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FOREIGN SHOCKS*

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Abstract

How and to what extent are small open economies affected by international shocks? I develop and estimate a medium scale DSGE model that addresses both questions. The model incorporates i) international markets for firm-to-firm trade in production inputs, and ii) producer heterogeneity where technology and price setting constraints vary across industries. Using Bayesian techniques on Canadian and US data, I document several macroeconomic regularities in the small open economy, all attributed to international disturbances. First, foreign shocks are crucial for domestic fluctuations at all forecasting horizons. Second, productivity is the most important driver of business cycles. Investment efficiency shocks on the other hand have counterfactual implications for international spillover. Third, the relevance of foreign shocks accumulates over time. Fourth, business cycles display strong co-movement across countries, even though shocks are uncorrelated and the trade balance is countercyclical. Fifth, exchange rate pass-through to aggregate CPI inflation is moderate, while pass-through at the sector level is positively linked to the frequency of price changes. Few of these features have been accounted for by existing open economy DSGE literature, but all are consistent with reduced form evidence. The model presented here offers a structural interpretation of the results.

Keywords: *DSGE, small open economy, international business cycles, Bayesian estimation.*

JEL Classification: *C11, E30, F41, F44.*

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1 INTRODUCTION

How and to what extent do business cycle shocks propagate across countries? These questions are fundamental in international macro, and of first order importance for welfare evaluation and policy making. But evidence provided by the literature is mixed. On the one side, a vast number of VAR studies find substantial cross-country spillover of shocks.¹ However, due to their reduced form nature, VAR models are largely silent regarding main disturbances and transmission channels at play. Estimated DSGE models, in contrast, facilitate formal identification of a rich set of structural innovations. But once confronted with data, these models have a hard time accounting for even moderate amounts of international spillover. Perhaps the most striking example is offered by Justiniano and Preston (2010), who document how an estimated New Keynesian model attributes virtually all business cycle fluctuations in Canada to domestic shocks.² Thus, existing literature faces a trade-off between structural interpretation and the need for reasonable results.

In this paper I revisit the role of international business cycle disturbances within a multi-sector open economy framework. To this end I develop and estimate an otherwise standard two-country New Keynesian model, but with i) international markets for firm-to-firm trade in production inputs, and ii) producer heterogeneity where firms operate in segmented markets and face different technological constraints. These modeling choices are motivated on two grounds: First, international input-output matrices reveal vast intermediate goods trade, both across diversified industries within countries, and across country borders. Table 1 reports the intermediate goods share of gross output in all OECD and BRICS countries where data were available. About 50% of gross output in most countries is sold to other firms as production inputs. Input shares are even higher in export and import data – about 60% of all trade between Canada and US is between firms. Thus, open economy models with only final goods abstract from most of the physical cross-country trade that actually takes place. Second, the combination of intermediate inputs and producer heterogeneity facilitates business cycle synchronization across countries. This is important, because the likelihood-based estimation procedure favors foreign shocks more if they can explain the strong degree of international co-movement found in data.

Usually, lack of international co-movement in DSGE models comes about due to the major role of asymmetric or country specific shocks, which create fluctuations in exchange rates and other relative prices. For instance, when the domestic terms of trade appreciates because of higher productivity abroad, domestic and foreign demand substitute away from domestically produced goods. This substitution effect is strong in most models, and works against income effects of declining real interest rates. The result is a minor role for foreign shocks in the estimated variance decomposition. But with firm-to-firm trade, I document how the terms of trade appreciation shifts importing firms' markup in the same direction as foreign markups. International markup synchronization dampens the substitution effect and reinforces the income effect, resulting in additional co-movement and stronger propagation of foreign shocks to tradable industries. Intersectoral linkages generate transmission to the rest of the domestic economy. For example, when

¹See e.g. Aastveit, Bjornland, and Thorsrud (2015), Crucini, Kose, and Otrok (2011), Kose, Otrok, and Prasad (2012), Kose, Otrok, and Whiteman (2003, 2008), Mumtaz, Simonelli, and Surico (2011).

²Similar findings are evident in e.g. Adolfson, Laséen, Lindé, and Villani (2007, 2008), Christiano, Trabandt, and Walentin (2011), Dib (2011), and Rabanal and Tuesta (2010). Schmitt-Grohé (1998) demonstrates that also the real business cycle model fails to account for international spillover.

Table 1: Intermediate trade in OECD and BRICS countries

<i>Country</i>	<i>Share</i>	<i>Country</i>	<i>Share</i>	<i>Country</i>	<i>Share</i>	<i>Country</i>	<i>Share</i>
<i>OECD</i>							
Australia	0.51	Finland	0.55	Korea	0.59	Slovenia	0.58
Austria	0.49	France	0.51	Luxembourg	0.63	Spain	0.52
Belgium	0.59	Germany	0.50	Netherlands	0.53	Sweden	0.53
Bulgaria	0.57	Greece	0.44	New Zealand	0.55	Switzerland	0.50
Canada	0.50	Hungary	0.62	Norway	0.47	Turkey	0.43
Czech Republic	0.63	Ireland	0.56	Poland	0.55	UK	0.53
Denmark	0.49	Italy	0.54	Portugal	0.54	US	0.46
Estonia	0.60	Japan	0.49	Slovakia	0.63	OECD	0.54
<i>BRICS</i>							
Brazil	0.51	China	0.64	India	0.48	Russia	0.49
South Africa	0.50	BRICS	0.52				

Note: Intermediate goods share of gross output (OECD data).

the price of manufactured goods declines, the supply of domestic service firms shifts out. This is because manufactured goods are important inputs in service production. It follows that even the supply of completely non-traded firms generally reacts to international shocks. Intersectoral firm-to-firm linkages are crucial as most of aggregate GDP is produced by domestic service firms with little direct exposure to foreign markets.

While Bergholt and Sveen (2014) explain basic mechanisms in a stylized environment, I extend the setup along several dimensions to facilitate a quantitative assessment, as in e.g. Adolfson et al. (2007). I estimate structural parameters using Bayesian techniques on 9 Canadian and 8 US time series, but restrict them to fit I-O data in both countries. I then conduct a broad evaluation of the open economy dimension of macroeconomic fluctuations in Canada. Several important results emerge: First, as in wide empirical literature, foreign shocks account for substantial variation in macroeconomic variables at all forecasting horizons (20-70%). Second, in a forecasting perspective the role of foreign shocks tends to build up over time, in line with VAR evidence (see e.g. Cushman and Zha (1997) and Justiniano and Preston (2010)). Third, while a cocktail of disturbances is responsible for macroeconomic fluctuations in the very short run, total factor productivity stands out as the most prominent type of shock over the business cycle. This contrasts the major role of investment efficiency shocks found in recently estimated models for closed economies (see Justiniano, Primiceri, and Tambalotti (2010, 2011)). I argue that these shocks have counterfactual implications for international synchronization patterns. Fourth, consistent with the empirical pass-through literature (e.g. Gopinath and Itskhoki (2010) and Gopinath, Itskhoki, and Rigobon (2010)) I find higher exchange rate pass-through in sectors with frequent price changes. This feature facilitates international spillover of shocks. Finally, when firm-to-firm trade and sectoral heterogeneity are taken out of the model, it assigns almost all business cycle fluctuations to domestic events.

The rest of the paper is organized as follows. A multi-sector DGSE model is described in Section 2. Section 3 presents data, calibration and posterior estimates. Main empirical results are reported in Section 4. In Section 5 I discuss how these results are facilitated by important transmission channels in the model. Section 6 summarizes the results from several counterfactual models while Section 7 concludes.

2 THE MODEL

I derive a general equilibrium system consisting of two blocks – “home” and “foreign”. Home is referred to as the domestic economy, while the rest of the world is captured by the foreign block. My focus is on the limiting case where home is small and has negligible influence on the world economy. A log-linear approximation around the non-stochastic steady state is presented below.³ To save space, I restrict attention to the domestic block.

2.1 HOUSEHOLDS

Consider a small open economy (SOE) with a measure one of symmetric households. The representative household consists of a continuum of members, with a fixed share μ_j working in each production sector $j \in [1, \dots, \mathcal{J}]$ in the domestic economy ($\sum_{j=1}^{\mathcal{J}} \mu_j = 1$). Household members consume, work and invest in order to maximize expected lifetime utility. The maximization problem is subject to a sequence of budget constraints, with revenues coming from returns on capital, labor income, dividends from ownership of firms, returns on domestic and foreign bonds, and government transfers. Optimality conditions for the representative household with respect to consumption, domestic and foreign bond holdings, capital and investment follow below, with prices being quoted in terms of consumption units:

$$\lambda_t = z_{U,t} - \frac{\sigma}{1 - \chi_C} (c_t - \chi_C c_{t-1}) \quad (1)$$

$$\lambda_t = \mathbb{E}_t(\lambda_{t+1}) + r_t - \mathbb{E}_t(\pi_{t+1}) \quad (2)$$

$$\lambda_t = \mathbb{E}_t(\lambda_{t+1}) + r_t^* - \mathbb{E}_t(\pi_{t+1} - \Delta e_{t+1}) - \epsilon_B nfa_t + z_{B,t} \quad (3)$$

$$q_t = -(r_t - \mathbb{E}_t(\pi_{t+1})) + \mathbb{E}_t([1 - \beta(1 - \delta)] r_{t+1}^k + \beta(1 - \delta) q_{t+1}) \quad (4)$$

$$p_{r,t}^i = q_t + z_{I,t} - \epsilon_I [(i_t - i_{t-1}) - \beta \mathbb{E}_t(i_{t+1} - i_t)] \quad (5)$$

The first equation aligns the shadow value of the budget constraint in period t , λ_t , with the marginal utility of aggregate consumption c_t . $\sigma > 0$ and $\chi_C \in [0, 1]$ govern the intertemporal elasticity of substitution and habit persistence in consumption, respectively. $z_{U,t}$ is a stationary shock to intertemporal preferences. Optimality conditions (2) and (3) equate the marginal utility of more consumption today with the expected present value of more future consumption, obtained by investing in domestic and foreign bonds. π_t and Δe_t are the CPI inflation rate and the nominal depreciation rate, respectively. Nominal interest rates on domestic and foreign bonds are denoted r_t and r_t^* , while nfa_t is the ratio of net foreign assets to GDP (measured in absolute deviations from steady state). $\epsilon_B > 0$ introduces a risk premium on foreign asset returns, as in Adolfson et al. (2007, 2008) and Christiano et al. (2011). If domestic households are net borrowers, they are charged a premium. If they are net lenders, they receive a lower return than foreign households. The risk premium also ensures that steady state is well-defined, see e.g. Schmitt-Grohé and Uribe (2003). $z_{B,t}$ denotes temporary deviations from interest rate parity, so-called risk premium shocks. The present value of one more unit of new capital, q_t , is characterized by equation (4). $r_t - \mathbb{E}_t(\pi_{t+1})$ is the expected real return (real interest rate) forgone by

³A detailed description of the full non-linear model is provided in the online appendix.

not investing in bonds, while r_t^k is the rental rate on capital in place. The parameters $\beta \in (0, 1)$ and $\delta \in [0, 1]$ denote the time discount factor and the capital depreciation rate, respectively. Finally, equation (5) determines optimal demand for aggregate investment goods. It effectively equates the relative investment price $p_{r,t}^i$ with the marginal gain of investment – the present value of capital net of investment adjustment costs. The latter is governed by $\epsilon_I \geq 0$, as in Christiano, Eichenbaum, and Evans (2005). $z_{I,t}$ is a stationary shock to the marginal efficiency of investment, a so-called MEI shock. The optimality conditions (1)-(5) summarize intertemporal household decisions in goods and asset markets. They are augmented with a capital accumulation equation of the form

$$k_{t+1} = (1 - \delta) k_t + \delta (z_{I,t} + i_t), \quad (6)$$

where k_t is capital operational in period t .

Next I turn to sectoral allocations. c_t and i_t are composite functions of sectoral consumption and investment goods, $c_{j,t}$ and $i_{j,t}$. In turn, these quantities are combinations of domestically produced ($c_{Hj,t}, i_{Hj,t}$) and imported ($c_{Fj,t}, i_{Fj,t}$) goods, respectively. At least some international trade takes place in all sectors. However, the trade intensity is sector specific, implying that import shares in c_t and i_t depend both on the import shares in each sector, and on the sector weights in aggregate demand baskets. Cost-minimization gives rise to a set of optimality conditions involving associated (real) price indexes, $p_{rj,t}$, $p_{rHj,t}$ and $p_{rFj,t}$:

$$\begin{aligned} c_{j,t} &= -\nu p_{rj,t} + c_t & i_{j,t} &= -\nu (p_{rj,t} - p_{r,t}^i) + i_t \\ c_{Hj,t} &= -\eta (p_{rHj,t} - p_{rj,t}) + c_{j,t} & i_{Hj,t} &= -\eta (p_{rHj,t} - p_{rj,t}) + i_{j,t} \\ c_{Fj,t} &= -\eta (p_{rFj,t} - p_{rj,t}) + c_{j,t} & i_{Fj,t} &= -\eta (p_{rFj,t} - p_{rj,t}) + i_{j,t} \end{aligned} \quad (7)$$

The elasticity of substitution between goods from different sectors is $\nu > 0$, while $\eta > 0$ denotes the elasticity of substitution between countries. Thus, households substitute their demand towards sectors and countries with relatively low prices. Up to first order, one can express aggregate CPI inflation π_t and investment goods inflation π_t^i as linear combinations of domestic sector prices:⁴

$$\pi_t = \sum_{j=1}^{\mathcal{J}} \xi_j \pi_{j,t} \quad \pi_t^i = \sum_{j=1}^{\mathcal{J}} \varpi_j \pi_{j,t} \quad p_{rj,t} = \alpha_j p_{rHj,t} + (1 - \alpha_j) p_{rFj,t}$$

The weights ξ_j , ϖ_j and α_j represent cost shares in steady state.

Sectoral labor markets are constructed similar to that in Erceg, Henderson, and Levin (2000), but I add a friction in the sense that labor cannot move freely between sectors or countries within the business cycle horizon.⁵ To fix ideas, consider the labor market in sector j . Firms buy labor services from a sector-specific labor union. In turn, the union provides these services by combining working hours from the μ_j household members employed in the sector. Among individual workers, only a randomly drawn fraction $1 - \theta_{wj}$ can adjust nominal wages optimally each period. Remaining workers index their wages partially to lagged CPI inflation. Nominal wage dynamics follow below:

$$\pi_{wj,t} = \beta \mathbb{E}_t (\pi_{wj,t+1}) + \iota_w (\pi_{t-1} - \beta \pi_t) + \kappa_{wj} (mrs_{j,t} - \omega_{j,t}) \quad (8)$$

⁴Note that sectoral prices are linked to aggregate CPI inflation by the identity $\pi_{j,t} = p_{rj,t} - p_{rj,t-1} + \pi_t$.

⁵Still, workers within each country do not have incentives to change sector occupation over time, as real wages are equal across sectors in steady state.

$\kappa_{wj} = \frac{(1-\theta_{wj})(1-\beta\theta_{wj})}{\theta_{wj}(1+\frac{1+\epsilon_w}{\epsilon_w}\varphi)}$ governs the responsiveness of $\pi_{wj,t}$ to time varying markups in the real wage $\omega_{j,t}$ over $mrs_{j,t}$, the marginal rate of substitution between hours worked and consumption. φ is the inverse Frisch elasticity of labor supply, while ϵ_w represents the steady state markup over competitive wages. $\iota_w \in [0, 1]$ is the degree of indexation among non-optimizing workers. The marginal rate of substitution is

$$mrs_{j,t} = z_{U,t} + z_{N,t} + \varphi n_{j,t} - \lambda_t, \quad (9)$$

where $z_{N,t}$ is referred to as a labor supply shock.

2.2 FIRMS

Each sector is populated by a continuum of profit-maximizing firms. Firms cannot change sectoral occupation over time, in analogy with labor.⁶ The individual firm produces differentiated consumption, investment and intermediate goods, which are sold in domestic and foreign markets. Production technology is Cobb-Douglas in materials, labor and capital, augmented with fixed costs. Gross output in sector j becomes

$$y_{j,t} = (1 + \epsilon_p) [z_{Aj,t} + \phi_j m_{j,t} + \psi_j n_{j,t} + (1 - \phi_j - \psi_j) k_{j,t}], \quad (10)$$

where $z_{Aj,t}$ is a sector-specific productivity shock, ϵ_p is the steady state price markup on differentiated goods, and $\phi_j, \psi_j, (\phi_j + \psi_j) \in (0, 1)$. A defining feature of the model is the presence of segmented markets for firm-to-firm trade. I follow Bergholt and Sveen (2014) and Bouakez, Cardia, and Ruge-Murcia (2009), and let $m_{j,t}$ be a composite of different materials produced in the different sectors. In principle, domestic production requires intermediate inputs from all firms in all industries in all countries.⁷ Bergholt and Sveen (2014) show how this setup facilitates sectoral interdependency, and therefore increases the potential role for international shocks in otherwise closed sectors such as services. Cost-minimization implies a set of optimality conditions for intermediate inputs:

$$\begin{aligned} m_{lj,t} &= -\nu (p_{rl,t} - p_{rj,t}^m) + m_{j,t} \\ m_{Hlj,t} &= -\eta (p_{rHl,t} - p_{rl,t}) + m_{lj,t} \\ m_{Flj,t} &= -\eta (p_{rFl,t} - p_{rl,t}) + m_{lj,t} \end{aligned} \quad (11)$$

In analogy with consumption and investment bundles, $m_{lj,t}$ denotes sector j 's demand for materials from sector l , while $m_{Hlj,t}$ and $m_{Flj,t}$ represent the domestic and imported components, respectively. $p_{rj,t}^m = \sum_{l=1}^J \zeta_{lj} p_{rl,t}$ is the composite price index associated with $m_{j,t}$. Importantly, the weights ζ_{lj} can be found from I-O matrices in each country. The system in (11) shows that optimal factor demand is directed towards those industries and countries with relatively low factor prices. Finally, material demand is high when other factors of production are relatively costly, as seen below:

$$m_{j,t} - n_{j,t} = \omega_{j,t} - p_{rj,t}^m \quad (12)$$

$$k_{j,t} - m_{j,t} = p_{rj,t}^m - r_t^k \quad (13)$$

⁶A free-entry condition prevents arbitrage opportunities of changing sectoral occupation in steady state.

⁷This gives rise to the internal feedback loop discussed below. In contrast, Carvalho and Nechio (2011) and Eyquem and Kamber (2013) assume that labor is the only factor used in intermediate goods production.

Producer prices are sticky à la Calvo (1983). Every period, each individual firm can set its price optimally with probability $1 - \theta_{pj}$. Remaining firms resort to a partial indexation rule. Nominal inflation dynamics for goods sold domestically and abroad follow:

$$\pi_{Hj,t} = \kappa_1 \mathbb{E}_t (\pi_{Hj,t+1}) + \kappa_2 \pi_{Hj,t-1} + \kappa_{j3} (rmc_{j,t} - p_{rHj,t} + z_{M,t}) \quad (14)$$

$$\pi_{Hj,t}^* = \kappa_1 \mathbb{E}_t (\pi_{Hj,t+1}^*) + \kappa_2 \pi_{Hj,t-1}^* + \kappa_{j3} (rmc_{j,t} - p_{rHj,t}^* + z_{M,t}) \quad (15)$$

The slope coefficients are defined as $\kappa_1 = \frac{\beta}{1+\beta\iota_p}$, $\kappa_2 = \frac{\iota_p}{1+\beta\iota_p}$, and $\kappa_{j3} = \frac{(1-\theta_{pj})(1-\beta\theta_{pj})}{\theta_{pj}(1+\beta\iota_p)}$, where $\iota_p \in [0, 1]$ is the degree of indexation among non-optimizing price setters. Intuitively, inflation comes about from time-varying markups in $p_{rHj,t}$ and $p_{rHj,t}^*$, the prices on domestic goods and exports, over marginal costs $rmc_{j,t}$. $z_{M,t}$ is referred to as a markup shock. Equation (15), with $\pi_{Hj,t}^*$ being expressed in international currency, follows from the assumption that export prices are set in buyer's currency – so called local currency pricing (LCP). I choose LCP rather than producer currency pricing (PCP) for two reasons. First, only 4% of Canadian exports to the US is priced in Canadian dollars. Second, PCP implies full pass-through from exchange rates into domestic inflation, at odds with the empirical pass-through literature (Gopinath et al., 2010). Marginal costs are

$$rmc_{j,t} = -z_{Aj,t} + \phi_j p_{rj,t}^m + \psi_j \omega_{j,t} + (1 - \phi_j - \psi_j) r_t^k. \quad (16)$$

Note for future reference that sector-level terms of trade are defined as domestic currency export-to-import price ratios, i.e. $\tau_{j,t} = p_{rHj,t}^* - p_{rFj,t}$.

2.3 DOMESTIC ABSORPTION AND GDP

Aggregate domestic absorption of sector j -goods is defined as the sum of consumption, investment and material components:

$$x_{j,t} = \gamma_j^c c_{j,t} + \gamma_j^i i_{j,t} + \sum_{l=1}^{\mathcal{J}} \gamma_{jl}^m m_{jl,t} \quad (17)$$

The coefficients γ_j^c , γ_j^i and γ_{jl}^m depend on the steady state and are defined in the appendix. I let $x_{Hj,t}$ be domestic absorption of domestically produced j -goods, and $x_{Fj,t}$ be the imported counterpart:

$$x_{Hj,t} = -\eta (p_{rHj,t} - p_{rj,t}) + x_{j,t} \quad (18)$$

$$x_{Fj,t} = -\eta (p_{rFj,t} - p_{rj,t}) + x_{j,t} \quad (19)$$

In analogy with domestic producer prices, imported inflation can be written as

$$\pi_{Fj,t} = \kappa_1^* \mathbb{E}_t (\pi_{Fj,t+1}) + \kappa_2^* \pi_{Fj,t-1} + \kappa_{j3}^* (rmc_{j,t}^* + s_t - p_{rFj,t} + z_{M,t}^*), \quad (20)$$

where $\kappa_1^* = \frac{\beta}{1+\beta\iota_p^*}$, $\kappa_2^* = \frac{\iota_p^*}{1+\beta\iota_p^*}$, and $\kappa_{j3}^* = \frac{(1-\theta_{pj}^*)(1-\beta\theta_{pj}^*)}{\theta_{pj}^*(1+\beta\iota_p^*)}$. s_t is the real exchange rate between the two countries, $rmc_{j,t}^*$ represents marginal costs abroad, and $z_{M,t}^*$ is an international markup shock. Similarly to domestic absorption of imports, one can define $x_{Hj,t}^*$ as global absorption of domestically produced j -goods:

$$x_{Hj,t}^* = -\eta (p_{rHj,t}^* - s_t - p_{rj,t}^*) + x_{j,t}^* \quad (21)$$

$p_{rj,t}^*$ and $x_{j,t}^*$ represent sector-specific prices and quantities in global markets. Market clearing implies that $y_{j,t} = \alpha_{xj} x_{Hj,t} + (1 - \alpha_{xj}) x_{Hj,t}^*$, where α_{xj} is the steady state share of domestic output that is supplied at home. GDP and trade balances at the sector level are derived according to the expenditure approach:

$$gdp_{j,t} = \gamma_j^1 (p_{rj,t} + x_{j,t}) + tb_{j,t} - \gamma_j^2 (p_{rj,t}^m + m_{j,t}) \quad (22)$$

$$tb_{j,t} = \gamma_j^{ex} (p_{rHj,t}^* + x_{Hj,t}^*) - \gamma_j^{im} (p_{rFj,t} + x_{Fj,t}) \quad (23)$$

The trade balance is expressed relative to sector GDP and in absolute deviation from steady state. γ_j^{ex} and γ_j^{im} represent sector-specific export/import-to-GDP ratios respectively, while γ_j^1 and γ_j^2 are found as solutions to the steady state of the model. Finally, by aggregating across sectors we get economy-wide GDP and trade balance:

$$gdp_t = \sum_{j=1}^{\mathcal{J}} \gamma_j gdp_{j,t} \quad \text{and} \quad tb_t = \sum_{j=1}^{\mathcal{J}} \gamma_j tb_{j,t} \quad (24)$$

The parameter γ_j is the steady state share of sector j in aggregate GDP. From the global economy's point of view, their debt is in zero net supply because the home economy engages in only a negligible part of the financial assets trade. Furthermore, I assume that foreign investors do not hold financial assets in the home economy.

2.4 MONETARY AND FISCAL POLICY

The model is closed with a specification of monetary and fiscal policy. I follow previous work in the DSGE literature (see e.g. Justiniano and Preston (2010); Lubik and Schorfheide (2007); Smets and Wouters (2007)) and assume that monetary policy can be approximated by a Taylor-type rule of the form

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (\rho_\pi \pi_t + \rho_y gdp_t + \rho_{dy} \Delta gdp_t + \rho_{de} \Delta e_t) + z_{R,t}. \quad (25)$$

$\rho_r, \rho_\pi, \rho_y, \rho_{dy}$ and ρ_{de} are policy coefficients, and $z_{R,t}$ is a monetary policy shock. Regarding fiscal policy, the government faces a period-by-period budget constraint with Ricardian taxes and newly issued government bonds on the income side, and public spending and maturing bonds on the expenditure side. Under the assumption that public debt is zero in steady state, one can write, up to a first order approximation, public spending as fully financed by (possibly time-varying) lump-sum taxes.

2.5 EXOGENOUS DISTURBANCES

I assume that all exogenous disturbances in the model follow a univariate AR(1) representation in log-linear form:

$$\mathbf{z}_t = \rho_z \mathbf{z}_{t-1} + \sigma_z \varepsilon_{z,t}, \quad \varepsilon_{z,t} \stackrel{i.i.d.}{\sim} N(0, 1) \quad (26)$$

$\mathbf{z}_t = [z_{U,t}, z_{N,t}, z_{B,t}, z_{I,t}, z_{M,t}, z_{R,t}, z_{A1,t}, \dots, z_{A\mathcal{J},t}]'$ is the vector of exogenous disturbances. ρ_z and σ_z are diagonal, and all non-zero elements in ρ_z are bounded between zero and one. Fluctuations in the foreign economy are subject to a similar set of disturbances, except that foreign risk premium shocks are negligible due to the small economy assumption.

3 ESTIMATION

Sector heterogeneity induces a non-symmetric equilibrium across different industries. I solve for the steady state analytically and use the solution to parameterize a log-linear approximation of the model. The steady state solution is provided in the appendix. Several model parameters are estimated using Bayesian techniques. This approach has been popularized by e.g. An and Schorfheide (2007), Geweke (1999), and Smets and Wouters (2003, 2007). Before discussing the results I describe data, the calibration, and priors.

3.1 DATA

To estimate the model I use HP-filtered quarterly data from Canada and US (1982Q4-2007Q4).⁸ Canada is treated as a prototype SOE, while US proxies the world economy. This country-pair has been analyzed in a number of two-country SOE-studies (see e.g. Justiniano and Preston (2010) and Schmitt-Grohé (1998)). I divide each economy into three sectors – the raw material sector, the manufacturing sector, and the service sector. These are classified according to the North American Industry Classification System (NAICS). Raw materials constitute NAICS industries 11-21, manufacturing 22-33, and services 41-56 and 71-72 respectively. The industries are exhaustive in the sense that they aggregate to privately produced GDP. Sector-level GDP series are interpolated as the raw data are available only at annual frequency. In addition, I use as observables quarterly consumption, investment, hours, CPI inflation, and policy rates from both countries, as well as the bilateral real exchange. This leaves me with a total of 17 time series used for estimation.⁹ Details about the data set are relegated to the appendix.

3.2 CALIBRATION AND PRIORS

A subset of the parameters is calibrated according to data and previous studies. Their values are reported in Table 2. Parameters not related to the multi-sector setup are set to common values in the literature (see e.g. Adolfson et al. (2007, 2008), Christiano et al. (2011), Justiniano and Preston (2010), and Smets and Wouters (2007)). Regarding ν , I choose a value of 0.5 based on Atalay (2013), who estimates sectoral substitution elasticities between 0.85 and essentially zero. $\nu = 1.5$ gives similar results. The remaining parameters are sector-specific, and these deserve further attention. To parameterize sector-specific steady state ratios I rely on the Canadian and US I-O matrices, obtained from the Structural Analysis Input Output (Total) Database constructed by OECD. The data reveal large differences across industries. For instance, while almost 70% of consumption goods is services, manufacturing firms produce the vast majority of investment goods. Raw materials, while only accounting for about 2% of aggregate consumption and investment in Canada, constitute 16% of GDP because of its exports and large supply of intermediates. Regarding trade, Canadian export-to-GDP ratios vary from 7% in the service sector to about 102% in the manufacturing sector. These sector differences represent

⁸Results are similar if data are linearly detrended.

⁹Raw data are collected from Federal Reserve Economic Database (FRED), Statistics Canada, and Bureau of Economic Analysis. They are available to the public and can be downloaded from <http://research.stlouisfed.org/fred2/>, <http://www.statcan.gc.ca/>, and <http://www.bea.gov/>.

Table 2: Calibration

β	Time discount factor	0.99	ν	Sectoral elasticity	0.5		
σ	Inverse intertemporal elasticity	1	δ	Capital depreciation	0.025		
φ	Inverse labor supply elasticity	2	ϵ_B	Risk premium elasticity	0.01		
ϵ_w, ϵ_p	Markup, labor and goods markets	1/7					
			<i>SOE</i>				
			(1)	(2)	(3)		
			<i>ROW</i>				
			(1)	(2)	(3)		
ϕ_j	Materials share, gross output	0.37	0.66	0.34	0.35	0.54	0.33
ψ_j	Labor share, gross output	0.12	0.21	0.32	0.10	0.22	0.29
$\gamma_j^{ex}, \gamma_j^{im}$	Trade share, sector GDP	0.67	1.02	0.07	–	–	–
ξ_j	Sector share, consumption	0.02	0.31	0.67	0.01	0.29	0.70
ϖ_j	Sector share, investment	0.02	0.85	0.13	0.03	0.77	0.20
		0.32	0.21	0.03	0.40	0.18	0.01
ζ_{lj}	Input-output matrix	0.38	0.61	0.32	0.33	0.58	0.28
		0.30	0.18	0.65	0.27	0.24	0.71

Note: Calibrated values in benchmark model. The sectors are (1) raw materials, (2) manufacturing, and (3) services. The two I-O matrices at the bottom display the fraction of total materials used in each sector that comes from each of the other sectors. Columns represent consumption (input), and rows production (output).

a key source of disaggregate heterogeneity in the model. Turning to data on materials, we see that substantial trade in intermediate goods takes place across sectors, as illustrated by the non-zero off-diagonal elements of the I-O matrices. For instance, the service sector in Canada buys about 32% of its materials from the manufacturing sector (which trade extensively in foreign markets). This is the sense in which trade across sectors provides indirect import in the model, and thereby serves as a potential amplification mechanism for foreign shocks.

The remaining parameters are estimated. I choose priors in the mid range of those used by Adolfson et al. (2007), Christiano et al. (2011), and Justiniano and Preston (2010), with identical distributions across countries on same parameters.¹⁰ The substitution elasticity between domestic and foreign goods is centered around 1 – above estimates by Corsetti, Dedola, and Leduc (2008), Gust, Leduc, and Sheets (2009), and Heathcote and Perri (2002), but below estimates by Adolfson et al. (2007). Regarding Calvo parameters for wage stickiness, I am not aware of any studies pointing to substantial sectoral differences. Thus, θ_{wj} is centered around 0.75 $\forall j$. Priors on sectoral price stickiness are inspired by a number of microeconomic studies, who show that raw materials and manufactured goods change prices much more frequently than service goods. For instance, looking at disaggregate US data, Bils and Klenow (2004), Bouakez et al. (2009) and Nakamura and Steinsson (2008) find virtually flexible prices in agricultural and raw materials, while estimated price durations in services range from 1.6 quarters (Bils and Klenow, 2004) to 9 quarters (Bouakez et al., 2009). I choose priors in the mid range of these estimates: Calvo parameters are set such that average price durations in raw materials, manufacturing and services are equal to 1.18, 1.25, and 5 quarters respectively. Priors for the seventeen structural shocks are comparable with e.g. Adolfson et al. (2007), although technology shocks

¹⁰Justiniano and Preston (2010), on the other hand, scale up priors on foreign shocks to twice the size of domestic shocks. This is done in order to induce a more important role for international business cycles.

Table 3: Prior and posterior distributions

		Prior	Posterior domestic			Posterior foreign		
		Prior(P1,P2)	Mode	Mean	5%-95%	Mode	Mean	5%-95%
χ_C	Habit	B(0.50,0.10)	0.61	0.60	0.48-0.71	0.56	0.60	0.48-0.72
ϵ_I	Inv. adj. cost	N(5.00,1.00)	0.77	1.06	0.52-1.57	2.59	3.04	1.48-4.51
η	H-F elasticity	G(1.00,0.15)	0.83	0.83	0.74-0.91	–	–	–
θ_{w1}		B(0.75,0.07)	0.76	0.73	0.61-0.86	0.75	0.74	0.61-0.87
θ_{w2}	Calvo wages	B(0.75,0.07)	0.38	0.38	0.27-0.48	0.75	0.74	0.62-0.86
θ_{w3}		B(0.75,0.07)	0.71	0.68	0.56-0.81	0.72	0.67	0.52-0.82
θ_{p1}		B(0.15,0.05)	0.11	0.13	0.06-0.20	0.21	0.22	0.17-0.26
θ_{p2}	Calvo prices	B(0.20,0.05)	0.14	0.15	0.09-0.21	0.30	0.31	0.25-0.36
θ_{p3}		B(0.80,0.07)	0.66	0.65	0.59-0.71	0.80	0.80	0.76-0.85
ι_w	Indexation, π_w	B(0.50,0.15)	0.30	0.32	0.13-0.51	0.52	0.51	0.26-0.74
ι_p	Indexation, π_p	B(0.50,0.15)	0.17	0.21	0.07-0.34	0.87	0.85	0.76-0.95
ρ_r	Smoothing, r	B(0.60,0.05)	0.73	0.74	0.69-0.78	0.76	0.76	0.72-0.80
ρ_π	Taylor, π	N(1.80,0.20)	1.95	2.00	1.72-2.27	1.70	1.75	1.51-1.99
ρ_y	Taylor, gdp	N(0.13,0.05)	0.03	0.04	0.02-0.06	0.08	0.09	0.05-0.12
ρ_{dy}	Taylor, Δgdp	N(0.13,0.05)	0.12	0.13	0.05-0.19	0.15	0.15	0.09-0.21
ρ_{de}	Taylor, Δe	N(0.10,0.05)	0.10	0.10	0.04-0.16	–	–	–
ρ_A	Technology	B(0.70,0.10)	0.90	0.89	0.84-0.93	0.90	0.90	0.86-0.93
ρ_R	Mon. pol.	B(0.70,0.10)	0.29	0.30	0.21-0.38	0.31	0.32	0.22-0.41
ρ_I	Investment	B(0.70,0.10)	0.51	0.50	0.36-0.63	0.40	0.41	0.31-0.52
ρ_U	Preferences	B(0.70,0.10)	0.41	0.43	0.27-0.57	0.59	0.56	0.40-0.72
ρ_N	Labor supply	B(0.70,0.10)	0.72	0.67	0.51-0.84	0.72	0.71	0.57-0.86
ρ_M	Markup	B(0.70,0.10)	0.50	0.51	0.38-0.64	0.54	0.54	0.43-0.65
ρ_B	UIP	B(0.70,0.10)	0.85	0.84	0.77-0.92	–	–	–
σ_{A1}		IG(0.20,2.00)	0.09	0.22	0.04-0.40	5.49	5.59	4.86-6.31
σ_{A2}	Sd technology	IG(0.50,2.00)	0.71	0.70	0.35-1.04	1.16	1.16	1.00-1.32
σ_{A3}		IG(0.20,2.00)	0.90	0.91	0.73-1.09	0.61	0.62	0.51-0.72
σ_R	Sd mon. pol.	IG(0.20,2.00)	0.25	0.26	0.22-0.30	0.11	0.12	0.10-0.13
σ_I	Sd investment	IG(0.50,2.00)	2.25	3.05	1.63-4.36	5.61	6.52	3.36-9.30
σ_U	Sd preferences	IG(0.20,2.00)	1.66	1.72	1.22-2.20	1.16	1.33	0.93-1.73
σ_N	Sd labor supply	IG(0.50,2.00)	0.24	0.59	0.11-1.41	0.23	0.42	0.12-0.76
σ_M	Sd markup	IG(0.50,2.00)	0.71	0.74	0.60-0.88	1.36	1.40	1.15-1.64
σ_B	Sd UIP	IG(0.20,2.00)	0.42	0.44	0.32-0.57	–	–	–
σ_{e1}		IG(0.20,1.00)	6.48	6.61	5.81-7.40	0.09	0.14	0.05-0.22
σ_{e2}	Sd mea. err.	IG(0.20,1.00)	3.27	3.32	2.90-3.74	2.98	3.02	2.66-3.38
σ_{e3}		IG(0.20,1.00)	0.73	0.75	0.64-0.87	0.13	0.14	0.09-0.19

Note: B denotes the beta distribution, N normal, G gamma, IG inverse gamma, P1 prior mean, P2 prior standard deviation. Posterior moments are computed from 500000 draws generated by the Random Walk Metropolis-Hastings algorithm, where the first 200000 are used as burn-in. The volatility of shocks is multiplied by 100 relative to the text.

in services are less volatile than in other sectors. This reflects estimates by Bouakez et al. (2009), who point to much less volatility in services. TFP differences used here are fairly conservative compared with their results. Finally, I include a measurement error in each of the observation equations linking observed GDP series to the model. This is motivated by the interpolation of sectoral GDP data, which might introduce certain high- or low-frequency properties not related to the business cycle. Measurement errors are assumed to be i.i.d. with prior standard deviations centered around 0.2. This is similar to the prior measurement errors on wages used by Justiniano, Primiceri, and Tambalotti (2013).

3.3 POSTERIOR ESTIMATES

To build the posterior parameter distribution, I simulate two Random Walk Metropolis-Hastings chains with 500000 draws per chain, starting at the posterior mode. The first 200000 draws are used as burn-in. I tune the scale of the jumping distribution and obtain acceptance ratios of about 0.3 in both chains. Posterior estimates are reported in Table 3. Most parameters are found to be in line with those found in previous studies, with notable exceptions discussed below. First, the posterior mode and mean of investment adjustment costs are significantly smaller in both countries than what is typically found in the DSGE literature, but still higher than microeconomic estimates (see Groth and Khan (2010)). This might be due to internal propagation in the model, a point which I will turn back to later. Second, the estimated price rigidities display large differences across sectors in both countries, with service sector prices being more sticky than prices in other sectors. This is consistent with a number of microeconomic studies as discussed earlier (e.g. Bils and Klenow (2004)), and cannot be accounted for by one-sector models à la Smets and Wouters (2007). A low Calvo parameter in manufacturing is perhaps also related to the inclusion of construction firms in that sector, as Bouakez et al. (2009) find that US construction prices are perfectly flexible. Third, there is much less indexation to previous prices and wages in Canada than in the US. This might have to do with the open economy dimension, as other parameters are fairly similar across countries. Also Justiniano and Preston (2010) report less indexation in Canada compared with the US. Finally, as in Lubik and Schorfheide (2007), I find some evidence of systematic response by monetary authorities to exchange rate fluctuations. Turning to the shock processes, we see that technology shocks are the most persistent, and that the most volatile disturbances in the model are productivity innovations in raw-material sectors and marginal efficiency of investment shocks. Moreover, productivity is substantially less volatile in the foreign service sector, in line with results in Bouakez et al. (2009). Finally, note that data are uninformative about some parameters, in particular those associated with labor supply shocks.

4 EMPIRICAL RESULTS

So far I have presented an estimated multi-sector DSGE model for Canada, a prototype SOE. This section documents the main empirical finding from the estimated model – the significance of foreign business cycle shocks for domestic variables. I restrict attention to Canadian GDP, consumption, investment, hours, CPI inflation, real wages, net exports, and the policy rate. Table 4 reports the variance decomposition of domestic forecast errors (FEVDs) at different forecasting horizons. The first column shows the importance of all foreign innovations combined. Remaining columns report contributions of individual disturbances.¹¹ Three results stand out. First, at all horizons a substantial fraction of the forecast error is attributed to foreign shocks. Second, their role in the variance decomposition tends to build up over time. Third, while a cocktail of disturbances is responsible for macroeconomic fluctuations in the very short run, foreign productivity shocks stand out as the prominent source of long run volatility. These findings are discussed next.

¹¹Shocks to the UIP condition are likely a mix of domestic and foreign events. Christiano et al. (2011) label UIP shocks as foreign, while Justiniano and Preston (2010) include them in the domestic block. I take a conservative view, and follow the latter definition throughout the analysis.

Table 4: Forecast error variance decomposition of foreign shocks (percent)

Variable	All foreign shocks	Decomposition							
		σ_{A1}^*	σ_{A2}^*	σ_{A3}^*	σ_R^*	σ_I^*	σ_U^*	σ_N^*	σ_M^*
Panel A: 1-quarter horizon									
GDP	21.38	6.68	0.67	0.00	0.01	7.37	0.09	0.00	6.55
Consumption	10.54	5.16	3.14	0.42	0.08	0.51	0.11	0.00	1.12
Investment	22.48	8.71	7.28	0.76	0.08	2.63	0.26	0.00	2.76
Hours	17.80	1.56	3.93	0.51	0.01	11.37	0.41	0.00	0.00
Interest	36.63	11.01	11.29	1.68	0.78	5.45	0.54	0.00	5.88
Inflation	41.19	15.26	12.13	1.28	0.20	4.48	0.43	0.00	7.40
Wage	47.48	21.52	14.60	1.46	0.25	0.85	0.30	0.00	8.50
Trade balance	37.88	2.98	7.25	1.49	0.23	24.52	1.06	0.00	0.34
Panel B: 4-quarter horizon									
GDP	47.76	22.88	14.76	1.49	0.16	1.95	0.13	0.00	6.39
Consumption	19.57	11.04	5.56	0.77	0.05	0.70	0.20	0.00	1.24
Investment	34.66	15.86	12.11	1.15	0.04	2.73	0.35	0.00	2.42
Hours	22.96	5.94	5.60	0.62	0.13	8.27	0.25	0.00	2.16
Interest	37.86	13.19	10.31	1.70	0.37	7.51	0.67	0.00	4.11
Inflation	40.92	15.31	11.70	1.26	0.22	4.66	0.47	0.00	7.29
Wage	53.25	28.73	17.16	1.93	0.11	0.33	0.33	0.00	4.66
Trade balance	34.32	1.93	2.89	0.62	0.14	27.03	0.81	0.00	0.91
Panel C: 8-quarter horizon									
GDP	52.83	26.66	18.00	1.99	0.10	1.91	0.15	0.00	4.02
Consumption	19.86	11.85	5.13	0.75	0.04	1.06	0.15	0.00	0.88
Investment	37.92	18.85	13.33	1.11	0.03	2.54	0.28	0.00	1.78
Hours	29.84	10.29	8.90	0.99	0.12	7.16	0.25	0.00	2.14
Interest	36.39	12.05	10.51	1.51	0.37	6.74	0.62	0.00	4.59
Inflation	41.88	15.31	12.03	1.24	0.23	5.05	0.50	0.00	7.52
Wage	53.39	29.68	17.27	2.07	0.06	1.35	0.22	0.00	2.74
Trade balance	31.96	1.76	3.66	0.95	0.15	23.73	0.73	0.00	0.98
Panel D: 20-quarter horizon									
GDP	63.11	29.13	22.87	2.09	0.08	5.92	0.11	0.00	2.92
Consumption	40.07	19.47	13.19	1.29	0.05	4.50	0.10	0.00	1.47
Investment	43.28	17.70	13.83	0.99	0.04	8.99	0.24	0.00	1.51
Hours	30.69	10.32	9.09	0.96	0.11	7.86	0.25	0.00	2.09
Interest	34.95	11.68	10.19	1.46	0.34	6.33	0.59	0.00	4.36
Inflation	42.48	15.57	12.30	1.32	0.22	5.15	0.49	0.00	7.43
Wage	66.96	32.45	23.75	2.28	0.06	6.33	0.13	0.00	1.95
Trade balance	33.54	2.11	4.33	1.22	0.13	23.98	0.72	0.00	1.04
Panel E: Long-run horizon									
GDP	73.86	29.35	30.46	2.25	0.08	9.37	0.10	0.00	2.24
Consumption	75.69	26.75	32.99	2.21	0.06	12.27	0.08	0.00	1.33
Investment	44.55	16.90	15.30	1.10	0.04	9.61	0.22	0.00	1.39
Hours	34.08	11.49	10.81	1.07	0.11	8.29	0.23	0.00	2.07
Interest	43.11	14.40	14.59	1.64	0.30	7.76	0.51	0.00	3.91
Inflation	46.09	16.53	14.46	1.42	0.21	5.98	0.47	0.00	7.01
Wage	83.15	31.51	35.55	2.52	0.06	11.96	0.10	0.00	1.44
Trade balance	32.25	1.97	4.35	1.25	0.12	22.90	0.66	0.00	1.01

Note: Calculated at the posterior mean. Note that when the forecasting horizon s becomes large, the contribution of a shock to the s step ahead forecast error converges to that shock's contribution to the unconditional volatility. Thus, Panel E reports each shock's contribution to long-run volatility.

4.1 ON THE ROLE OF FOREIGN SHOCKS

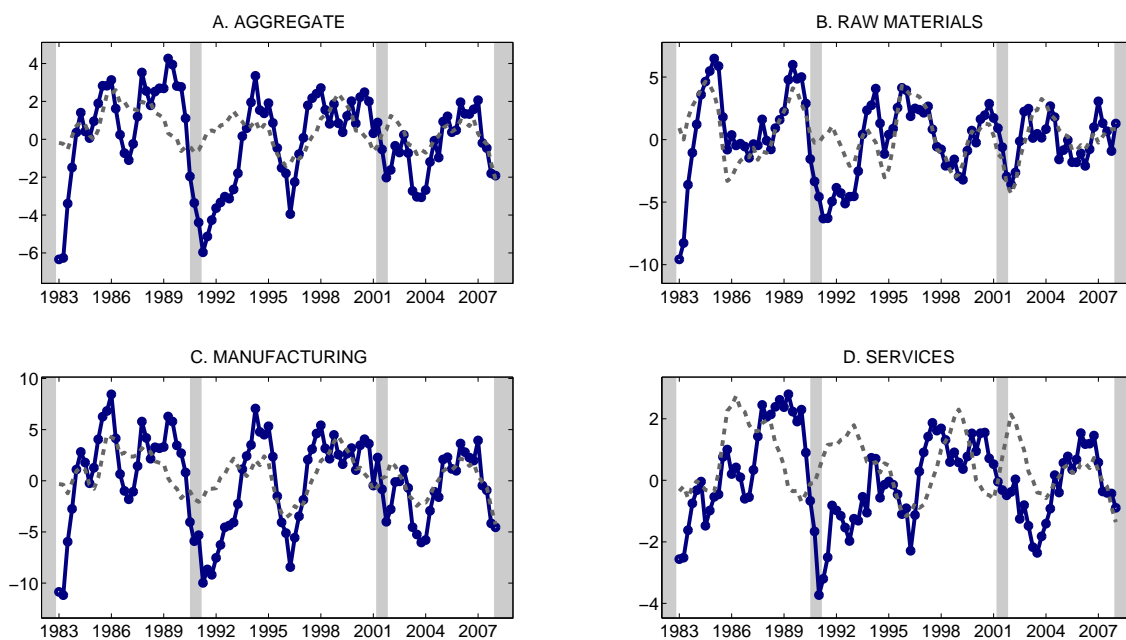
Are foreign shocks important for macroeconomic volatility in small open economies? The model presented here answers “yes” when confronted with Canadian and US data. This is in line with ample empirical evidence. For instance, Kose et al. (2003) estimate a FAVAR model with separate world, region, and country specific factors. They report that the world and region factors combined explain about 45-75% of the volatility in Canadian GDP, consumption and investment. Similar results are obtained in VAR studies of different countries and sample periods, and with alternative identifying assumptions regarding shocks. Recent examples include Crucini et al. (2011), Kose et al. (2008), Kose et al. (2012), and Mumtaz et al. (2011). Estimated SOE-DSGE models, in contrast, have a hard time accounting for international business cycle transmission. Let us take GDP as an example: Justiniano and Preston (2010), using a benchmark model, find that foreign shocks explain about 1% of the fluctuations in Canadian GDP at all forecasting horizons. Adolfson et al. (2007) estimate a medium scale model on Swedish data, and report that foreign shocks explain between 9% (1 quarter) and 1% (20 quarters) of Swedish GDP.¹² Christiano et al. (2011) extend the Swedish model to include financial frictions and unemployment, and find that 8% of GDP is explained by a set of five foreign disturbances (including UIP shocks) within the 8-quarter horizon. The limited role for foreign shocks seems to hold also in DSGE models for large economies (see Jacob and Peersman (2013)). As an illustration of the importance of international business cycle spillover, Figure 1 plots quarterly Canadian GDP in data and in the model when only foreign shocks are included. Consider first aggregate GDP. A significant share of the movements is explained by foreign shocks, and their importance rises over the sample as the initial discrepancy attributed to pre-sample conditions dies out. Further decomposition into sectoral variables suggests a tendency of more variation being explained by foreign disturbances in the raw-material sector than in manufacturing, while manufacturing seems more prone to foreign shocks than services.¹³

The second result, that foreign variance shares are increasing in the forecasting horizon, is consistent with a number of empirical studies as well. Justiniano and Preston (2010) estimate a VAR model and report that foreign shocks explain 22% of Canadian GDP at the 1-quarter horizon, 44% at the 4-quarter horizon, and 76% in the long run. The numbers in Table 4 closely resemble those findings. Also Cushman and Zha (1997) and Aastveit et al. (2015) use VARs to document higher foreign variance shares at longer horizons. However, the DSGE model allows us to take one step further and ask, within a structural framework, why foreign variance shares rise over time. The clue lies in estimated properties of TFP. Table E.2 in the appendix reports the FEVD of domestic shocks. In the very short run (1 quarter), no single shock is the major driver of the selected set of macroeconomic variables. Rather, innovations to different variables are caused by different disturbances. For example, GDP is driven both by shocks to service productivity, the interest rate, and the marginal efficiency of investment (MEI). Consumption and investments are explained well by preference and MEI shocks respectively, while the trade balance is captured by risk-premium and MEI shocks, as in Jacob and Peersman (2013).

¹²These numbers are found in a working paper version (see Adolfson, Laséen, Lindé, and Villani (2005)).

¹³The figure indicates that the two largest recessions in the sample (1981-1982 and 1990-1992) had little to do with international events. Foreign shocks in the first case are probably hidden in pre-sample conditions. In the latter case the recession was indeed far more severe in Canada (see Cross (2011) and Voss (2009)).

Figure 1: GDP in data and in the model with only foreign shocks



Note: GDP in data (blue) and IR-smoothed counterfactual series excluding domestic shocks (gray). Counterfactuals are calculated based on the posterior mean. Shaded bars denote NBER-defined US recessions.

For the unconditional volatility of macroeconomic variables, it is clear that productivity plays a major role. All foreign and domestic TFP shocks combined are responsible for about 70-80% of aggregate volatility in GDP, consumption and wages, and about half of the movements in inflation and interest rates.¹⁴ Arguably, the increasing importance of foreign productivity over time can be traced to the estimated autoregressive process for TFP. Productivity innovations are relatively long-lasting events, explaining why they account for substantial shares of the forecasting errors at longer horizons. A fundamental question is why data prefer productivity-driven business cycles. Section 5 sheds more light on this issue. For now, I stress the model’s ability to match the international synchronization patterns found in data.

4.2 INTERNATIONAL CO-MOVEMENT

Most macroeconomic models have a hard time accounting for the high degree of business cycle co-movement across countries (see Backus, Kehoe, and Kydland (1992) for an early illustration). Thus, international business cycle synchronization is one important empirical dimension in which the model can and should be evaluated. In Table 5 I report contemporaneous correlations between Canadian variables and their US counterparts. The first column shows the data moments, remaining columns report model implied correlations – both at the aggregate level and within sectors across countries. Clearly, the model suggests substantial business cycle co-movement, both at aggregate and dis-aggregate levels. For instance, the correlation between US and Canadian GDP is about 0.75, compared with 0.78 in data. But the fit is not perfect. International consumption co-movement is over-

¹⁴Similar results are found for the US variables (not shown).

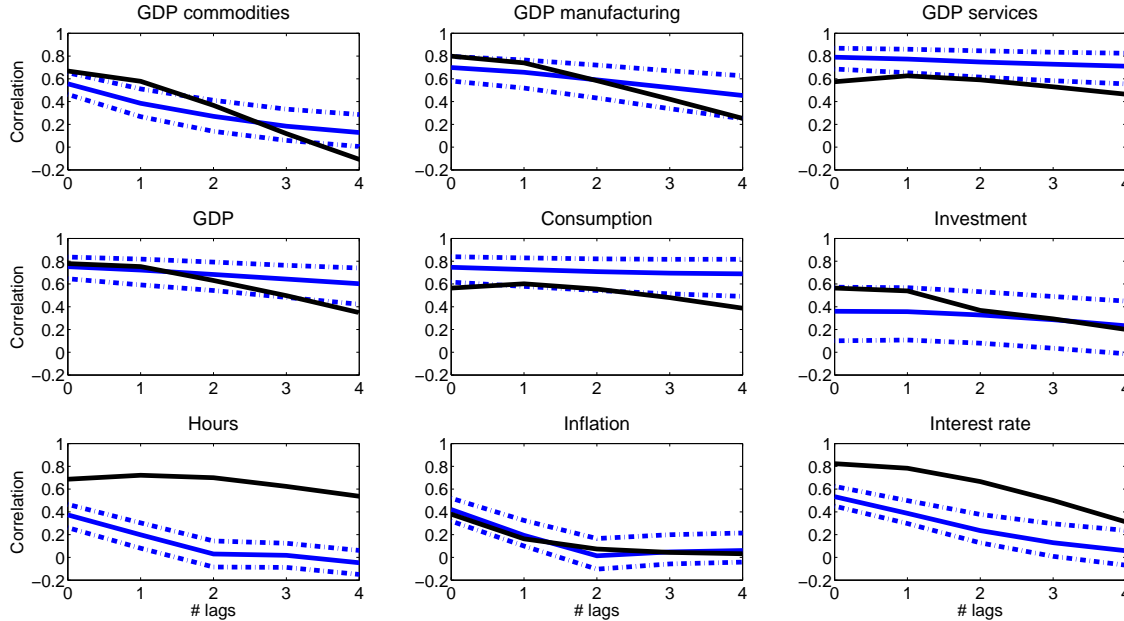
Table 5: Cross-country correlations (scaled by 100)

Variable	Data	Model							
		Aggregate		Commodities		Manufacturing		Services	
		Med.	5%-95%	Med.	5%-95%	Med.	5%-95%	Med.	5%-95%
GDP	77.9	75.2	64.6-83.7	55.6	46.4-65.4	69.9	58.1-79.7	79.0	68.6-86.9
Consumption	56.4	74.6	61.6-83.9	82.7	75.9-88.1	80.9	69.4-88.3	69.4	55.1-79.9
Investment	56.4	36.0	10.1-57.2	60.8	35.3-76.6	35.5	9.4-56.7	32.9	8.4-53.4
Hours	68.7	37.3	26.1-46.5	46.9	32.2-58.8	37.2	23.1-47.7	27.9	18.0-36.6
Inflation	37.9	42.1	31.8-52.0	93.0	90.3-94.9	58.3	50.5-64.9	20.5	8.3-37.3
Interest rate	82.3	53.4	45.0-62.3	–	–	–	–	–	–

Note: Cross-country correlations in data and the simulated model (median and 90% highest probability density intervals). Model moments take into account parameter and sample uncertainty, and are based on draws from the posterior distribution.

stated and the model predicts too little co-movement in hours and interest rates. These discrepancies arise because the likelihood-based estimation procedure attempts to fit the entire autocovariance structure of the data. This is a massive challenge given the model’s tight restrictions, even more so when cross-country moments are part of the system. Autocorrelation functions for the Canadian observables versus their foreign counterparts are plotted in Figure 2. Consistent with Table 5, the model is able to replicate autocorrelations in data fairly well. The fit is particularly impressive for inflation, while the ability to match hours is worse and deteriorates at longer lags. Overall, it seems clear that the model

Figure 2: Autocorrelation plots



Note: Plots of $Corr(y_t, y_{t-s}^*)$ for $s = 0, \dots, 4$ lags, with y and y^* being vectors of domestic and foreign variables. Data (black) versus the model (blue) (median and 90% highest probability density intervals). Model moments take into account parameter and sample uncertainty, and are based on draws from the posterior distribution.

Table 6: Short-run pass-through rates (scaled by 100)

<i>Price measure</i>	π_t	π_t^i	$\pi_{j,t}$	$\pi_{Fj,t}$	$\pi_{Hj,t}$	$\pi_{Hj,t}^*$	$\pi_{j,t}^m$	$\tau_{j,t}$
Aggregate	12.47	26.17	–	–	–	–	–	–
Commodities	–	–	35.50	64.80	14.09	31.69	23.73	-33.10
Manufacturing	–	–	29.45	53.34	16.77	36.41	26.13	-16.93
Services	–	–	3.92	8.00	3.72	79.68	13.04	71.68

Note: Pass-through rates scaled by 100, calculated based on the posterior mean. $\pi_{Hj,t}^*$ in the table is expressed in terms of domestic currency, in contrast to the text.

can account for substantial business cycle synchronization across countries. At least, the numbers reported here represent major improvements compared with the corresponding correlation coefficients estimated by Justiniano and Preston (2010), which are close to zero.

4.3 A NOTE ON EXCHANGE RATE PASS-THROUGH

Exchange rate pass-through is defined as the response of prices to a change in the nominal exchange rate. High pass-through implies transmission of international business cycle shocks via the exchange rate channel. However, empirical studies suggest only a weak link between domestic prices and exchange rate fluctuations – a partial exchange rate disconnect. This is also true in the current model. Yet, I will argue that sectoral differences in exchange rate pass-through, coupled with the use of intermediate goods in production, greatly facilitates international spillover and co-movement.

This section documents the degree of pass-through implied by the estimated model. To this end I calculate price responses to a UIP shock. Arguably this shock is exogenous to model fundamentals, making it comparable with exchange rate fluctuations analyzed in reduced form studies. Results are shown in Table 6, where price responses are expressed relative to the exchange rate innovation (i.e. $\Delta price_t / \Delta e_t$). On impact, a one percent shock to the risk premium causes the nominal exchange rate to depreciate by about 1.55%, while inflation rises by 0.19%. Short-run pass-through to CPI inflation follows as $\frac{\Delta \pi_t}{\Delta e_t} = 12.5\%$. The corresponding pass-through to aggregate investment prices is 26.2%. Aggregate results mask significant sector heterogeneity: Pass-through to sectoral market prices (third column) is 35.5% in raw materials and 29.5% in manufacturing, compared with only 3.9% in services. In turn, these numbers are weighted averages of inflation in imported and domestically produced goods' prices. The exchange rate depreciation reduces the markup of importing firms. Their desire to stabilize the markup causes higher imported inflation (fourth column). This is standard. What is less standard is the significant pass-through to prices for domestically produced goods (fifth column). This comes about from trade in materials, as some of the exchange rate depreciation passes through to material prices (seventh column). The resulting drop in domestic firms' markup leads to higher domestic inflation. The exchange rate effect on $\pi_{Hj,t}$ adds to overall pass-through and can be high if domestic prices are changed frequently. More generally, the model predicts an inverse relationship between price stickiness and pass-through because firms with frequent price changes are more likely to respond to the exchange rate. This explains the bulk of pass-through variation across sectors. High pass-through to

flexible prices is also a robust feature in data, as shown by Gopinath and Itskhoki (2010).

Without stretching the results too far, a brief comparison with empirical pass-through regressions is warranted. Campa and Goldberg (2005) estimate the short-run (quarterly) exchange rate pass-through to aggregate import prices in Canada, and find an elasticity of 75%. Gopinath et al. (2010) report a lower number, about 35%. Table 6 provides results somewhere in between, about 53-65% in the trade-intensive sectors manufacturing and raw materials. Finally, Goldberg and Campa (2010) estimate pass-through elasticities of the CPI in 21 developed economies (Canada is not included) and find an average elasticity of about 15%, close to the value of 13% in Table 6.

5 TRANSMISSION CHANNELS

In this section I describe the mechanisms leading to transmission of business cycle fluctuations across countries, and analyze the role of different shocks. I point out an important feedback loop that comes about from the intersectoral linkages. Its main implication is synchronization of firms' markup across sectors and countries. In turn, markup synchronization leads to co-movement of domestic and foreign prices and quantities, a key feature of the data. Shocks that generate markup synchronization are good candidates for international business cycle co-movement, and favored by the likelihood-based estimation procedure. First, I describe these mechanisms in more detail. Second, I study impulse responses to shed light on the dynamic effects of selected shocks.

5.1 THE ROLE OF INTERMEDIATE TRADE AND SECTOR HETEROGENEITY

To better understand how intermediate trade and sector heterogeneity affect the domestic exposure to international business cycles, I proceed in three steps. First, note that the laws of motion for $p_{rHj,t}$ and $p_{rHj,t}^*$ (the prices for domestic sector goods sold at home and exported, respectively) can be written as follows:

$$\begin{aligned} p_{rHj,t} &= \theta_{pj} (p_{rHj,t-1} - \pi_t + \iota_p \pi_{Hj,t-1}) + (1 - \theta_{pj}) \bar{p}_{rHj,t} \\ p_{rHj,t}^* &= \theta_{pj} (p_{rHj,t-1}^* + \Delta e_t - \pi_t + \iota_p \pi_{Hj,t-1}^*) + (1 - \theta_{pj}) \bar{p}_{rHj,t}^* \end{aligned} \quad (27)$$

Both prices above are quoted in domestic currency and in terms of consumption goods. The two equations state that prices for domestically produced goods are linear combinations of the lagged price level (and some terms associated with indexation and exchange rate changes) and the new prices set by firms who re-optimize in the current period, $\bar{p}_{rHj,t}$ and $\bar{p}_{rHj,t}^*$. If optimal new prices rise, we get inflationary pressure on the sector averages $p_{rHj,t}$ and $p_{rHj,t}^*$ as well. The second step is to note that the forward-looking nature of the dynamics described above is captured by two optimality conditions for newly set prices (markup shocks are abstracted from):

$$\begin{aligned} \bar{p}_{rHj,t} &= p_{rHj,t} + (1 - \beta\theta_{pj}) \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (rmc_{j,s} - p_{rHj,s}) \\ \bar{p}_{rHj,t}^* &= p_{rHj,t}^* + (1 - \beta\theta_{pj}) \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (rmc_{j,s} - p_{rHj,s}^*) \end{aligned} \quad (28)$$

These two equations illustrate that the profit-maximizing price, from the individual firm's point of view, is a linear combination of i) the sector-specific average and ii) current and expected future deviations in the price markup over marginal costs. In the limit as θ_{pj} goes to zero, the expectation sums disappear.¹⁵ However, when $\theta_{pj} > 0$, then all innovations that increase (decrease) real marginal costs relative to producer prices cause temporary upward (downward) pressure on $\bar{p}_{rHj,t}$ and $\bar{p}_{rHj,t}^*$. This takes us to the third step, the introduction of intermediate trade and sector heterogeneity. The linearized real marginal cost in sector j can be written as follows:

$$\begin{aligned} rmc_{j,t} &= -z_{Aj,t} + \phi_j \sum_{l=1}^{\mathcal{J}} \zeta_{lj} p_{rl,t} + \psi_j \omega_{j,t} + (1 - \phi_j - \psi_j) r_t^k \\ &= -z_{Aj,t} + \phi_j \sum_{l=1}^{\mathcal{J}} \zeta_{lj} [\alpha_l p_{rHl,t} + (1 - \alpha_l) p_{rFl,t}] + \psi_j \omega_{j,t} + (1 - \phi_j - \psi_j) r_t^k \end{aligned} \quad (29)$$

The first line shows that costs are directly affected by market prices $p_{rl,t}$ in all domestic industries $l \in \mathcal{J}$, because intermediate trade takes place across sectors. The second line demonstrates that costs depend on import prices $p_{rFl,t}$, set by firms in foreign sectors. This is true as long as the domestic absorption parameters α_j are less than one. Importantly, $p_{rFl,t}$ can be represented by a system similar to (27)-(29). Thus, all shocks that affect sectoral marginal costs in the foreign economy will in principle show up in equation (29).

Three important observations immediately follow from the system (27)-(29). First, intermediate trade introduces co-movement between domestic and foreign producer prices: A rise (fall) in any import price $p_{rFl,t}$ directly reduces (increases) the markup of domestic firms, resulting in rising (declining) domestic producer prices $p_{rHj,t}$ and $p_{rHj,t}^*$ (equation (27)). Second, the model features an important feedback loop. That is, the first round rise (fall) in $p_{rHj,t}$ further increases (reduces) domestic sector j 's costs, because $p_{rHj,t}$ shows up in (29). There is a similar feedback loop involving foreign producer prices and foreign marginal costs. Third, the initial impulse propagates across sectors as long as $\zeta_{lj} > 0$ for some $l \neq j$. Thus, foreign shocks can hit some industries in the SOE, notably those with high trade intensity, and then propagate to others via intermediate trade. The latter kind of spillover is governed by the off-diagonals of the I-O matrix, and allows even relatively non-traded sectors to be affected by international disturbances. The setup presented here nests as special cases some common approaches in the literature, including models with i) one sector ($\mathcal{J} = 1$),¹⁶ ii) no intermediate trade ($\phi_j = 0$), and iii) no foreign trade ($\alpha_j = 1$). However, all these dimensions matter for the transmission of foreign shocks. Obviously, if $\alpha_j = 1 \forall l \in \mathcal{J}$, then economic activity in the SOE is completely unrelated to the rest of the world. If $\phi_j = 0$, then there is one less source of co-movement in producer prices (the one described above), and hence one less mechanism that induces business cycle spillover. If $\mathcal{J} = 1$, then the entire transmission has to take place at the aggregate level without sectoral reallocations. In contrast, the multi-sector model presented here allows industries with limited international trade to be affected via cross-sectoral intermediate market linkages. Thus, even fluctuations in completely non-traded industries can in principle be driven by business cycle shocks abroad.

¹⁵Optimal prices without price setting rigidities and markup shocks satisfy $p_{rHj,t} = p_{rHj,t}^* = rmc_{j,t} \forall t$.

¹⁶Or alternatively, no (ex ante) sector heterogeneity ($\phi_j = \phi, \gamma_{lj} = \gamma, \alpha_j = \alpha$).

5.2 THE SCOPE FOR INTERNATIONAL SYNCHRONIZATION

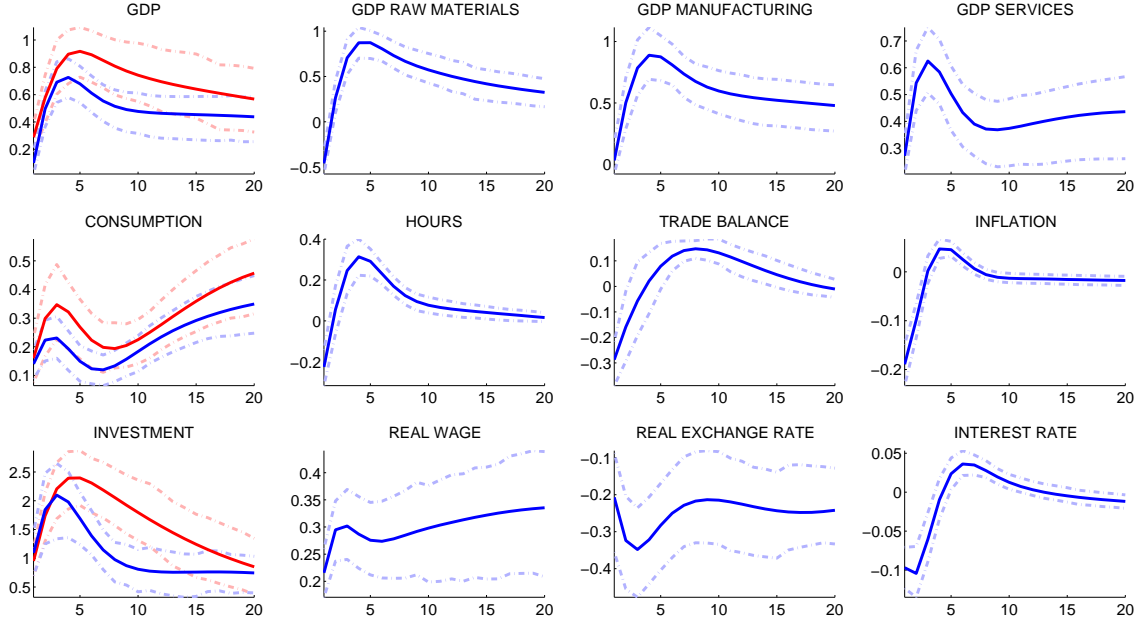
To better understand the role of foreign disturbances for domestic business cycles, I analyze the impulse responses of domestic variables to selected international shocks. I focus on two points: First, I explain in detail why foreign productivity shocks generate business cycle co-movement in the current framework. Second I argue that some demand shocks, which are found important in closed economy studies, have counterfactual implications in a context where the model is asked to fit data from different countries.

5.2.1 DYNAMIC EFFECTS OF PRODUCTIVITY SHOCKS

Figure 3 shows impulse responses to a productivity shock in foreign manufacturing. This shock is, according to the model, one of the main sources of macroeconomic volatility in Canada. A first thing to note is the striking co-movement in GDP, consumption and investment across countries. What is going on here? Consider first the foreign variables. As expected, higher productivity abroad raises foreign GDP, consumption, and investment. The set of frictions in the model, in particular sticky prices and monopolistic competition, implies less working hours and declining foreign interest rates (not shown). All these effects are well known from the textbook one-sector, closed economy model. Regarding spillover to the SOE, lower imported inflation in manufacturing reduces interest rates and facilitates co-movement between domestic and foreign absorption of manufactured goods. But at the same time there is expenditure switching away from domestic manufacturing firms. This kind of substitution effect works against the positive income effect of low real rates. In previously estimated models, these are the two main forces at play. Moreover, these models generate little co-movement in GDP, hours, and other supply side variables across countries, suggesting major expenditure switching.

In contrast, the multi-sector structure presented here provides us with a rich story about additional mechanisms at work. First, lower imported inflation in manufacturing creates substitution of final demand towards all manufactured goods, including those that are produced domestically. Second, cheaper manufactured goods also reduce domestic firms' expenditures on materials. This is seen from equation (29). In fact, producer costs decline in all domestic industries because also non-manufacturing producers use manufactured goods extensively as input. Profit-maximizing behavior then induces domestic firms across the economy to reduce their prices, and overall domestic inflation declines even further. That triggers another round of cheaper intermediate goods, causing another round of price reductions, and so on. The result is a dynamic, open economy version of the feedback loop emphasized by e.g. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). How important this feedback loop is for each sector depends on the sectoral trade intensity, the degree of price stickiness, and the share of intermediate inputs in production. But common to all sectors is the fact that cheaper domestic goods limit the initial expenditure-switching towards imports. In total, the sectoral linkages described here prevent much of the substitution effect and reinforces the income effect on demand towards domestic firms, implying a large rise in aggregate domestic activity and value added. Productivity shocks in foreign manufacturing are particularly powerful for this purpose. Manufacturing has high trade intensity and relatively flexible price setting, allowing a strong reaction in manufactured prices to shocks. Finally, manufactured goods are important inputs in services, the largest sector in the economy.

Figure 3: A productivity shock in foreign manufacturing



Note: Foreign (red) and domestic (blue) Bayesian impulse responses to a productivity shock in foreign manufacturing (one standard deviation). Posterior mean (solid) and 90% highest posterior probability intervals (dotted). The trade balance is in absolute deviations from steady state relative to GDP.

5.2.2 DYNAMIC EFFECTS OF DEMAND SHOCKS

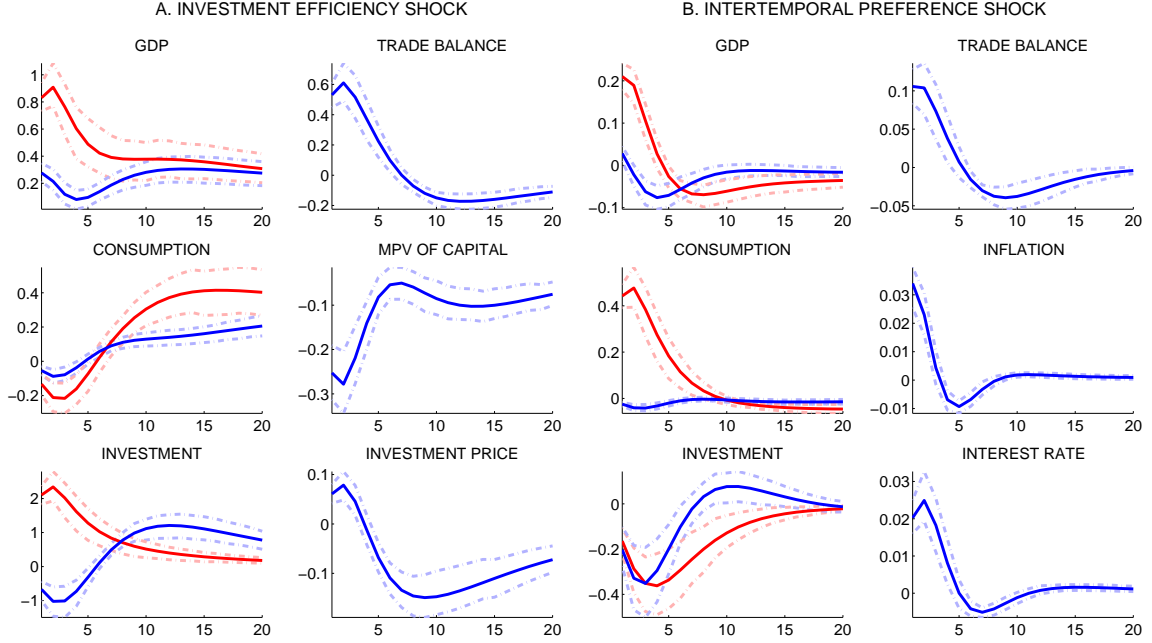
Figure 4 shows responses following foreign shocks to the marginal efficiency of investment (MEI) and to intertemporal preferences, respectively. Let us start with the MEI shock. It temporarily increases the amount of capital transformed from each investment unit, and thereby raises the relative return to capital investments. This induces foreigners to invest more, at the cost of consumption in the first periods (see Furlanetto, Natvik, and Seneca (2013) for an analysis of this issue). The net effect is a positive shift in aggregate demand and higher inflation. In the SOE, the foreign MEI shock generates responses in several variables that are qualitatively similar to those in the foreign economy. That is, due to rising import prices, aggregate inflation and interest rates in the SOE increase. These responses bring down domestic consumption and makes production more expensive. Yet, high foreign investment demand is expansionary for domestic raw material and manufactured goods producers, who export investment goods intensively. But the MEI shock cannot explain international synchronization of all variables – it creates strong divergence in investment patterns across the two countries. To see why, note that sectoral absorption of investments can be written as follows (where we abstract from domestic MEI shocks):

$$i_{j,t} = -\nu (p_{rj,t} - p_{r,t}^i) + i_{t-1} + \frac{1}{\epsilon_I} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (q_s - p_{r,s}^i)$$

Thus, sectoral investments are linked to the relative sector price $p_{rj,t}$, and via aggregate investment demand, to the expected path for real returns to investments, $\{q_s - p_{r,s}^i\}_{s=t}^{\infty}$.¹⁷

¹⁷Investment adjustment costs are priced into $p_{r,t}^i$. Without adjustment costs and domestic MEI shocks, the equation collapses to $q_s = p_{r,s}^i \forall s$.

Figure 4: Foreign demand shocks



Note: Foreign (red) and domestic (blue) Bayesian impulse responses to i) a foreign MEI shock (columns 1-2) and ii) a foreign preference shock (columns 3-4). Posterior mean (solid) and 90% highest posterior probability intervals (dotted). The trade balance is in absolute deviations from steady state relative to GDP.

Intuitively, sectoral and aggregate investment demand is high when the value of current and future capital exceeds the cost of capital accumulation. Investment co-movement across countries must, therefore, come about from synchronization of expected capital returns. However, in the case of a foreign MEI shock, higher import prices in the SOE raises $p_{r,t}^i$, while higher real interest rates reduce the present value of installed capital.¹⁸ These effects are shown in Figure 4, and give rise to a large wedge between domestic and foreign investment activity that is not typically seen in data.

A similar story applies to foreign preference shocks, except that now the story is about international consumption patterns. Sectoral consumption demand in the SOE can be written as follows (where we abstract from domestic preference shocks):

$$c_{j,t} = -\nu p_{r,j,t} + \chi_C c_{t-1} - \frac{1 - \chi_C}{\sigma} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (r_s - \pi_{s+1})$$

Consumption co-movement across countries must, therefore, come about from synchronization of the entire real interest rate path. But rising consumption demand abroad leads to higher imported inflation and, for any standard monetary policy rule, to increased real interest rates in the SOE. A more general lesson follows from this analysis: It is difficult, in model economies with trade-offs arising from constantly binding resource constraints, to obtain widespread co-movement as a consequence of asymmetric or country-specific demand shocks.

¹⁸The link between q_t and real interest rates is found by solving the linearized optimality condition for capital forward to obtain $q_t = \mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\delta))^{s-t} [-(r_s - \pi_{s+1}) + (1 - \beta(1-\delta)) r_{s+1}^k]$. Thus, an increase in current or future expected real interest rates reduce the value of capital.

Table 7: Foreign variance decompositions – Alternative models

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1Q-LR	1Q-LR	1Q-LR	1Q-LR	1Q-LR	1Q-LR	1Q-LR
GDP	21.4-73.9	27.8-71.2	25.6-65.5	45.6-66.4	61.5-76.4	35.6-67.5	15.6-66.1
Consumption	10.5-75.7	17.7-72.0	1.0-75.5	3.6-69.3	2.4-85.1	1.3-72.2	9.4-67.9
Investment	22.5-44.6	21.1-40.7	13.1-24.8	5.3-26.5	12.3-32.4	9.1-25.2	16.4-41.1
Hours	17.8-34.1	6.6-31.2	8.9-23.4	8.6-18.6	36.8-41.4	9.7-25.3	20.0-31.1
Interest rate	36.6-43.1	32.3-36.2	16.3-60.5	10.6-28.3	4.0-34.5	4.1-28.3	35.5-40.1
Inflation	41.2-46.1	38.0-41.4	25.0-62.9	18.6-25.9	7.5-17.3	0.4-19.2	39.1-43.5
Real wage	47.5-83.2	45.3-82.3	32.2-86.6	11.3-50.9	22.4-88.6	21.7-73.1	41.5-75.7
Trade balance	37.9-32.3	30.4-35.2	27.4-23.9	35.7-30.1	42.32-27.2	15.6-12.6	33.8-31.2

Note: Forecast error variance decomposition (see Table 4 for details). Models: (1) baseline model, (2) no habits, (3) no investment adjustment costs, (4) flexible wages, (5) flexible prices, (6) real business cycle model, (7) unitary elasticity of substitution between domestic goods and imports.

6 SENSITIVITY ANALYSIS

The model comes with a number of real and nominal frictions. These are usually included in order to improve the fit to data. This section documents the role of foreign shocks in re-estimated versions of the model without key frictions and transmission mechanisms.

6.1 THE ROLE OF FRICTIONS

First, I abstract from habit formation by calibrating $\chi_C = \chi_C^* = 0$. Consumption habits are often included in order to reproduce the hump-shaped consumption responses to shocks found in VAR models. Second, I abstract from investment adjustment costs by setting $\epsilon_I = \epsilon_I^* = 0$. This leads to more volatile investment dynamics and effectively equates the marginal value of capital with the investment price. Third, I assume full labor market flexibility. That is, I set $\theta_{wj} = \theta_{wj}^* = 0.001$ and $\gamma_w = \gamma_w^* = 0$. This specification imposes that real wages are equated with the marginal rate of substitution between hours and consumption. Fourth, I do the same for prices ($\theta_{pj} = \theta_{pj}^* = 0.001$ and $\gamma_p = \gamma_p^* = 0$), implying marginal costs equal to producer prices in all sectors. Fifth, I estimate a real business cycle version of the model without habits or investment adjustment costs, and with full wage and price flexibility. Finally, I fix the elasticity of substitution between domestic and foreign goods to one ($\eta = 1$). In simpler models with perfect international risk sharing and no capital, this specification implies balanced trade at all times (see Galí and Monacelli (2005) for an example). The role of η is interesting in this analysis because Bergholt and Sveen (2014) find that parameter to be important for the international propagation of productivity shocks. The remaining parameters (both calibrated and priors for the estimated) are treated as before in all of the models. Results are reported in Table 7, where attention is restricted to foreign variance shares at the 1-quarter horizon and the long run. Across all specifications the following results hold: (i) foreign shocks are important for domestic business cycle fluctuations, (ii) foreign shocks are responsible for larger variance shares at longer forecasting horizons, and (iii) productivity shocks are the main drivers of business cycle fluctuations. They matter less for consumption and investment in the very short run (1 quarter) when habits or wage and price stickiness are abstracted from, but quickly gain importance as the forecasting horizon expands.

Table 8: Counterfactual model – Business cycle predictions

	Panel A:	Panel B:					Panel C:	
	$100\rho_{y,y^*}$	1Q	4Q	8Q	20Q	LR	$\Delta\pi/\Delta e$	
GDP	22.7	1.89	3.11	3.52	6.22	10.33	π_t	7.86
Consumption	24.9	0.94	1.22	1.93	5.47	16.43	$\pi_{F,t}$	36.77
Investment	1.0	1.37	1.37	1.28	2.69	3.42	$\pi_{H,t}$	-2.27
Hours	8.3	1.72	2.15	2.18	2.26	2.36	$\pi_{H,t}^*$	57.55
Inflation	12.9	2.76	2.60	2.64	2.70	3.03	τ_t	20.78
Interest rate	14.5	3.73	3.08	3.04	3.00	4.02		
Trade balance	–	8.14	10.03	10.17	10.21	10.16		

Note: See Table 4-Table 6 for details.

6.2 A COUNTERFACTUAL, SYMMETRIC MODEL ECONOMY

Finally I ask whether the role of foreign shocks survives in a context without factor trade and sector heterogeneity. To this end, I estimate the particular version of the model when $\mathcal{J} = 1$ and $\phi = 0$. The model now becomes a fairly standard DSGE model for a small open economy.¹⁹ Calibrated values are set as follows: First, I rescale labor and capital shares in both economies to keep the constant returns to scale assumption based on the numbers in Table 2. This gives $\psi = 0.543$. Second, I calibrate trade shares in GDP by subtracting the intermediate input share of imports in each sector, and then calculating the aggregate (sector GDP weighted) import share in the economy. The resulting import share in GDP is 0.26 ($\alpha = 0.7405$). The remaining calibrated values are chosen as before. Also the prior distributions are as in the baseline model, except that price and wage stickiness have prior modes equal to 0.7, while the aggregate TFP shock has a mode equal to 0.2. Business cycle predictions from the counterfactual model are provided in Table 8. Parameter estimates are reported in the appendix. Consider first the posterior mean of the model implied cross-country correlations (Panel A). For all variables under consideration, they fall to less than half of those in the baseline model. The decline is particularly large for investment. Still, the degree of co-movement is higher than that found by Justiniano and Preston (2010). Part of difference is attributed to the inclusion of investment, which is abstracted from in their study. When higher foreign productivity takes down international prices, domestic investment (and capital) is stimulated by cheaper imports. Turning to the decomposition of shocks (Panel B), we see that foreign shocks become nearly irrelevant for most domestic variables within the business cycle. They explain less than 7% of the variation in all variables except the trade balance within the 5-year horizon. This is about one-tenth of the shares attributed to foreign shocks in the baseline model (Table 4). In the long run, foreign shocks account for 2-17% of the macroeconomic volatility in the SOE, far below typical estimates in the VAR literature (but in the ballpark of existing medium scale DSGE models). Clearly, this model version is not equipped to address the role of foreign shocks. Implied pass-through rates are provided in Panel C. The short-run pass-through to CPI drops from 12.5% to 7.9% – still a fairly high number, given that exporters in the model price their goods in local currency. The main reason is the estimated low degree of price stickiness, with a posterior centered around 0.5.

¹⁹Obviously, with the counterfactual consequence that sectoral responses to all business cycle shocks are symmetric. Bouakez et al. (2009) study implications of imposing such symmetry in a closed economy.

7 CONCLUDING REMARKS

I ask how and to what extent international business cycle disturbances cause macroeconomic fluctuations in small open economies. To shed some light on these questions, I construct and estimate a medium scale small open economy model with several shocks and frictions typically used in the DSGE literature. The model is embedded with i) trade in intermediate goods between firms, and ii) sectoral producer heterogeneity. These extensions to the workhorse one-sector open economy model are sufficient to reconcile DSGE theory with data along international dimensions. When the model is fitted to Canadian and US data, a set of important empirical results emerge: First, foreign shocks explain a major share of macroeconomic fluctuations in the SOE. Second, posterior estimates emphasize the role of productivity, in the sense that technology shocks, not investment efficiency fluctuations, are the major drivers of business cycles. Third, foreign shocks become increasingly important over longer forecasting horizons. Fourth, the model generates substantial business cycle synchronization, even though trade balances are counter-cyclical and shocks are uncorrelated. Fifth, exchange rate pass-through is moderate, with sectoral pass-through depending on the frequency of price changes. While these results are consistent with reduced form literature such as VAR and FAVAR studies, they are typically not captured by estimated open economy DSGE models.

The model presented here allows us to gain insight about the mechanisms that cause these results. An important implication of intermediate trade is that it synchronizes producer prices and costs in the cross-section of firms, both within and across borders. This helps in generating co-movement in an environment with producer heterogeneity and otherwise segmented markets. Foreign shocks in particular can enter the SOE through some industries exposed to international trade, and then propagate to others via domestic factor markets. Synchronized producer prices across sectors and countries generate substantial international co-movement in i) current and future real interest rates, which determines consumption, and ii) the expected path of capital returns, a key statistic for investment decisions. However, synchronization of real interest rates comes at the cost of too high consumption co-movement across countries. I find that foreign technology shocks are particularly well-suited for international business cycle synchronization. These are also relatively persistent, an important reason why foreign shocks become essential at longer forecasting horizons. Foreign investment efficiency shocks, on the other hand, cause international divergence in the present value of capital and investment. Investment is positively correlated across countries, implying that the likelihood-based estimation procedure attributes a smaller role to investment efficiency shocks.

One limitation of the present model is the lack of meaningful interactions between financial markets and the macroeconomy. Indeed, the recent financial crisis has demonstrated the potential importance of financial frictions for international business cycles. By now, there is a large (and growing) literature on financial frictions in closed economies. Yet for many, if not for most small open economies, the crisis was associated with foreign events. Therefore, an interesting topic for future research is the propagation of financial distress across countries, e.g. an open economy extension of the market frictions studied by Christiano, Motto, and Rostagno (2014). But for such an analysis to make sense, one should be equipped with a model that can account for macroeconomic spillover as well. This paper offers a constructive step towards that end.

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ONLINE APPENDIX,
FOREIGN SHOCKS (BERGHOLT, 2015)

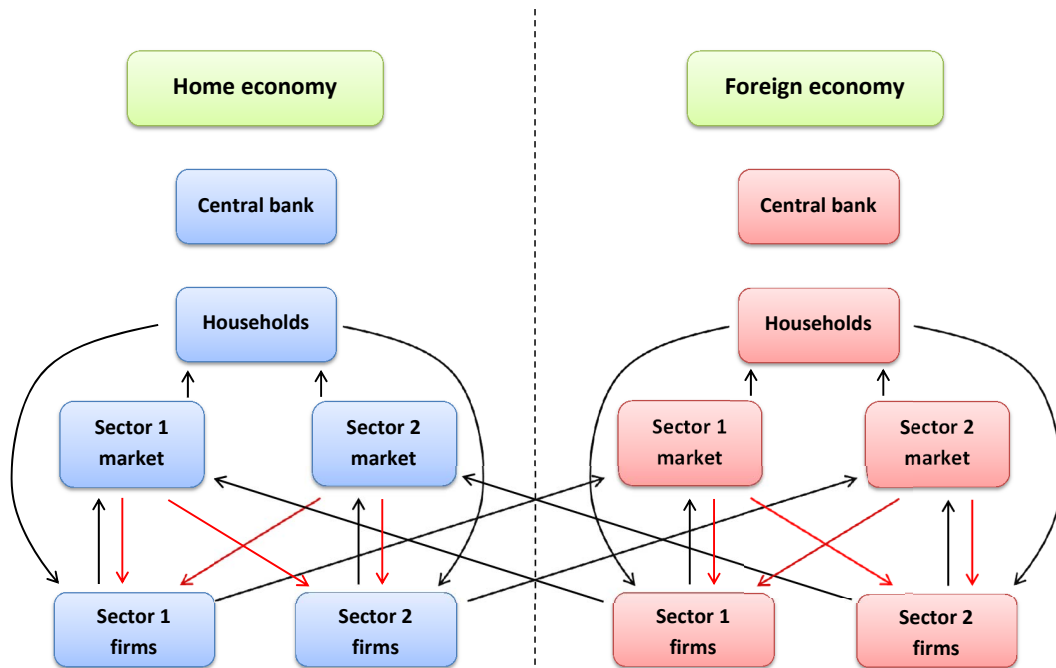
A THE FULL MODEL

I establish a general equilibrium system consisting of two blocks (referred to as home and foreign), where the home block is a small scale version of its foreign counterpart. The foreign block is thought of as the rest of the world. I first derive optimality conditions in a general setting where the home economy is arbitrarily large compared to the rest of the world. However, my focus is on the limiting case where the relative size of the home economy goes to zero. General equilibrium is therefore evaluated for this special case. The approach allows me to model the foreign block of the model as a closed economy version of the domestic block. To save space, I only derive the domestic block below.

A.1 ILLUSTRATIVE MODEL OVERVIEW

Figure A.1 summarizes the relevant transaction channels in the model when $\mathcal{J} = 2$. Households buy consumption and investment goods (in all domestic markets), and enjoy leisure. This is financed by labor and capital income, dividends, and transfers. Firms in each sector hire labor, capital and buy materials, to produce consumption goods, investment goods, and production goods (sold as materials to other firms). Domestic supply chains are highlighted by red arrows. The central bank stabilizes inflation.

Figure A.1: A bird's view of the model economies when $\mathcal{J} = 2$



Note: Two-sector version of the model economies. The vertical line represents the country border. Arrows summarize the trade flows (quantities), and supply chain channels are highlighted in red.

A.2 THE NON-LINEAR MODEL

In this section I provide a detailed characterization of the model economy at Home.

A.2.1 HOUSEHOLDS

Household member h working in sector j maximizes lifetime utility given at time t by

$$\mathcal{U}_{j,t}(h) = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Z_{U,s} [U_{j,s|t-i}(h) - V_{j,s|t-i}(h)],$$

where $U_{j,t|t-i}(h)$ is period t utility of consumption, and $V_{j,t|t-i}(h)$ period t disutility of labor, for a member that was last able to re-optimize the wage i periods ago. $\beta \in (0, 1)$ is a time discount factor. Components of period utility are specified in period t as follows:²⁰

$$\begin{aligned} U_{j,t|t-i}(h) &= \frac{(1 - \chi_C)^\sigma (C_{t|t-i}(h) - \chi_C C_{t-1})^{1-\sigma}}{1 - \sigma} \\ V_{j,t|t-i}(h) &= Z_{N,t} \chi_N \frac{L_{j,t|t-i}(h)^{1+\varphi}}{1 + \varphi} \end{aligned} \quad (\text{A.1})$$

Given wage re-optimization i periods ago, $C_{t|t-i}(h)$ denotes period t consumption while $L_{j,t|t-i}(h)$ denotes hours worked for household member h . $Z_{U,t}$ and $Z_{N,t}$ represent stationary shocks to intertemporal preferences and the labor supply, respectively. I assume the existence of a complete set of tradable Arrow securities within each economy. This makes consumption independent of the wage history, i.e. $C_{t|t-i}(h) = C_{t|t}(h) \equiv C_t(h)$. Because the representative household is of measure one, household member h consumption is also aggregate consumption ($C_t(h) = C_t$). I drop the h -subscript whenever possible from now on.

Households buy consumption goods, invest in capital, accumulate domestic and foreign bond assets, and sell labor services to domestic firms. Maximization of lifetime utility is subject to a sequence of budget constraints. In period t , the budget constraint takes the following form:

$$\begin{aligned} P_t C_t + P_t^i I_t + B_{H,t+1} + \mathcal{E}_t B_{F,t+1}^* + \mathbb{E}_t \{ Z_{t,t+1} D_{t+1} \} \\ \leq D_t + W_{j,t}(h) L_{j,t}(h) + R_t^k P_t K_t + P_t D_t + R_{t-1} B_{H,t} + R_{t-1}^* \Upsilon_{t-1} \mathcal{E}_t B_{F,t}^* - P_t T_t \end{aligned} \quad (\text{A.2})$$

Domestic households pay a premium on the return on foreign bonds given by $\Upsilon_t = \exp[-\epsilon_B (A_t - A)] Z_{B,t}^F$, where $A_t = \frac{\mathcal{E}_t B_{F,t+1}^*}{P_t \text{GDP}} = \mathcal{S}_t \frac{B_{F,t+1}^*}{P_t^* \text{GDP}}$ is real net foreign asset holdings as share of steady state GDP. $Z_{B,t}^F$ captures deviations from uncovered interest rate parity and is referred to as a risk premium shock. Investment in capital is subject to the following capital accumulation equation:

$$K_{t+1} = (1 - \delta) K_t + Z_{I,t} \left[1 - F \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (\text{A.3})$$

The adjustment cost function F satisfies $F' \geq 0$, $F'' \geq 0$, and $F(1) = F'(1) = 0$.

²⁰The term involving consumption is scaled by $(1 - \chi_C)^\sigma$ to render the steady state independent of the estimated habit coefficient χ_C .

First I describe optimal demand schedules at the disaggregate level in the SOE. The economy consists of \mathcal{J} different industries or sectors. Final consumption and investment aggregates are composites of consumption and investment goods from each of the different sectors:

$$C_t = \left[\sum_{j=1}^{\mathcal{J}} \xi_j^{\frac{1}{\nu_c}} C_{j,t}^{\frac{\nu_c-1}{\nu_c}} \right]^{\frac{\nu_c}{\nu_c-1}} \quad I_t = \left[\sum_{j=1}^{\mathcal{J}} \varpi_j^{\frac{1}{\nu_i}} I_{j,t}^{\frac{\nu_i-1}{\nu_i}} \right]^{\frac{\nu_i}{\nu_i-1}} \quad (\text{A.4})$$

For given levels of consumption and investment, the optimal demand for inputs from sector j is given by the following downward sloping demand schedules:

$$C_{j,t} = \xi_j \left(\frac{P_{j,t}}{P_t} \right)^{-\nu_c} C_t \quad I_{j,t} = \varpi_j \left(\frac{P_{j,t}}{P_t^i} \right)^{-\nu_i} I_t \quad (\text{A.5})$$

Corresponding consumer and investment price indexes are $P_t = \left[\sum_{j=1}^{\mathcal{J}} \xi_j P_{j,t}^{1-\nu_c} \right]^{\frac{1}{1-\nu_c}}$ and $P_t^i = \left[\sum_{j=1}^{\mathcal{J}} \varpi_j P_{j,t}^{1-\nu_i} \right]^{\frac{1}{1-\nu_i}}$. The domestic sector markets are populated by domestic and foreign suppliers. In each of these markets there is trade in private and public consumption goods, investment goods, and intermediate production goods. Demand for consumption $C_{j,t}$ and investment $I_{j,t}$ in sector j are constructed according to a nested CES structure:

$$\begin{aligned} C_{j,t} &= \left[\bar{\alpha}_j^{\frac{1}{\eta}} C_{Hj,t}^{\frac{\eta-1}{\eta}} + (1 - \bar{\alpha}_j)^{\frac{1}{\eta}} C_{Fj,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} & I_{j,t} &= \left[\bar{\alpha}_j^{\frac{1}{\eta}} I_{Hj,t}^{\frac{\eta-1}{\eta}} + (1 - \bar{\alpha}_j)^{\frac{1}{\eta}} I_{Fj,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ C_{Hj,t} &= \left[\int_0^1 C_{Hj,t}(f)^{\frac{1}{1+\epsilon_{p,t}}} df \right]^{1+\epsilon_{p,t}} & I_{Hj,t} &= \left[\int_0^1 I_{Hj,t}(f)^{\frac{1}{1+\epsilon_{p,t}}} df \right]^{1+\epsilon_{p,t}} \\ C_{Fj,t} &= \left[\int_0^1 C_{Fj,t}(f)^{\frac{1}{1+\epsilon_{p,t}^*}} df \right]^{1+\epsilon_{p,t}^*} & I_{Fj,t} &= \left[\int_0^1 I_{Fj,t}(f)^{\frac{1}{1+\epsilon_{p,t}^*}} df \right]^{1+\epsilon_{p,t}^*} \end{aligned}$$

$C_{Hj,t}$ and $I_{Hj,t}$ are indexes of all the consumption and investment goods $C_{Hj,t}(f)$ and $I_{Hj,t}(f)$, made by each domestic firm $f \in [0, 1]$. $C_{Fj,t}$ and $I_{Fj,t}$ are corresponding indexes of all consumption and investment goods $C_{Fj,t}(f)$ and $I_{Fj,t}(f)$, imported from each firm f in the foreign economy. $\epsilon_{p,t}$ is a time-varying markup on domestically produced goods, while $\epsilon_{p,t}^*$ is the markup on imported goods. η is the substitution elasticity between goods from different countries. Sector-level quantities in the foreign block, denoted $C_{j,t}^*$ and $I_{j,t}^*$ respectively, are constructed by equivalent systems. Deep production parameters however are allowed to vary across economies. $\bar{\alpha}_j$ and $\bar{\alpha}_j^*$ in particular, which measure the weights of domestic products in the production of final goods, are defined as

$$\bar{\alpha}_j = 1 - (1 - \varsigma)(1 - \alpha_j) \text{ and } \bar{\alpha}_j^* = 1 - \varsigma(1 - \alpha_j^*).$$

The relative size of the home economy compared to the foreign block is denoted $\varsigma \in [0, 1]$, while the degrees of bias toward domestic products in sector j are captured by $\alpha_j \in [0, 1]$ and $\alpha_j^* \in [0, 1]$.²¹ For future reference, note that both $C_{j,t}$ and $I_{j,t}$ consist of both domestic and imported goods. However, import shares vary across sectors, so aggregate import

²¹This setup encompasses some interesting special cases, including i) complete autarky ($\alpha_j = \alpha_j^* = 1$), ii) perfectly integrated markets ($\alpha_j = \alpha_j^* = 0$), and iii) the limiting case of a small open economy ($\varsigma \rightarrow 0$).

shares in C_t and I_t depend on the sectoral weights ξ_j and ϖ_j . Cost-minimizing allocations between domestic and imported products, and between single products from each country's sector j , are given in the home economy by the following system of optimality conditions:

$$\begin{aligned}
C_{Hj,t} &= \bar{\alpha}_j \left(\frac{P_{Hj,t}}{P_{j,t}} \right)^{-\eta} C_{j,t}, & I_{Hj,t} &= \bar{\alpha}_j \left(\frac{P_{Hj,t}}{P_{j,t}} \right)^{-\eta} I_{j,t}, \\
C_{Fj,t} &= (1 - \bar{\alpha}_j) \left(\frac{P_{Fj,t}}{P_{j,t}} \right)^{-\eta} C_{j,t}, & I_{Fj,t} &= (1 - \bar{\alpha}_j) \left(\frac{P_{Fj,t}}{P_{j,t}} \right)^{-\eta} I_{j,t}, \\
C_{Hj,t}(f) &= \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} C_{Hj,t}, & I_{Hj,t}(f) &= \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} I_{Hj,t}, \\
C_{Fj,t}(f) &= \left(\frac{P_{Fj,t}(f)}{P_{Fj,t}} \right)^{-\frac{1+\epsilon_{p,t}^*}{\epsilon_{p,t}^*}} C_{Fj,t}, & I_{Fj,t}(f) &= \left(\frac{P_{Fj,t}(f)}{P_{Fj,t}} \right)^{-\frac{1+\epsilon_{p,t}^*}{\epsilon_{p,t}^*}} I_{Fj,t}.
\end{aligned} \tag{A.6}$$

The foreign economy allocates consumption and investment goods according to similar first-order conditions. The corresponding price indexes in the SOE follow as

$$\begin{aligned}
P_{j,t} &= [\bar{\alpha}_j P_{Hj,t}^{1-\eta} + (1 - \bar{\alpha}_j) P_{Fj,t}^{1-\eta}]^{\frac{1}{1-\eta}}, \\
P_{Hj,t} &= \left[\int_0^1 P_{Hj,t}(f)^{-\frac{1}{\epsilon_{p,t}}} df \right]^{-\epsilon_{p,t}}, \\
P_{Fj,t} &= \left[\int_0^1 P_{Fj,t}(f)^{-\frac{1}{\epsilon_{p,t}^*}} df \right]^{-\epsilon_{p,t}^*}.
\end{aligned}$$

Next I describe optimality conditions with respect to $C_t(h)$, $I_t(h)$, $K_{t+1}(h)$, $B_{H,t+1}(h)$, and $B_{F,t+1}^*(h)$. Let $\Lambda_t(h) \beta^t$ be the (period t) Lagrangian multiplier on equation (A.2), and $\Lambda_t(h) \beta^t \mathcal{Q}_t(h)$ be the multiplier for (A.3). The Lagrangian at time t for household member h working in sector j is stated below, where I abstract from Arrow securities and government transfers:

$$\begin{aligned}
&\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} Z_{U,s} \left[\frac{(1 - \chi_C)^\sigma (C_s(h) - \chi_C C_{s-1})^{1-\sigma}}{1 - \sigma} - Z_{N,s} \chi_N \frac{L_{j,s|s-i}(h)^{1+\varphi}}{1 + \varphi} \right] \right. \\
&- \sum_{s=t}^{\infty} \Lambda_s(h) \beta^{s-t} \left[C_s(h) + \frac{P_s^i}{P_s} I_s(h) + \frac{B_{H,s+1}(h)}{P_s} + \frac{\mathcal{E}_s B_{F,s+1}^*(h)}{P_s} - \frac{W_{j,s|s-i}(h)}{P_s} L_{j,s|s-i}(h) \right] \\
&+ \sum_{s=t}^{\infty} \Lambda_s(h) \beta^{s-t} \left[R_s^k K_s(h) + \mathcal{D}_s(h) + \left(R_{s-1} \frac{B_{H,s}(h)}{P_{s-1}} + R_{s-1}^* \Upsilon_{s-1} \frac{\mathcal{E}_s B_{F,s}^*(h)}{P_{s-1}} \right) \Pi_s^{-1} \right] \\
&\left. - \sum_{s=t}^{\infty} \Lambda_s(h) \beta^{s-t} \mathcal{Q}_s(h) \left[K_{s+1}(h) - (1 - \delta) K_s(h) - Z_{I,s} \left[1 - F \left(\frac{I_s(h)}{I_{s-1}(h)} \right) \right] I_s(h) \right] \right\}
\end{aligned}$$

Optimality conditions in period t with respect to consumption, domestic and foreign bond holdings, capital and investment, follow below. To simplify the notation, I drop the reference to household h :

$$\Lambda_t = Z_{U,t} (1 - \chi_C)^\sigma (C_t - \chi_C C_{t-1})^{-\sigma} \tag{A.7}$$

$$\mathbb{E}_t (R_t^{-1}) = \beta \mathbb{E}_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{-1} \right) \quad (\text{A.8})$$

$$\mathbb{E}_t (R_t^{*-1}) = \beta \mathbb{E}_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{-1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \Upsilon_t \right) \quad (\text{A.9})$$

$$\mathcal{Q}_t = \beta \mathbb{E}_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} [R_{t+1}^k + (1 - \delta) \mathcal{Q}_{t+1}] \right) \quad (\text{A.10})$$

$$\begin{aligned} \frac{P_t^i}{P_t} &= \mathcal{Q}_t Z_{I,t} \left[1 - F \left(\frac{I_t}{I_{t-1}} \right) - F' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] \\ &\quad + \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \mathcal{Q}_{t+1} Z_{I,t+1} F' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \end{aligned} \quad (\text{A.11})$$

Equation (A.7) states that the marginal utility of consumption should be equated with Λ_t , the shadow value of the budget constraint. Equation (A.8), the optimality condition for domestic bond holdings, defines the optimal intertemporal consumption path by equating the marginal utility loss from less consumption today with the marginal utility gain from more consumption in the next period. The (nominal) stochastic discount factor is $\beta \mathbb{E}_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{-1} \right)$. Equation (A.9), together with (A.8), makes the household indifferent between domestic and foreign bond holdings at the margin. \mathcal{Q}_t in (A.10) can be interpreted as the present value of an additional unit of operational capital in the next period. It is equal to the discounted sum of next period's capital returns and the next period's present value of capital net of depreciation. Finally, equation (A.11) equates the relative price for investment goods with the gain by an additional unit of capital today. One more unit of capital saves $Z_{I,t} \left[1 - F \left(\frac{I_t}{I_{t-1}} \right) - F' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right]$ units of investments today, and also reduces expected adjustment costs tomorrow by $\mathbb{E}_t Z_{I,t+1} F' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2$ units.

Next, I move to the labor market in sector j . I construct sectoral labor markets similar to that in Erceg et al. (2000), but add a friction in the sense that workers cannot move freely between sectors. However, all household members work the same number of hours and receive an identical real wage in the long run. Denote the measure of household members working in sector j by $\mu_j \in (0, 1)$, where $\sum_{j=1}^{\mathcal{J}} \mu_j$, the measure of workers in the economy, is normalized to unity. A competitive labor bundler buys hours from all the household members employed in the sector, and combines these hours into an aggregate labor service $N_{j,t}$. This aggregate takes the form

$$N_{j,t} = \left[\left(\frac{1}{\mu_j} \right)^{\frac{\epsilon_w}{1+\epsilon_w}} \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} L_{j,t}(h) \frac{1}{1+\epsilon_w} dh \right]^{1+\epsilon_w}, \quad (\text{A.12})$$

where $\bar{\mu}_j = \sum_{l=1}^j \mu_l$ denotes the total mass of workers employed in sectors $1, \dots, j$. The labor bundler sells his aggregate to all the firms in sector j , charging $W_{j,t}$. He chooses demand for each labor variety to maximize profits, given by

$$W_{j,t} N_{j,t} - \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} W_{j,t}(h) L_{j,t}(h) dh.$$

The optimal number of hours purchased from household member h is

$$L_{j,t}(h) = \frac{1}{\mu_j} \left(\frac{W_{j,t}(h)}{W_{j,t}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}} N_{j,t} = \left(\frac{W_{j,t}(h)}{W_{j,t}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}} L_{j,t}, \quad (\text{A.13})$$

where $L_{j,t} = \frac{N_{j,t}}{\mu_j}$ is defined as the average effective labor hours per worker in the sector.

The wage index is $W_{j,t} = \left[\frac{1}{\mu_j} \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} W_{j,t}(h)^{-\frac{1}{\epsilon_w}} dh \right]^{-\epsilon_w}$. Market clearing implies $N_{j,t} = \int_0^1 N_{j,t}(f) df$, where $N_{j,t}(f)$ is the amount of labor rented to firm f in sector j . Total hours worked in sector j is

$$\int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} L_{j,t}(h) dh = \frac{1}{\mu_j} N_{j,t} \Delta_{wj,t} = \mu_j L_{j,t},$$

where it has been used that $\Delta_{wj,t} = \int_{\bar{\mu}_{j-1}}^{\bar{\mu}_j} \left(\frac{W_{j,t}(h)}{W_{j,t}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}} dh = \mu_j$ holds up to a first order.

Hours worked per person in the entire economy follows as $L_t = \sum_{j=1}^{\mathcal{J}} \mu_j L_{j,t} = N_t$. Each period, only a fraction $1 - \theta_{wj}$ of the household members working in sector j can re-optimize wages. The remaining $1 - \theta_{wj}$ household members index wages according to the indexation rule $W_{j,t}(h) = W_{j,t-1}(h) \Pi_{t-1}^{\epsilon_w} \Pi^{1-\epsilon_w}$. Let $\bar{W}_{j,t}(h)$ denote the optimal wage for a household member h that is able to re-optimize in period t . The wage in period $s > t$ for a member that was last able to re-optimize in period t is then found by backward substitution:

$$W_{j,s|t}(h) = W_{j,s-1|t}(h) \Pi_{s-1}^{\epsilon_w} \Pi^{1-\epsilon_w} = \bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\epsilon_w} \Pi^{1-\epsilon_w} \quad (\text{A.14})$$

For this household member equation (A.13) can be written as

$$L_{j,s|t}(h) = \left(\frac{\bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\epsilon_w} \Pi^{1-\epsilon_w}}{W_{j,s}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w}} L_{j,s}. \quad (\text{A.15})$$

Finally, the Calvo restriction on nominal wage changes implies that

$$W_{j,s+1}(h) = \begin{cases} \bar{W}_{j,s+1}(h) & \text{with probability } 1 - \theta_{wj} \\ W_{j,s}(h) \Pi_s^{\epsilon_w} \Pi^{1-\epsilon_w} & \text{with probability } \theta_{wj} \end{cases}.$$

A household member who is able to reset the wage in period t , will therefore choose the optimal wage $\bar{W}_{j,t}(h)$ in order to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_{wj})^{s-t} \left[-Z_{U,s} Z_{N,s} \chi_N \frac{L_{j,s|t}(h)^{1+\varphi}}{1+\varphi} + \Lambda_s \frac{W_{j,s|t}(h)}{P_s} L_{j,s|t}(h) \right]$$

subject to equations (A.14)-(A.15). The relevant first order condition for this problem is

$$0 = \mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_{wj})^{s-t} \Lambda_s \frac{L_{j,s}(h)}{P_s} \left[\bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\epsilon_w} \Pi^{1-\epsilon_w} - (1 + \epsilon_w) MRS_{j,s|t}(h) P_s \right], \quad (\text{A.16})$$

where $MRS_{j,s|t}(h) = -\frac{u_{j,L}(C_{s,L_{j,s|t}}(h))}{\Lambda_s}$ is the marginal rate of substitution (between consumption and labor) in period s , given a wage last set in period t . Equation (A.16) collapses to $\Omega_{j,t}(h) = (1 + \epsilon_w) MRS_{j,t}(h)$ in the limiting case with flexible wages, where $\Omega_{j,t}(h) = \frac{W_{j,t}}{P_t}$ is the real wage. This holds for all workers, so $\Omega_{j,t}(h) = \Omega_{j,t} \forall h$ in this case. In the more general case with nominal wage stickiness, one can combine (A.16) with the equation linking individual and aggregate marginal rates of substitution,

$$MRS_{j,s|t}(h) = \left(\frac{\bar{W}_{j,t}(h) \prod_{i=1}^{s-t} \Pi_{s-i}^{\epsilon_w} \Pi^{1-\epsilon_w}}{W_{j,s}} \right)^{-\frac{1+\epsilon_w}{\epsilon_w} \varphi} MRS_{j,s},$$

and the law of motion for aggregate wages in sector j ,

$$W_{j,t} = \left[\theta_{wj} (W_{j,t-1} \Pi_{t-1}^{\epsilon_w} \Pi^{1-\epsilon_w})^{-\frac{1}{\epsilon_w}} + (1 - \theta_{wj}) \bar{W}_{j,t}^{-\frac{1}{\epsilon_w}} \right]^{-\epsilon_w},$$

to derive the New Keynesian wage Phillips curve.

A.2.2 FIRMS

In this section I describe the domestic production process in detail. Output of domestic firm f in sector j is given by a Cobb-Douglas production function augmented with fixed costs:

$$Y_{j,t}(f) = Z_{Aj,t} M_{j,t}(f)^{\phi_j} N_{j,t}(f)^{\psi_j} K_{j,t}(f)^{1-\phi_j-\psi_j} - \Phi_j, \quad (\text{A.17})$$

where $M_{j,t}(f)$, $N_{j,t}(f)$ and $K_{j,t}(f)$ are firm f 's use of materials, labor and capital respectively. Φ_j is a fixed production cost that will be calibrated to ensure zero profit in steady state. Constant returns to scale in variable output implies $\phi_j, \psi_j, (\phi_j + \psi_j) \in (0, 1)$. $Z_{Aj,t}$ is sector-specific productivity.

Intermediate trade is modeled as in Bouakez et al. (2009) and Bergholt and Sveen (2014). Monopolistic firms in sector j buy a composite of different materials produced in the different sectors. The materials input aggregate in sector j is given by

$$M_{j,t} = \left[\sum_{l=1}^{\mathcal{J}} \zeta_{lj}^{\frac{1}{\nu_m}} M_{lj,t}^{\frac{\nu_m-1}{\nu_m}} \right]^{\frac{\nu_m}{\nu_m-1}}, \quad (\text{A.18})$$

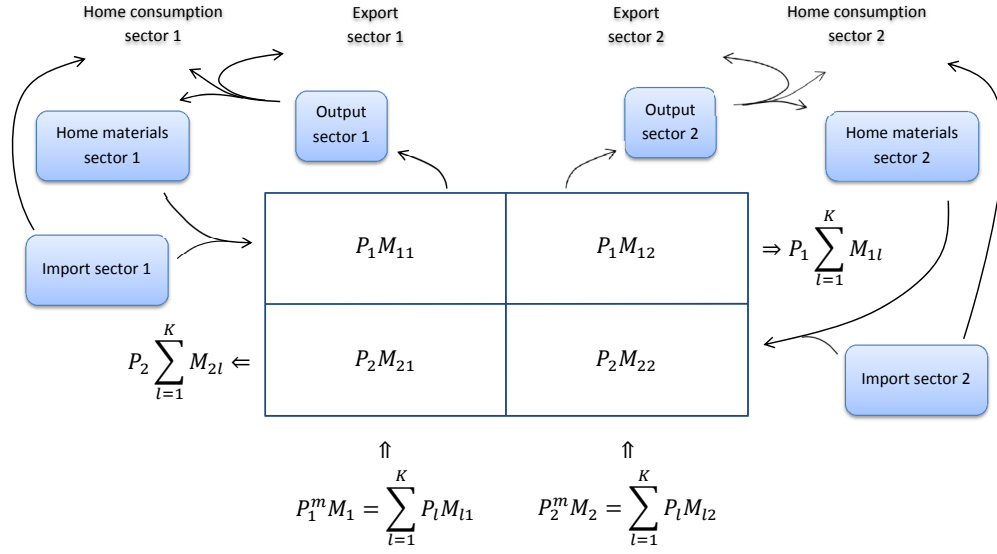
where $\sum_{l=1}^{\mathcal{J}} \zeta_{lj} = 1$ and $\zeta_{lj} \in (0, 1)$. The materials are distributed such that $M_{j,t} = \int_0^1 M_{j,t}(f) df$. Optimal demand for materials from sector l follows as

$$M_{lj,t} = \zeta_{lj} \left(\frac{P_{l,t}}{P_{j,t}^m} \right)^{-\nu_m} M_{j,t}, \quad (\text{A.19})$$

where $P_{j,t}^m = \left[\sum_{l=1}^{\mathcal{J}} \zeta_{lj} P_{l,t}^{1-\nu_m} \right]^{\frac{1}{1-\nu_m}}$ is the relevant price index for intermediate inputs in sector j .

A detailed sketch of the input-output matrix in the domestic economy is provided in Figure A.2 for the case $\mathcal{J} = 2$. The first column shows the total material costs in sector 1, where $M_{11,t}$ and $M_{21,t}$ are the quantities firms in this sector are buying from sectors 1 and

Figure A.2: The I-O matrix for domestic markets when $\mathcal{J} = 2$



Note: Input-output matrix in a two-sector version of our model economy. Arrows summarize the trade flows.

2, respectively. In the same way, sector 2 material costs are the sum of the elements in the second column. The first row then denotes the total value of materials sold from sector 1 to itself ($P_{1,t}M_{11,t}$) and sector 2 ($P_{1,t}M_{12,t}$), respectively. More generally, firms in sector j take as inputs the materials composite and labor service specific to that sector, and produces output sold to i) domestic households, ii) domestic firms, and iii) foreign households and firms. Given the flows of intermediate goods across domestic producer markets, one can find cost minimizing allocations between domestic and imported intermediates, and between single intermediates from each country's sector j , as follows:

$$\begin{aligned}
 M_{Hlj,t} &= \bar{\alpha}_l \left(\frac{P_{Hl,t}}{P_{l,t}} \right)^{-\eta} M_{lj,t}, & M_{Flj,t} &= (1 - \bar{\alpha}_l) \left(\frac{P_{Fl,t}}{P_{l,t}} \right)^{-\eta} M_{lj,t}, \\
 M_{Hlj,t}(f) &= \left(\frac{P_{Hl,t}(f)}{P_{Hl,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} M_{Hlj,t}, & M_{Flj,t}(f) &= \left(\frac{P_{Fl,t}(f)}{P_{Fl,t}} \right)^{-\frac{1+\epsilon_{p,t}^*}{\epsilon_{p,t}^*}} M_{Flj,t}.
 \end{aligned}$$

Next I describe the general profit-maximization problem that emerges once intermediate goods have been allocated. Price setting by domestic and foreign firms is subject to monopoly supply power and sticky prices in a way analogous to the labor market. Firms set prices à la Calvo (1983) and Yun (1996), but export goods are priced in local currency (LCP). Denote prices set by domestic producer f in sector j by $P_{Hj,t}(f)$ and $P_{Hj,t}^*(f)$ respectively, where the first is on goods sold at home and the second on exported goods. Let $1 - \theta_{pj}$ denote the probability that a given producer is able to reset his prices. The fraction θ_{pj} of firms that is not able to re-optimize prices, update them according to the indexation rules $P_{Hj,t}(f) = P_{Hj,t-1}(f)\Pi_{Hj,t-1}^{\epsilon_{p,t}}\Pi_{Hj}^{1-\epsilon_{p,t}}$ and $P_{Hj,t}^*(f) = P_{Hj,t-1}^*(f)\Pi_{Hj,t-1}^{\epsilon_{p,t}^*}\Pi_{Hj}^{1-\epsilon_{p,t}^*}$, where $\Pi_{Hj,t} = \frac{P_{Hj,t}}{P_{Hj,t-1}}$ and $\Pi_{Hj,t}^* = \frac{P_{Hj,t}^*}{P_{Hj,t-1}^*}$ are gross inflation rates. Let $\bar{P}_{Hj,t}(f)$ and $\bar{P}_{Hj,t}^*(f)$ denote optimal prices for a firm f that is able to re-optimize in period t . Prices

for a firm that was last able to re-optimize $s - t$ periods ago are found by backward substitution:

$$P_{Hj,s|t}(f) = P_{Hj,s-1|t}(f)\Pi_{Hj,s-1}^{\iota_p}\Pi_{Hj}^{1-\iota_p} = \bar{P}_{Hj,t}(f)\prod_{i=1}^{s-t}\Pi_{Hj,s-i}^{\iota_p}\Pi_{Hj}^{1-\iota_p} \quad (\text{A.20})$$

$$P_{Hj,s|t}^*(f) = P_{Hj,s-1|t}^*(f)\Pi_{Hj,s-1}^{*\iota_p}\Pi_{Hj}^{*1-\iota_p} = \bar{P}_{Hj,t}^*(f)\prod_{i=1}^{s-t}\Pi_{Hj,s-i}^{*\iota_p}\Pi_{Hj}^{*1-\iota_p} \quad (\text{A.21})$$

Define domestic and foreign absorption of output, produced by firm f in sector j , as follows:

$$X_{Hj,t}(f) = C_{Hj,t}(f) + I_{Hj,t}(f) + \sum_{l=1}^{\mathcal{J}} M_{Hjl,t}(f) + G_{Hj,t}(f)$$

$$\tilde{X}_{Hj,t}^*(f) = \tilde{C}_{Hj,t}^*(f) + \tilde{I}_{Hj,t}^*(f) + \sum_{l=1}^{\mathcal{J}} \tilde{M}_{Hj,t}^*(f) + \tilde{G}_{Hj,t}^*(f)$$

These quantities are in per capita terms as seen from the small open economy. The individual firm then chooses a plan $\mathcal{P}_{j,t}(f)$ for production, supply, prices, and inputs,

$$\mathcal{P}_{j,t}(f) = \left\{ \begin{array}{l} \{Y_{j,s}(f), X_{Hj,s}(f), \tilde{X}_{Hj,s}^*(f)\}_{s=t}^{\infty} \\ \{P_{Hj,s}(f), P_{Hj,s}^*(f)\}_{s=t}^{\infty} \\ \{M_{j,s}(f), N_{j,s}(f), K_{j,s}(f)\}_{s=t}^{\infty} \end{array} \right\},$$

to maximize an expected discounted dividend stream given by

$$\mathbb{E}_t \sum_{s=t}^{\infty} \mathcal{Z}_{t,s} P_s \mathcal{D}_{j,s}(f).$$

Dividends and total costs in period s , in terms of consumption goods, are given by

$$\mathcal{D}_{j,s}(f) = P_{rHj,s}(f) X_{Hj,s}(f) + P_{rHj,s}^*(f) \tilde{X}_{Hj,s}^F(f) - TC_{rj,s}(f)$$

and

$$TC_{rj,s}(f) = P_{rj,s}^m M_{j,s}(f) + \Omega_{j,s} N_{j,s}(f) + R_s^k K_{j,s}(f),$$

respectively. The stochastic discount factor is defined as $\mathcal{Z}_{t,s} = \beta^{s-t} \frac{\Lambda_s}{\Lambda_t} \frac{P_t}{P_s}$, the real price on materials as $P_{rj,s}^m = \frac{P_{j,s}^m}{P_s}$, while $P_{rHj,s}^*(f) = \frac{\mathcal{E}_s P_{Hj,s}^*(f)}{P_s}$ is the domestic currency price of exports of f -goods. \mathcal{E}_s is the nominal exchange rate between the domestic and the foreign currency. Profit-maximization is subject to a set of constraints:

$$X_{Hj,s}(f) + \tilde{X}_{Hj,s}^F(f) = Y_{j,s}(f)$$

$$Y_{j,s}(f) = Z_{Aj,s} M_{j,s}(f)^{\phi_j} N_{j,s}(f)^{\psi_j} K_{j,s}(f)^{1-\phi_j-\psi_j} - \Phi_j$$

$$X_{Hj,s}(f) = \left(\frac{P_{Hj,s}(f)}{P_{Hj,s}} \right)^{-\frac{1+\epsilon_{p,s}}{\epsilon_{p,s}}} X_{Hj,s}$$

$$\begin{aligned}\tilde{X}_{Hj,s}^* (f) &= \left(\frac{P_{Hj,s}^* (f)}{P_{Hj,s}^*} \right)^{-\frac{1+\epsilon_{p,s}}{\epsilon_{p,s}}} \tilde{X}_{Hj,s}^* \\ P_{Hj,s+1} (f) &= \begin{cases} \bar{P}_{Hj,s+1} (f) & \text{with probability } 1 - \theta_{pj} \\ P_{Hj,s} (f) \Pi_{Hj,s}^{\prime p} \Pi_{Hj}^{1-\prime p} & \text{with probability } \theta_{pj} \end{cases} \\ P_{Hj,s+1}^* (f) &= \begin{cases} \bar{P}_{Hj,s+1}^* (f) & \text{with probability } 1 - \theta_{pj} \\ P_{Hj,s}^* (f) \Pi_{Hj,s}^{*\prime p} \Pi_{Hj}^{*1-\prime p} & \text{with probability } \theta_{pj} \end{cases}\end{aligned}$$

The first constraint is a market clearing condition, the second a technological constraint, and the third and fourth the demand schedules faced by firm j . Domestic and foreign absorption of domestically produced sector j goods are defined as follows:

$$\begin{aligned}X_{Hj,t} &= C_{Hj,t} + I_{Hj,t} + \sum_{l=1}^{\mathcal{J}} M_{Hjl,t} + G_{Hj,t} \\ \tilde{X}_{Hj,t}^* &= \tilde{C}_{Hj,t}^* + \tilde{I}_{Hj,t}^* + \sum_{l=1}^{\mathcal{J}} \tilde{M}_{Hj,t}^* + \tilde{G}_{Hj,t}^*\end{aligned}$$

Optimality conditions with respect to $Y_{j,t}(f)$, $X_{Hj,t}(f)$, $\tilde{X}_{Hj,t}^*(f)$, $M_{j,t}(f)$, $N_{j,t}(f)$, and $K_{j,t}(f)$, are stated below. $\Xi_j(f)$, $MC_j(f)$, $\Gamma_j(f)$ and $\Gamma_j^*(f)$ represent the Lagrangian multipliers on the constraints.

$$\Xi_{j,t}(f) = MC_{j,t}(f) \quad (\text{A.22})$$

$$\Gamma_{j,t}(f) = P_{Hj,t}(f) - MC_{j,t}(f) \quad (\text{A.23})$$

$$\Gamma_{j,t}^F(f) = \mathcal{E}_t P_{Hj,t}^F(f) - MC_{j,t}(f) \quad (\text{A.24})$$

$$MC_{j,t}(f) = \frac{P_{j,t}^m}{MPM_{j,t}(f)} \quad (\text{A.25})$$

$$MC_{j,t}(f) = \frac{W_{j,t}}{MPL_{j,t}(f)} \quad (\text{A.26})$$

$$MC_{j,t}(f) = \frac{R_t^k P_t}{MPK_{j,t}(f)} \quad (\text{A.27})$$

The marginal products of material, labor and capital for firm f in sector j are denoted $MPM_{j,t}(f)$, $MPL_{j,t}(f)$, and $MPK_{j,t}(f)$ respectively. Optimality conditions (A.25)-(A.27) can be summarized by two equations determining the optimal use of relative inputs:

$$\begin{aligned}\frac{M_{j,t}(f)}{N_{j,t}(f)} &= \frac{\phi_j \Omega_{j,t}}{\psi_j P_{rj,t}^m} \\ \frac{N_{j,t}(f)}{K_{j,t}(f)} &= \frac{\psi_j R_t^k}{1 - \phi_j - \psi_j \Omega_{j,t}}\end{aligned}$$

Next I state the optimality conditions with respect to $P_{Hj}(f)$ and $P_{Hj}^*(f)$:

$$0 = E_t \sum_{s=t}^{\infty} (\theta_{pj})^{s-t} \mathcal{Z}_{t,s} X_{Hj,s}(f) \left[\bar{P}_{Hj,t}(f) \prod_{i=1}^{s-t} \Pi_{Hj,s-i}^{\prime p} \Pi_{Hj}^{1-\prime p} - (1 + \epsilon_{p,s}) MC_{j,s}(f) \right] \quad (\text{A.28})$$

$$0 = E_t \sum_{s=t}^{\infty} (\theta_{pj})^{s-t} \mathcal{Z}_{t,s} X_{Hj,s}^*(f) \mathcal{E}_s \left[\bar{P}_{Hj,t}^*(f) \prod_{i=1}^{s-t} \Pi_{Hj,s-i}^{*\prime p} \Pi_{Hj}^{*1-\prime p} - (1 + \epsilon_{p,s}) MC_{j,s}(f) \right] \quad (\text{A.29})$$

In the limiting case with flexible prices, these first-order conditions collapse to $\frac{P_{Hj,t}}{P_t} = \frac{\mathcal{E}_t P_{Hj,t}^*}{P_t} = (1 + \epsilon_{p,t}) RMC_{j,t}$ for all firms. The law of one price holds period by period in this case. It is clear from equation (A.17) and (A.25)-(A.27) that all firms in sector j face the same marginal cost. The real marginal cost can be written as

$$RMC_{j,t} = \frac{1}{Z_{Aj,t}} \left(\frac{P_{rj,t}^m}{\phi_j} \right)^{\phi_j} \left(\frac{\Omega_{j,t}}{\psi_j} \right)^{\psi_j} \left(\frac{R_t^k}{1 - \phi_j - \psi_j} \right)^{1 - \phi_j - \psi_j}, \quad (\text{A.30})$$

where $RMC_{j,t} = \frac{MC_{j,t}}{P_t}$ measures costs in terms of consumption goods. The staggered price setting structure combined with partial indexation implies that prices of domestically produced goods can be written as follows:

$$P_{Hj,t} = \left[\theta_{pj} \left(P_{Hj,t-1} \Pi_{Hj,t-1}^{\prime p} \Pi_{Hj}^{1-\prime p} \right)^{-\frac{1}{\epsilon_{p,t}}} + (1 - \theta_{pj}) \bar{P}_{Hj,t}^{-\frac{1}{\epsilon_{p,t}}} \right]^{-\epsilon_{p,t}}$$

$$P_{Hj,t}^* = \left[\theta_{pj} \left(P_{Hj,t-1}^* \Pi_{Hj,t-1}^{*\prime p} \Pi_{Hj}^{*1-\prime p} \right)^{-\frac{1}{\epsilon_{p,t}}} + (1 - \theta_{pj}) \bar{P}_{Hj,t}^{*-\frac{1}{\epsilon_{p,t}}} \right]^{-\epsilon_{p,t}}$$

Finally, one can combine these with the optimality conditions for prices to derive two New Keynesian price Phillips curves for domestic goods and exports. The sectoral import price $P_{Fj,t}$ follows the same type of law of motion as those described above.

A.2.3 MONETARY AND FISCAL POLICY

Monetary authorities are assumed to follow an extended Taylor-rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\rho_\pi} \left(\frac{GDP_t}{GDP} \right)^{\rho_y} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\rho_{dy}} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \right)^{\rho_e} \right]^{1-\rho_r} Z_{R,t} \quad (\text{A.31})$$

$Z_{R,t}$ is a monetary policy shock. Fiscal authorities face a period-by-period budget constraint of the form

$$P_t^g G_t + R_{t-1} B_{H,t} = B_{H,t+1} + P_t T_t$$

I assume that public debt is zero in steady state. This implies that government spending is fully financed by lump-sum taxes up to a first-order approximation.

A.2.4 MARKET CLEARING AND AGGREGATION

Trade between the world economy and the SOE becomes negligible from the world economy's point of view when $\varsigma \rightarrow 0$. Previously I defined $\tilde{X}_{Hj,t}^*(f)$ as home firm f 's export units per home capita. Similarly, let $X_{Hj,t}^*(f)$ denote home firm f 's export units per foreign capita. These two are linked via the identity $\tilde{X}_{Hj,t}^*(f) = \frac{1-\varsigma}{\varsigma} X_{Hj,t}^*(f)$. When (A.6) and the relevant optimality condition for foreign imports are evaluated in the limit

as $\varsigma \rightarrow 0$, we get the following system of trade demand schedules in the small open economy:

$$X_{Hj,t} = \alpha_j \left(\frac{P_{Hj,t}}{P_{j,t}} \right)^{-\eta} X_{j,t}, \quad (\text{A.32})$$

$$X_{Fj,t} = (1 - \alpha_j) \left(\frac{P_{Fj,t}}{P_{j,t}} \right)^{-\eta} X_{j,t}, \quad (\text{A.33})$$

$$\tilde{X}_{Hj,t}^* = \frac{1 - \varsigma}{\varsigma} X_{Hj,t}^* = (1 - \alpha_j^*) \left(\frac{P_{Hj,t}^*}{P_{j,t}^*} \right)^{-\eta} X_{j,t}^* \quad (\text{A.34})$$

Here total absorption in the domestic sector j market is defined as

$$X_{j,t} = C_{j,t} + I_{j,t} + \sum_{l=1}^{\mathcal{J}} M_{jl,t} + G_{j,t}. \quad (\text{A.35})$$

Moreover, it is clear from the expressions for $X_{Fj,t}^*$ and $X_{Hj,t}^*$ in the foreign block, as well as the export demand schedule $\tilde{X}_{Fj,t}^H$, that $\varsigma \rightarrow 0$ implies

$$X_{Fj,t}^* = \bar{\alpha}_j^* \left(\frac{P_{Fj,t}^*}{P_{j,t}^*} \right)^{-\eta} X_{j,t}^* = X_{j,t}^*, \quad (\text{A.36})$$

$$X_{Hj,t}^* = (1 - \bar{\alpha}_j^*) \left(\frac{P_{Hj,t}^*}{P_{j,t}^*} \right)^{-\eta} X_{j,t}^* = 0, \text{ and} \quad (\text{A.37})$$

$$\tilde{X}_{Fj,t}^H = \frac{\varsigma}{1 - \varsigma} X_{Fj,t} = 0. \quad (\text{A.38})$$

The first line uses $\lim_{\varsigma \rightarrow 0} P_{Fj,t}^* = P_{j,t}^*$. Aggregate output in sector j is

$$Y_{j,t} = \int_0^1 Y_{j,t}(f) df = X_{Hj,t} \Delta_{Hj,t} + \tilde{X}_{Hj,t}^* \Delta_{Hj,t}^*, \quad (\text{A.39})$$

where the two relative price dispersion terms $\Delta_{Hj,t} = \int_0^1 \left(\frac{P_{Hj,t}(f)}{P_{Hj,t}} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} df$ and $\Delta_{Hj,t}^* = \int_0^1 \left(\frac{P_{Hj,t}^*(f)}{P_{Hj,t}^*} \right)^{-\frac{1+\epsilon_{p,t}}{\epsilon_{p,t}}} df$ are equal to one up to a first order. Nominal gross sales in sector j is $P_{Hj,t} X_{Hj,t} + \mathcal{E}_t P_{Hj,t}^* \tilde{X}_{Hj,t}^*$. Real value added, which is the nominal value added denominated by the CPI, can be written in three different, but model consistent ways:

$$\begin{aligned} GDP_{j,t} &= \frac{P_{Hj,t}}{P_t} X_{Hj,t} + \frac{\mathcal{E}_t P_{Hj,t}^*}{P_t} \tilde{X}_{Hj,t}^* - \frac{P_{j,t}^m}{P_t} M_{j,t} \\ &= \Omega_{j,t} N_{j,t} + R_t^k K_{j,t} + \mathcal{D}_{j,t} \\ &= \frac{P_{j,t}}{P_t} (C_{j,t} + I_{j,t} + G_{j,t}) + TB_{j,t} + \frac{1}{P_t} \left(P_{j,t} \sum_{l=1}^{\mathcal{J}} M_{jl,t} - P_{j,t}^m M_{j,t} \right) \end{aligned} \quad (\text{A.40})$$

The first line defines GDP in sector j according to the output approach, i.e. as the value of gross output minus the value of intermediate consumption. The second line measures

GDP according to the income approach. A no-arbitrage condition implies that real dividends from a portfolio of stocks in sector j , $\mathcal{D}_{j,t} = \int_0^1 \mathcal{D}_{j,t}(f) df$, is zero in the steady state. The last line in (A.40) uses the expenditure approach, where one calculates the integral of all domestic demand functions. The trade balance in sector j is given by

$$TB_{j,t} = \frac{\mathcal{E}_t P_{Hj,t}^*}{P_t} \tilde{X}_{Hj,t}^* - \frac{P_{Fj,t}}{P_t} X_{Fj,t}. \quad (\text{A.41})$$

Economy-wide GDP is defined as $GDP_t = \sum_{j=1}^{\mathcal{J}} GDP_{j,t}$. Thus, one can aggregate the second line of equation (A.40) over all j . The result is $GDP_t = \Omega_t N_t + R_t^k K_t + \mathcal{D}_t$. A more familiar expression is found by combining this with the representative household's budget constraint, which must hold with equality. Then we get

$$\begin{aligned} GDP_t &= C_t + \frac{P_t^i}{P_t} I_t + \frac{\mathcal{E}_t B_{F,t+1}^*}{P_t} - R_{t-1}^* \Upsilon_t \frac{\mathcal{E}_t B_{F,t}^*}{P_t} + \frac{B_{H,t+1}}{P_t} - R_{t-1} \frac{B_{H,t}}{P_t} + T_t \\ &= C_t + \frac{P_t^i}{P_t} I_t + \frac{P_t^g}{P_t} G_t + TB_t, \end{aligned} \quad (\text{A.42})$$

where the last line follows from the budget constraint of the government and the current account identity

$$TB_t = \frac{\mathcal{E}_t B_{F,t+1}^*}{P_t} - R_{t-1}^* \Upsilon_{t-1} \frac{\mathcal{E}_t B_{F,t}^*}{P_t}. \quad (\text{A.43})$$

The identity simply states that positive trade balances are used to accumulate foreign assets. Another way to derive (A.42) is by summing the last line in (A.40) over all j and noting that $TB_t = \sum_{j=1}^{\mathcal{J}} TB_{j,t}$. From the foreign economy's point of view, their debt is in zero net supply because the home economy engages in only a negligible part of the financial assets trade. Furthermore, I assume that foreign investors do not hold financial assets in the home economy. Equilibrium in the foreign bonds market is finally represented by a modified uncovered interest rate parity condition, found by combining (A.8) and (A.9):

$$\mathbb{E}_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{-1} \left(R_t - \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} R_t^* \Upsilon_t \right) \right\} = 0 \quad (\text{A.44})$$

The foreign economy is characterized by a similar system of equations, except that trade constitutes a negligible part of economic activity. This completes the description of the model.

B STEADY STATE AND THE LOG-LINEAR MODEL

In this section, I provide i) the full steady state system of the model, ii) a recursive solution for the steady state under the restrictions of unitary relative prices and balanced trade, and iii) the complete log-linearized model.

B.1 THE FULL STEADY STATE SYSTEM

Denote the steady state level of any variable without the t -subscript, e.g. the steady state level of X_t as X . The steady state equilibrium system for the small open economy follows below. The world economy is modeled as a closed economy version of the model described above, and has a similar steady state (not shown):

$$\begin{aligned}
C^{*-\sigma} &= \mathcal{S}C^{-\sigma} \\
1 &= \sum_{j=1}^{\mathcal{J}} \xi_j P_{rj}^{1-\nu_c} \\
\Omega &= (1 + \epsilon_w) \chi_N C^\sigma L^\varphi \\
\sum_{j=1}^{\mathcal{J}} \mu_j &= 1 \\
I &= \delta \sum_{j=1}^{\mathcal{J}} K_j \\
R^k &= \mathcal{Q} (\beta^{-1} - (1 - \delta)) \\
P_r^{i1-\nu_i} &= \sum_{j=1}^{\mathcal{J}} \varpi_j P_{rj}^{1-\nu_i} \\
\mathcal{Q} &= P_r^i \\
C_j &= \xi_j P_{rj}^{-\nu_c} C \\
\sum_{j=1}^{\mathcal{J}} \xi_j &= 1 \\
I_j &= \varpi_j \left(\frac{P_{rj}}{P_r^i} \right)^{-\nu_i} I \\
\sum_{l=1}^{\mathcal{J}} \varpi_j &= 1 \\
M_{lj} &= \zeta_{lj} \left(\frac{P_{rl}}{P_{rj}^m} \right)^{-\nu_m} M_j \\
\sum_{l=1}^{\mathcal{J}} \zeta_{lj} &= 1 \\
P_{rj}^{m1-\nu_m} &= \sum_{l=1}^{\mathcal{J}} \zeta_{lj} P_{rl}^{1-\nu_m} \\
\mu_j L &= N_j \\
X_j &= C_j + I_j + \sum_{l=1}^{\mathcal{J}} M_{jl} + G_j \\
Y_j &= X_{Hj} + X_{Hj}^* \\
P_{rHj} Y_j &= P_{rj}^m M_j + \Omega N_j + R^k K_j \\
Y_j &= M_j^{\phi_j} N_j^{\psi_j} K_j^{1-\phi_j-\psi_j} - \Phi_j \\
\frac{N_j}{K_j} &= \frac{\psi_j}{1 - \phi_j - \psi_j} \frac{R^k}{\Omega} \\
\frac{M_j}{N_j} &= \frac{\phi_j}{\psi_j} \frac{\Omega}{P_{rj}^m} \\
RMC_j &= \frac{P_{rj}^m \phi_j \Omega \psi_j R^k^{1-\phi_j-\psi_j}}{Z_j \phi_j^{\phi_j} \psi_j^{\psi_j} (1 - \phi_j - \psi_j)^{1-\phi_j-\psi_j}} \\
P_{rHj} &= (1 + \epsilon_p) RMC_j \\
P_{rFj} &= (1 + \epsilon_p) RMC_j^F \mathcal{S} \\
P_{rj}^{1-\eta} &= \alpha_j P_{rHj}^{1-\eta} + (1 - \alpha_j) P_{rFj}^{1-\eta} \\
X_{Hj} &= \alpha_j \left(\frac{P_{rHj}}{P_{rj}} \right)^{-\eta} X_j \\
X_{Fj} &= (1 - \alpha_j) \left(\frac{P_{rFj}}{P_{rj}} \right)^{-\eta} X_j \\
X_{Hj}^* &= (1 - \alpha_j^*) \left(\frac{P_{rHj}}{P_{rj}^F} \right)^{-\eta} \mathcal{S}^\eta X_j^* \\
GDP_j &= P_{rHj} Y_j - P_{rj}^m M_j \\
GDP &= \sum_{j=1}^{\mathcal{J}} GDP_j \\
TB_j &= P_{rHj} X_{Hj}^* - P_{rFj} X_{Fj} \\
TB &= \sum_{j=1}^{\mathcal{J}} TB_j
\end{aligned}$$

$$\mathcal{T}_j = \frac{P_{rHj}}{P_{rFj}}$$

B.2 RECURSIVE SOLUTION OF STEADY STATE VARIABLES

Next, I derive an analytical solution for the steady state system above. I restrict the analysis to an equilibrium with balanced trade, zero public spending, and relative prices equal to unity, i.e. with all nominal prices growing at the same rate Π . First I solve recursively for steady state in the foreign economy. Second, I use that solution as input to find steady state in the small open economy. This “block recursive” approach to the steady state solution is necessary because several foreign variables affect domestic steady state values. The domestic solution is described below.

B.2.1 THE FOREIGN (CLOSED) ECONOMY

The real interest rate is defined as $\frac{R}{\Pi} = \beta^{-1}$. We get steady state investment and material prices from $P_{rj}^* = 1$, $\sum_{j=1}^{\mathcal{J}} \varpi_j^* = 1$, and $\sum_{j=1}^{\mathcal{J}} \zeta_{lj}^* = 1$:

$$P_r^{i*} = \left(\sum_{j=1}^{\mathcal{J}} \varpi_j^* P_{rj}^{*1-\nu_i} \right)^{\frac{1}{1-\nu_i}} = 1$$

$$P_{rj}^{m*} = \left(\sum_{l=1}^{\mathcal{J}} \zeta_{lj}^* P_{rl}^{*1-\nu_m} \right)^{\frac{1}{1-\nu_m}} = 1$$

From (A.10):

$$Q^* = P_r^{i*} = 1$$

From (A.9):

$$R^{k*} = Q^* [\beta^{-1} - (1 - \delta)] = \beta^{-1} - (1 - \delta)$$

From $P_{rj}^* = 1$ and (A.28):

$$RMC_j^* = \frac{1}{1 + \epsilon_p}$$

Without loss of generality I normalize $C^* = 1$. Thus, steady state variables are measured in consumption units. Foreign consumption at the sector level follows:

$$C_j^* = \xi_j^* C^* = \xi_j^* \quad \forall j$$

Next, I set out to derive sector-level output. Note first that the large economy assumption implies $Y_j^* = X_j^* \forall j$. Second, using (the foreign economy versions of) equations (A.3), (A.5), (A.19), and (A.26), we can write $I^* = \delta \sum_{j=1}^{\mathcal{J}} K_j^*$, $I_j^* = \varpi_j^* I^*$, $K_j^* = \frac{1-\phi_j^*-\psi_j^*}{R^{k*}} Y_j^*$, and $M_j^* = \phi_j^* Y_j^*$. Combining these expressions with the market clearing condition $Y_j^* = C_j^* + I_j^* + \sum_{l=1}^{\mathcal{J}} M_{jl}^*$, the foreign economy’s production network follows in compact form as

$$\mathbf{Y}^* = \mathbf{C}^* + \mathbf{\Psi}^* \mathbf{Y}^*,$$

where $\mathbf{Y}^* = [Y_1^*, \dots, Y_{\mathcal{J}}^*]'$ is the output vector and $\mathbf{C}^* = [C_1^*, \dots, C_{\mathcal{J}}^*]'$ is the final consumption vector. The (j, l) 'th element of the $\mathcal{J} \times \mathcal{J}$ -matrix Ψ^* is equal to

$$\Psi_{jl}^* = \zeta_{jl}^* \phi_l^* + \varpi_j^* \delta \frac{1 - \phi_l^* - \psi_l^*}{\beta^{-1} - 1 + \delta}.$$

Standard matrix manipulation therefore gives us the following solution for gross output:

$$\mathbf{Y}^* = \tilde{\Psi}^* \mathbf{C}^*,$$

with $\tilde{\Psi}^* = [\mathbf{1} - \Psi^*]^{-1}$ being referred to as the steady state influence matrix. $\mathbf{1} - \Psi^*$ is invertible under mild conditions, at least given the high level of aggregation considered here.²² Moreover, $\det(\mathbf{1} - \Psi^*)$ is generally positive. Next, one can combine the solution for Y_j^* with (A.17), (A.19), (A.25), (A.27) to get

$$\begin{aligned} M_j^* &= \phi_j^* Y_j^* \\ M_{lj}^* &= \zeta_{lj}^* M_j^* \\ K_j^* &= \frac{1 - \phi_j^* - \psi_j^*}{\beta^{-1} - 1 + \delta} Y_j^* \end{aligned}$$

Aggregate capital and investment, and sector-level investment demand, follow below:

$$\begin{aligned} K^* &= \sum_{j=1}^{\mathcal{J}} K_j^* \\ I^* &= \delta K^* \\ I_j^* &= \varpi_j^* I^* \end{aligned}$$

To derive the real wage level, I sum up budget constraints across individual households and impose the no-arbitrage condition $\Omega_j^* = \Omega^*$ for the labor market. The result is $C^* + I^* = \Omega^* L^* + R^{k^*} K^*$, or

$$\Omega^* = \frac{C^* - (\beta^{-1} - 1) K^*}{L^*}.$$

Taking L^* as given, a restriction follows for the shift parameter χ_N^* from households' steady state labor supply:

$$\chi_N^* = \frac{\Omega^*}{(1 + \epsilon_w) C^{*\sigma^*} L^{*\varphi^*}}$$

Finally, sectoral productivity level, labor input, employment and markup (over variable costs) can be found from equations (A.17), (A.26), (A.30) and the identity $L^* = \frac{N_j^*}{\mu_j^*}$:

$$Z_j^* = (1 + \epsilon_p) \left(\frac{1}{\phi_j^*} \right)^{\phi_j^*} \left(\frac{\Omega^*}{\psi_j^*} \right)^{\psi_j^*} \left(\frac{R^{k^*}}{1 - \phi_j^* - \psi_j^*} \right)^{1 - \phi_j^* - \psi_j^*}$$

²²A necessary and sufficient restriction for non-singularity is that none of the following are true: i) $\phi_j^* = 1 \forall j$, and ii) $\zeta_{jj}^* = \phi_j^* = 1$ for some j . For the small open economy, these expressions write i) $\alpha_j \phi_j = 1 \forall j$, and ii) $\alpha_j = \zeta_{jj} = \phi_j = 1$ for some j . Proofs are available from the author upon request.

$$\begin{aligned}\Phi_j^* &= \epsilon_p Y_j^* \\ N_j^* &= \frac{\psi_j^*}{\Omega^*} Y_j^* \\ \mu_j^* &= \frac{N_j^*}{L^*}\end{aligned}$$

For completeness, note that $\sum_{j=1}^{\mathcal{J}} \mu_j^* = 1$, $\sum_{j=1}^{\mathcal{J}} N_j^* = L^*$, and that sectoral and aggregate GDP write as follows:

$$\begin{aligned}GDP_j^* &= (1 - \phi_j^*) Y_j^* \\ GDP^* &= \sum_{j=1}^{\mathcal{J}} GDP_j^* = C^* + I^*\end{aligned}$$

This completes the foreign block. Next I derive steady state in the small open economy.

B.2.2 THE SMALL OPEN ECONOMY

From $P_{rHj} = P_{rFj} = 1$ we get

$$P_{rj} = [\alpha_j P_{rHj}^{1-\eta} + (1 - \alpha_j) P_{rFj}^{1-\eta}]^{\frac{1}{1-\eta}} = 1$$

Thus, the relative prices for investments and sector-level material inputs become

$$\begin{aligned}P_r^i &= \left(\sum_{j=1}^{\mathcal{J}} \varpi_j P_{rj}^{1-\nu_i} \right)^{\frac{1}{1-\nu_i}} = 1 \\ P_{rj}^m &= \left(\sum_{j=1}^{\mathcal{J}} \zeta_{lj} P_{rl}^{1-\nu_m} \right)^{\frac{1}{1-\nu_m}} = 1\end{aligned}$$

The solutions for \mathcal{Q} , R^k and $RM C_j$ are found following the procedures used in the foreign block. The real exchange rate is unity due to the assumption of unitary real import prices and $P_{rFj} = (1 + \epsilon_p) RM C_j^* \mathcal{S}$, the optimality condition for foreign firms:

$$\mathcal{S} = \frac{P_{rFj}}{(1 + \epsilon_p) RM C_j^*} = 1$$

The steady state UIP condition $\mathcal{S} = \left(\frac{C}{C^*}\right)^\sigma$ then implies that $C = C^* = 1$. Moreover, balanced steady state trade in the small open economy implies that $Y_j = X_j = C_j + I_j + \sum_{l=1}^{\mathcal{J}} M_{jl}$. Thus, C_j , Y_j , K_j , M_j , M_{lj} , K , I , I_j , Ω , χ_N , $RM C_j$, Z_j , N_j and μ_j are found in that order, and in the same manner as their foreign counterparts. Similarly, under the assumption of balanced trade we get sectoral and aggregate GDP as follows:

$$\begin{aligned}GDP_j &= (1 - \phi_j) Y_j \\ GDP &= \sum_{j=1}^{\mathcal{J}} GDP_j = C + I\end{aligned}$$

Given data on export-to-GDP ratios $\frac{X_{Hj}^*}{GDP_j}$, we now also have X_{Hj}^* . The model is completed with the following trade block:

$$\begin{aligned} X_{Fj} &= X_{Hj}^* \\ X_{Hj} &= Y_j - X_{Fj} \\ \alpha_j &= \frac{X_{Hj}}{X_j} \\ \alpha_j^* &= 1 - \frac{X_{Hj}^*}{X_j^*} \end{aligned}$$

B.3 LOG-LINEARIZED SYSTEM

Define small case variables as log-deviations from steady state, e.g. $x_t \equiv \ln\left(\frac{X_t}{\bar{X}}\right)$. $100x_t$ is interpreted as the percentage deviation in a neighborhood around the steady state. First, I define some price identities:

$$\begin{aligned} \pi_t &= \ln\left(\frac{P_t}{P_{t-1}}\Pi^{-1}\right) & p_{rHj,t}^* &= \ln\left(\frac{\mathcal{E}_t P_{Hj,t}^*}{P_t}\right) & \pi_{wj,t} &= \ln\left(\frac{W_{j,t}}{W_{j,t-1}}\Pi^{-1}\right) \\ \pi_{j,t} &= \ln\left(\frac{P_{j,t}}{P_{j,t-1}}\Pi^{-1}\right) & \pi_{Fj,t} &= \ln\left(\frac{P_{Fj,t}}{P_{Fj,t-1}}\Pi^{-1}\right) & \omega_{j,t} &= \ln(\Omega_{j,t}) \\ p_{rj,t} &= \ln\left(\frac{P_{j,t}}{P_t}\right) & p_{rFj,t} &= \ln\left(\frac{P_{Fj,t}}{P_t}\right) & \pi_t^i &= \ln\left(\frac{P_t^i}{P_{t-1}^i}\Pi^{-1}\right) \\ \pi_{Hj,t} &= \ln\left(\frac{P_{Hj,t}}{P_{Hj,t-1}}\Pi^{-1}\right) & p_{rj,t}^* &= \ln\left(\frac{P_{j,t}^*}{P_t^*}\right) & p_{r,t}^i &= \ln\left(\frac{P_t^i}{P_t}\right) \\ p_{rHj,t} &= \ln\left(\frac{P_{Hj,t}}{P_t}\right) & \pi_{j,t}^m &= \ln\left(\frac{P_{j,t}^m}{P_{j,t-1}^m}\Pi^{-1}\right) & s_t &= \ln\left(\frac{\mathcal{E}_t P_t^*}{P_t}\right) \\ \pi_{Hj,t}^* &= \ln\left(\frac{P_{Hj,t}^*}{P_{Hj,t-1}^*}\Pi^{-1}\right) & p_{rj,t}^m &= \ln\left(\frac{P_{j,t}^m}{P_t}\right) \end{aligned}$$

The log-linearized system of equations that constitutes the home block follows below:

$$c_{j,t} = -\nu_c p_{rj,t} + c_t \tag{B.1}$$

$$\lambda_t = z_{U,t} - \frac{\sigma}{1 - \chi_C} (c_t - \chi_C c_{t-1}) \tag{B.2}$$

$$\lambda_t = \mathbb{E}_t(\lambda_{t+1}) + r_t - \mathbb{E}_t(\pi_{t+1}) \tag{B.3}$$

$$\lambda_t = \mathbb{E}_t(\lambda_{t+1}) + r_t^* - \mathbb{E}_t(\pi_{t+1}) + \mathbb{E}_t(\Delta e_{t+1} + v_t) \tag{B.4}$$

$$v_t = -\epsilon_B a_t + z_{B,t} \tag{B.5}$$

$$i_{j,t} = -\nu_i (p_{rj,t} - p_{r,t}^i) + i_t \tag{B.6}$$

$$q_t = -r_t + \mathbb{E}_t(\pi_{t+1} + [1 - \beta(1 - \delta)]r_{t+1}^k + \beta(1 - \delta)q_{t+1}) \tag{B.7}$$

$$q_t = -z_{I,t} + p_{r,t}^i + \epsilon_I [(i_t - i_{t-1}) - \beta \mathbb{E}_t(i_{t+1} - i_t)] \tag{B.8}$$

$$k_{t+1} = (1 - \delta)k_t + \delta(z_{I,t} + i_t) \tag{B.9}$$

$$k_t = \sum_{j=1}^{\mathcal{J}} \gamma_j^k k_{j,t} \tag{B.10}$$

$$p_{r,t}^i = \sum_{j=1}^{\mathcal{J}} \varpi_j p_{rj,t} \quad (\text{B.11})$$

$$\pi_{wj,t} = \omega_{j,t} - \omega_{j,t-1} + \pi_t \quad (\text{B.12})$$

$$\pi_{wj,t} = \beta \mathbb{E}_t (\pi_{wj,t+1}) + \iota_w (\pi_{t-1} - \beta \pi_t) + \kappa_{wj} (mrs_{j,t} - \omega_{j,t}) \quad (\text{B.13})$$

$$mrs_{j,t} = z_{U,t} + z_{N,t} + \varphi n_{j,t} - \lambda_t \quad (\text{B.14})$$

$$\pi_t = \sum_{j=1}^{\mathcal{J}} \xi_j \pi_{j,t} \quad (\text{B.15})$$

$$\pi_{j,t} = p_{rj,t} - p_{rj,t-1} + \pi_t \quad (\text{B.16})$$

$$\pi_{j,t} = \alpha_j \pi_{Hj,t} + (1 - \alpha_j) \pi_{Fj,t} \quad (\text{B.17})$$

$$p_{rj,t}^m = \sum_{l=1}^{\mathcal{J}} \zeta_{lj} p_{rl,t} \quad (\text{B.18})$$

$$\pi_{Hj,t} = p_{rHj,t} - p_{rHj,t-1} + \pi_t \quad (\text{B.19})$$

$$\pi_{Hj,t} = \kappa_1 \mathbb{E}_t (\pi_{Hj,t+1}) + \kappa_2 \pi_{Hj,t-1} + \kappa_{j3} (rmc_{j,t} - p_{rHj,t} + z_{M,t}) \quad (\text{B.20})$$

$$\pi_{Hj,t}^* = p_{rHj,t}^* - p_{rHj,t-1}^* + \pi_t - \Delta e_t \quad (\text{B.21})$$

$$\pi_{Hj,t}^* = \kappa_1 \mathbb{E}_t (\pi_{Hj,t+1}^*) + \kappa_2 \pi_{Hj,t-1}^* + \kappa_{j3} (rmc_{j,t} - p_{rHj,t}^* + z_{M,t}) \quad (\text{B.22})$$

$$rmc_{j,t} = -z_{Aj,t} + \phi_j p_{rj,t}^m + \psi_j \omega_{j,t} + (1 - \phi_j - \psi_j) r_t^k \quad (\text{B.23})$$

$$\pi_{Fj,t} = p_{rFj,t} - p_{rFj,t-1} + \pi_t \quad (\text{B.24})$$

$$\pi_{Fj,t} = \kappa_1 \mathbb{E}_t (\pi_{Fj,t+1}) + \kappa_2 \pi_{Fj,t-1} + \kappa_{j3} (rmc_{j,t}^* + s_t - p_{rFj,t} + z_{M,t}^*) \quad (\text{B.25})$$

$$\tau_{j,t} = p_{rHj,t}^* - p_{rFj,t} \quad (\text{B.26})$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (\rho_\pi \pi_t + \rho_y gdp_t + \rho_{\Delta y} \Delta gdp_t + \rho_e \Delta e_t) + z_{R,t} \quad (\text{B.27})$$

$$x_{j,t} = \gamma_j^c c_{j,t} + \gamma_j^i i_{j,t} + \sum_{l=1}^{\mathcal{J}} \gamma_{jl}^m m_{jl,t} \quad (\text{B.28})$$

$$m_{l,t} = -\nu_m (p_{rl,t} - p_{rj,t}^m) + m_{j,t} \quad (\text{B.29})$$

$$m_{j,t} - n_{j,t} = \omega_{j,t} - p_{rj,t}^m \quad (\text{B.30})$$

$$n_{j,t} - k_{j,t} = r_t^k - \omega_{j,t} \quad (\text{B.31})$$

$$x_{Hj,t} = -\eta (p_{rHj,t} - p_{rj,t}) + x_{j,t} \quad (\text{B.32})$$

$$x_{Fj,t} = -\eta (p_{rFj,t} - p_{rj,t}) + x_{j,t} \quad (\text{B.33})$$

$$x_{Hj,t}^* = -\eta (p_{rHj,t}^* - s_t - p_{rj,t}^*) + x_{j,t}^* \quad (\text{B.34})$$

$$y_{j,t} = \alpha_{xj} x_{Hj,t} + (1 - \alpha_{xj}) x_{Hj,t}^* \quad (\text{B.35})$$

$$y_{j,t} = (1 + \epsilon_p) [z_{Aj,t} + \phi_j m_{j,t} + \psi_j n_{j,t} + (1 - \phi_j - \psi_j) k_{j,t}] \quad (\text{B.36})$$

$$gdp_{j,t} = \gamma_j^1 (p_{rj,t} + x_{j,t}) + tb_{j,t} - \gamma_j^2 (p_{rj,t}^m + m_{j,t}) \quad (\text{B.37})$$

$$gdp_t = \sum_{j=1}^{\mathcal{J}} \gamma_j gdp_{j,t} \quad (\text{B.38})$$

$$tb_{j,t} = \gamma_j^{ex} (p_{rHj,t}^* + x_{Hj,t}^*) - \gamma_j^{im} (p_{rFj,t} + x_{Fj,t}) \quad (\text{B.39})$$

$$tb_t = \sum_{j=1}^{\mathcal{J}} \gamma_j tb_{j,t} \quad (\text{B.40})$$

$$a_t = \frac{1}{\beta} a_{t-1} + tb_t + \frac{\sum_{j=1}^{\mathcal{J}} (\gamma_j^{ex} - \gamma_j^{im}) \gamma_j}{\beta - 1} (r_{t-1}^* + v_{t-1} + \Delta e_t - \pi_t) \quad (\text{B.41})$$

$$z_{Aj,t} = \rho_A z_{Aj,t-1} + \varepsilon_{Aj,t} \quad (\text{B.42})$$

$$z_{I,t} = \rho_I z_{I,t-1} + \varepsilon_{I,t} \quad (\text{B.43})$$

$$z_{U,t} = \rho_U z_{U,t-1} + \varepsilon_{U,t} \quad (\text{B.44})$$

$$z_{N,t} = \rho_N z_{N,t-1} + \varepsilon_{N,t} \quad (\text{B.45})$$

$$z_{M,t} = \rho_M z_{M,t-1} + \varepsilon_{M,t} \quad (\text{B.46})$$

$$z_{R,t} = \rho_R z_{R,t-1} + \varepsilon_{R,t} \quad (\text{B.47})$$

$$z_{B,t} = \rho_B z_{B,t-1} + \varepsilon_{B,t} \quad (\text{B.48})$$

Structural composite parameters that follow from the steady state solution:

$$\gamma_j^1 = \frac{1 - (1 - \phi_j) (\gamma_j^{ex} - \gamma_j^{im})}{1 - \phi_j}$$

$$\gamma_j^2 = \frac{\phi_j}{1 - \phi_j}$$

$$\gamma_j^c = 1 - \gamma_j^i - \sum_{l=1}^{\mathcal{J}} \gamma_j^l$$

$$\gamma_j^i = \frac{\delta (1 - \phi_j - \psi_j)}{[\beta^{-1} - (1 - \delta)] [1 - (1 - \phi_j) (\gamma_j^{ex} - \gamma_j^{im})]}$$

$$\gamma_j^m = \frac{\zeta_{jl} \phi_l \frac{\mu_l}{\psi_l}}{[1 - (1 - \phi_j) (\gamma_j^{ex} - \gamma_j^{im})] \frac{\mu_j}{\psi_j}}$$

$$\gamma_j^k = \frac{(1 - \phi_j - \psi_j) \frac{\mu_j}{\psi_j}}{\sum_{l=1}^{\mathcal{J}} (1 - \phi_l - \psi_l) \frac{\mu_l}{\psi_l}}$$

$$\gamma_j = \frac{(1 - \phi_j) \frac{\mu_j}{\psi_j}}{\sum_{l=1}^{\mathcal{J}} (1 - \phi_l) \frac{\mu_l}{\psi_l}}$$

$$\alpha_{xj} = 1 - \gamma_j^{ex} (1 - \phi_j)$$

$$\alpha_j = \frac{\alpha_{xj}}{\alpha_{xj} + \gamma_j^{im} (1 - \phi_j)}$$

C THE ESTIMATION PROCEDURE

In this appendix, I explain the estimation procedure in detail. A more general introduction to Bayesian estimation and Markov Chain methods can be found in e.g. Koop (2003) and Bauwens, Lubrano, and Richard (1999).

C.1 PRELIMINARIES

Before estimation I log-linearize all optimality conditions and resource constraints around the non-stochastic steady state of the model (see Appendix B). The full linear model is solved numerically for the rational expectations solution by means of a generalized Schur decomposition (see Klein (2000)). The resulting policy function is finally combined with data to form a state-space representation:

$$\tilde{y}_t = \mathcal{A}\tilde{y}_{t-1} + \mathcal{B}\varepsilon_{1,t} \tag{C.1}$$

$$y_t^* = \mathcal{C}\tilde{y}_t + \varepsilon_{2,t} \tag{C.2}$$

$$\mathbb{E}(\varepsilon_{1,t}) = \mathbb{E}(\varepsilon_{2,t}) = \mathbb{E}(\varepsilon_{1,t}\varepsilon'_{2,s}) = 0 \quad \forall (s, t)$$

$$\mathbb{E}(\varepsilon_{1,t}\varepsilon'_{1,s}) = \mathcal{M}\delta_{ts}$$

$$\mathbb{E}(\varepsilon_{2,s}\varepsilon'_{2,t}) = \mathcal{N}\delta_{ts}$$

\tilde{y}_t denotes the time t vector of all the choice variables (the policy function), y_t^* the vector of observables of sample size T , and $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ the vectors of structural shocks and measurement errors. $\mathcal{A} = \mathcal{A}(\Theta)$ and $\mathcal{B} = \mathcal{B}(\Theta)$ are matrices with known functions of the structural parameters Θ . \mathcal{C} is a selection matrix that extracts the vectors of observables from \tilde{y}_t . δ_{ts} is the Kronecker delta. Equations (C.1) and (C.2) are referred to as the transition and measurement equations, respectively, and are used as basic building blocks during estimation.

C.2 BAYES' RULE AND THE LIKELIHOOD FUNCTION

The purpose of Bayesian estimation is to combine prior beliefs with information from the data to characterize a posterior distribution of structural parameters Θ . Denote the prior density by $p(\Theta)$, and the likelihood function of Θ given data by $\mathcal{L}(\Theta|y_T^*, \dots, y_1^*) \equiv p(y_T^*, \dots, y_1^*|\Theta)$. Bayes' theorem then allows us to write the posterior density as

$$\begin{aligned} p(\Theta|y_T^*, \dots, y_1^*) &= \frac{p(y_T^*, \dots, y_1^*|\Theta)p(\Theta)}{\int p(y_T^*, \dots, y_1^*|\Theta)p(\Theta) d\Theta} \\ &\propto p(y_T^*, \dots, y_1^*|\Theta)p(\Theta) \equiv \mathcal{K}(\Theta|y_T^*, \dots, y_1^*), \end{aligned} \tag{C.3}$$

where the integral is a constant that corresponds to the marginal data density. All posterior moments of interest can be computed given $\mathcal{K}(\Theta|y_T^*, \dots, y_1^*)$, which is referred to as the posterior kernel. However, this object must be approximated numerically, as no analytical solution is available for the likelihood function. To this end we consider a general likelihood function which can be factorized recursively to yield

$$p(y_T^*, \dots, y_1^*|\Theta) = p(y_1^*|\Theta) \prod_{t=2}^T p(y_t^*|y_{t-1}^*, \dots, y_1^*, \Theta).$$

In the case of a Normal likelihood function, the log likelihood follows as

$$\begin{aligned}\ln p(y_T^*, \dots, y_1^* | \Theta) &= \ln p(y_1^* | \Theta) + \sum_{t=2}^T \ln p(y_t^* | y_{t-1}^*, \dots, y_1^*, \Theta) \\ &= -\frac{Tn}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^T \ln |\Sigma_t| - \frac{1}{2} \sum_{t=1}^T x_t^{*'} \Sigma_t^{-1} x_t^*,\end{aligned}\quad (\text{C.4})$$

where $x_t^* = y_t^* - \mathbb{E}(y_t^* | y_{t-1}^*, \dots, y_1^*, \Theta)$ is the one-step-ahead prediction error of the data and $\Sigma_t = \mathbb{E}(x_t^* x_t^{*'})$ is the conditional variance of the prediction error. The log likelihood can be evaluated given information about x_t^* and Σ_t .

C.3 THE KALMAN FILTER

We use the Kalman filter to derive x_t^* and Σ_t . Define $y_{t|t-1}^* \equiv \mathbb{E}(y_t^* | y_{t-1}^*, \dots, y_1^*, \Theta)$, $\tilde{y}_{t|t-1} \equiv \mathbb{E}(\tilde{y}_t | y_{t-1}^*, \dots, y_1^*, \Theta)$, and $\mathcal{P}_{t|t-1} \equiv \mathbb{E}\left[(\tilde{y}_{t|t} - \tilde{y}_{t|t-1})(\tilde{y}_{t|t} - \tilde{y}_{t|t-1})'\right]$. It follows from this notation and equation (C.1) that

$$\tilde{y}_{t|t-1} = \mathcal{A}\tilde{y}_{t-1|t-1} \quad (\text{C.5})$$

$$\mathcal{P}_{t|t-1} = \mathcal{A}\mathcal{P}_{t-1|t-1}\mathcal{A}' + \mathcal{B}\mathcal{M}\mathcal{B}' \quad (\text{C.6})$$

These two are referred to as prediction equations in the Kalman filter. Furthermore, equation (C.2) implies $y_{t|t-1}^* = \mathcal{C}\tilde{y}_{t|t-1}$. Thus, using (C.2) we can write

$$x_t^* = y_t^* - y_{t|t-1}^* = y_t^* - \mathcal{C}\tilde{y}_{t|t-1} \quad (\text{C.7})$$

$$\Sigma_t = \mathbb{E}\left((y_t^* - y_{t|t-1}^*)(y_t^* - y_{t|t-1}^*)'\right) = \mathcal{C}\mathcal{P}_{t|t-1}\mathcal{C}' + \mathcal{N}. \quad (\text{C.8})$$

The filter is completed with the time t updates $\tilde{y}_{t|t}$ and $\mathcal{P}_{t|t}$. To this end we use the identity $\tilde{y}_t = \tilde{y}_{t|t-1} + (\tilde{y}_t - \tilde{y}_{t|t-1})$, and combine (C.2) with (C.7) to get $x_t^* = \mathcal{C}(\tilde{y}_t - \tilde{y}_{t|t-1}) + \epsilon_{2,t}$. It follows from these expressions and the definition of $\mathcal{P}_{t|t-1}$ that

$$\begin{pmatrix} x_t^* \\ \tilde{y}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ \tilde{y}_{t|t-1} \end{pmatrix}, \begin{pmatrix} \Sigma_t & \mathcal{C}\mathcal{P}_{t|t-1} \\ \mathcal{P}_{t|t-1}\mathcal{C}' & \mathcal{P}_{t|t-1} \end{pmatrix}\right).$$

The rule for the conditional Normal distribution (see e.g. Bauwens et al. (1999), Theorem A.12 p. 299) allows us to write the distribution of \tilde{y}_t given x_t^* and past data as

$$\mathcal{L}(\tilde{y}_t | x_t^*, y_{t-1}^*, \dots, y_1^*, \Theta) = N(\tilde{y}_{t|t-1} + \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}x_t^*, \mathcal{P}_{t|t-1} - \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}\mathcal{C}\mathcal{P}_{t|t-1}).$$

Note that x_t^* contains the time t information y_t^* . Thus, $\tilde{y}_{t|t}$ and $\mathcal{P}_{t|t}$ are just the two moments above:

$$\tilde{y}_{t|t} = \tilde{y}_{t|t-1} + \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}x_t^* \quad (\text{C.9})$$

$$\mathcal{P}_{t|t} = \mathcal{P}_{t|t-1} - \mathcal{P}_{t|t-1}\mathcal{C}'\Sigma_t^{-1}\mathcal{C}\mathcal{P}_{t|t-1} \quad (\text{C.10})$$

These are referred to as the updating equations. The system (C.5)-(C.6) and (C.9)-(C.10) constitute the Kalman filter. The recursive nature of the filter allows us to successively obtain x_t^* and Σ_t , where the output from (C.9) and (C.10) is used as input in (C.5) and (C.6) the next period.²³ Once the series $\{x_t^*\}_{t=1}^T$ and $\{\Sigma_t\}_{t=1}^T$ are in place, we are ready to evaluate the log-likelihood function (C.4) for any given parameter vector Θ .

²³Starting values $\tilde{y}_{1|0}$ and $\mathcal{P}_{1|0}$ are set to the unconditional mean and variance of \tilde{y} , and are calculated using equation (C.1).

C.4 THE POSTERIOR DISTRIBUTION AND THE RWMH ALGORITHM

The last step of the estimation procedure is to obtain estimates of posterior moments of interest, in particular measures of central tendency and variability. For that we need to characterize the entire posterior distribution of Θ . It is analytically intractable, so Markov Chain Monte Carlo (MCMC) techniques are used for this purpose. The posterior mode, denoted Θ_m , is first found by numerical optimization of $\mathcal{K}(\Theta|y_T^*, \dots, y_1^*)$. I use a Metropolis-Hastings-type optimization routine to find the mode (see below). The variance of Θ_m is calculated from the inverse of the negative Hessian evaluated at Θ_m :

$$\Sigma_m = (E[H(\Theta_m)])^{-1} = \left(- \left[\frac{\partial^2 \ln(\mathcal{K}(\Theta|y_T^*, \dots, y_1^*))}{\partial \Theta \partial \Theta'} \right]_{\Theta=\Theta_m} \right)^{-1}$$

The variance of each estimate in Θ_m is just the diagonal elements of Σ_m .

The posterior distribution is simulated using the Random Walk Metropolis Hastings (RWMH) algorithm. The general idea, in the words of Canova (2007) is “to specify a transition kernel for a Markov Chain such that starting from some initial value and iterating a number of times, we produce a limiting distribution which is the target distribution we need to sample from”. The RWMH algorithm is stated below:

1. Choose starting point $\Theta^{(0)}$ (I use the posterior mode). For $s = 1, \dots, S$, run a loop over steps 2-4.
2. Draw a proposal $\hat{\Theta}^{(s)}$ from the jumping distribution

$$\mathcal{J}(\hat{\Theta}^{(s)}|\Theta^{(s-1)}) = N(\Theta^{(s-1)}, c\Sigma_m),$$

where Σ_m is the covariance matrix evaluated at the posterior mode and c is a scaling factor of the covariance matrix.

3. Compute $\mathcal{K}(\hat{\Theta}^{(s)}|y_T^*, \dots, y_1^*)$ and the acceptance ratio defined as

$$r = \frac{p(\hat{\Theta}^{(s)}|y_T^*, \dots, y_1^*)}{p(\Theta^{(s-1)}|y_T^*, \dots, y_1^*)} = \frac{\mathcal{K}(\hat{\Theta}^{(s)}|y_T^*, \dots, y_1^*)}{\mathcal{K}(\Theta^{(s-1)}|y_T^*, \dots, y_1^*)}.$$

4. Accept the proposal $\hat{\Theta}^{(s)}$ with probability $\min(r, 1)$. Set $\Theta^{(s)} = \hat{\Theta}^{(s)}$ if $\hat{\Theta}^{(s)}$ is accepted, and $\Theta^{(s)} = \Theta^{(s-1)}$ otherwise.
5. Build a histogram of the retained values of Θ . Let this be the final approximation of the posterior distribution.

Step 4 implies that we accept all draws that make us move to a more dense part of the posterior. However, we also accept some draws with lower density. The idea is to frequently visit the region of the parameter space with high probability, while at the same time visit as much as possible of the space. Common practice in the literature is to set the scaling factor c such that the acceptance ratio lies somewhere between 20% and 40%. I tune c to get an acceptance ratio of around 30%. Finally, step 5 provides us with an estimate of the full posterior distribution which can be used for Bayesian inference.

D DATA

A number of macroeconomic time series are used to construct quarterly data in both economies for (sector-level and aggregate) GDP, private consumption expenditures, private investment, the nominal interest rate, inflation, hours, and the real exchange rate. The data used for estimation are constructed as follows: Sector-level GDP series, which in the raw data are observed at an annual frequency, are interpolated to obtain quarterly series using piecewise cubic Hermite interpolating polynomials. GDP, consumption and investment expenditures are all deflated by the implicit CPI deflator to make the series model consistent. Investment is calculated as the sum of private gross fixed capital formation and change in stocks. CPI inflation is constructed as the ratio between current and lagged CPI deflator. Interest rates are divided by 4 to recast them into quarterly numbers. Hours worked (per week) in Canada is divided by total number of employed persons to get weekly hours per person. This makes the variable comparable with US hours. The real exchange rate is defined as the nominal exchange rate times the ratio of US CPI to Canadian CPI. GDP, consumption, investment and hours are divided by the labor force to render the variables model consistent. All variables except for the interest rates are logged and multiplied by 100 before estimation. All variables except for interest rates are also seasonally adjusted at the source. Data are HP-filtered in the benchmark estimation to remove non-stationary trends. Raw data are taken from the datasets listed below:

Table D.1: Data set/series – Raw data

<i>Variable</i>	<i>Canada</i>	<i>United States</i>
GDP (sector and aggregate)	CANSIM 379-0023	GDPbyInd_VA_NAICS
Consumption	CANPFCEQDSMEI	USAPFCEQDSMEI
Gross fixed capital formation	CANSIM 380-0068	GDPI
Change in stocks	CANSIM 380-0069	GDPI
Implicit CPI deflator	CANPCEDEFQISNAQ	USAPCEDEFQISNAQ
Implicit GDP deflator	CANGDPDEFQISMEI	USAGDPDEFQISMEI
Interest rate	INTGSTCAM193N	FEDFUNDS
Hours	CANSIM 383-0008	PRS85006023
Employment (females)	CANEMPFEMQDSMEI	USAEMPFEMQDSMEI
Employment (males)	CANEMPMALQDSMEI	USAEMPMALQDSMEI
Labor force	CANLFTOTQDSMEI	USALFTOTQDSMEI
Exchange rate	EXCAUS	–

Domestic variables are defined as follows before detrending (LF denotes the labor force):

- $\bullet \log(GDP_{j,t}) = \log\left(\frac{C379-0023_t}{PCEDEF_t LFTOT_t}\right)$
- $\bullet \log(N_t) = \log\left(\frac{C383-0008_t (EMPFEM_t + EMPMAL_t)}{LFTOT_t}\right)$
- $\bullet \log(C_t) = \log\left(\frac{PFCE_t}{PCEDEF_t LFTOT_t}\right)$
- $\bullet \log(R_t) = \frac{INTGST_t}{400}$
- $\bullet \log\left(\frac{P_t^i}{P_t} I_t\right) = \log\left(\frac{C380-0068_t + C380-0069_t}{PCEDEF_t LFTOT_t}\right)$
- $\bullet \log(\Pi_t) = \log\left(\frac{PCEDEF_t}{PCEDEF_{t-1}}\right)$
- $\bullet \log(\mathcal{S}_t) = \log\left(\frac{EXCAUS_t USAPCEDEF_t}{CANPCEDEF_t}\right)$

E ADDITIONAL TABLES

Table E.1: Sectoral variance decomposition of foreign shocks (%)

<i>Variable</i>	<i>Sector</i>	<i>All foreign shocks</i>	<i>Decomposition</i>							
			σ_{A1}^*	σ_{A2}^*	σ_{A3}^*	σ_R^*	σ_I^*	σ_U^*	σ_N^*	σ_M^*
			Long-run horizon							
GDP	1	62.30	10.37	31.03	2.94	0.17	11.92	0.32	0.00	5.55
	2	65.88	32.67	22.27	1.94	0.07	7.17	0.11	0.00	1.65
	3	81.75	32.37	33.34	2.20	0.07	11.69	0.10	0.00	1.98
Consumption	1	91.52	55.32	23.27	1.74	0.05	9.98	0.10	0.00	1.06
	2	84.44	29.21	37.48	2.41	0.08	13.24	0.12	0.00	1.90
	3	68.42	23.82	30.00	2.02	0.05	11.30	0.06	0.00	1.17
Investment	1	54.31	31.12	11.72	1.03	0.03	9.10	0.21	0.00	1.09
	2	45.18	16.95	15.74	1.12	0.04	9.67	0.22	0.00	1.45
	3	38.77	14.84	12.57	0.97	0.03	9.15	0.20	0.00	1.02
Hours	1	60.23	21.48	15.45	1.73	0.15	16.95	0.48	0.00	3.99
	2	36.41	12.18	11.06	1.28	0.11	9.65	0.31	0.00	1.82
	3	24.56	10.14	7.26	0.59	0.08	4.88	0.11	0.00	1.52
Inflation	1	88.91	78.72	2.55	0.87	0.10	2.99	0.32	0.00	3.36
	2	54.92	15.63	19.96	1.49	0.39	3.73	0.51	0.00	13.21
	3	22.00	6.79	7.24	0.87	0.02	6.58	0.16	0.00	0.36
Wage	1	69.14	16.06	39.28	3.02	0.06	8.80	0.16	0.00	1.75
	2	74.08	30.79	29.85	2.26	0.06	9.75	0.09	0.00	1.28
	3	88.67	32.12	38.56	2.62	0.07	13.68	0.10	0.00	1.51
Trade balance	1	46.91	6.00	8.47	1.74	0.13	22.12	0.56	0.00	7.88
	2	31.76	1.75	3.96	1.12	0.11	23.66	0.55	0.00	0.61
	3	18.21	0.73	2.39	3.09	0.39	6.17	4.46	0.00	0.97
Materials	1	74.70	20.19	32.91	3.12	0.16	11.68	0.22	0.00	6.41
	2	81.21	46.10	23.26	1.94	0.07	7.72	0.11	0.00	2.01
	3	76.20	30.13	30.80	2.06	0.08	11.16	0.08	0.00	1.89
Terms of trade	1	93.55	88.68	1.07	0.48	0.01	2.53	0.07	0.00	0.71
	2	58.03	1.68	40.75	1.84	0.14	7.72	0.14	0.00	5.77
	3	36.96	0.76	5.98	14.89	0.52	13.60	0.12	0.00	1.09

Table E.2: Forecast error variance decomposition of domestic shocks (%)

Variable	All domestic shocks	Decomposition								
		σ_{A1}	σ_{A2}	σ_{A3}	σ_R	σ_I	σ_U	σ_N	σ_M	σ_B
Panel A: 1-quarter horizon										
GDP	78.62	0.02	10.05	24.25	15.08	24.23	4.29	0.01	0.21	0.49
Consumption	89.46	0.01	1.19	8.49	8.81	1.61	62.60	0.00	1.83	4.93
Investment	77.52	0.00	3.68	1.56	1.37	54.14	0.26	0.00	1.99	14.52
Hours	82.20	0.01	2.54	14.68	17.40	25.46	4.34	0.04	15.36	2.37
Interest	63.37	0.01	3.23	2.44	13.09	8.12	0.85	0.00	10.55	25.08
Inflation	58.81	0.02	6.64	8.39	14.65	3.85	0.47	0.00	12.54	12.24
Wage	52.52	0.02	6.52	13.09	1.90	0.03	0.21	0.14	19.19	11.42
Trade balance	62.12	0.00	0.67	1.20	0.19	19.75	1.10	0.00	2.38	36.84
Panel B: 4-quarter horizon										
GDP	52.24	0.01	6.50	19.13	4.60	13.05	1.26	0.01	0.38	7.31
Consumption	80.43	0.01	1.58	24.41	7.29	2.50	35.43	0.00	1.37	7.83
Investment	65.34	0.01	6.11	2.13	0.66	31.32	0.41	0.00	1.16	23.55
Hours	77.04	0.01	2.08	6.52	19.70	28.00	3.20	0.12	14.63	2.78
Interest	62.14	0.01	1.88	9.78	7.55	14.38	1.24	0.00	5.38	21.92
Inflation	59.08	0.01	4.86	10.30	17.81	4.20	0.47	0.00	9.64	11.79
Wage	46.75	0.01	3.77	22.42	0.97	0.43	0.10	0.16	7.40	11.50
Trade balance	65.68	0.00	1.45	1.04	0.57	20.52	0.49	0.00	2.50	39.10
Panel C: 8-quarter horizon										
GDP	47.17	0.01	6.74	19.77	2.86	8.84	0.74	0.01	0.24	7.96
Consumption	80.14	0.01	1.73	38.11	5.77	2.14	24.60	0.01	0.95	6.82
Investment	62.08	0.01	8.45	2.56	0.49	23.61	0.47	0.01	0.86	25.63
Hours	70.16	0.01	2.58	5.97	17.83	24.52	2.72	0.16	12.44	3.93
Interest	63.61	0.01	1.74	15.19	6.73	14.38	1.20	0.00	4.77	19.58
Inflation	58.12	0.01	4.72	10.56	17.27	4.25	0.46	0.00	9.39	11.46
Wage	46.61	0.01	3.81	27.43	0.53	1.00	0.07	0.11	4.19	9.47
Trade balance	68.04	0.00	2.56	1.79	0.52	17.73	0.47	0.00	2.14	42.83
Panel D: 20-quarter horizon										
GDP	36.89	0.01	6.04	16.35	1.92	6.00	0.54	0.01	0.20	5.83
Consumption	59.93	0.01	3.10	30.13	3.20	5.35	13.38	0.01	0.52	4.24
Investment	56.72	0.01	7.47	2.18	0.43	21.32	0.39	0.00	0.90	24.03
Hours	69.31	0.01	2.63	6.08	17.08	24.03	2.61	0.17	12.02	4.70
Interest	65.05	0.01	1.77	17.84	6.31	14.63	1.15	0.00	4.46	18.89
Inflation	57.52	0.01	4.68	10.61	16.87	4.41	0.45	0.00	9.17	11.32
Wage	33.04	0.01	3.55	20.79	0.26	1.05	0.11	0.05	2.02	5.19
Trade balance	66.46	0.00	2.45	1.67	0.47	17.98	0.40	0.00	2.09	41.40
Panel E: Long-run horizon										
GDP	26.14	0.01	4.20	11.17	1.31	4.08	0.38	0.00	0.15	4.85
Consumption	24.31	0.00	1.84	10.87	1.13	2.15	4.79	0.00	0.23	3.30
Investment	55.45	0.01	7.10	2.15	0.39	19.73	0.35	0.00	0.83	24.90
Hours	65.92	0.01	2.78	5.92	15.91	22.58	2.45	0.15	11.20	4.92
Interest	56.89	0.01	1.84	15.54	5.39	12.71	1.00	0.00	3.81	16.58
Inflation	53.91	0.01	4.41	9.93	15.76	4.13	0.43	0.00	8.57	10.68
Wage	16.85	0.00	1.90	9.63	0.12	0.52	0.09	0.02	0.94	3.62
Trade balance	67.75	0.00	2.76	1.76	0.45	16.98	0.37	0.00	1.95	43.48

Table E.3: Priors and posterior results – Structural parameters

	Prior distribution			Posterior distribution				
	Type	Mean	Std.	Mode	Std.	Mean	5%	95%
χ_C	B	0.500	0.100	0.439	0.060	0.459	0.339	0.580
ϵ_I	N	5.000	1.000	0.247	0.088	0.334	0.162	0.496
η	G	1.000	0.150	0.930	0.066	0.947	0.830	1.061
θ_w	B	0.700	0.075	0.343	0.047	0.370	0.248	0.484
θ_p	B	0.700	0.075	0.483	0.033	0.494	0.429	0.559
l_w	B	0.500	0.150	0.331	0.118	0.388	0.148	0.614
l_p	N	0.500	0.150	0.110	0.059	0.143	0.044	0.239
ρ_r	N	0.600	0.050	0.660	0.023	0.671	0.610	0.731
ρ_π	N	1.800	0.200	1.835	0.154	1.869	1.599	2.142
ρ_y	N	0.125	0.050	0.025	0.012	0.032	0.013	0.050
ρ_{dy}	B	0.125	0.050	0.189	0.038	0.186	0.112	0.256
ρ_e	N	0.100	0.050	0.049	0.027	0.055	0.002	0.106
χ_C^*	B	0.500	0.100	0.429	0.067	0.457	0.327	0.587
ϵ_I^*	B	5.000	1.000	0.173	0.059	0.225	0.110	0.334
θ_w^*	B	0.700	0.075	0.257	0.051	0.296	0.209	0.372
θ_p^*	B	0.700	0.075	0.512	0.033	0.526	0.467	0.587
l_w^*	B	0.500	0.150	0.282	0.102	0.331	0.121	0.541
l_p^*	B	0.500	0.150	0.108	0.059	0.139	0.044	0.235
ρ_r^*	B	0.600	0.050	0.672	0.028	0.679	0.623	0.737
ρ_π^*	N	1.800	0.200	1.851	0.133	1.878	1.626	2.137
ρ_y^*	N	0.125	0.050	0.019	0.012	0.025	0.010	0.041
ρ_{dy}^*	N	0.125	0.050	0.234	0.028	0.241	0.173	0.309
ρ_A	B	0.700	0.100	0.881	0.032	0.875	0.823	0.930
ρ_R	B	0.700	0.100	0.256	0.045	0.270	0.187	0.350
ρ_I	B	0.700	0.100	0.583	0.057	0.563	0.449	0.681
ρ_U	B	0.700	0.100	0.483	0.082	0.489	0.344	0.638
ρ_N	B	0.700	0.100	0.720	0.084	0.692	0.530	0.851
ρ_M	B	0.700	0.100	0.506	0.079	0.488	0.357	0.620
ρ_B	B	0.700	0.100	0.824	0.050	0.807	0.728	0.893
ρ_A^*	B	0.700	0.100	0.900	0.049	0.886	0.823	0.954
ρ_R^*	B	0.700	0.100	0.309	0.050	0.324	0.229	0.415
ρ_I^*	B	0.700	0.100	0.657	0.053	0.638	0.534	0.745
ρ_U^*	B	0.700	0.100	0.555	0.075	0.552	0.400	0.705
ρ_N^*	B	0.700	0.100	0.721	0.091	0.689	0.526	0.847
ρ_M^*	B	0.700	0.100	0.679	0.089	0.629	0.492	0.769

Note: B stands for Beta, N Normal, G Gamma. The two last columns report 90% posterior probability bands obtained from the MCMC simulation. See Table E.4 for the marginal data density.

Table E.4: Priors and posterior results – Shocks

	Prior distribution			Posterior distribution				
	Type	Mean	Std.	Mode	Std.	Mean	5%	95%
$100\sigma_A$	IG	0.20	2.00	0.928	0.057	0.940	0.827	1.052
$100\sigma_R$	IG	0.20	2.00	0.271	0.027	0.273	0.226	0.320
$100\sigma_I$	IG	0.50	2.00	1.021	0.261	1.290	0.791	1.747
$100\sigma_U$	IG	0.20	2.00	1.085	0.133	1.185	0.894	1.475
$100\sigma_N$	IG	0.50	2.00	0.233	0.190	0.406	0.121	0.711
$100\sigma_M$	IG	0.50	2.00	1.215	0.189	1.321	1.033	1.602
$100\sigma_B$	IG	0.20	2.00	0.479	0.104	0.522	0.350	0.693
$100\sigma_A^*$	IG	0.20	2.00	0.667	0.045	0.677	0.599	0.755
$100\sigma_R^*$	IG	0.20	2.00	0.154	0.016	0.156	0.128	0.184
$100\sigma_I^*$	IG	0.50	2.00	0.645	0.124	0.767	0.521	0.998
$100\sigma_U^*$	IG	0.20	2.00	0.806	0.117	0.891	0.657	1.115
$100\sigma_N^*$	IG	0.50	2.00	0.234	0.216	0.398	0.118	0.708
$100\sigma_M^*$	IG	0.50	2.00	0.732	0.094	0.785	0.624	0.949
$100\sigma_e$	IG	0.20	1.00	1.540	0.100	1.570	1.382	1.751
$100\sigma_e^*$	IG	0.20	1.00	0.908	0.062	0.921	0.815	1.027
MDD				-1522.111		-1510.582		

Note: IG stands for Inverse Gamma 1. The two last columns report 90% posterior probability bands obtained from the MCMC simulation. The marginal data density (MDD) is estimated using i) a Laplace approximation based on the posterior mode, and ii) the modified harmonic mean estimator based on draws from the simulated Markov chains.