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Fire sales, indirect contagion and systemic stress testing

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Abstract

We present a framework for quantifying the impact of fire sales in a network of financial institutions with common asset holdings, subject to leverage or capital constraints. Asset losses triggered by macro-shocks may interact with one-sided portfolio constraints, such as leverage or capital constraints, resulting in liquidation of assets, which in turn affects market prices, leading to contagion of losses and possibly new rounds of fire sales when portfolios are marked to market.

Price-mediated contagion occurs through common asset holdings, which we quantify through liquidity-weighted overlaps across portfolios. Exposure to price-mediated contagion leads to the concept of indirect exposure to an asset class, as a consequence of which the risk of a portfolio depends on the matrix of asset holdings of other large and leveraged portfolios with similar assets.

Our model provides an operational stress testing method for quantifying the systemic risk arising from these effects. Using data from the European Banking Authority, we examine the exposure of the EU banking system to price-mediated contagion. Our results indicate that, even with optimistic estimates of market depth, moderately large macro-shocks may trigger fire sales which may then lead to substantial losses across bank portfolios, modifying the outcome of bank stress tests. Price-mediated contagion leads to a heterogeneous cross-sectional loss distribution across banks, which cannot be replicated simply by applying a macro-shock to bank portfolios in absence of fire sales.

Unlike models based on 'leverage targeting', which assume symmetric reactions to gains or losses, our approach is based on the asymmetric interaction of portfolio losses with one-sided constraints, distinguishes between insolvency and illiquidity and leads to substantially different loss estimates in stress scenarios.

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Contents

1 Introduction **3**
- 1.1 The need for macroprudential stress tests . . . 3
- 1.2 Summary and main findings . . . 5
- 1.3 Outline . . . 6

2 Modeling spillover effects from fire sales **7**
- 2.1 Balance sheets and portfolio constraints . . . 7
- 2.2 Stress scenarios . . . 8
- 2.3 Deleveraging . . . 9
- 2.4 Market impact and price-mediated contagion . . . 11
- 2.5 Feedback loops, insolvency and illiquidity . . . 14
- 2.6 Comparison with "leverage targeting" . . . 15

3 A systemic stress test of the European banking system **16**
- 3.1 Data . . . 16
- 3.2 Market impact and market depth . . . 18
- 3.3 Portfolio overlaps . . . 21
- 3.4 Stress scenarios . . . 24
- 3.5 Systemic stress test of European banks: results . . . 26

4 Indirect exposures **33**
- 4.1 Notional vs effective exposures . . . 33
- 4.2 Indirect exposures: empirical evidence . . . 35
- 4.3 Relevance of indirect contagion for bank stress tests . . . 38
- 4.4 Why indirect exposures cannot be reproduced in single-bank stress tests . . . 41

5 Implications for macroprudential stress testing and regulation **43**

A Appendix **48**
- A.1 Sources for market data . . . 48
- A.2 EBA: data identifiers and residual exposures . . . 49

1 Introduction

1.1 The need for macroprudential stress tests

In October 2008, the IMF's estimate of losses in the US subprime mortgage sector was on the order of 500 bn USD, a large loss but still lower than, say, the loss from the Dot-com bubble burst Hellwig (2009). However, as the US subprime crisis developed into a full-blown global financial crisis, losses spilled over into other asset classes, sectors and countries and ballooned into trillions of dollars Hellwig (2009), exceeding the losses of the Dot-com bubble by an order of magnitude.

Supervisory stress tests for banks, which have become a cornerstone of financial regulation, have focused on examining the resilience of bank balance sheets to severe stress scenarios. However, as the above example illustrates, loss amplification mechanisms which may compound initial losses may be just as important for understanding the nature of systemic risk.

Indeed, as pointed out in the Basel Committee on Banking Supervision's recent report, stress tests conducted by bank supervisors still lack a genuine macroprudential component Basel Committee on Banking Supervision (2015). The report identifies the key missing ingredients as "endogenous reactions and feedback effects to initial stress". As noted by ECB Vice-President Vitor Constâncio Anderson (2016), in the current approach to bank stress tests "*no bank reaction is considered. It would be far more realistic to assume that market participants could react to adverse conditions, rather than assuming passive bank behaviour throughout the entire stress test period. Bank behaviour or reaction could take the form of deleveraging, straight capital increases or working out of non-performing loans.*"

There is ample empirical evidence that such deleveraging occurred on a large scale in 2008, leading to cross-asset contagion and amplification of losses in the financial system Kashyap et al. (2008); Brunnermeier and Pedersen (2009); Khandani and Lo (2011); Manconi et al. (2012); Cont and Wagalath (2016). The well-documented occurrence of fire sales during market downturns Ellul et al. (2011); Coval and Stafford (2007); Shleifer and Vishny (2011); Jotikasthira et al. (2012) is not a coincidence: portfolio constraints -capital, leverage or liquidity constraints- that financial institutions are subject to forces them to deleverage when these constraints are breached as a result of losses, leading to fire sales of assets Kyle and Xiong (2001); Cont and Wagalath (2013). Similar large-scale deleveraging is also foreseeable in future stress scenarios, and foreseen by financial institutions themselves: "*If we are unable to raise needed funds in the capital markets (including through offerings of equity and regulatory capital securities), we may need to liquidate unencumbered assets to meet our liabilities. In a time of reduced liquidity, we may be unable to sell some of our assets, or we may need to sell assets at depressed prices, which in either case could adversely affect our results of operations and financial condition*" Credit Suisse (2015).

Fire sales generate endogenous risk Shin (2010) and can act as channel of loss contagion across asset classes and across financial institutions holding these assets Cont and Wagalath (2016); Caccioli et al. (2014). Unlike direct contagion through counterparty exposures Cont et al. (2013), fire-sales spillovers are mediated by prices and thus defy limits on counterparty exposures and institutional ring-fencing measures. As noted by Glasserman and Young (2014), following the introduction of large exposure limits and collateral requirements, the likelihood of direct contagion through counterparty exposures has diminished in the banking system.

It is therefore important for supervisors to include in macro-stress testing frameworks used for assessing bank capital adequacy the impact of fire sales and the deleveraging of portfolios in stress scenarios. This is especially relevant given that, post-crisis, supervisory stress tests have set a binding constraint for bank capital adequacy.[1]

Fire sales and the resulting destabilizing feedback effects have been extensively studied in the literature Kyle and Xiong (2001); Shleifer and Vishny (2011) from a conceptual viewpoint. The challenge is to develop a quantitative framework versatile enough to be taken to empirical data and used in an operational macro-stress testing framework to quantify the endogenous risk and spillover effects arising from fire sales.

The goal of the present work is to address this challenge, by proposing a modeling framework for quantifying the exposure of the financial system to the endogenous losses and feedback effects resulting from fire sales in a macro-stress scenario. We provide a detailed discussion of the model, its use for the design of *systemic stress tests*, and the results obtained by applying the methodology to EU bank portfolios.

Previous attempts by regulators to account indirectly for the impact of fire sales in bank stress tests include the use of (exogenously specified) liquidation costs or increasing the severity of shocks in single-bank stress tests to account for possible loss amplification due to feedback from fire sales. These adjustments may mimick the severity of potential losses which may result from fire sales but fail to capture key cross-sectional features of fire-sales spillovers, such as the contagion across asset classes and the heterogeneous distribution of fire-sales losses across financial institutions and across asset classes of varying liquidity.

More recently, Greenwood et al. (2015); Duarte and Eisenbach (2013) have proposed a stress testing approach incorporating the impact on asset prices of deleveraging in bank portfolios, based on the assumption of *leverage-targeting* Adrian and Shin (2010) i.e. that, in response to a shock, financial institutions rebalance their portfolios to maintain a constant leverage, which leads to a linear deleveraging rule in reaction to market shocks. This approach was used to analyze fire-sales spillovers in the EU banking system by Greenwood et al. (2015) and in the US banking system by Duarte and Eisenbach (2013). Both studies find evidence of potentially large exposures of the banking system to contagion via fire sales. This approach has been explored by supervisors as a possible method for incorporating fire sales in macro stress tests Henry et al. (2013); Cappiello et al. (2015). There is some empirical evidence that in the medium term large financial institutions maintain fairly stable levels of leverage Adrian and Shin (2010) but it is not clear why the same institutions would enforce such leverage targets in the short term, especially in stress scenarios where this could entail high liquidation costs.

We propose a different approach for modeling fire sales, based on the premise that deleveraging by financial institutions occurs in reaction to losses in their portfolios Kyle and Xiong (2001); Cont and Wagalath (2013). This deleveraging may be the result of investor redemptions for funds, as evidenced in Ellul et al. (2011); Coval and Stafford (2007); Shleifer and Vishny (2011); Jotikasthira et al. (2012); but for regulated financial institutions such as banks, large scale deleveraging is mainly driven by portfolio constraints – capital, leverage or liquidity constraints – which may be breached when large losses occur. We have focused for simplicity on leverage constraints, but the model is easily extendable to multiple constraints on portfolios. Given the one-sided nature of these constraints, such institutions react asymmetrically to large losses and large gains Ang

[1]See the Section "Process and Requirements after CCAR 2016: `https://www.federalreserve.gov/newsevents/press/bcreg/bcreg20160629a1.pdf`.

et al. (2006). The asymmetry and the threshold nature of deleveraging differentiates our approach from models based on leverage targeting, leading to quite different outcomes.

Deleveraging by financial institutions impacts market prices and, when portfolios are marked to market, leads to further losses which may in turn trigger further deleveraging. We quantify this impact and the resulting endogenous risk, paying attention to the estimation of market impact parameters; the magnitude of these parameters, and their heterogeneity across asset classes, is shown to greatly influence the results of the stress tests.

We use these ingredients to design a systemic stress testing framework for banking systems. Application of the method to the EU banking system shows that this approach may lead to outcomes which are substantially different from single-bank stress tests. We emphasize in particular the concept of *indirect exposure*, which we show to be relevant for bank stress testing.

1.2 Summary and main findings

We present a framework for quantitative modeling of fire sales in a network of financial institutions with common asset holdings, subject to leverage or capital constraints. Asset sales may be triggered in reaction to external shocks to asset values when portfolios are subject to capital or leverage constraints; the market impact of these asset sales then leads to contagion of mark-to-market losses to other portfolios, which may in turn be led to deleverage if their constraints are breached. In contrast to balance sheet contagion which arises through direct bilateral exposures, this *price-mediated contagion* occurs through common asset holdings, even in absence of direct linkages between financial institutions. The resulting feedback loop may lead to loss amplification, *systemic risk* and large-scale instability of the financial system. Our model provides a method for quantifying the exposure of the financial system to these effects, which we apply to data on European banks. This leads to several interesting findings.

- **Existence of a tipping point**: we show the existence of a critical macro-shock level beyond which fire sales trigger considerable contagion. The level of this critical shock depends on the institutions' leverage, as well as the concentration, commonality and liquidity of their asset holdings. In the European banking system, this critical shock size is found to correspond to large, but not extreme, losses in asset values.

- **Magnitude and heterogeneity of losses due to fire sales**: we find that fire sales contribute significantly to system-wide losses in stress scenarios, accounting for more than 35% of the total losses and between 20 to 40% of system bank equity. These results are significant enough to modify the outcome of bank stress tests.

  Moreover, while the total system-wide loss can always be replicated in a stress test without fire sales by applying a larger shock to assets, the heterogeneous cross-sectional distribution of losses across banks cannot be reproduced in absence of fire sales by simply applying a larger macro-shock.

- **Importance of gain-loss asymmetry and the threshold nature of fire sales:** We argue that fire sales arise when portfolio constraints such as leverage, liquidity or capital ratios are breached as a result of large portfolio losses. The *one-sided* nature of these constraints leads to an asymmetric reaction of banks to gains and losses, which differentiates our model from the 'leverage targeting' model used in some

previous studies of fire sales Adrian and Shin (2010); Duarte and Eisenbach (2013); Greenwood et al. (2015). Comparison with stress tests based on leverage targeting shows substantial differences in outcomes: leverage targeting models overestimate the magnitude of fire sales, especially at smaller shock levels, but underestimate the acceleration (convexity) of fire sales with increasing shock size present in the threshold model.

- **Distinction between insolvency and illiquidity:** unlike previous models of contagion, which have mainly focused on modeling insolvency, our model distinguishes between failure due to insolvency and failure due to illiquidity. We observe that, while insolvency is the dominant mode of failure of banks in scenarios associated with extremely large initial shocks, illiquidity is the dominant mode of failure in scenarios associated with moderate shocks which are nevertheless large enough to trigger fire sales.

- **Indirect exposures:** As a result of fire-sales spillovers, a portfolio's exposure to an asset class in a stress scenario may be larger than its notional exposure. This naturally leads to the notion of *indirect exposure*, which is scenario-dependent, and can be quantified using our model. One striking finding is that many EU banks have significant (indirect) exposures to asset classes they do not hold, such as commercial and residential mortgages in EU countries where they do not issue loans.

- **Sensitivity to market depth across asset classes:** Calibration to data on market prices, trading volumes and turnover reveals a significant dispersion in market depth across asset classes. We show that ignoring this heterogeneity of market depth across asset classes leads to a considerable bias in the estimation of fire-sales losses in stress tests. This highlights the importance of conducting a rigorous sensitivity analysis on liquidity estimates.

- **Second-round effects are significant**: fire sales may lead to a feedback loop which generates market losses and further fire sales across other financial institutions. Ignoring these feedback effects and the corresponding second- and higher-round deleveraging may lead to a significant underestimation of system-wide losses.

These findings have many implications for risk management in financial institutions and for the monitoring systemic risk in the financial sector. In particular, they underline the need for a systemic approach to stress testing and the necessity of macroprudential tools for tackling the risks resulting from price-mediated contagion. We believe the model presented here provides a useful tool for monitoring of system-wide and bank-level exposure to price-mediated contagion.

1.3 Outline

The paper is structured as follows. Section 2 introduces our modeling framework and outlines a method for systemic stress testing in presence of fire sales. Section 3 describes the application of this stress testing approach to the European banking system using data from the European Banking Authority (EBA) and details our empirical findings. In Section 4, we introduce the concept of *indirect exposure* resulting from fire-sales spillovers and illustrate the magnitude of indirect exposures in the European banking system. Section 5 discusses implications of our findings for systemic stress testing, risk management in institutions and macroprudential policy.

2 Modeling spillover effects from fire sales

2.1 Balance sheets and portfolio constraints

We consider a stylized model of the financial sector, with multiple financial institutions whose portfolios may contain holdings across multiple asset classes. We will sometimes refer to these institutions as 'banks' although, as we will explain below, it is relevant to include non-banks in the scope of the model. Each institution holds two types of assets:

1. Illiquid assets: These are portfolio holdings which are either not easily marketable or would be subject to a deep discount if they were to be sold during a stress scenario; we assume that they are not subject to fire sales. This category includes non-securitized loans, commercial and residential mortgage exposures and retail exposures.[2]

2. Securities: These are positions in marketable securities which may be liquidated over a short time scale if necessary. Sovereign bonds, corporate bonds and derivatives exposures are included in this category. Each asset class μ in this category is characterized by a market depth parameter D_μ, whose estimation we will describe in detail below. The market liquidity of assets within this group can vary strongly and our model accounts for this heterogeneity.

Financial institutions are labeled by $i = 1..N$, security (types) by $\mu = 1..M$, illiquid asset classes by $\kappa = 1..K$. Securities holdings (in euros) of institution i are denoted $(\Pi^{i,\mu}, \mu = 1..M)$ and holdings in illiquid assets are denoted $(\Theta^{i,\kappa}, \kappa = 1..K)$.

The capital (equity) of financial institution i is denoted C^i and $I_i = \sum_{\kappa=1}^{K} \Theta^{i,\kappa}$ is the total value of illiquid assets. The leverage ratio of the institution is then given by

$$\lambda^i(\Pi, C, I) = \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + I_i}{C^i} = \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + \sum_{\kappa=1}^{K} \Theta^{i,\kappa}}{C^i} \leq \lambda_{\max}, \tag{1}$$

where the upper bound $\lambda_{\max} > 1$ corresponds to a regulatory leverage constraint, as for instance required by Basel 3. We consider here for simplicity that the leverage ratio will be the binding constraint for financial institutions' capital, but the model may be easily adapted to include more than one constraint: for instance, if we introduce regulatory risk weights w_μ for each asset class, the constraint on the ratio of capital to risk weighted assets may be expressed as

$$\frac{\sum_{\mu=1}^{M} w_\mu \Pi^{i,\mu} + \sum_{\kappa=1}^{K} w_\kappa \Theta^{i,\mu}}{C^i} \leq R_{\max}.$$

Other portfolio constraints, such as liquidity ratios, lead to similar one-sided linear inequalities.

The state of the financial system is summarized by the matrices of holdings Π, Θ and the capital levels C. We will now describe how this system evolves when subject to macroeconomic stress.

[2]This category further includes assets that do not suffer any direct or indirect losses, are not available for deleveraging, but still contribute to balance sheet size and leverage (e.g. intangible assets).

2.2 Stress scenarios

We consider stress scenarios described through the percentage loss $\epsilon_\kappa \in [0, 100\%]$ in values for each asset class κ, as in the approach used in regulatory bank stress tests. For simplicity, and to emphasize contagion across asset classes, we will consider shocks to illiquid assets in the examples below, but the model perfectly accommodates stress scenarios with heterogeneous shocks across all asset classes. The initial loss of bank i in the stress scenario $\epsilon = (\epsilon_\kappa, \kappa = 1..K)$ is given by

$$< \Theta^i, \epsilon >= \sum_{\kappa=1}^{K} \epsilon_\kappa \Theta^{i,\kappa}. \tag{2}$$

This results in a loss (i.e. a reduction) in the value of illiquid holdings, reducing it to

$$I_i(\epsilon) := I_i - < \Theta^i, \epsilon >$$

and a corresponding decrease in equity:

$$C_0^i(\epsilon) = (C^i - < \Theta^i, \epsilon >)_+. \tag{3}$$

The leverage of bank i then increases to

$$\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) = \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + I_i(\epsilon)}{C_0^i(\epsilon)} = \frac{\sum_{\mu=1}^{M} \Pi^{i,\mu} + I_i - < \Theta^i, \epsilon >}{(C^i - < \Theta^i, \epsilon >)_+}. \tag{4}$$

If this value exceeds the leverage constraint $\lambda_{\max}$ then the institution needs to deleverage, i.e. sell some assets.[3] This occurs if the loss level exceeds the threshold

$$< \Theta^i, \epsilon > \quad \geq \frac{C^i \left(\lambda_{\max} - \lambda^i(\Pi, C, I)\right)}{\lambda_{\max} - 1}, \tag{5}$$

which is proportional to the capital buffer of i or its distance from the leverage constraint prior to the shock. The amount of loss an institution can absorb before being led to deleverage is equal to its capital buffer *in excess* of regulatory requirements (Hellwig (2009)).

In the case where the shock is applied to a single asset class κ, institution i is led to deleverage when the (percentage) loss ϵ_κ exceeds the level

$$\epsilon^*_{i,\kappa}(\Pi, C, I) = \frac{C^i(\lambda_{\max} - \lambda^i(\Pi, C, I))}{(\lambda_{\max} - 1)\Theta^{i,\kappa}}. \tag{6}$$

This threshold is clearly different across institutions, depending on their initial leverage, capital buffer and holdings in the illiquid asset subject to losses. The model hence explicitly accounts for the heterogeneity in the individual bank's resilience to losses.

Hence, any stress scenario ϵ which falls outside of the (convex) set

$$\mathcal{S}(\Pi, C, \Theta) = \left\{ \epsilon \in [0,1]^K \,\middle|\, \forall i = 1..N, < \Theta^i, \epsilon > \quad \leq \frac{C^i(\lambda_{\max} - \lambda^i(\Pi, C, I))}{(\lambda_{\max} - 1)}, \right\}. \tag{7}$$

[3] If one allowed for negative capital by removing the positive part in (3), one would need to distinguish the additional case of negative leverage, which we avoid here. We also note that as $I_i(\epsilon) > 0$ for all reasonable shocks (c.f. Table 2), the ratio is well defined.

will lead to deleveraging by one or more financial institutions. The convex set $\mathcal{S}(\Pi, C, \Theta)$ corresponds to a 'safety zone' of stress scenarios which all banks can withstand; the larger this safety zone, the more resilient is the network to losses.

We assume that no major deleveraging occurs inside this zone. This is different from 'leverage targeting' models such as Greenwood et al. (2015), where portfolios react (symmetrically) to arbitrarily small shocks.

The size of the safety zone depends on the capital levels C_i of the financial institutions but also on their capital buffers in excess of requirements. If the vector ϵ of shocks to assets lies outside the safety zone, then one or more institutions are led to deleverage their portfolios. We now describe how this deleveraging is modeled.

2.3 Deleveraging

If (and only if) the magnitude of losses in asset values is such that the leverage constraint is breached for institution i, it deleverages a proportion Γ^i of its portfolio in order to restore its leverage ratio to a leverage target $\lambda_b \leq \lambda_{\max}$. This leads to the following equation for $\Gamma^i \in [0,1]$:

$$\frac{(1-\Gamma^i)\sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon)}{C_0^i(\epsilon)} = \lambda_b \tag{8}$$

Thus, in response to an external shock ϵ, institution i needs to deleverage a fraction

$$\begin{aligned}\Gamma^i(\Pi, C_0(\epsilon)) &= \left(\frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon) - \lambda_b C_0^i(\epsilon)}{\sum_{\mu=1}^M \Pi^{i,\mu}} \wedge 1\right) \mathbf{1}_{\frac{\sum_{\mu=1}^{\mathbf{M}} \mathbf{\Pi^{i,\mu}} + \mathbf{I_i}(\epsilon)}{\mathbf{C_0^i}(\epsilon)} > \lambda_{\max}} \\ &= \left(\frac{C_0^i(\epsilon)(\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) - \lambda_b)}{\sum_{\mu=1}^M \Pi^{i,\mu}} \wedge 1\right) \mathbf{1}_{\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) > \lambda_{\max}},\end{aligned} \tag{9}$$

of its marketable assets, where $\Pi_0, C_0, I(\epsilon)$ are given by (3) In terms of the *initial* capital C and the illiquid holdings Θ, this can be written as

$$\Gamma^i(\Pi, C) = \left(\frac{(\lambda_b - 1) < \Theta^i, \epsilon > + C^i(\lambda^i(\Pi, C, I) - \lambda_b)}{\sum_{\mu=1}^M \Pi^{i,\mu}} \wedge 1\right) \mathbf{1}_{\frac{\sum_{\mu=1}^{\mathbf{M}} \mathbf{\Pi^{i,\mu}} + \mathbf{I} - <\mathbf{\Theta^i}, \epsilon>}{(\mathbf{C} - <\mathbf{\Theta^i}, \epsilon>)_+} > \lambda_{\max}}. \tag{10}$$

Unlike the state variables Π, C which evolve as deleveraging occurs, Θ and ϵ (and $I(\epsilon)$), which represent respectively the holdings in illiquid assets and the initial shocks to these assets, are static parameters.

As in Greenwood et al. (2015); Duarte and Eisenbach (2013), we assume that banks delever their marketable assets proportionally. This proportional deleveraging assumption is supported by empirical studies on asset sales of large financial institutions Getmansky et al. (2016), Schaanning (2017). An alternative would be to assume a pecking order of liquidation, or that banks determine their liquidation policy by maximizing expected liquidation value Braouezec and Wagalath (2017). Greenwood et al. (2015); Duarte and Eisenbach (2013) perform robustness tests on the order of liquidation, finding that it reduces the magnitude of fire-sales losses.

Figure 1 (circles) displays the dependence of the deleveraging ratio (10) on the shock level ϵ for a portfolio holding a single class of illiquid asset: this ratio is zero for shocks lower than the threshold ϵ_i^* corresponding to the breach of the leverage constraint, then increases linearly thereafter. There is a discontinuity at the onset of deleveraging, which

depends on the size of the capital buffer which the bank intends to rebuild by deleveraging. In the case where the leverage constraint is saturated after deleveraging, i.e. $\lambda_b = \lambda_{\max}$, the dependence on the shock size is continuous and convex (solid line). In the example shown in Figure 1 we have assumed $\lambda_b = 0.95\lambda_{\max}$, which corresponds to a safety buffer 5% below the leverage constraint.

Previous studies on fire sales have assumed instead a *linear* dependence of the volume of deleveraging with respect to the shock size (dotted line): this is the leverage targeting model, which we will discuss further in Section 2.6.

Unlike the leverage targeting model, our model gives rise to deleveraging only if the shock level exceeds a (bank-specific) threshold ϵ_i^*. Once this threshold is reached, the volume of deleveraging increases linearly with $(\epsilon - \epsilon_i^*)_+$ until no more marketable assets are available for sale (i.e. $\Gamma^i = 1$). At this point, although the institution is still solvent, it may become illiquid. The model thus leads to a natural distinction between failures due to insolvency, which may occur if the initial loss in asset values is large enough, and failures due to illiquidity, which may occur further down the road if the liquidation of marketable assets fails to raise enough liquidity.

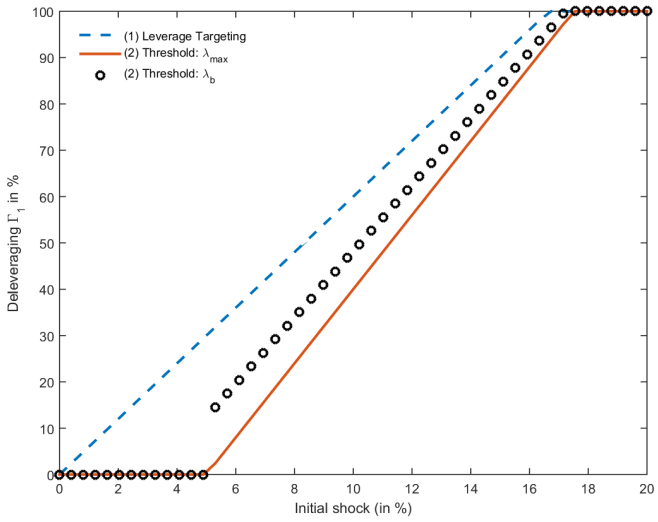

Figure 1: Volume of asset sales (% of marketable assets) as a function of the percentage loss in value of illiquid assets for a portfolio with 20% in illiquid assets and initial leverage of 25, and a leverage constraint of 33. Leverage targeting leads to a linear response (dashed line), whereas our assumption of deleveraging to comply with a leverage constraint leads to no asset sales for shocks smaller than a threshold and a linear increase above the threshold (solid line). Finally, if we assume that the bank deleverages to restore a non-zero capital buffer we obtain the discontinuous response function (circles).

Summing across all institutions $j = 1..N$ in the network yields the total volume of asset sales (in monetary units) in the stress scenario ϵ:

$$q^\mu(\epsilon, \Pi, C) = \sum_{j=1}^{N} \Gamma^j(\Pi, C_0(\epsilon))\Pi^{j,\mu}$$

for the asset class μ. In the case $\lambda_b = \lambda_{\max}$, the functions $\Gamma^j(\Pi, C_0(\epsilon))$ are convex with respect to ϵ over a large range of values, i.e. in the region $\max_i \Gamma^i \leq 1$. In this range,

the aggregate volume of asset sales $q^\mu(\epsilon, \Pi, C)$ exhibits a convex dependence in ϵ, which leads to a 'multiplier effect': as more severe stress scenarios (larger ϵ) are considered, the marginal response in terms of deleveraging also increases. An example, based on data from the EU banking system (see next section) is shown in Figure 2.

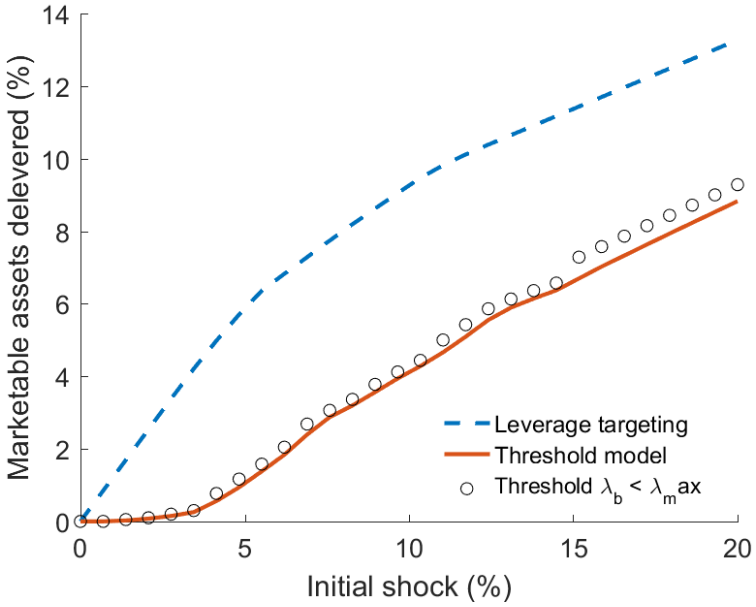

Figure 2: Volume of asset sales (% of marketable assets) across EU banks in reaction to a scenario in which banks realize losses of ϵ percent of the notional value on Spanish residential and commercial real estate exposures (horizontal axis). The Basel 3 leverage constraint $\lambda_{\max} = 33$ is used in this example. The solid red line corresponds to $\lambda_{\max} = \lambda_b$, the circles correspond to $\lambda_b = 0.95\lambda_{\max}$.

2.4 Market impact and price-mediated contagion

If this volume of deleveraging represents a sizeable fraction of the market depth, it may have a non-negligible impact on the market price of these assets and lead to a price decline, whose magnitude ΔS^μ is an increasing function of q^μ:

$$\frac{\Delta S^\mu}{S^\mu} = -\Psi_\mu(q^\mu) \tag{11}$$

where $\Psi_\mu : \mathbb{R} \to [0, 1]$ is a continuous, increasing and concave function with $\Psi_\mu(0) = 0$, which we call the *market impact function* for asset class μ. Ψ_μ may be thought of as an inverse demand function. Assuming Ψ_μ is a smooth function, linearizing for small volumes yields

$$\Psi_\mu(q) \simeq \frac{q}{D_\mu} \quad \text{where} \quad D_\mu = \frac{1}{\Psi'_\mu(0)}$$

is a measure of *market depth* for asset class μ. Naturally this quantity depends on the asset μ, as the same dollar liquidation volume q will have a different price impact depending on the asset class. A simple one-parameter specification often used in practice is $\Psi_\mu(q) = \psi(q/D_\mu)$ where $\psi(.)$ is an increasing function with $\psi'(0) = 1$. We discuss parametric specifications of Ψ_μ and their estimation in Section 3.2.

We can now describe the processes which occur at (the k-th round of) deleveraging. Denote by $\Pi_{k-1}, C_{k-1}, S_{k-1}$ respectively the holdings in marketable assets, the equity and the (vector of) asset prices after $k-1$ rounds of deleveraging. At round k:

1. Each institution j deleverages by selling a proportion $\Gamma^j(\Pi_{k-1}, C_{k-1})$ of its marketable assets, leading to an aggregate amount $q_k^\mu = \sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1})\Pi_{k-1}^{j,\mu}$ of sales in asset class μ.

2. The market impact of asset sales results in a decline in market prices, moving the market price to
$$S_k^\mu = S_{k-1}^\mu \left(1 - \Psi_\mu\left(q_k^\mu\right)\right). \tag{12}$$

3. This decline in price changes the market value of holdings in asset class μ to

$$\begin{aligned}\Pi_k^{i,\mu} &:= \Pi(\Pi_{k-1}, C_{k-1}) \qquad (13)\\ &= \underbrace{\left(1 - \Gamma^i(\Pi_{k-1}, C_{k-1})\right)}_{\text{Remainder after deleveraging by } i} \overbrace{\Pi_{k-1}^{i,\mu}}^{\text{Previous value}} \underbrace{\left(1 - \Psi_\mu\left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1})\Pi_{k-1}^{j,\mu}\right)\right)}_{\text{Price impact on remaining holdings}}.\end{aligned}$$

This generates two types of losses for portfolio i. First, the price moves due to the market impact of fire sales, which leads to a mark-to-market loss given by

$$\begin{aligned}M^i(\Pi_{k-1}, C_{k-1}) &= \sum_{\mu=1}^M \left((1 - \Gamma^i(\Pi_{k-1}, C_{k-1}))\Pi_{k-1}^{i,\mu} - \Pi_k^{i,\mu}\right) \qquad (14)\\ &= (1 - \Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \Pi_{k-1}^{i,\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1})\Pi_{k-1}^{j,\mu}\right).\end{aligned}$$

A second source of loss, not accounted for in previous studies, stems from the fact that assets are not liquidated at the current market price but at a discount: this 'implementation shortfall', as it is called in the literature on optimal trade execution Almgren and Chriss (2000)) corresponds to the difference between the market price at the time of sale and the volume-weighted average price (VWAP) during liquidation. This VWAP lies somewhere between the pre- and post-fire-sales prices. We model it as a weighted average with weights $(1-\alpha, \alpha), \alpha \in [0,1]$ of the pre- and post-fire-sales prices, where $\alpha = 0$ corresponds to zero implementation shortfall, and $\alpha = 1$ corresponds to full implementation shortfall (assets liquidated at post-fire sales price). This leads to the general formula for the implementation shortfall:

$$\begin{aligned}R^i(\Pi_{k-1}, C_{k-1}) &= \sum_{\mu=1}^M \left[\Gamma_k^i \Pi_{k-1}^{i,\mu} - \left((1-\alpha)\Gamma_k^i \Pi_{k-1}^{i,\mu} + \alpha\Gamma_k^i \Pi_{k-1}^{i,\mu}(1 - \Psi_\mu(q_k^\mu))\right)\right]\\ &= \alpha\Gamma^i(\Pi_{k-1}, C_{k-1}) \sum_{\mu=1}^M \Pi_{k-1}^{i,\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1})\Pi_{k-1}^{j,\mu}\right). \qquad (15)\end{aligned}$$

where we wrote Γ_k^i as shorthand for $\Gamma^i(\Pi_{k-1}, C_{k-1})$. In the empirical examples below, we will use $\alpha = \frac{1}{2}$, which corresponds to a VWAP midway between the pre- and post-fire-sales

prices. In Cont and Schaanning (2017) it is shown that $\alpha \geq \frac{1}{2}$ is a plausible assumption, also from a modeling perspective.

Summing (15) with (14) yields the total loss of portfolio i at the k-th round of deleveraging:

$$\begin{aligned} L^i(\Pi_{k-1}, C_{k-1}) &= M^i(\Pi_{k-1}, C_{k-1}) + R^i(\Pi_{k-1}, C_{k-1}) \\ &= \big(1 - (1-\alpha)\Gamma^i(\Pi_{k-1}, C_{k-1})\big) \sum_{\mu=1}^{M} \Pi_{k-1}^{i,\mu} \Psi_\mu \left(\sum_{j=1}^{N} \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j,\mu} \right). \end{aligned} \tag{16}$$

This loss reduces the equity of institution i by the same amount:

$$C_k^i = \big(C_{k-1}^i - L^i(\Pi_{k-1}, C_{k-1})\big)_+ \tag{17}$$

Linearizing the market impact function Ψ_μ yields

$$L^i(\Pi_{k-1}, C_{k-1}) \approx (1 - (1-\alpha)\Gamma^i) \sum_{j=1}^{M} \underbrace{\sum_{\mu=1}^{N} \frac{\Pi_{k-1}^{i,\mu} \Pi_{k-1}^{j,\mu}}{D_\mu}}_{\Omega_{ij}(\Pi_{k-1})} \Gamma^j = (1 - (1-\alpha)\Gamma^i) \sum_{j=1}^{M} \Omega_{ij}(\Pi_{k-1}) \Gamma^j, \tag{18}$$

which shows that the magnitude of fire-sales spillovers from institution i to institution j is proportional to the *liquidity-weighted overlap* Ω_{ij} between portfolios i and j Cont and Wagalath (2013):

$$\Omega_{ij}(\Pi) := \sum_{\mu=1}^{M} \frac{\Pi^{i,\mu} \Pi^{j,\mu}}{D_\mu}. \tag{19}$$

The matrix of portfolio overlaps

$$\Omega(\Pi) = \Pi D^{-1} \Pi^\top, \tag{20}$$

where D is the diagonal matrix of market depths D_μ, can be viewed as a weighted adjacency matrix of the underlying network, linking portfolios through their common exposures. We will further analyze the properties of this matrix in Section 3.

In summary, an initial loss in asset values may trigger a feedback loop, schematically represented in Figure 3, in which, at each iteration, portfolio deleveraging leads to fire sales, leading to price declines and mark-to-market losses which may in turn trigger further fire sales. The state variables representing the matrix Π of portfolio holdings in marketable assets and the equity levels $C = (C_i, i = 1..N)$ are initialized as described in equations (3)-(4) (for $k = 0$) and updated at each round of deleveraging

$$(\Pi_k, C_k) = f(\Pi_{k-1}, C_{k-1}), \tag{21}$$

where Π_k is defined in (13) and

$$C_k^i = \big(C_{k-1}^i - L^i(\Pi_{k-1}, C_{k-1})\big)_+ \tag{22}$$

where the loss L^i is defined in (16).

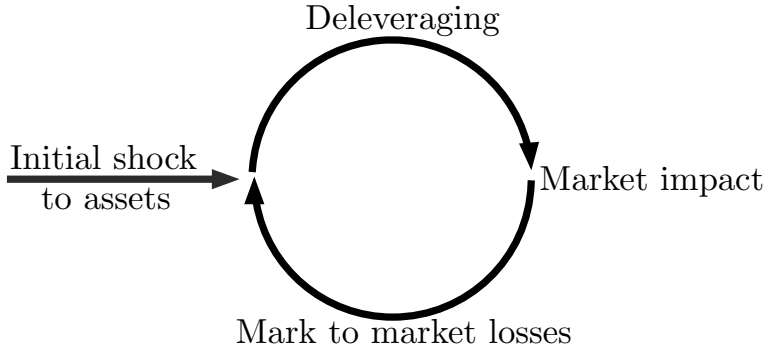

Figure 3: An initial loss in asset values may generate a feedback loop which may lead to multiple rounds of deleveraging and further declines in asset values.

2.5 Feedback loops, insolvency and illiquidity

The iteration described above continues in principle as long as at least one institution is in breach of its leverage/ capital constraint after losses due to deleveraging are accounted for. However, in the (realistic) situation where we assume that institutions build a non-zero buffer beyond the minimal capital requirements (i.e. $\lambda_b < \lambda_{\max}$ in the notation of Section 2.3), this *fire-sales cascade* terminates after a finite number T of iterations Cont and Schaanning (2017). As we will discuss below, this is not the case in leverage targeting models, which lead to infinite fire-sales cascades.

Along the way, some institutions may become insolvent: this occurs if at any point in the iterations the loss $L^i(\Pi_k, C_k)$ exceeds the capital C_k. Then instution i becomes insolvent and does not play any further role in subsequent rounds. Another type of failure which may occur along the cascade is failure due to *illiquidity*: this occurs when an institution has sold all of its marketable assets and is left with no further liquid assets. This may occur even though the institution is still solvent.

This distinction between failure due to insolvency and failure due to illiquidity is highly relevant in practice. In fact, one can note that this was precisely the scenario that occurred in the failure of Bear Stearns and Lehman Brothers.[4] In contrast to most default risk models and previous studies on fire sales, our model distinguishes between these two causes of failure and highlights the fact that institutions can fail even when they have positive equity.

In contrast to models of default contagion, contagion of losses across institutions occurs not just at default but actually *before* the default of an institution, and its scope is not limited to counterparties. Deleveraging by distressed institutions, which is precisely aimed at *preventing* their default, is in fact what triggers this contagion.

Denoting by T the length of the cascade, the fire-sales loss for bank i triggered by the stress scenario ϵ is given by

$$FLoss(i, \epsilon) = \sum_{k=1}^{T} L^i(\Pi_{k-1}, C_{k-1}). \tag{23}$$

and the total system-wide fire-sales loss in this scenario is

$$SLoss(\epsilon) = \sum_{i=1}^{N} FLoss(i, \epsilon). \tag{24}$$

[4]See letter by the then SEC chairman Christopher Cox to the Basel Committee on Banking Supervision `https://www.sec.gov/news/press/2008/2008-48.htm`.

Note that the fire-sales loss (23) does not include the initial loss which triggers the deleveraging: in absence of deleveraging and price-mediated contagion $FLoss(i) = SLoss = 0$. Table 1 summarizes the notations of the model and references the equations where they are defined.

Variable	**Notation**	**Defined in**
Financial institutions	$i, j = 1..N$	-
Asset class: illiquid assets	$\kappa = 1..K$	Section 2.1
Asset class: marketable assets	$\mu = 1..M$	Section 2.1
Number of iterations (rounds)	k	-
State variables (in EUR)		
Marketable assets	$\Pi^{i,\mu}$	Section 2.1
Capital	C^i	Section 2.1
Parameters		
Illiquid asset (in EUR)	$\Theta^{i,\kappa}$	Section 2.1
Initial shock (in %)	ϵ_κ	(4)
Market depth for asset class μ	D_μ	(13)
Key quantities		
Deleveraging proportion at round k	Γ_k^i	(9)
Leverage of institution i	λ^i	(1)
Fire-sales loss (k-th round)	$L^i(\Pi_{k-1}, C_{k-1})$	(16)
Fire-sales loss for bank i (all rounds)	$FLoss(i, \epsilon)$	(23)
System-wide fire-sales loss	$SLoss(\epsilon)$	(24)

Table 1: Overview of model notations.

2.6 Comparison with "leverage targeting"

Recent empirical studies on fire-sales spillovers Duarte and Eisenbach (2013); Greenwood et al. (2015) have explored a different mechanism for fire sales, based on the idea that banks maintain a 'leverage target', which leads them to rebalance their portfolios in a procyclical manner following changes in asset values. An oft-cited argument to support this model is the empirical correlations between quarterly changes in asset size and debt size for banks Adrian and Shin (2010, 2014).

While both models incorporate the idea of market impact of deleveraging and the resulting endogenous portfolio losses and contagion effects, they differ in some important ways:

1. *The threshold nature of fire sales*: in the leverage targeting model, bank (de)leveraging in response to arbitrarily small changes in asset values, regardless of their capital or liquidity buffers, generates fire-sales losses even for low market stress levels. In our model, deleveraging only occurs when losses are large enough to trigger portfolio constraints: for shocks below this critical level, there is no deleveraging. By assuming that all institutions constantly respond to arbitrarily small changes in asset values, the leverage targeting model overestimates the magnitude of deleveraging in response to small shocks. This is illustrated in Figure 2, which compares the overall deleveraging across EU banks in response to losses on exposures to the Spanish housing market.

2. *Dependence of deleveraging on magnitude of losses*: Leverage targeting implies a volume of deleveraging linear in the size of the portfolio loss; since the volume of deleveraging is capped at 100% of assets, this leads to a *concave* dependence of the volume of asset sales on the shock size. By contrast, for small to moderate shocks, the volume of deleveraging has a convex dependence on the loss size in our model, as shown in Section 2.3 and illustrated in the example of Figure 2: deleveraging accelerates as we increase the shock size to more extreme levels, leading to a 'multiplier effect', absent in the leverage targeting model.

3. *Finite length of fire-sales cascades:* The assumption of leverage targeting leads to an infinite sequence of iterations which never cease since at each round further mark-to-market losses are generated endogenously, which leads to a deviation from the target leverage and in turn generates new asset sales or purchases. In stress tests, one then needs to choose an ad-hoc number of iterations to compute the loss. Although losses converge as we iterate this cascade, in general estimates of fire-sales losses depend on the actual number of iterations that chosen in a simulation.

   By contrast, as shown in Cont and Schaanning (2017), in a threshold model with a capital buffer $\lambda_b < \lambda_{\max}$, the fire-sales cascade always terminates after a finite number of iterations, typically 5 to 10 rounds in most empirical examples, as shown in the next section.

The consequences of these differences are explored in more detail in the next section, where we compare the results of stress tests performed using the two approaches, and in the companion paper Cont and Schaanning (2017). To implement the leverage targeting model in our stress test, we simply replace the deleveraging function by

$$\Gamma^i(\Pi, C_0(\epsilon)) = \left(\frac{\sum_{\mu=1}^M \Pi^{i,\mu} + I_i(\epsilon) - \lambda_b C_0^i(\epsilon)}{\sum_{\mu=1}^M \Pi^{i,\mu}} \wedge 1 \right).$$

Only marketable assets are assumed to be available for deleveraging. This assumption is different from Duarte and Eisenbach (2013); Greenwood et al. (2015), where deleveraging is applied to the entire portfolio.

3 A systemic stress test of the European banking system

We now describe how the model may be used to perform a systemic stress test, in order to quantify the exposure of the banking system to fire-sales spillovers, and apply the framework to data on the European banking system.

3.1 Data

Our empirical study is based on data from the European Banking Authority (EBA), which provides information, collected in 2011 and 2016, on notional exposures of 90 European banks across 148 asset classes.[5] Holdings are given by asset class and geographical region.

[5]This dataset was also used in the study by Greenwood et al. (2015), and facilitates comparison with the literature.

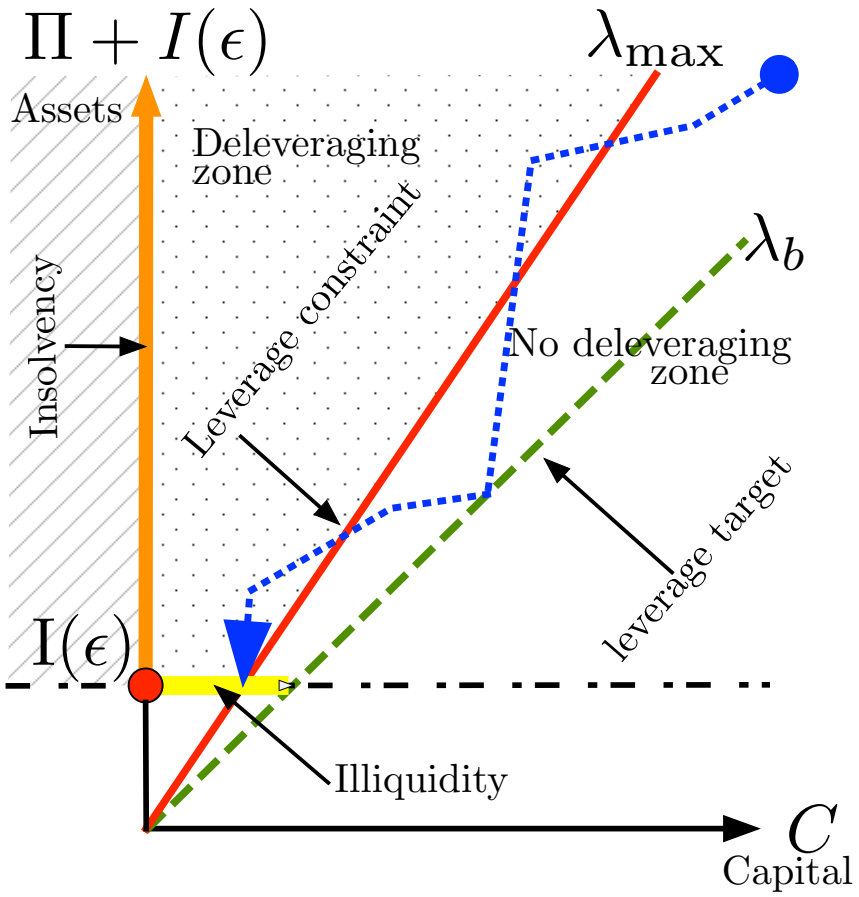

Figure 4: Evolution of asset values (vertical axis) and capital (horizontal axis) in a fire-sales cascade. The solid red line corresponds to the leverage constraint, the dashed green line denotes a target leverage corresponding to an excess capital buffer, and the dotted blue line is the path of a sample portfolio. Losses in asset values erode the equity, moving the portfolio closer to the origin; when the portfolio crosses the red line corresponding to the leverage constraint, deleveraging occurs: the institution tries to reconstitute a buffer by returning to the target leverage (dotted line). The market impact of these asset sales leads to further losses and displaces the state to the left; if it crosses the red line again, a new round of deleveraging follows etc. An institution becomes insolvent when it reaches the boundary $C = 0$ (vertical axis) and illiquid when it reaches the boundary $\Pi = 0$ (horizontal dashed line).

Asset classes are specified in Table 2. Greenwood et al. (2015) assumed all assets to be available for liquidation; we only consider a subset to be marketable, i.e. available for liquidation at short notice. We identify four classes of marketable assets ("securities"), which may be liquidated in a stress scenario; the other asset classes are classified as illiquid assets.[6] Assets are further labelled by 37 geographical regions, which correspond to the 27 countries of the EU (i.e. without Croatia at the time) plus the United States (US), Norway (NO), Iceland (IS), Liechtenstein (LI), Japan (JP), Asia (A1), Other non-EEA non-emerging countries (E3), Eastern Europe non-EEA (E5), Middle and South America (M1) and Rest of the world (R5). Hence, with the four marketable asset classes and 37 geographical regions, the matrix of marketable assets Π is given by a 90×148 matrix.

[6]Table 10 in the Appendix, we provide the data identifiers that allow correspondence with the EBA dataset.

The illiquid asset holdings are given by a 90×75 matrix Θ. This corresponds to 74 asset classes for commercial and residential mortgage exposures respectively in the 37 regions and a 75th entry consisting of all remaining illiquid asset holdings.[7]

Illiquid assets
Residential mortgage exposures
Commercial real estate exposure
Retail exposures: Revolving credits, SME, other
Indirect sovereign exposures in the trading book
Defaulted exposures
Residual exposure (cf. Appendix Table 10)
Securities / marketable assets
Corporate bonds
Sovereign debt
Direct sovereign exposures in derivatives
Institutional client exposures: interbank, CCPs,...

Table 2: Asset classes used for the stress test.

3.2 Market impact and market depth

A key assumption in models of fire-sales spillovers concerns the impact of asset liquidations on market prices. This may be summarized in the choice of a market impact function Ψ_μ, which defines the correspondence between the liquidation size q (in monetary units) and the relative price change for each asset class μ: $\frac{\Delta S^\mu}{S^\mu} = -\Psi_\mu(q)$. An adequate choice for Ψ_μ should be increasing, concave, satisfy $\Psi_\mu(0) = 0$ and lead to non-negative prices.

Common specifications are the linear model Kyle (1985); Bertsimas and Lo (1998), Almgren and Chriss (2000); Obizhaeva (2012); Cont et al. (2014)

$$\Psi_\mu(q) = \frac{q}{D_\mu} \quad \text{with} \qquad D_\mu = c\frac{ADV_\mu}{\sigma_\mu} \tag{25}$$

where ADV_μ is the average daily trading volume (in EUR), σ_μ the daily volatility (in %) of the asset, c a coefficient close to 0.5, estimated from transactions data, and the square root model Bouchaud (2010)

$$\Psi_\mu(q) = c\ \sigma_\mu \quad \sqrt{\frac{q}{ADV_\mu}} \tag{26}$$

We note that both trading volume and volatility are associated with a liquidation horizon τ, taken in most studies to be daily by default. If we assume the liquidation horizon τ to be longer than a day, then the market depth parameter needs to be adjusted. In the linear impact model, the adjustment is:

$$D_\mu(\tau) = c\ \frac{ADV_\mu \tau}{\sigma_\mu \sqrt{\tau}} = c\ \frac{ADV_\mu}{\sigma_\mu} \times \sqrt{\tau}. \tag{27}$$

[7]Our representation differs slightly from Greenwood et al. (2015), who considered 42 asset classes consisting of the 37 sovereign exposures by geographical region and five further classes, aggregated across all geographical regions: "commercial real estate", "mortages", "corporate loans", "small and medium enterprise loans" and "retail revolving credit lines".This leads to a less granular model compared to ours as we distinguish assets both by type and country.

This adjustment is important, and corresponds to the intuitive observation that liquidating the same portfolio over a longer horizon reduces impact. The liquidation horizon τ may be interpreted as the time window the banks dispose of to comply with portfolio constraints. In the case study below we will use $\tau = 20$ days.

By contrast, in the square root model, the impact of a transaction is invariant to a change in the liquidation horizon since the denominator and the numerator in (26) scale in the same way. This leads to the counterintuitive (and, we believe, incorrect) conclusion that impact is insensitive to the rate of liquidation. For this reason, we refrain from using the square-root model in the sequel.

In a linear impact model, asset classes are differentiated according to their market depth D_μ. Greenwood et al. (2015) assume a uniform depth $D_\mu = 10^{13}$ (EUR) for all asset classes. This homogeneity assumption is not supported by empirical studies on market impact Bouchaud (2010); Obizhaeva (2012); Cont et al. (2014), which indicate that market impact varies widely across assets. Ignoring this heterogeneity may lead to biased results, overestimating losses in more liquid asset classes while underestimating losses in less liquid asset classes.

Duarte and Eisenbach (2013) use haircuts and repo rates for determining the liquidity of different asset classes; this does introduce some heterogeneity across asset classes but the relation between these quantities and market impact is not clear. For instance, haircuts may simply reflect the volatility of an asset, rather than its liquidity or market depth.

We use a direct, data-driven approach to the modeling of market impact. To estimate the market depth parameters for each asset class using (27), we

- estimate volatility parameters σ_μ using daily returns of S&P sector indices[8].
- obtain average daily volume estimates ADV_μ from annual volume data provided by the US Treasury and various central banks (Appendix A.1).

As noted above, we use $\tau = 20$, which corresponds to a liquidation horizon of 4 weeks, a fairly lenient assumption. Cont and Wagalath (2016) and Obizhaeva (2012) find $c \approx 0.33$. Ellul et al. (2011) find $c \approx 0.2 - 0.3$ for US corporate bonds under fire-sales pressure by insurance companies. An important difference is that in Ellul et al. (2011) the bonds are being liquidated due to the bond issuer's credit rating being downgraded, while in our analysis, we assume that the fire sales are exogenous and not linked to the security issuers. We have used $c = 0.4$ here.

ADV for US corporate bonds was obtained from SIFMA (see Appendix A.1). For European bonds, we simply use the same sovereign-to-corporate ratio of ADV as observed in the US ($567.81bn/269.8bn \approx 0.48$) to estimate the ADV of the corporate and "institutional" asset classes for European corporate bonds.

For some asset classes, data on trading volume are unavailable (or difficult to obtain). We work around this issue by estimating, based on OECD data, the following regression model for the relationship between average daily volume (ADV) and outstanding notional

$$\log ADV_\mu := c_1 \log (N_\mu) + c_0 + \varepsilon_\mu \tag{28}$$

where N_μ denotes outstanding notional, and using it to estimate volume for the remaining asset classes. Table 3 shows the results of this regression analysis.

[8] `http://us.spindices.com/indices/fixed-income/sp-eurozone-sovereign-bond-index`
`http://us.spindices.com/index-family/us-treasury-and-us-agency/all`
`http://us.spindices.com/indices/fixed-income/sp-500-investment-grade-corporate-bond-index`

Coefficient	US treas	US corp	UK	DE	ES
c_1:	0.53***	0.64***	0.56***	0.48	0.94***
std. dev.	0.13	0.15	0.05	0.30	0.14
c_0:	0.63	-1.1**	-0.35**	0.4	-1.35***
std. dev.	0.55	0.55	0.14	0.99	0.37
adj. R^2	0.39	0.60	0.93	0.15	0.76
n	19	19	11	10	15

Table 3: Logarithmic regression of bond trading volume on outstanding notional (Equation 28).

Figure 5 shows the distribution of market depth estimates for all asset classes in the EBA dataset, on a logarithmic scale. The histogram reveals considerable heterogeneity in the cross-sectional distribution of market depth, with 4 orders of magnitude separating the most liquid from the least liquid assets.

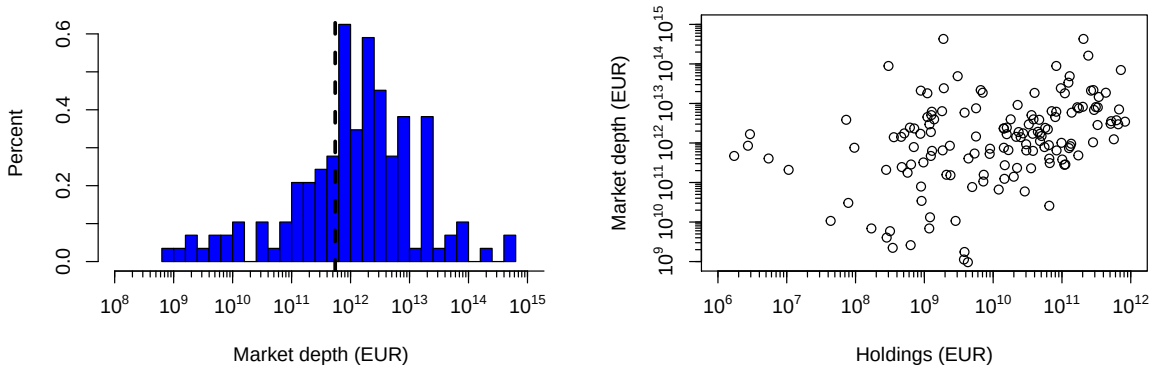

Figure 5: Left: Histogram of estimated market depths for the 148 asset classes in the EBA dataset. The dashed vertical line indicates the holdings-weighted average depth given by (31).Right: Scatter plot of depth vs. holdings.

Table 4 provides values for average daily volumes (ADV) and market depths for the asset classes representing the largest holdings. These values are used as base values; we will later perform a sensitivity analysis of our results with respect to changes in these values.

Extrapolation to large volumes When applied to large transaction volumes, linear or square-root impact models may lead to negative prices. In Greenwood et al. (2015) this was addressed by capping the loss at 100%: $\Psi_\mu(q) = \min\left\{1, \frac{q}{D_\mu}\right\}$. Cifuentes et al. (2005) use an exponential specification

$$\Psi_\mu(q) = 1 - \exp\left(-\frac{q}{D_\mu}\right) \tag{29}$$

which also ensures that prices remain non-negative but gives a concave impact. Nevertheless, prices can get arbitrarily close to zero in both of these models. However, it is

Asset class (sovereign unless specified)	ADV (bn EUR)	Market depth D_μ (10^{12} EUR)	Impact of 10 bn
US	567.8	428.8	0.2332
US (corp.)	269.8	70.2	1.424
IT	28.82	21.8	4.585
ES	28.0	21.1	4.737
DE	24.7	18.7	5.345
GB	24	18.1	5.522
FR	10	7.58	13.18
GR	8.17	6.18	16.16
SE	3.92	2.96	33.67
PT	2.27	1.71	58.14

Table 4: Average daily trading volume and estimated market depth over $\tau = 20$ days, for the largest holdings in the EBA dataset. The impact in basis points are also given for a liquidation of 10 bn EUR over $\tau = 20$ days.

realistic to assume that long before the price level reaches zero, arbitrageurs will step in to purchase assets subject to fire sales at a discount Shleifer and Vishny (1992). To capture this effect we introduce a price floor $B_\mu > 0$ and consider a two-parameter level-dependent price impact function:

$$\Psi_\mu(q, S) := \left(1 - \frac{B_\mu}{S}\right)\left(1 - \exp(-\frac{q}{\delta_\mu})\right). \tag{30}$$

B_μ determines how far the price can fall in a fire-sales scenario. Note that this is a lower bound for the price and in a given stress test the price may not actually fall to this level. In the empirical examples below, we set B_μ at 50 % of the market price levels. Choosing

$$\delta_\mu = \left(1 - \frac{B_\mu}{S_0^\mu}\right) D_\mu$$

makes the specification (30) compatible with the linear specification (25) for small volumes.

3.3 Portfolio overlaps

As shown in Eq. (18), the transmission of fire-sales losses from portfolio i to j depends on the liquidity-weighted overlap between portfolio i and j:

$$\Omega_{ij}(\Pi) := \sum_{\mu=1}^{M} \frac{\Pi^{i,\mu}\Pi^{j,\mu}}{D_\mu}.$$

The matrix Ω of liquidity-weighted overlaps thus plays an important role in the transmission of fire-sales losses, which may be viewed as a contagion process on a network of financial institutions in which the link from i to j is weighted according to the liquidity-weighted overlap Ω_{ij}. We call this network the *indirect contagion network*. Similar network structures were explored by Braverman and Minca (2016); Guo et al. (2015) for

mutual funds. Figure 6 displays the indirect contagion network for European banks as implied by EBA data collected in 2011. The nodes correspond to different banks, with node size proportional to balance sheet size, and edges correspond to non-zero portfolio overlaps, with edge widths proportional to the logarithm of the liquidity-weighted overlap Ω_{ij}.

Figure 7 (left) shows the distribution of Ω_{ij} in this network. The peak at zero reflects the fact that many pairs of banks have no common asset holdings (so, zero overlap), i.e. the network is sparse. On the other hand, the values are dispersed over five orders of magnitude, which illustrates the heterogeneity of the network.

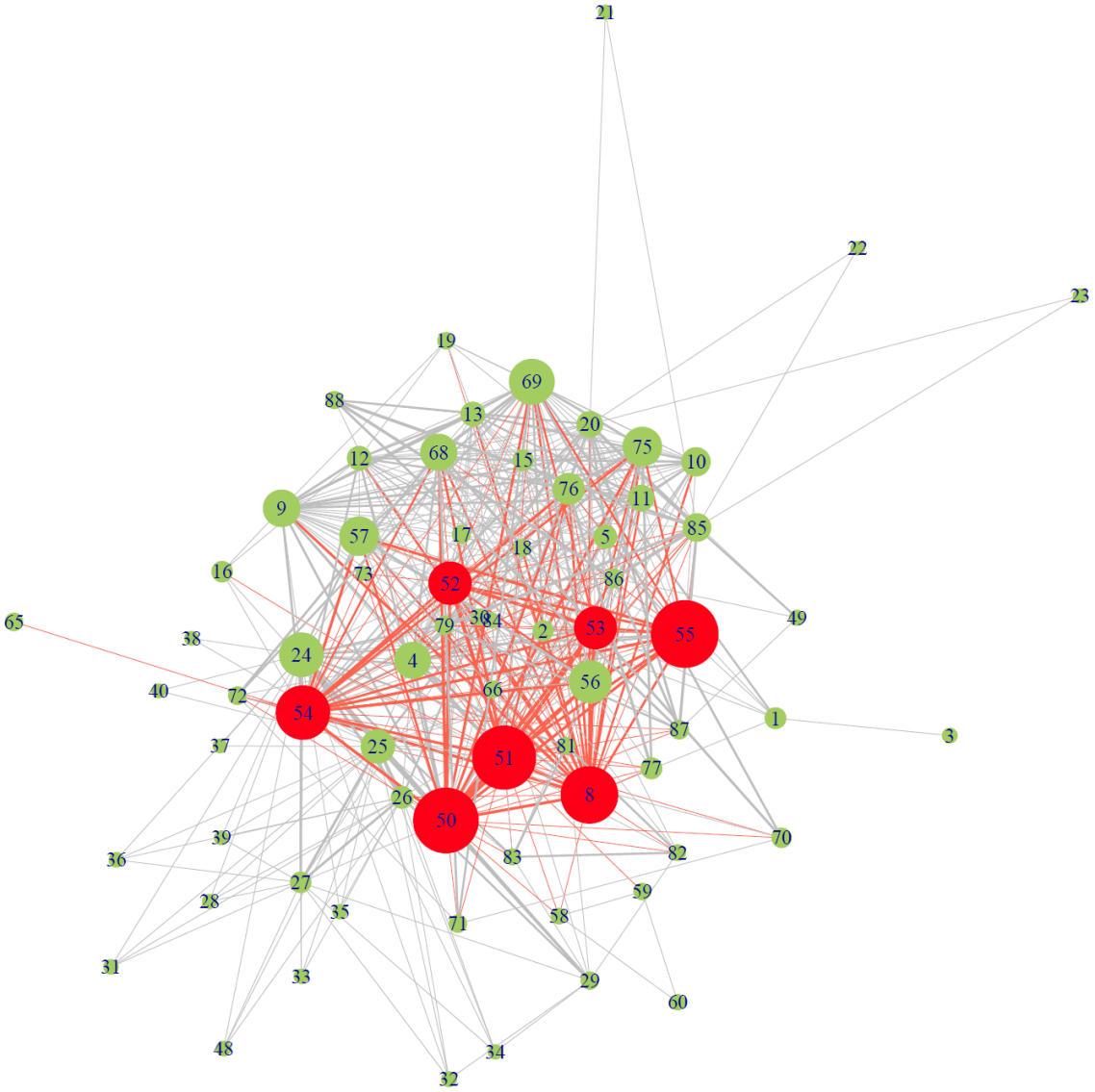

Figure 6: The core of the European *indirect* contagion network: Node sizes are proportional to balance sheet size. Edge widths are proportional to the liquidity-weighted overlap. Red nodes correspond to the banks with highest loading in the first principal component of the portfolio overlap matrix Ω.

The overlap matrix Ω also gives a glimpse of the nature of 'second-round' contagion effects in this network. The element (i, j) of the matrix Ω^2 may be interpreted as the

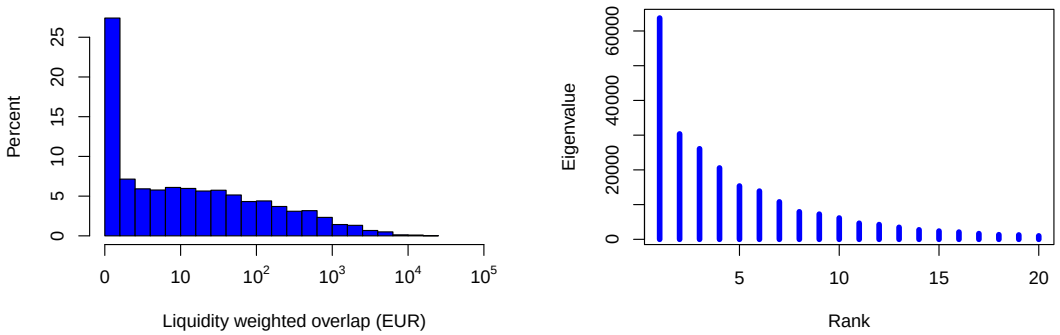

Figure 7: Left: histogram of liquidity-weighted portfolio overlaps across EU banks (2011). Right: Ranked eigenvalues of the matrix Ω of liquidity-weighted portfolio overlaps.

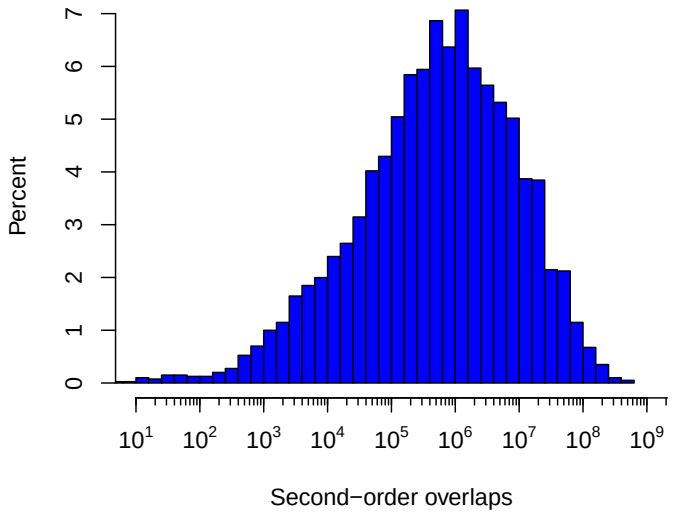

Figure 8: Histogram of 'second-order overlaps' (coefficients of the matrix Ω^2).

(maximal) indirect contagion from i to j channeled through other nodes:

$$\Omega^2(i,j) = \sum_k \Omega_{ik}\Omega_{kj}.$$

Figure 8 shows the histogram of these 'second-order overlaps'. Unlike Figure 7, where we see a large fraction of zero overlaps, here we see that all coefficients are strictly positive: this means that, although many pairs of portfolios have zero overlaps, second-round effects may potentially cause spillovers from *any* institution to any other! This observation shows that second-round effects should not be ruled out a priori and may greatly increase the scope of contagion in the network.

A quantitative way of examining the heterogeneity of this network is to compute the eigenvalues and eigenvectors of Ω. The right-hand plot of Figure 7 shows the ranked eigenvalues of Ω. The first few eigenvalues clearly dominate the others by an order of

magnitude. This means that the overlap in the portfolio network can be characterised quite well by a low-dimensional factor model for portfolio holdings. We discuss this point further in a companion paper Cont and Schaanning (2017).

3.4 Stress scenarios

We define stress scenarios, similar to regulatory stress tests, in terms of percentage shocks across asset classes[9], given by a vector $\epsilon = (\epsilon_\kappa, \kappa = 1..K) \in [0,1]^K$ where ϵ_κ is the percentage loss (stress level) applied to asset class κ. We have chosen four scenarios in the examples below to illustrate the properties of the model:

1. Losses on residential and commercial real estate exposures in Spain (Region: ES);
2. Losses on residential real estate exposures in Northern and Western Europe (Regions: GB, BE, NO, SE);
3. Losses on commercial real estate exposures in Southern Europe (Regions: IT, GR, ES, PT);
4. Losses on commercial real estate exposures in Eastern Europe (Regions: CZ, EE, HU, LV, LT, PL, RO, SK, RU, BG, E3 (= other non-EEA non-emerging countries);

In each scenario, we increase the shock to the affected asset classes gradually from 0% to 20%; the other asset classes undergo no initial loss. To emphasise cross-asset contagion, we have chosen stress scenarios where initial losses affect only illiquid assets in these examples, but scenarios can be extended to include initial shocks to marketable assets.

Calibration of stress scenario severities In our parameterisation a shock $\epsilon = 5\%$ to asset class κ represents a loss equal to 5% of the notional exposure to this asset class. This is commensurate with bank losses during severe housing crises[10].

The EBA stress test report states (p. 10) that the net loss in the stress scenario reduces the average capital ratio from 8.9 % down to 7.7 %[11]. This net stress-test loss in the EBA scenario, is precisely the initial loss that our model uses as input to trigger a (potential) fire-sales cascade. Taking as an example the Spanish bank Santander, which had EUR 594 bn in risk-weighted assets (RWA), a 1.2% loss of RWA in capital corresponds exactly to an initial loss of EUR 7.13 bn. From the dataset we know that Santander holds EUR 82 bn direct exposures to Spanish residential and commercial mortgages. Hence an initial shock of 8.7% in our Scenario 1 corresponds exactly to the severity of the EBA stress-test loss for Santander. Our 20% shock in this scenario corresponds to an initial loss that is about 2.3 times the severity of the EBA stress test. Figure 9 shows that for different Spanish banks the initial shock size to Spanish residential and commercial exposures which generates losses equal to 1.2 percent of RWA lies between 1.17 % and 8.7%. Taking the weighted (by RWA) average shock, one obtains an initial shock of 4.7%;

[9] The design of such scenarios is far from trivial and outside the scope of this presentation. On this occasion, we note that ironically the 2016 EBA stress test did not feature a "Brexit" scenario.

[10] See a comparison of losses in percentage of gross lending across different countries' housing crises in Chart 6 of the Norges Bank Staff Memo:`http://static.norges-bank.no/pages/104012/Staff_Memo_5-2015_eng.pdf`

[11] See `http://www.eba.europa.eu/documents/10180/15935/EBA_ST_2011_Summary_Report_v6.pdf`.

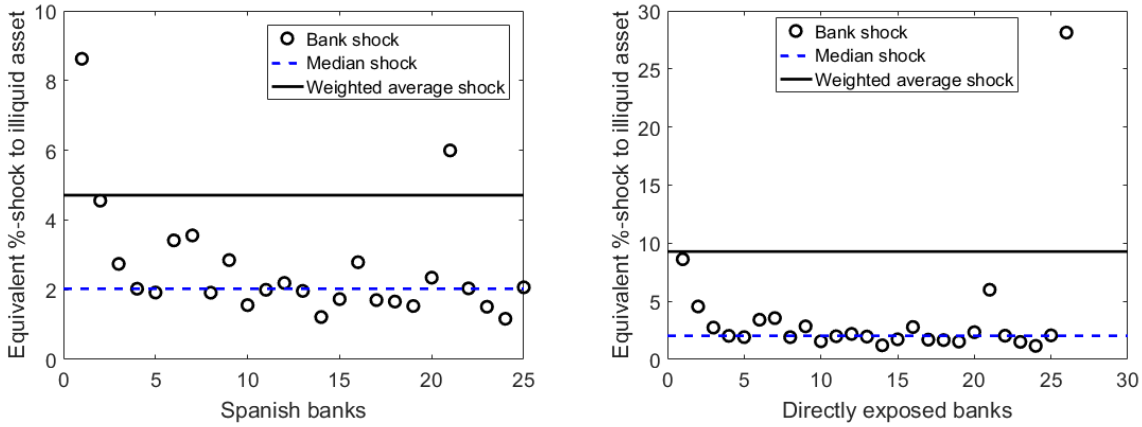

Figure 9: Level of initial stress ϵ applied to the Spanish residential and commercial real estate exposures which, for each bank, generates an initial loss equal to 1.2 % RWA. Left: for Spanish banks. Right: For banks with direct exposures to Spanish real estate in excess of EUR 1 bn.

Correspondence of stress scenario intensities	Scen. 1	Scen. 2	Scen. 3	Scen. 4
Weighted average shock size that creates EBA-equivalent initial losses (regional sample)	4.71%	6.38 %	36.5%	NA
Weighted average shock that creates EBA-equivalent initial losses ("direct exp. > 25% capital" sample)	9.28 %	9.14 %	24.0 %	33.5%**

Table 5: Initial stress level which leads to an average initial loss of 1.2% of RWA, (corresponding to the EBA stress scenario intensity), for each scenario. **Only six (small) banks have exposure in excess of 25% of their capital to these asset classes.

the median shock is 2.03%. If instead of considering Spanish banks only, we consider banks with direct exposures in excess of 25% of their capital, we obtain the right plot. In this case, smaller Spanish banks with different business models have dropped out of the sample. The weighted average shock required to trigger losses equal to 1.2% of RWA in this case is equal to 9.3%. Overall, for this scenario, a reasonably severe regulatory stress test would thus fall into the range of 4 - 10%.

We perform the same computations for scenarios 2, 3 and 4, and report the results in Table 5 to provide a mapping of our scenario intensities to the official EBA stress scenario intensity.

Finally, for completeness we note that in Greenwood et al. (2015) and Duarte and Eisenbach (2013) the initial shocks are given by a 50% fall in GIIPS debt, and a 1% reduction of all assets, respectively. This corresponds to initial losses of EUR 343 bn and EUR 233 bn on a system wide level, and RWAs of 5.3 % and 2.07% for all affected banks, respectively. In comparison to our scenarios, as well as the EBA official scenario, these initial shocks are thus quite large. Greenwood et al. (2015) and Duarte and Eisenbach (2013) consider one specific initial shock and do not vary the intensity of the stress.

Initial Losses	EBA	Greenwood et al. (2015)	Duarte and Eisenbach (2013)*
Average bank loss in % of RWA	1.2%	7.63% (GIIPS banks) 7.17% (large exposure banks) 5.25 % (all banks)	2.07% (all banks)

Table 6: Magnitude of the initial loss in percent of RWA.

3.5 Systemic stress test of European banks: results

We now report the results of our systemic stress test for the European banking system. To perform the stress test, we use a leverage constraint $\lambda_{\max} = 33$, which corresponds to the Basel 3 leverage constraint. An alternative would be to use as constraint the ratio of capital to risk-weighted assets (RWA), or both. Given the limitations of our dataset, we have opted for the former, but the same exercise may be conducted with both constraints. We note that for EU banks it is mostly the case that leverage, not the ratio of capital to RWA, is the binding constraint.

As discussed in Section 2.3, we assume a target leverage slightly below the constraint $\lambda_b = 0.95\lambda_{\max} = 31.3$ in order to have a buffer. We have conducted the stress test for various values $0.9\lambda_{\max} \leq \lambda_b \leq \lambda_{\max}$; results are similar across this range and for brevity we only report them for $\lambda_b = 0.95\lambda_{\max}$. In most examples, fire sales die out after a few rounds; in all cases we have limited the iterations to 20 rounds and report losses for each round and the total loss for all rounds.

Magnitude of fire-sales losses. Figure 10 shows the contribution of fire-sales losses to total system-wide loss in the stress test in Scenario 1. We observe that when stress levels exceed 4-5%, fire-sales losses cannot be neglected and may account for up to 80% of the total loss. This is definitely an argument for including fire sales and deleveraging effect in a stress test.

Figure 11 represents the magnitude of fire-sales losses $SLoss(\epsilon)$ as a percentage of bank equity for the four different stress scenarios. What is striking is the existence of a *tipping point* which separates a stability zone corresponding to small shocks from a systemic risk zone, where the magnitude of losses represents a considerable fraction of bank equity. The existence of such tipping points is characteristic of the collective behavior of networks whose components are subject to thresholds Granovetter (1978).

Another striking feature of Figure 11 is that while the 4 scenarios lead to different outcomes in our model, they lead to similar outcomes in the leverage-targeting model (represented by the dashed lines). This is a systematic difference between the two models, which we will discuss further (see also Cont and Schaanning (2017)).
These results are based on market depth parameters estimated as described in Section 3.2. To explore the sensitivity of the results to the market depth, we change the liquidation horizon τ from 1 day to 100 days, which, as observed from (27), has the effect of scaling the market depth parameters by $\sqrt{\tau}$. This dependence of results on the liquidation horizon may be alternatively seen as a sensitivity analysis with respect to the market depth parameter for a fixed horizon τ.

Figure 12 analyzes the dependence of system-wide fire-sales losses $SLoss(\epsilon)$ to this scaling. We observe the sensitivity of the result on the market depth level, which underlines the importance of a proper estimation procedure for these parameters. We can

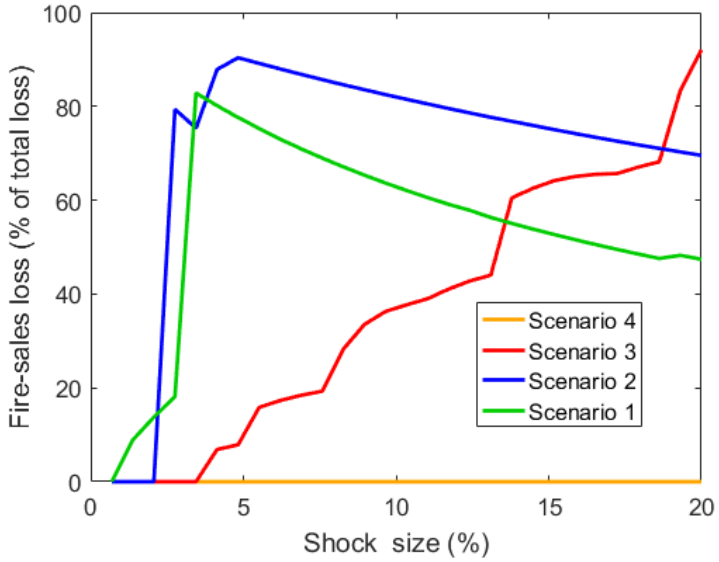

Figure 10: Contribution of fire-sales losses to total system-wide loss, as a function of the initial stress level.

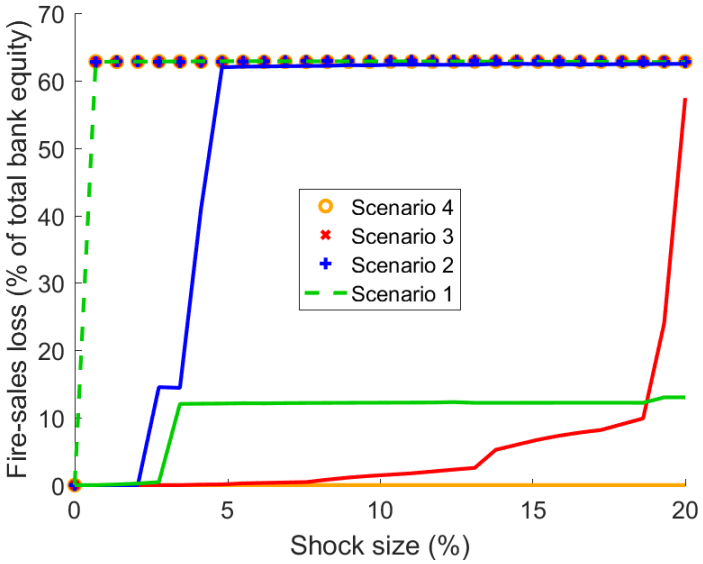

Figure 11: fire-sales losses (as % of bank equity) in the four stress scenarios as a function of the initial shock size. The fourth scenario does not generate fire sales in the threshold model. The losses in the leverage targeting model are displayed by dashed lines.

distinguish three regimes: (i) no fire sales (losses) for small shocks $\epsilon < \min_i \epsilon_i^*$, (ii) significant fire-sales losses on the order of 10-15% of bank equity for large stress levels in presence of large market depth, (iii) a region of *high systemic risk*, where fire-sales losses deplete a large proportion of bank equity. The boundary between the regimes may be attained either by increasing the stress level ϵ or by decreasing the market depth. In both cases, there is a tipping point above/below which large-scale deleveraging is triggered.

Figure 13 displays a quite different picture for the leverage targeting model: here we observe a very high level of losses (more than 30% of bank equity) at any level of market depth and shock level. Moreover, fire-sales losses are essentially independent of the initial

shock size. This shows that losses in the leverage targeting model are not driven by the initial shock size but by the rebalancing generated by the leverage targeting rule. These results also indicate that any study on the magnitude of fire sales should include a rigorous sensitivity analysis with respect to the market depth parameters.

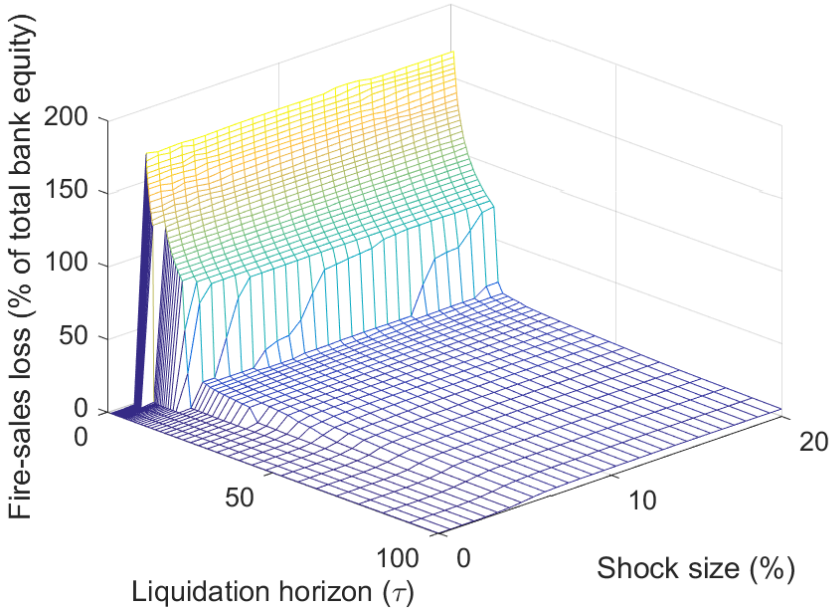

Figure 12: The fire-sales losses as a function of the initial shock and the market depth (scaled via the liquidation horizon) for the threshold model. We can clearly distinguish three regions: (i) no fire sales; (ii) price-mediated contagion with losses between 10 - 15% of total bank equity, (iii) region of high systemic risk: all banks default (i.e. absent regulatory intervention, large scale contagion is to be expected). This shows that the market depth has a highly significant impact on the estimated losses and the extent of contagion.

Distribution of fire-sales losses across banks. We now turn to a more detailed examination of losses induced by fire sales, at the level of individual institutions.

Figure 14 compares bank-level fire-sales losses with losses under the leverage targeting model, using the same market depth estimates. fire-sales losses are observed to be much larger for some banks in the leverage targeting model in comparison with the threshold model; note the logarithmic scale in this plot. This figure shows that fire-sales losses are not only larger on a system-wide level in the leverage targeting model (as seen in 13), but are also higher at the individual bank level. Moreover, our model distinguishes the three scenarios by severity, while fire-sales losses are very similar in the three scenarios for the leverage targeting model.

Impact of heterogeneity in market depth. Greenwood et al. (2015) assume the same market depth parameter for all asset classes, whereas our estimation, shown in Figure 5, yields a heterogeneous distribution of market depth across asset classes. To investigate the influence of this heterogeneity, we compare bank-level losses with losses in

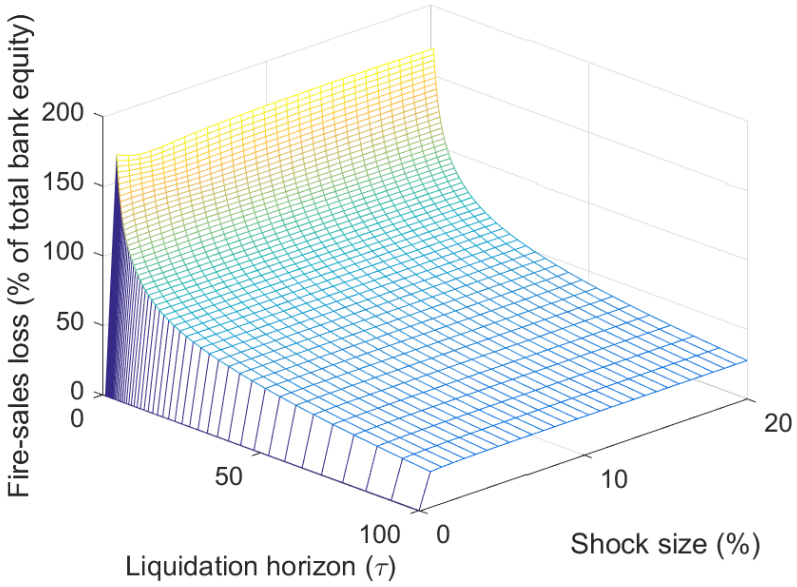

Figure 13: The fire-sales losses as a function of the initial shock and the market depth, scaled via the liquidation horizon, for the leverage targeting model. The leverage targeting model predicts large-scale losses for all combinations of shock sizes and market depths.

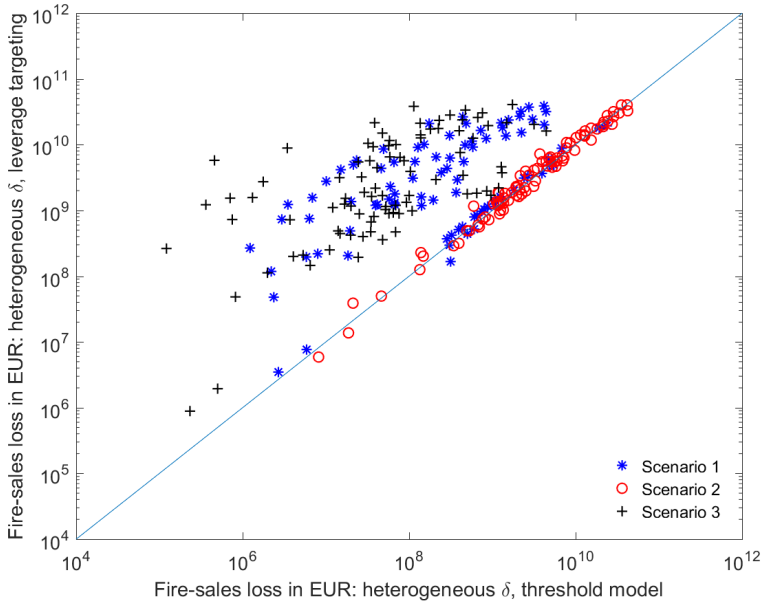

Figure 14: Bank-level fire-sales losses, compared with losses under the leverage targeting model, using the same market depth estimates (logarithmic scale). Loss estimates are higher in the leverage targeting model by several orders of magnitude.

a model with uniform market depth $\overline{D}$ computed as a holdings-weighted average:

$$\sum_{i,\mu} \frac{\Pi_{i,\mu}}{\overline{D}} = \sum_{i,\mu} \frac{\Pi_{i,\mu}}{D_\mu}. \tag{31}$$

The result, displayed in Figure 15, clearly shows a huge impact of heterogeneity in market depth when estimating fire-sales losses: clearly, institutions are differentiated according

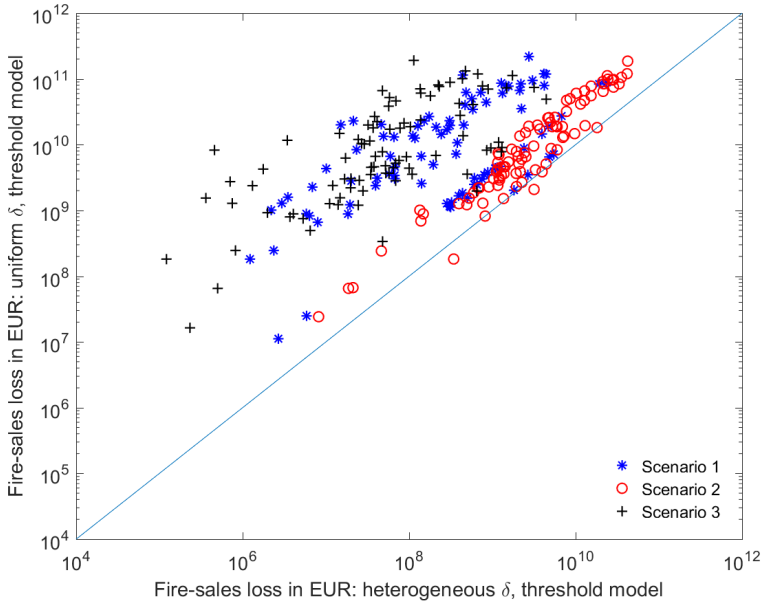

Figure 15: Bank-level fire-sales losses computed using heterogeneous market depth parameters vs uniform market depth. The use of a uniform market depth for all assets increases fire-sales loss estimates considerably.

to the liquidity of their holdings, sometimes by three orders of magnitude. This clearly shows that bank stress tests with fire sales require a careful estimation of market impact/ market depth parameters for each asset class.

Figure 16 shows that the same experiment in the leverage targeting model leads to quite different results: heterogeneity of market depth does not seem to have much impact in this case. The reason is that the leverage targeting model overestimates fire-sales losses, which are so large that they trigger the insolvency of many large leveraged institutions in the first rounds of deleveraging. These defaulted institutions then cease to further contribute to the loss estimates, which then do not have any further dependence on market depth parameters.

For completeness, we also compare in Figure 17 our results with the ones obtained in the leverage targeting model using a uniform market impact parameter $\overline{D}$ for all assets, as in Greenwood et al. (2015). The models agree in the 'meltdown' region where all banks default, but for intermediate stress levels the difference between loss estimates in the two models is substantial, up to several orders of magnitude (note the logarithmic scales in Figure 17). As discussed above, the main difference in the loss estimates arises from the deleveraging rule (leverage targeting vs one-sided leverage constraint) but the difference in market depth parameters accentuates the difference.

Bank failures: illiquidity and insolvency. Most theoretical models of financial contagion have focused exclusively either on insolvency or on illiquidity as the cause of bank failure, while bank stress tests have traditionally focused exclusively bank solvency. As discussed in Section 2.5, our model allows for both possibilities: a bank may become insolvent due to asset losses, or become illiquid when all marketable assets have been sold. The model thus allows to examine which is the principal mode of failure in the stress test.

Note that this distinction is not available in Greenwood et al. (2015); Duarte and

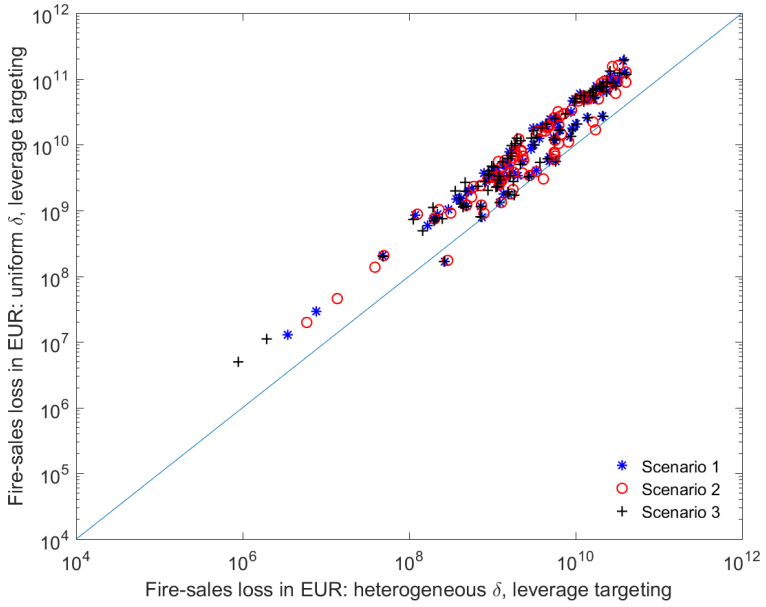

Figure 16: Impact of heterogeneity in market depth in the leverage targeting model.

Eisenbach (2013): their assumption that all assets are available for liquidation entails that the only situation where an institution becomes illiquid is when there are no more assets available for liquidation, which implies that it is also insolvent. Our implementation of the leverage targeting model distinguishes between marketable and illiquid assets, to allow a meaningful comparison with our model.

We now proceed to analyse the number of failures respectively due to illiquidity and insolvency as a function of the initial stress level. The left (resp. right) panel in Figure 18 displays the number of banks which become insolvent (resp. illiquid) after a given number of rounds of deleveraging, as a function of the initial stress level. We observe that insolvency is far from being the only mode of failure: depending on stress levels, 10 to 20 banks are expected to default due to illiquidity, yet remain solvent.
Figure 19 shows respectively the number of failures due to insolvency (left) and those due to illiquidity (right) as a function of the liquidation horizon (market depth scaling factor) and the initial shock size. We can see that while the number of insolvent banks increases with the stress level, the number of failures due to illiquidity is non-monotone: it is maximal in the range of relevant shocks (between 5% and 10%) but then decreases as the shock level increases and insolvency becomes the main mode of default.

Second and higher round effects. We end this section by discussing the impact of further rounds of deleveraging. Although in theory deleveraging may continue for many rounds (and, in the leverage targeting model, for an infinite number of rounds), previous empirical studies have mostly focused on a single round of deleveraging. (Greenwood et al., 2015, Appendix B) find that, with their parameter choices, iterating the cascade leads to all banks defaulting; we confirm this feature of the leverage targeting model in Figure 13 for more general parameter combinations. Duarte and Eisenbach (2013) do not find evidence for significant higher round effects for US banks. A possible explanation for this may be their choice of parameters (large market depth, and/or large initial shock). We attempt to clarify these findings by exploring systematically the impact of second or

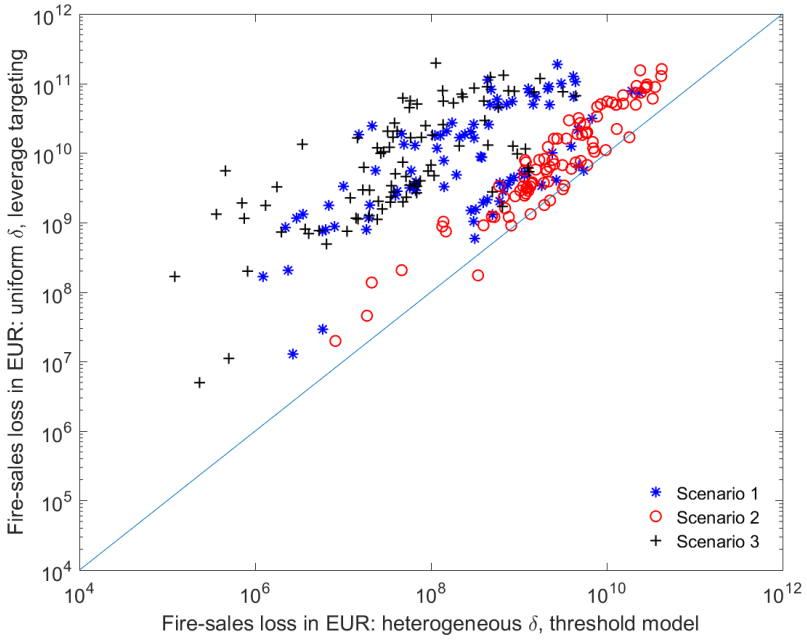

Figure 17: Bank-level fire-sales losses in the leverage targeting model with uniform market depth differ by several orders of magnitude from those in the threshold model using heterogeneous market depths.

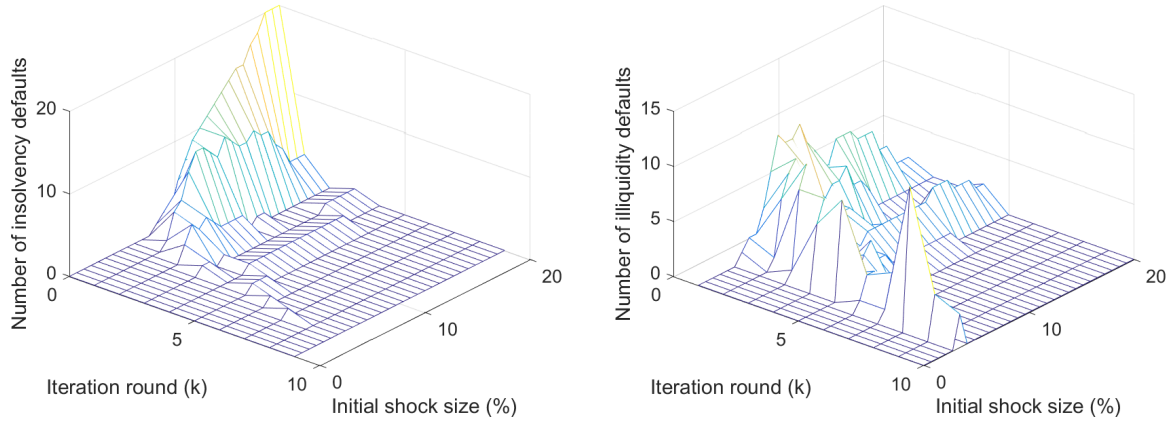

Figure 18: Left: Number of insolvent banks as a function of number of rounds of deleveraging and initial stress level. Right: Number of illiquid banks as a function of number of rounds of deleveraging and initial stress level.

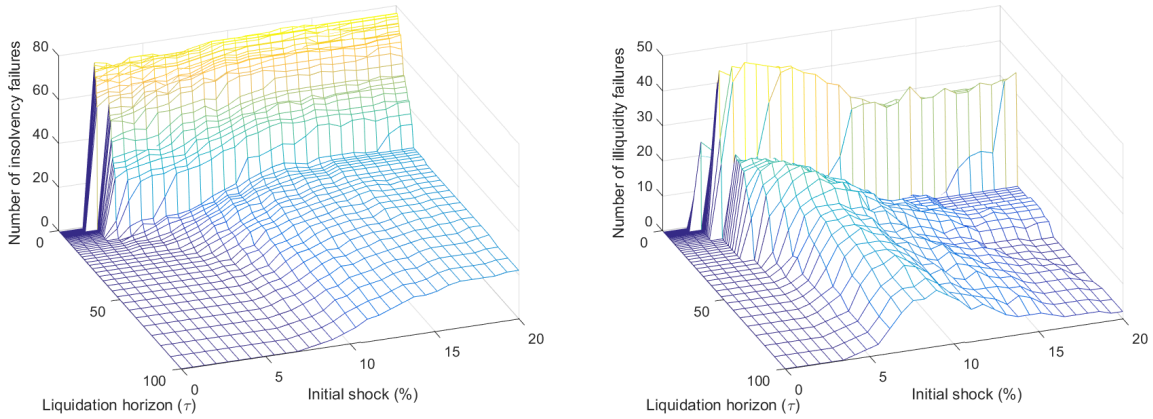

Figure 19: Number of bank failures due to insolvency (left) and illiquidity (right), as a function of the initial stress level and liquidation horizon (market depth).

higher rounds of deleveraging.

First, as noted in Section 3.3, even though the matrix Ω of portfolio overlaps is sparse, the matrix Ω^2 of second-order overlaps is dense, which implies that second round spillovers can potentially lead to contagion from *any* institution to any other, quite unlike what happens at the first round. By comparing the distribution of liquidity-weighted overlaps in Figure 7 to second-round overlaps in Figure 8, we see that some shocks need *at least* two rounds to propagate from one bank to another.

Second, as observed in Figure 18, many bank failures occur only at higher rounds, especially failures due to illiquidity. Only examining a single round of the feedback loops thus leads to an underestimation of the number and severity of bank failures. The left panel in Figure 20 shows the fire-sales loss at each round k on a log scale for the estimated market depth with $\tau = 20$.

The right panel in Figure 20 shows the ratio of total loss (20 rounds) to the first round loss, as a function of the initial stress level. The non-monotone feature of this dependence shows that fire-sales contagion is most important for moderate stress levels, which trigger deleveraging but are not extreme enough to generate insolvency at the first round. In this case, second and higher rounds considerably change the outcome in terms of total loss level and number of bank failures.

The results are even starker for the leverage targeting model: subsequent rounds lead to an increase by a factor 10 to 40 of estimated fire-sales losses, especially when the initial shock is small. This makes the "number of rounds" an important implicit parameter in the leverage targeting model.

4 Indirect exposures

4.1 Notional vs effective exposures

The starting point in portfolio risk analysis is the notion of exposure to an asset class, usually quantified by the notional volume of holdings, in monetary units, in that asset class. In absence of contagion, losses in a stress scenario for this asset class will be a linear function of the percentage shock to the asset value, the proportionality coefficient being this notional exposure.

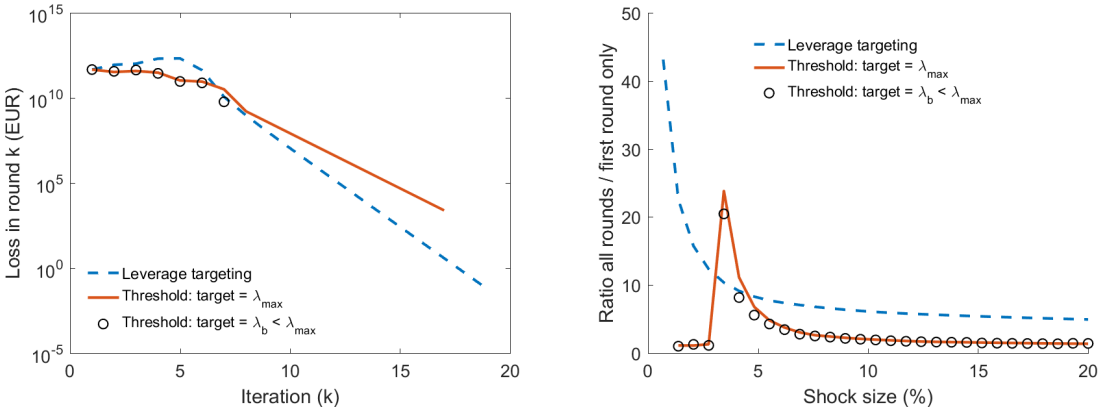

Figure 20: Left: fire-sales losses as a function of the iteration step at the estimated market depth ($\tau = 20$) for the leverage targeting model (dashed), and the threshold models with $\lambda_b = \lambda_{\max}$ (full) and $\lambda_b < \lambda_{\max}$ (circles). Right: Ratio of total loss to first round loss, as a function of initial stress level.

However, the endogenous risk arising from fire-sales may amplify this initial loss in a non-linear way, leading to a total loss that is higher than the one given by the notional exposure, leading to an *effective exposure* higher than the notional one. Furthermore, spillover effects from fire-sales may further increase this loss through indirect contagion. The end result is that the (marginal) ratio of a portfolio's loss in an asset class to shocks affecting this asset class may be *higher* than its notional exposure. We capture this effect by defining the notion of *indirect exposure* and quantifying it using our model.

Denoting as above by $\Theta^{i,\kappa}$ the holdings of institution i in the (illiquid) asset class κ, consider a stress scenario in which the asset class κ depreciates by ϵ_κ. Then institution i has a direct loss $\epsilon_\kappa \Theta^{i,\kappa}$ and, as long as the shock size is small, no deleveraging occurs and the latter also represents the total loss, which increases linearly with ϵ_κ.

However, as stress levels increase, feedback effects and price-mediated contagion may amplify the initial losses and result in a loss $Loss(i, \epsilon_\kappa) = \epsilon_\kappa \Theta^{i,\kappa} + FLoss(i, \epsilon) > \epsilon_\kappa \Theta^{i,\kappa}$. The *effective exposure* to the asset class κ accounts for these additional losses and is defined as

$$E^{i,\kappa}(\epsilon_\kappa) := \frac{\text{Loss}(i, \epsilon_\kappa)}{\epsilon_\kappa} = \underbrace{\Theta^{i,\kappa}}_{\text{Notional exposure}} + \underbrace{\frac{FLoss(i, \epsilon_\kappa)}{\epsilon_\kappa}}_{\text{Indirect exposure}}. \tag{32}$$

Unlike the notional exposure, the *indirect exposure* depends on the shock size ϵ_κ and, more importantly, on the configuration – size, leverage – of *other* financial institutions with common asset holdings. Clearly, unlike the first term in (32), the second term cannot be computed by examining the portfolio of i alone and depends on the entire network of overlapping portfolios which contribute to fire-sales losses. Given the discussion in the previous section, this should not come as a surprise, but it clearly departs from the common assumption that notional exposures of a portfolio are sufficient to quantify its risk. Here, the risk of a portfolio cannot be quantified in isolation, but depends on the network of overlapping portfolios and the constraints that these portfolios face.

Interestingly, a financial institution may even have a non-zero (indirect) exposure to an asset class which it does not hold in its portfolio, i.e. the second term in (32) may be non-zero even if the first term is zero. Consider for example a bank A which does not hold any subprime asset-backed securities, but has common holdings with a bank B with large

holdings in subprime ABS. A loss in subprime ABS entails no direct loss to A but, if large enough, may force B to deleverage by selling its marketable assets, also held by A, leading to mark-to-market losses for A. This means that A will have an indirect exposure to large losses in subprime ABS, an asset it does not hold! In this example, deleveraging by B only occurs if subprime losses are significant, so A has no exposure to a *small* depreciation in subprime ABS: the indirect exposure of A is clearly scenario-dependent. However, the threshold beyond which spillover from B to A occurs depends on the leverage, size and composition of B's portfolio, which is unknown to A. This example, far from being a curiosity, is in fact illustrative of the mechanism which led to amplification and contagion of losses from the subprime asset class to the entire global financial system in 2007-2008 Longstaff (2010).

4.2 Indirect exposures: empirical evidence

To assess how large such indirect exposures may be in the European banking system, we use the results of the previous section to examine the magnitude of indirect exposures of European banks to residential and commercial mortgages. A first observation is that most European banks tend to have very few mortgage loans outside of their own country, so direct exposure to foreign residential and commercial mortgage is zero in most cases, except for a few multinational banks. Given the large volume of mortgages on bank portfolios, it is generally assumed that domestic house prices are the main risk factor for commercial bank portfolios. However, as we shall see now, European banks also have substantial indirect exposures to residential and commercial real estate asset classes in *other* European countries where they do not issue mortgages.

Figure 21 displays the (total) loss for two banks in our sample, HSBC and Santander, in the scenario where stress is applied to the Spanish real estate sector (Scenario 1). Santander, a major Spanish bank, holds a lot of Spanish mortgages on its portfolio, representing a notional exposure of EUR 82.7 bn, which corresponds to the slope of the loss for shocks less than 3%, where no deleveraging occurs. HSBC, on the other hand, holds very few Spanish mortgages (less than EUR 0.5 bn) so its notional exposure to this asset class is much lower.

As losses on Spanish real estate exposures exceed 3.5%, some Spanish banks start deleveraging and indirect losses lead to a sharp increase in the loss level. For a 5% shock level, the total loss for Santander sharply increases to around EUR 25 bn, of which around EUR 22 bn are indirect fire-sales losses and 3bn are direct losses. This corresponds to an indirect exposure exceeding EUR 440 bn, which is significantly larger than its actual direct exposure! In this regime, the indirect exposure is an important source of losses. This deleveraging by Spanish banks then affects HSBC through price-mediated contagion: as the shock level exceeds 5%, the fire-sales loss for HSBC starts to become important. For a 5.5% shock level, HSBC's losses through price-mediated contagion are equal to 2.73 bn EUR, which corresponds to an indirect exposure to Spanish real estate close to EUR 44 bn. Note that in the case of HSBC, the indirect loss corresponds to 100 times its notional exposure to this asset class! This example suggests that in stress scenarios which are sufficiently severe to trigger fire sales, actual losses can be much higher than what is suggested by notional exposures. The difference corresponds to indirect exposures.

The right-hand graph in Figure 21 displays the magnitude of indirect exposures corresponding to these losses. We see that the indirect exposure of Santander, which mainly stems from its own deleveraging and deleveraging by other Spanish banks with similar as-

sets, sharply increases when the shock size reaches the threshold of 3%, then peaks around 4%: as the stress level increases beyond a certain level, some banks become insolvent and cease to contribute to fire-sales spillovers.

As illustrated in the right panel in Figure 21, these indirect exposures clearly depend on the severity of the stress scenario considered, the bank's initial exposures, and the exposure and constraints of other portfolios holding similar marketable assets. An implication is that one cannot mimic the impact of indirect contagion by simply applying a higher stress level for all banks, as is implicitly done in current supervisory stress tests.

Table 7 reports indirect exposures to Spanish real estate for several other European banks. Comparing with their notional exposures, we observe that non-Spanish banks have a substantial indirect exposure to Spanish real estate, which is not revealed by inspecting their notional exposures to this asset class, which are between ten and a hundred times smaller. This indirect exposure is due to the overlap in marketable assets with banks that are directly exposed to the Spanish housing market.

These observations are not particular to the Spanish real estate sector. Other examples, which we do not report here for the sake of brevity, show that in the European banking system a stress scenario affecting the commercial or residential real estate sector of a given country may generate significant losses arising from indirect exposures for foreign banks.

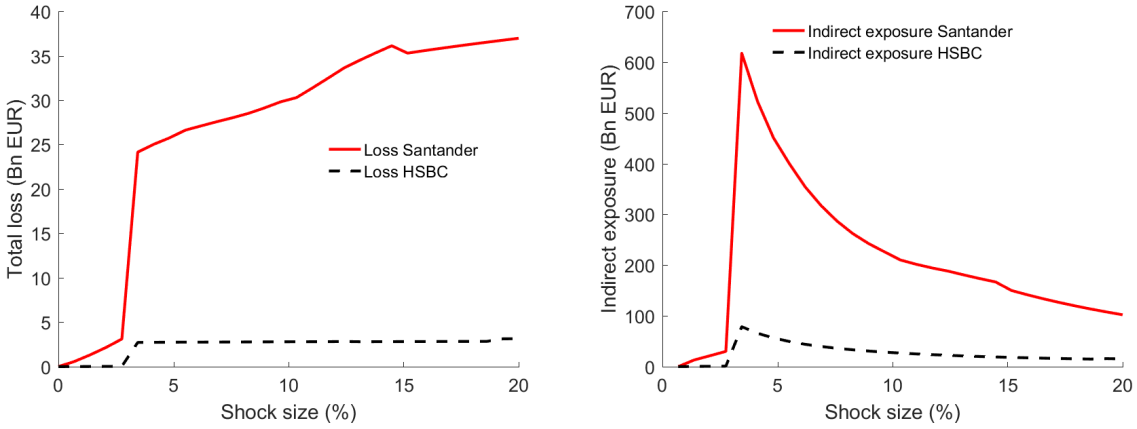

Figure 21: Left: Losses arising from a depreciation in Spanish real estate, for HSBC and Santander, as a function of the depreciation level (horizontal axis). Right: The indirect exposures corresponding to these losses.

The concept of indirect exposures may also be defined at the country level. Figure 22 shows the indirect exposures of European banks (aggregated by country) to the Spanish housing market. Shocks below 1% do not appear to trigger fire sales, and do not generate indirect exposures.

As the stress level is increased, fire sales are triggered and spillover losses materialise; when the initial shock size exceeds 3%, contagion increases fire-sales losses by two to three orders of magnitude. What is striking is that these thresholds – 1%, 3% – are not large, suggesting that indirect exposures and losses arising from price-mediated contagion cannot be ignored in stress tests. The left panel of Figure 22 reveals that, after Spanish banks, Portuguese and British banks have the largest indirect exposure to the Spanish housing market. The right panel of 22 shows these indirect exposures. Table 8 reports fire sales losses and estimated indirect exposures for different stress levels for the countries highlighted in Figure 22.

Bank	Bank equity (EUR bn)	Fire-sales loss (EUR bn) Stress level 1%	Fire-sales loss (EUR bn) Stress level 5.5%	Fire-sales loss (EUR bn) Stress level 10%
Santander	42.0	0.826	22.0	22.2
BBVA	24.9	0.771	18.8	20.0
Deutsche Bank	37.6	0.045	3.00	3.07
HSBC	86.90	0.024	2.73	2.76
RBS	59.0	0.069	4.36	4.42
Nordea	19.1	0	0.17	0.19
Bank	Direct exposure (EUR bn)	Indirect exposure (EUR bn) Stress level 1%	Indirect exposure (EUR bn) Stress level 5.5%	Indirect exposure (EUR bn) Stress level 10%
Santander	82.7	30.0	354	202
BBVA	82.5	28.0	302	177
Deutsche Bank	8.88	1.63	48.4	27.8
HSBC	0.452	0.86	43.9	25.0
RBS	2.68	2.50	70.2	40.1
Nordea	0.003	0.05	2.76	1.69

Table 7: Capital, direct exposures and fire-sales losses and indirect exposures for three selected shock sizes. When price-mediated contagion occurs, fire-sales losses erode a considerable fraction of the equity. This corresponds to indirect exposures that often exceed the direct exposures.

Overall, the direct exposure of European banks to the Spanish residential and commercial real estate sector is EUR 740 bn. When averaged both across all banks and all shock sizes (between 0 and 20%), the indirect exposures amount in this example to 160% of the direct exposures, which is non-negligible. To be clear, in the event of a meltdown of the Spanish housing sector, non-Spanish European banks will, on average, be affected for up to 160% of their actual holdings in Spanish mortgages.

This clearly means that housing prices in one European country can strongly affect banks of other European countries, an issue which can only be tackled by macroprudential policies applied at a Europe-wide level.[12]

These indirect exposure figures, as all other results obtained in the stress tests, are sensitive to assumptions on market depth, or, alternatively, on the liquidation horizon. Figure 23 shows the fire sales losses of the UK banking system to the Spanish housing market. Reading the graph from front to back, we see that a contraction of market liquidity can greatly exacerbate fire-sales losses: there is a tipping point at which losses increase from EUR 10 bn to EUR 100 bn. Figure 24 shows the estimated indirect exposure of the UK banking system to the Spanish housing market as a function of the stress level and the liquidation horizon (or market depth scaling factor). In the case of the UK, the main asset classes contributing to this indirect exposure to Spanish residential and commercial mortgages are via portfolio overlaps in Spanish, UK, US, and Asian corporate bonds as well as Spanish, Asian, non-emerging EEA, and US government bonds.

Indirect exposures and the losses arising from them provide a more palpable explanation of how an initial loss of about USD 500 bn in the US subprime sector was able to

[12]On this point, we note that Brexit does not act as a barrier for price-mediated contagion.

balloon into trillions of dollars of losses across multiple asset classes during the financial crisis.

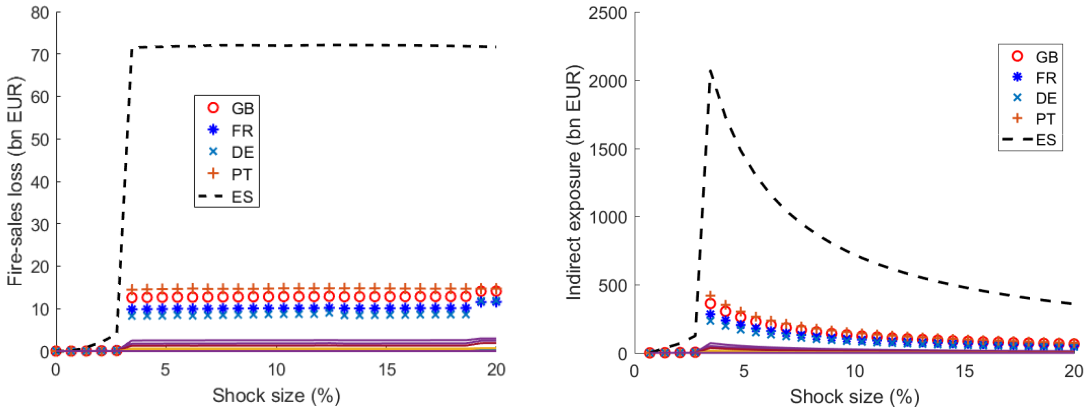

Figure 22: Indirect exposures of European banks, aggregated by country, to the Spanish housing market.

Country	Total bank equity (EUR bn)	Fire-sales loss (EUR bn) Stress level: 1%	Fire-sales loss (EUR bn) Stress level: 2%	Fire-sales loss (EUR bn) Stress level: 3.5%
ES	22.8	0	2.14	71.6
GB	7.9	0	0.08	12.6
FR	15.9	0	0.09	9.88
DE	2.4	0	0.07	8.23
PT	697.0	0	0.03	14.5
Country	Direct exposure (EUR bn)	Indirect exposure (EUR bn) Stress level: 1%	Indirect exposure (EUR bn) Stress level: 2%	Indirect exposure (EUR bn) Stress level: 3.5%
ES	22.8	0	0.6	7
GB	7.9	0	0.02	0.51
FR	15.9	0	0.03	0.43
DE	2.4	0	0.02	0.37
PT	697.0	0	0.01	0.13

Table 8: Fire-sales losses and indirect exposures at country level for the most heavily exposed countries.

4.3 Relevance of indirect contagion for bank stress tests

Given the magnitude of indirect contagion, it is not surprising that accounting for indirect losses can modify the outcome of bank stress tests. To show this more clearly, we compare the outcomes of a bank-level stress test, where the (direct) loss resulting from initial stress is compared to its capital, with that of a systemic stress test, where we also account for fire-sales spillover which may follow from the initial loss.

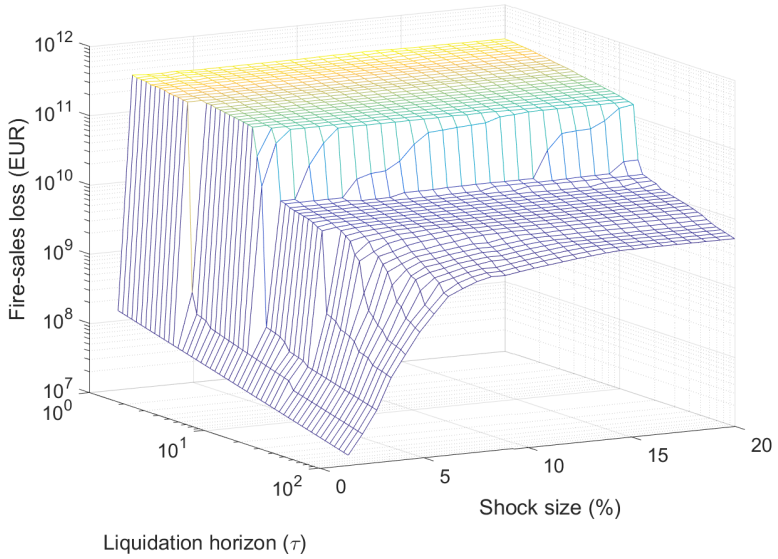

Figure 23: The fire-sales loss (in EUR) of the UK banking system to the Spanish residential and commercial mortgage market as a function of the stress level and liquidation horizon (market depth). Even with optimistic market depth estimates, when the initial shock is above 3.5%, indirect losses are non-negligible.

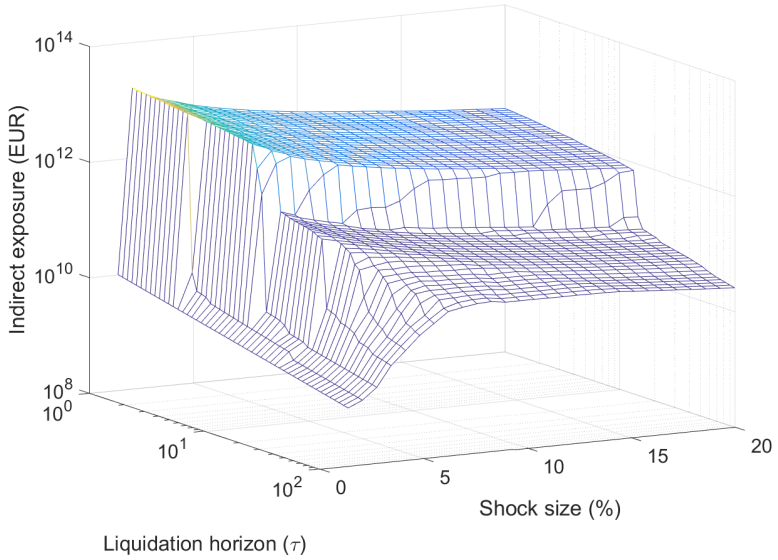

Figure 24: The indirect exposure (in EUR) of the UK banking system to the Spanish residential and commercial mortgage market as a function of the stress level and liquidation horizon (market depth).

Figure 25 plots, as a function of the shock size, the number of banks (out of 90 banks) in the sample which have enough capital to sustain the initial losses but become insolvent after a single round of fire-sales (for the standard horizon $\tau = 20$). For moderate stress levels (5-10%) we identify up to 10 banks (in scenario 1), reported in Table 9, which pass the bank-level stress test but fail the systemic stress test.

Figure 26 shows the number of banks that pass the bank-level stress test but fail the systemic stress test as a function of the initial stress level and the liquidation horizon.

We see that the discrepancy between the outcomes of the systemic and single-bank stress test is precisely in the regions corresponding to moderate to high stress levels typically used in supervisory stress tests.

These results illustrate that the contagious losses from fire sales can be of a sufficiently large magnitude to change the outcome of bank stress tests.

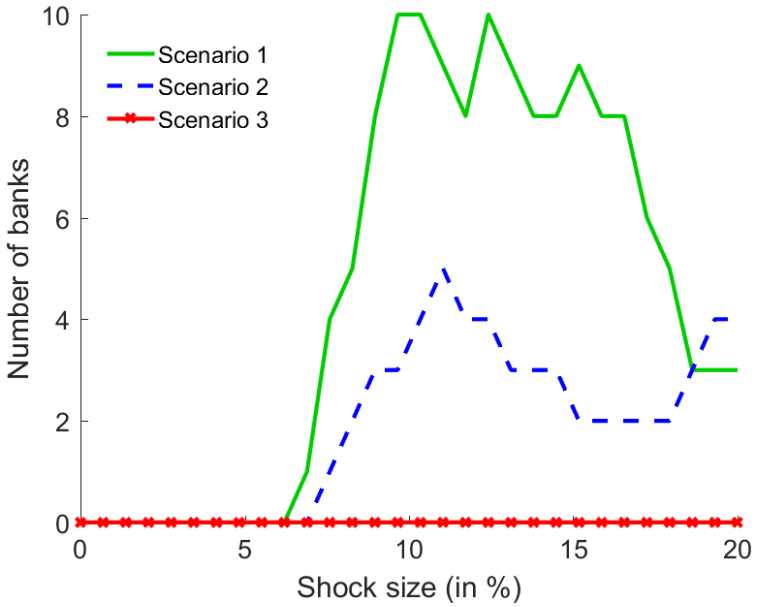

Figure 25: Number of banks with capital sufficient to withstand the initial stress, but failing to withstand the losses due to indirect contagion after a single round of deleveraging.

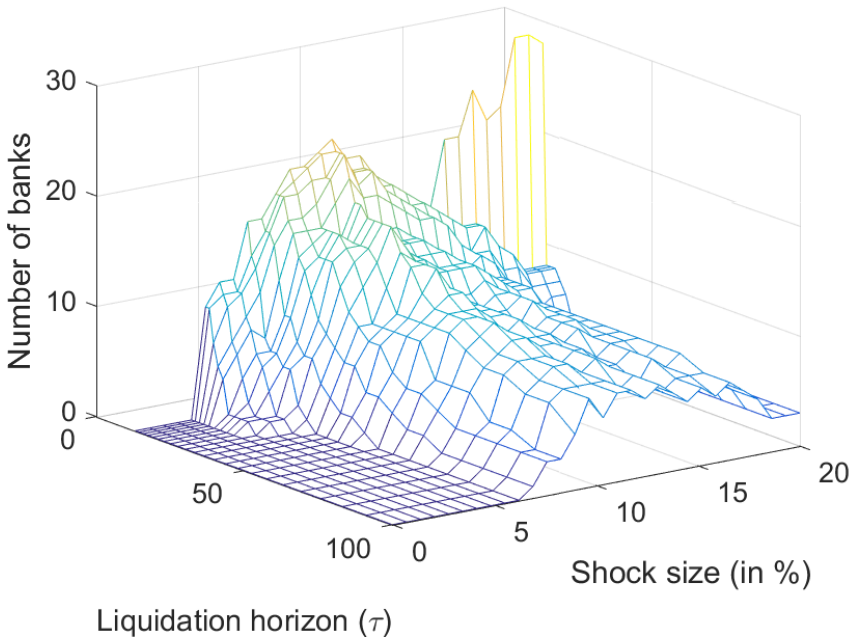

Figure 26: Number of banks with capital sufficient to withstand the initial stress, but which fail to withstand losses due to indirect contagion as a function of the initial shock.

Scenario	Banks failing systemic stress test after 2 rounds of fire sales
1	BFA-BANKIA Caja de ahorros y pensiones de Barcelona Banco popular Español Banco de Sabadell Caixa d'Estivalis de Catalunya, Tarragona i Manresa Caixa de Aforros de Galicia, Vigo, Ourense e Pontevedra Groupe BMN Bankinter Caja de Ahorros y M.P. de Zaragoza, Aragon y Rioja Banco Pastor
2	DnB NOR SEB Svenska Handelsbanken Swedbank

Table 9: EU banks that pass the single-bank stress test for a stress level of 10% but fail the systemic stress test after a single round of fire sales is accounted for.

4.4 Why indirect exposures cannot be reproduced in single-bank stress tests

An argument sometimes advanced to avoid moving to a systemic stress testing framework is that while fire-sales effects are important and contribute to amplifying losses, their effect can be mimicked in bank-level stress tests by applying more severe shocks, without simulating fire sales in a detailed manner. Let us now examine the validity of this claim using the above results.

First, we note that the total loss across all banks in the systemic stress test can be obviously reproduced in a macro stress test without fire sales by simply scaling the initial shock size by an adequate factor. However, in absence of fire sales, bank losses are proportional to the banks notional exposure to the asset class subject to stress, whereas the loss in the systemic stress test is proportional to the *effective exposure*, so bank-level losses will not be reproduced correctly under this adjustment.

Figure 27 compares bank-level losses in the systemic stress test with fire sales (vertical axis) to losses of the same banks in a stress test without fire sales, but with the severity of the shock chosen so that the total system-wide loss is equal in the two stress tests (horizontal axis). As is clearly visible from this figure, the distribution of losses across banks is fundamentally different in these two cases. No amount of scaling at the level of macro-shocks defining the stress scenario can adjust for the cross-sectional heterogeneity of indirect exposures. Thus, even if the stress level is adjusted to make the total loss in the banking system equal in both stress tests, the allocation of losses across banks is essentially different once fire sales are accounted for. This is a strong argument for including a proper model of fire sales into any macroprudential stress testing framework.

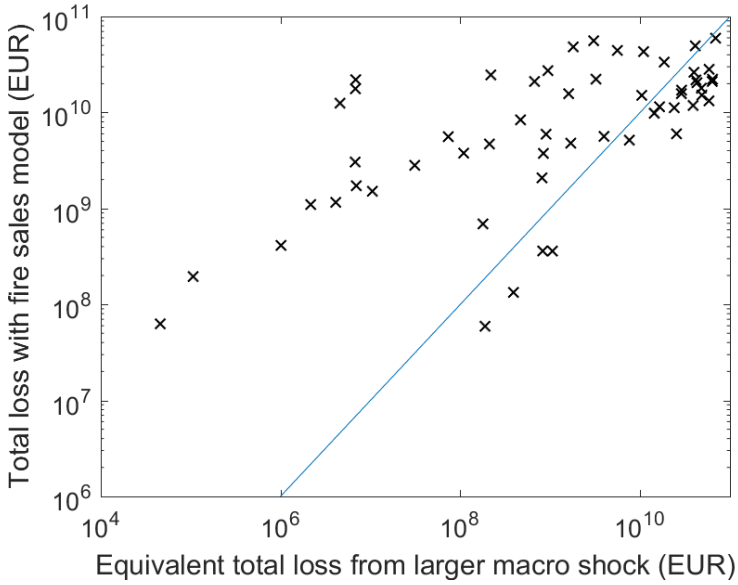

Figure 27: Scatter plot of the loss in a stress scenario against the exposure to the illiquid assets of the stress scenario: It is not possible to account for fire sales in a stress scenario by using a larger initial macro shock. As the graph shows, even when the shock is scaled such that the same total loss is generated, the distribution of losses across banks will be fundamentally different.

5 Implications for macroprudential stress testing and regulation

We have presented a stress testing framework for quantifying the endogenous risk exposure of the banking system to fire sales and feedback effects which may arise from macro-shocks to the financial system. Our findings provide quantitative evidence for the importance of *endogenous risk* Danielsson et al. (2004); Shin (2010); Cont and Wagalath (2016) in the financial system and have implications for systemic stress testing, macroprudential policy and risk management in financial institutions:

1. *Need for a systemic stress testing framework for capital adequacy*

   Theoretical studies Danielsson et al. (2004); Shin (2010); Pedersen (2009); Cont and Wagalath (2013) have repeatedly pointed out the importance of *endogenous risk* for financial stability. Fire sales are arguably an extreme example of endogenous risk and pro-cyclical behavior, yet they are not systematically integrated into bank stress testing methodologies.

   Our quantitative findings join a list of previous studies in pointing out the importance and magnitude of fire sales and price-mediated contagion as a risk amplification mechanism for systemic risk. Even under benign assumptions on market liquidity, the magnitude of exposures arising from this channel is too large to ignore; losses arising from this channel can dominate other types of risk exposures in certain risk scenarios. More importantly, we have shown that risk exposures of financial institutions arising from fire sales cannot be replicated in single-institution stress tests, even after scaling to extreme stress levels. This pleads against a widespread approach which consists in simply applying a (constant) discount to asset values in stress tests to account for liquidation costs: we have argued that this discount is endogenous and strongly depends on the degree of leverage and concentration of asset holdings across financial institutions.

   These observations, which are based on public data and may be readily replicated by regulators, plead for a systemic approach to bank stress testing which properly accounts for price-mediated contagion. We have presented the building blocks of an operational framework for estimating such endogenous effects and incorporating them into a macroprudential stress testing framework.

   Role of the liquidation horizon: Our analysis shows that the magnitude of fire sales losses is sensitive to the liquidation horizon. Allowing financial institutions in difficulty a longer horizon to liquidate attenuates the impact of fire sales.

3. *Indirect exposures as tools for risk management*:

   As pointed out by Ellul et al. (2014) "forward-looking institutions that rationally internalize the probability of fire sales are incentivized to adopt a more prudent investment strategy during normal times, which leads to a safer portfolio entering the crisis". One of the obstacles to internalizing the risk of fire sales is that its proper assessment requires some knowledge of the concentration of asset holdings across financial institutions, which is typically only available to regulators.

   One of the by-products of our stress testing approach is the ability to compute the *indirect exposures* of an institution to various asset classes (Section 4). Communicating to an institution the magnitude of its indirect exposures to various asset

classes, as evaluated in the systemic stress test, allows the institution to have a better assessment of its risk and provides incentives to reduce such exposures.

4. *The role of mark-to-market accounting rules*:

   Previous studies Allen and Carletti (2008); Ellul et al. (2014) have focused on mark-to-market accounting as the channel of transmission of losses in fire-sales contagion. Allen and Carletti (2008) present arguments against the use of mark-to-market accounting for determining solvency during crisis periods; indeed, such temporary suspensions were used in 2009 by US banks. Jotikasthira et al. (2015) use the example of the insurance sector to caution against this recommendation.

   Our model offers a perspective on this debate: having distinguished between marketable ('Level I') assets and illiquid assets, we note that a suspension of mark-to-market accounting rules is only likely to affect the latter. Suspension of mark-to-market accounting for illiquid assets may indeed affect the first step in our iteration, in which losses to illiquid assets trigger the initial deleveraging. But once deleveraging by a set of institutions takes place, the subsequent losses are not accounting losses, but market losses in 'Level I' securities whose magnitude is not affected by accounting conventions. Thus, temporary suspension of market accounting rules may reduce the perimeter of institutions affected by an initial stress to some illiquid asset class, but once fire sales affect liquid 'Level I' securities, all institutions holding them will be affected by market losses.

5. *Implications for the interaction between banks and non-banks*:

   Price-mediated contagion is not limited to banks; any institution exposed to fire-sales risk or redemption risk and having common asset holdings with banks may play a role in channeling losses to the banking sector. Indeed, there is ample empirical evidence of fire sales by asset managers Coval and Stafford (2007); Jotikasthira et al. (2012) and insurance companies Ellul et al. (2011).

   Current attempts to monitor the interaction between the banking sector and non-bank financial institutions mainly focus on direct exposures and liabilities between banks and non-banks Grillet-Aubert et al. (2016). Given that a large fraction of financial assets are held by non-banks – large asset managers, pension funds, and insurance companies – the scope for indirect contagion from 'non-banks' to the banking sector through the fire-sales channel exists and its proper assessment calls for a system-wide stress test iextended to major non-banks. Extending the present model to include non-banks would require a careful analysis of the mechanisms which would lead such institutions to shed assets: redemption risk for asset managers and asset-liability mismatch for pension funds are plausible avenues to consider Getmansky et al. (2016); Calimani et al. (2016).

6. *Need for transnational coordination on stress testing and macroprudential policy*: Price-mediated contagion defies institutional ring-fencing and national borders. Our estimated magnitudes for indirect cross-country exposures in Europe, as shown in Table 7, illustrates this point. Unlike direct exposures, which may be limited through various capital restrictions, portfolio overlaps and indirect exposures are difficult to put limits on: any such restrictions would amount to limiting international diversification of bank portfolios. Any meaningful systemic stress test should account for the magnitude of these cross-country indirect exposures, and thus cannot

be conducted at the level of a single country and calls for transnational coordination of macroprudential policies.

References

Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437, 2010. URL https://doi.org/10.1016/j.jfi.2008.12.002.

Tobias Adrian and Hyun Song Shin. Procyclical leverage and value-at-risk. *Review of Financial Studies*, 27(2):373–403, 2014. URL https://doi.org/10.1093/rfs/hht068.

Franklin Allen and Elena Carletti. Mark-to-market accounting and liquidity pricing. *Journal of Accounting and Economics*, 45(2–3):358–378, 8 2008. URL https://doi.org/10.1016/j.jacceco.2007.02.005.

Robert Almgren and Neil Chriss. Optimal execution of portfolio transactions. *Journal of Risk*, 3:5–39, 2000. URL https://doi.org/10.21314/JOR.2001.041.

Ronald W. Anderson. Stress testing and macroprudential regulation: A transatlantic assessment. *Systemic Risk Center, Financial Markets Group & CEPR Press*, 2016. URL http://voxeu.org/sites/default/files/Stress_testing_eBook.pdf.

Andrew Ang, Joseph Chen, and Yuhang Xing. Downside risk. *Review of Financial Studies*, 19(4):1191–1239, 2006. URL https://doi.org/10.1093/rfs/hhj035.

Basel Committee on Banking Supervision. Making supervisory stress tests more macroprudential: Considering liquidity and solvency interactions and systemic risk. *BIS Working Paper*, 2015. URL http://www.bis.org/bcbs/publ/wp29.pdf.

Dimitris Bertsimas and Andrew W. Lo. Optimal control of execution costs. *Journal of Financial Markets*, 1(1):1–50, 1998. URL https://doi.org/10.1016/j.jfineco.2011.03.020.

Jean-Philippe Bouchaud. Price impact. *Encyclopedia of Quantitative Finance*, 2010. URL https://doi.org/10.1002/9780470061602.eqf18006.

Yann Braouezec and Lakshithe Wagalath. Risk-based capital requirements and optimal liquidation in a stress scenario. *Review of Finance*, to appear, 2017. URL https://ssrn.com/abstract=2781964.

Anton Braverman and Andreea Minca. Networks of common asset holdings: Aggregation and measures of vulnerability. *Statistics and Risk Modeling, Forthcoming*, 2016. URL www.ssrn.com/abstract=2379669.

Markus K. Brunnermeier and Lasse Heje Pedersen. Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238, 2009. URL https://doi.org/10.1093/rfs/hhn098.

Fabio Caccioli, Munik Shrestha, Cristopher Moore, and J. Doyne Farmer. Stability analysis of financial contagion due to overlapping portfolios. *Journal of Banking and Finance*, 46:233 – 245, 2014. URL https://doi.org/10.1016/j.jedc.2014.09.041.

Susanna Calimani, Grzegorz Halaj, and Dawid Zochowski. Simulating fire-sales in banking and shadow banking system. *Mimeo*, 2016.

Lorenzo Cappiello, Linda Fache Rousova, and Mattia Montagna. Systemic risk, contagion and financial networks. *European Central Bank Special Feature*, pages 147–158, 2015. URL https://www.ecb.europa.eu/pub/fsr/shared/pdf/sfcfinancialstabilityreview201511.en.pdf.

Rodrigo Cifuentes, Gianluigi Ferrucci, and Hyun Song Shin. Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566, 2005. URL https://doi.org/10.1162/jeea.2005.3.2-3.556.

Rama Cont and Eric Schaanning. Fire sales spillovers, price-mediated contagion and systemic risk. *Working paper*, 2017.

Rama Cont and Lakshithe Wagalath. Running for the exit: Distressed selling and endogenous correlation in financial markets. *Mathematical Finance*, 23:718–741, 2013. URL https://doi.org/10.1111/j.1467-9965.2011.00510.x.

Rama Cont and Lakshithe Wagalath. Fire sales forensics: Measuring endogenous risk. *Mathematical Finance*, 26:835–866, 2016. URL https://doi.org/10.1111/mafi.12071.

Rama Cont, Amal Moussa, and Edson Bastos Santos. Network structure and systemic risk in banking systems. *Handbook of Systemic Risk*, pages 327 – 367, 2013. URL https://doi.org/10.1017/CBO9781139151184.018.

Rama Cont, Arseniy Kukanov, and Sasha Stoikov. The price impact of order book events. *Journal of Financial Econometrics*, 12:47–88, 2014. URL https://doi.org/10.1093/jjfinec/nbt003.

Joshua Coval and Erik Stafford. Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics*, 86(2):479 – 512, 2007. URL https://doi.org/10.1016/j.jfineco.2006.09.007.

Credit Suisse. Credit Suisse annual report. 2015. URL https://www.credit-suisse.com/media/assets/corporate/docs/about-us/investor-relations/financial-disclosures/financial-reports/csg-ar-2015-en.pdf.

Jon Danielsson, Hyun-Song Shin, and Jean-Pierre Zigrand. The impact of risk regulation on price dynamics. *Journal of Banking & Finance*, 28(5):1069–1087, 5 2004. URL https://doi.org/10.1016/S0378-4266(03)00113-4.

Fernando Duarte and Thomas M. Eisenbach. Fire sale spillovers and systemic risk. *Federal Reserve Bank of New York Staff Report*, 645, 2013. URL https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr645.pdf.

Andrew Ellul, Chotibhak Jotikasthira, and Christian T. Lundblad. Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics*, 101(3):596 – 620, 2011. URL https://doi.org/10.1016/j.jfineco.2011.03.020.

Andrew Ellul, Chotibhak Jotikasthira, Christian T. Lundblad, and Yihui Wang. Mark-to-market accounting and systemic risk: evidence from the insurance industry. *Economic Policy*, 29(78):596 – 620, 2014. URL https://doi.org/10.1111/1468-0327.12030297-341.

Mila Getmansky, Giulio Girardi, Kathleen Weiss Hanley, Stanislava Nikolova, and Loriana Pelizzon. Portfolio similarity and asset liquidation in the insurance industry. *SSRN Working Paper*, 2016. URL https://ssrn.com/abstract=2749691.

Paul Glasserman and H. Peyton Young. How likely is contagion in financial networks? *Journal of Banking and Finance*, pages 383–399, 2014.

Mark Granovetter. Threshold models of collective behavior. *American Journal of Sociology*, 83:489–515, 1978. URL http://www.jstor.org/stable/2778111.

Robin Greenwood, Augustin Landier, and David Thesmar. Vulnerable banks. *Journal of Financial Economics*, 115(3):471 – 485, 2015. URL https://doi.org/10.1016/j.jfineco.2014.11.006.

Laurent Grillet-Aubert, Jean-Baptiste Haquin, Clive Jackson, Neill Killeen, and Christian Weistroffer. Assessing shadow banking – non-bank financial intermediation in Europe. ESRB Occasional Paper 10, European Systemic Risk Board, July 2016. URL https://www.esrb.europa.eu/pub/pdf/occasional/20160727_occasional_paper_10.en.pdf.

Weilong Guo, Andreea Minca, and Li Wang. The topology of overlapping portfolio networks. *Statistics and Risk Modeling*, 33(3-4):139–155, 2015. URL https://doi.org/10.1515/strm-2015-0020.

Martin F. Hellwig. Systemic Risk in the Financial Sector: An Analysis of the Subprime-Mortgage Financial Crisis. *De Economist*, 157(2):129–207, 2009. URL https://doi.org/10.1007/s10645-009-9110-0.

J. Henry, C. Kok, A. Amzallag, P. Baudino, I. Cabral, M. Grodzicki, M. Gross, G. Halaj, M. Kolb, M. Leber, C. Pancaro, M. Sydow, A. Vouldis, M. Zimmermann, and D. Zochowski. A macro stress testing framework for assessing systemic risks in the banking sector. Occasional Paper Series 152, European Central Bank, 2013. URL https://www.ecb.europa.eu/pub/pdf/scpops/ecbocp152.pdf.

Chotibhak Jotikasthira, Christian T. Lundblad, and Tarun Ramadorai. Asset fire sales and purchases and the international transmission of funding shocks. *Journal of Finance*, 67(6):2015–2050, 2012. URL https://doi.org/10.1111/j.1540-6261.2012.01780.x.

Chotibhak Jotikasthira, Christian T. Lundblad, and Yihui Wang. Is historical cost accounting a panacea? Market stress, incentive distortions, and gains trading. *Journal of Finance*, 70(6):2489–2538, 2015. URL https://doi.org/10.1111/jofi.12357.

Anil Kashyap, Raghuram Rajan, and Jeremy C. Stein. Rethinking capital regulation. In *Maintaining Stability in a Changing Financial System*, pages 431–71. Federal Reserve Bank of Kansas City, 2008. URL https://www.kansascityfed.org/~/media/files/publicat/sympos/2008/kashyaprajanstein031209.pdf.

Amir E. Khandani and Andrew W. Lo. What happened to the quants in august 2007? evidence from factors and transactions data. *Journal of Financial Markets*, 14(1):1–46, 2011. URL https://doi.org/10.1016/j.finmar.2010.07.005.

Albert S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):pp. 1315–1335, 1985. URL http://www.jstor.org/stable/1913210.

Albert S Kyle and Wei Xiong. Contagion as a wealth effect. *The Journal of Finance*, 56 (4):1401–1440, 2001. URL https://doi.org/10.1111/0022-1082.00373.

Francis Longstaff. The subprime credit crisis and contagion in financial markets. *Journal of Financial Economics*, 97(3):436–450, September 2010. URL https://doi.org/10.1016/j.jfineco.2010.01.002.

Alberto Manconi, Massimo Massa, and Ayako Yasuda. The role of institutional investors in propagating the crisis of 2007–2008. *Journal of Financial Economics*, 104(3):491 – 518, 2012. URL https://doi.org/10.1016/j.jfineco.2011.05.011.

Anna Obizhaeva. Liquidity estimates and selection bias. *Working paper*, 2012. URL http://ssrn.com/abstract=1178722.

Lasse H. Pedersen. When Everyone Runs for the Exit. *The International Journal of Central Banking*, 5:177–199, 2009. URL http://www.ijcb.org/journal/ijcb09q4a10.pdf.

Eric Schaanning. Fire sales and systemic risk in financial networks. *PhD thesis, Imperial College London*, 2017.

Hyun S. Shin. *Risk and Liquidity*. Oxford University Press, 2010.

Andrei Shleifer and Robert Vishny. Fire sales in finance and macroeconomics. *Journal of Economic Perspectives*, 25(1):29–48, 2011. URL https://doi.org/10.1257/jep.25.1.29.

Andrei Shleifer and Robert W. Vishny. Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance*, 47(4):1343–1366, 1992. URL https://doi.org/10.1111/j.1540-6261.1992.tb04661.x.

A Appendix

A.1 Sources for market data

US treasuries: http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/SF-US-SF-Trading-Volume-SIFMA.xls
US corporate bonds: http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/Corporate-US-Corporate-Trading-Volume-SIFMA.xls
DE treasuries: http://www.deutsche-finanzagentur.de/en/institutional-investors/secondary-market/
UK treasuries: http://www.dmo.gov.uk/index.aspx?page=Gilts/Turnover
ES treasuries: http://www.tesoro.es/sites/default/files/estadisticas/18.pdf

FR treasuries: http://www.aft.gouv.fr/rubriques/trading-volume_109.html
IT treasuries: http://www.dt.tesoro.it/export/sites/sitodt/modules/documenti_en/debito_pubblico/presentazioni_studi_relazioni/3_3_2000_13_43_The-Italian-Treasury-pdf BE treasuries: http://www.debtagency.be/fr_products_olo_volume.htm
SE treasuries: http://www.riksbank.se/Documents/Rapporter/Finansmarknaden/2014/rap_finansm_140829_eng.pdf
PT treasuries: http://www.igcp.pt/fotos/editor2/2015/Estatisticas/12_Transacies_medias_diarias_OT_e_BT_Dez15_1.pdf
GR treasuries: http://www.bankofgreece.gr/Pages/en/Markets/HDAT/statistics.aspx

A.2 EBA: data identifiers and residual exposures

Model variable	EBA dataset identifier
Assets	
Illiquid assets Θ	
Residential mortgage exposures ($\epsilon_\kappa \geq 0$)	33013
Commercial real estate exposures ($\epsilon_\kappa \geq 0$)	33018
Retail: Revolving exposures ($\epsilon_\kappa \equiv 0$)	33015
Retail: SME exposures ($\epsilon_\kappa \equiv 0$)	33016
Retail: other exposures ($\epsilon_\kappa \equiv 0$)	33017
Indirect sovereign exp. in the trading book ($\epsilon_\kappa \equiv 0$)	34017
Defaulted exposures ($\epsilon_\kappa \equiv 0$)	33020
Remaining exposures* ($\epsilon_\kappa \equiv 0$)	-
Securities Π	
Institutional client exposures	33010
Corporate exposures	33011
Sovereign exposures	34013 , 34014 , 34015
Direct sovereign exposures in derivatives	34016
Liabilities	
Tier 1 capital	30014
Debt	-

Table 10: Mapping of EBA data to model variables.

At the time the data was collected, banks were following Basel II guidelines, which corresponds to 8% ratio of capital to risk-weighted assets (RWA), and the Basel 3 leverage constraint was not yet in place. Some banks in the sample have leverage higher than the Basel III limit of 33. In order to avoid fire sales in absence of a shock, we scale these banks' capital levels to bring their leverage within the interval of $[29.7, 31.35] = 33 \times [90\%, 95\%]$ as shown in Figure 28.[13]

Correcting for data inconsistencies. The EBA data provides information on notional exposures of each bank to 148 asset classes.[14] Bank BE005 records a zero value

[13] Similarly, Greenwood et al. (2015) cap the leverage at 30 in their analysis.

[14] For more details on the regulatory definition of "exposure" cf. the EBA methodological note: https://www.eba.europa.eu/documents/10180/15932/EBA-ST-2011-004-Detailed-Methodological-Note_1.pdf as well

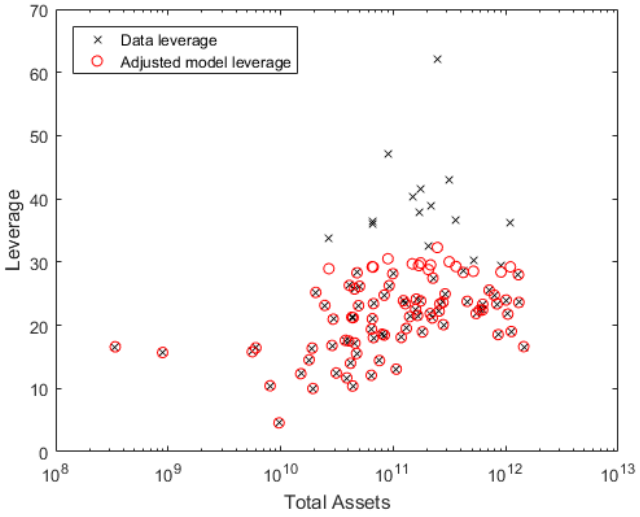

Figure 28: Leverage of banks in the EBA dataset and adjusted leverage used as model input: By adding capital to the banks with leverage above 33, the leverage of these banks is brought slightly below 33.

for total exposures despite positive exposures in the individual asset classes. As a proxy for this bank, we use information from the "Total assets after the effects of mandatory restructuring plans"[15], which is usually quite close to the values of "total exposures" across the dataset.

Another consistency issue is that the individual exposures in the dataset do not always sum up to the total exposure figure. Indeed, EBA explains in a footnote to the "total exposure" data that: *"Total exposures is the total EAD according to the CRD definition based on which the bank computes RWA for credit risk. Total exposures, in addition to the exposures broken down by regulatory portfolios in this table* [corresponding to the four asset classes in the portfolio Π above] *include EAD for securitisation transactions, counterparty credit risk, sovereigns, guaranteed by sovereigns, public sector entities and central banks"*. Due to this, the sum of balance sheet items deviates from the "total exposures" information recorded in the dataset. In order to correct for this deviation we add a "remaining exposures" item to the illiquid assets category. The average size of the negative correction terms is 5.9% of the corresponding balance sheet sizes. Double counting is thus a minor issue. The average size of the positive correction terms is 13.4%. This average is inflated by five small banks that are outliers and have correction terms above 50%. Excluding these outliers, the average of the correction terms reduces to 9%. The average size of the correction term over the entire data set is 8.9%. On average, we thus underestimate the size of the banks' balance sheets. This may only bias results in the sense of underestimating the impact of fire sales. We adjust for this with the "other illiquid assets" category, as they play no role in the fire sale cascade.

as the identifier 33021 in the dataset.

[15]This information is recorded under identifier 30029 in the EBA dataset.