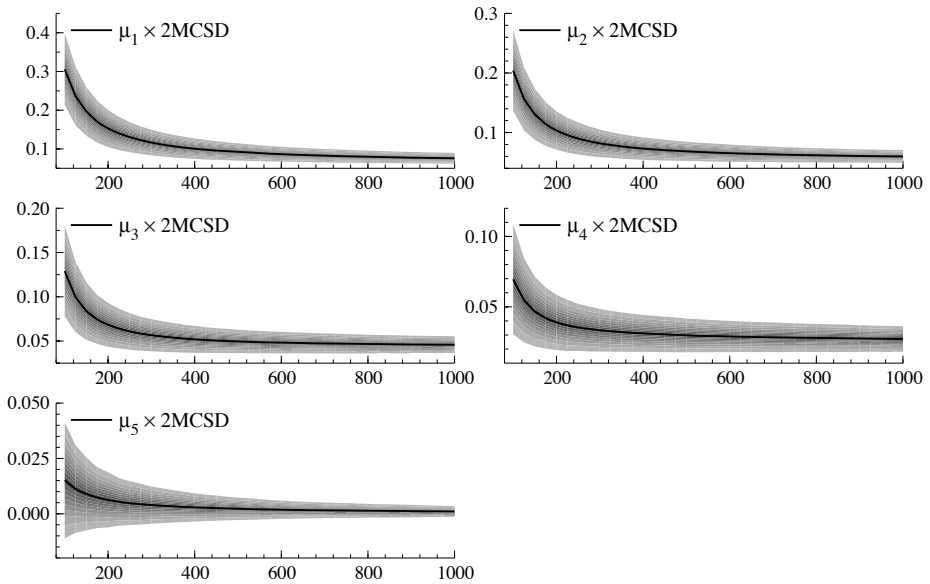

Figure 18: Cointegration eigenvalues. Mean plus/minus two Monte Carlo standard deviation across 5000 Monte Carlo replications.
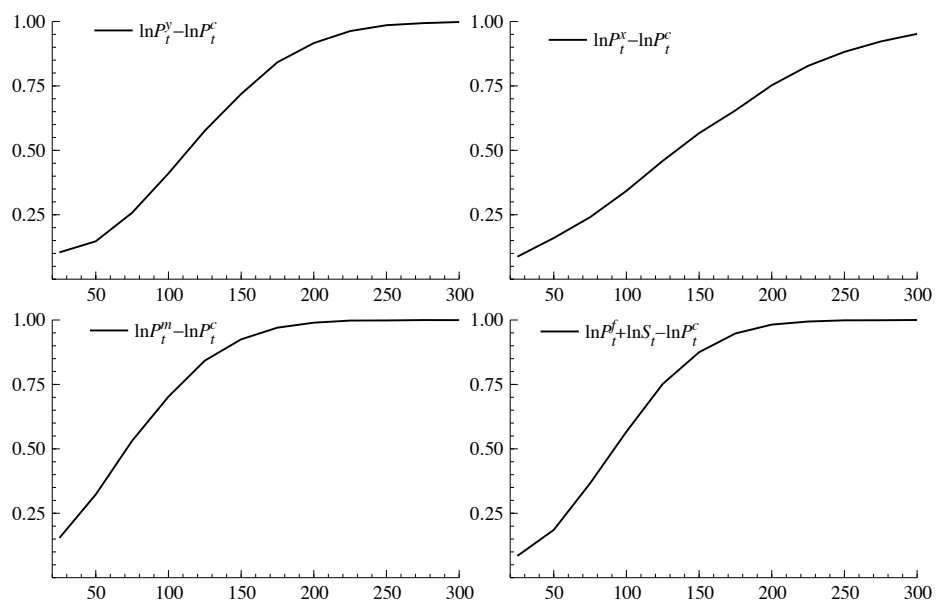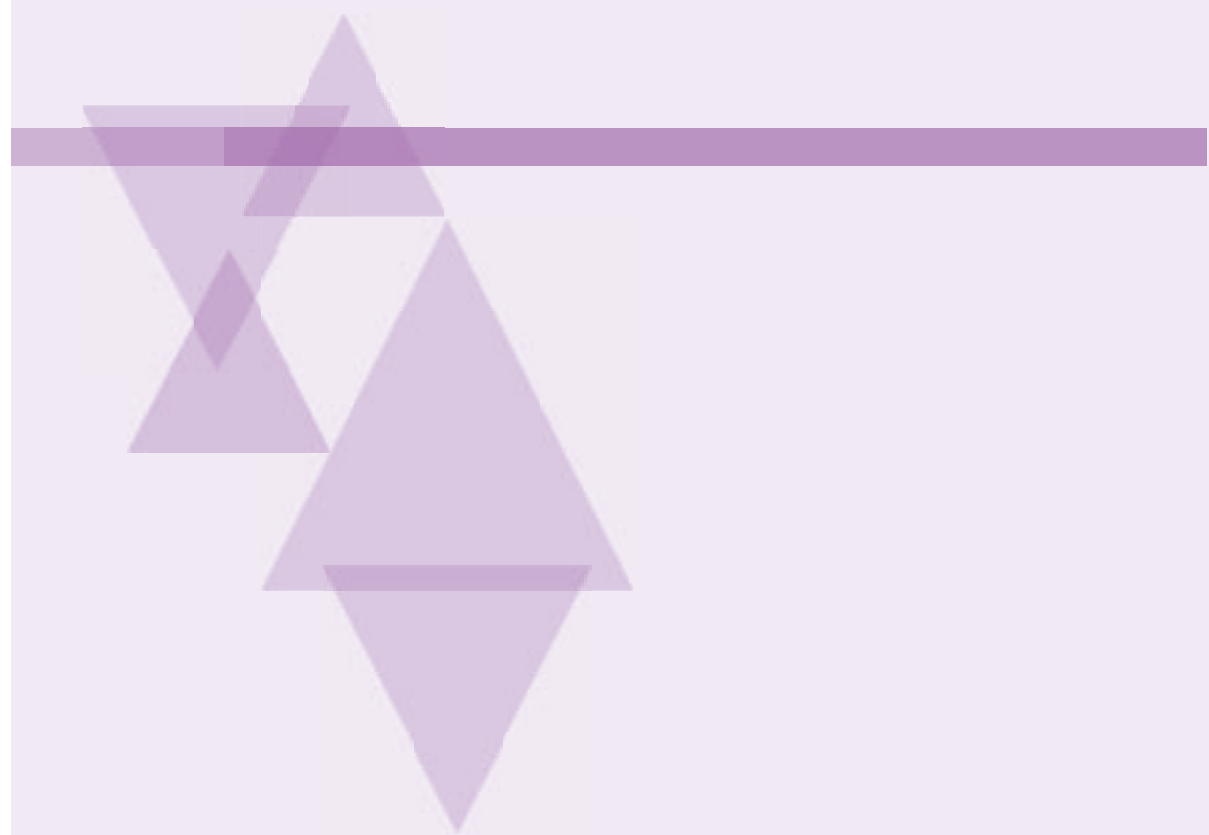
Figure 19: Recursive rejection frequencies ADF test. Two lags + constant term. Results based on 5000 Monte Carlo replications. 5% significance level.