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AUTHORS:

CLAUDIA FORONI,
PIERRE GUÉRIN AND
MASSIMILIANO
MARCELLINO

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Using Low Frequency Information for Predicting High Frequency Variables *

Claudia Foroni † Pierre Guérin‡ Massimiliano Marcellino§

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Abstract

We analyze how to incorporate low frequency information in models for predicting high frequency variables. In doing so, we introduce a new model, the reverse unrestricted MIDAS (RU-MIDAS), which has a periodic structure but can be estimated by simple least squares methods and used to produce forecasts of high frequency variables that also incorporate low frequency information. We compare this model with two versions of the mixed frequency VAR, which so far had been only applied to study the reverse problem, that is, using the high frequency information for predicting low frequency variables. We then implement a simulation study to evaluate the relative forecasting ability of the alternative models in finite samples. Finally, we conduct several empirical applications to assess the relevance of quarterly survey data for forecasting a set of monthly macroeconomic indicators. Overall, it turns out that low frequency information is important, particularly so when it is just released.

Keywords: Mixed-Frequency VAR models, temporal aggregation, MIDAS models.

JEL Classification Code: E37, C53.

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†Norges Bank

‡Bank of Canada

§Bocconi University, IGIER and CEPR

1 Introduction

A large and increasing literature is dealing with models that explicitly account for data of different frequencies, especially in a forecasting context. Different classes of models have been developed to tackle the different sampling frequencies at which macroeconomic and financial indicators are available. First, a common choice is to cast the model in state space form and use the Kalman filter for estimation and forecasting (see, among many others, Mariano and Murasawa (2003), Giannone et al. (2008), Aruoba et al. (2009)). Alternatively, in a Bayesian context, the estimation of models with mixed-frequency data has been studied, for example, by Eraker et al. (2015) and Schorfheide and Song (2015). Second, Ghysels (2014) introduced a different class of mixed-frequency VAR model, in which the vector of dependent variables includes both high-frequency and low-frequency variables, with the former stacked according to the timing of the data release (see also Blasques et al. (2014) for an application in the context of a small-scale factor model). The model can be estimated by simple OLS but stacking increases the number of regressors. Bayesian methods can be used to handle this issue when the number of degrees of freedom is very small. Finally, in a univariate context, the MIDAS approach by Ghysels et al. (2006) directly links low-frequency to high-frequency data. MIDAS models have been extensively used in macroeconomic forecasting (see, e.g., Clements and Galvao (2008) and Clements and Galvao (2009) for early contributions). MIDAS models, being non-linear, require a form of NLS estimation, which increases substantially the computational costs in specifications with more than one high frequency explanatory variable. The unrestricted MIDAS, U-MIDAS, model of Foroni et al. (2013b) can instead be estimated by simple OLS and therefore handle several high frequency explanatory variables. However, it works particularly well only when the frequency mismatch is low, for example in the quarterly / monthly case.

The attention in the literature so far has mostly been on how to exploit high-frequency information to improve forecasts of low-frequency variables. One notable exception is Dal Bianco et al. (2012), who analyze how macroeconomic fundamentals can improve the forecasts of the euro-dollar exchange rate at the weekly frequency. In doing so, they estimate a mixed-frequency VAR cast in state-space form à la Mariano and Murasawa (2010).

The aim of our paper is to study in a systematic way how to incorporate low-frequency information in forecasting models for high-frequency variables. We propose a new model where the high-frequency dependent variable is dynamically linked with the low-frequency explanatory variable(s). We denote this model as Reverse Unrestricted MIDAS (RU-MIDAS), since it is the counterpart of the U-MIDAS model of Foroni et al. (2013b). In particular, we also discuss how RU-MIDAS regressions can be derived in a general linear dynamic framework, following similar steps to the analysis presented in Foroni et al. (2013b) for the U-MIDAS regression. Next, this new model is analyzed in comparison with two other standard mixed-frequency methods: a mixed-frequency VAR cast in state-space form (see, e.g., Mariano and Murasawa (2010)), in which the low-frequency series are treated as high-frequency variables with missing observations, and the mixed-frequency VAR suggested by Ghysels (2014), where the high-frequency and low-frequency variables are stacked together in the vector of dependent variables. Both methods are well known but, as mentioned, they have been mostly applied to the analysis of low-frequency variables, while here we focus on predicting high-frequency variables.

We then evaluate the forecasting ability of the different models in a Monte Carlo study, focusing on the monthly / quarterly case that is particularly relevant for macroeconomic applications. We consider different data generating processes (DGPs), based on bivariate VARs either at high frequency or at mixed-frequency. We find that the forecasting performance for the monthly variable of all the models we consider drastically improves right after the release of the quarterly variable. In the same vein, the forecasting performance deteriorates as we move away from the release of the quarterly variable and the monthly AR benchmark becomes difficult to beat. Moreover, there is no clear ranking of the different low-to-high frequency models, since their relative performance varies across horizons and specifications.

Finally, to assess the performance of the alternative methods in empirical applications, we evaluate the relevance of surveys available only at quarterly frequency to forecast the corresponding monthly variables. It is well documented in the literature that expectations contained in survey data help in improving the accuracy of the forecasts for key macroeconomic variables (see, for example, Chun (2011), Chernov and Mueller (2012) and Stark (2010)). In particular, we use the Survey of Professional Forecasters (SPF), which is recognized as a very accurate survey (see Stark (2010), who performs an extensive evaluation of the SPF). Moreover, surveys of professional forecasters receive a lot of attention from both market participants and policy makers as they are often seen as a good proxy of agents' expectations and future economic outcomes. Ghysels and Wright (2009) use mixed-frequency data models to obtain high-frequency estimates of survey releases, and find they provide helpful estimates for future economic activity. In contrast, in this paper, we directly use the information of the quarterly survey (that is, without conducting any prior disaggregation) to improve the forecasts of important monthly U.S. variables: CPI inflation, industrial production growth and real personal consumption expenditures.

As a second application, we assess whether the SPF has predictive power for exchange rates. The literature on exchange rates is vast and generally agrees on the limited use of macroeconomic fundamentals to predict exchange rates, a feature also known as the Meese and Rogoff puzzle (see Meese and Rogoff (1983b), Engel and West (2005) and Rime et al. (2010) among others). Surveys instead represent expected future fundamentals, which can perform better in forecasting than the current macroeconomic fundamentals. In particular, we evaluate whether expectations on the three-month T-Bill rate and CPI inflation have predictive power for the trade-weighted average of the foreign exchange value of the U.S. Dollar against major currencies. This choice of predictors is motivated by the purchasing power parity and the uncovered interest rate parity relations (see Rossi (2013) for a review of the literature on exchange rate forecasting).[1]

In our empirical applications, we obtain satisfactory results from the use of low frequency (quarterly) survey information, particularly in forecasting monthly inflation and industrial production, at almost every horizon. Moreover, we succeed in outperforming the random walk benchmark in forecasting the monthly nominal U.S. trade-weighted exchange rate, especially when surveys on inflation are employed. However, it remains difficult to find a single preferred high frequency model, confirming the evidence from the Monte Carlo experiments.

The paper is organized as follows. In Section 2, we introduce the RU-MIDAS model. In Section 3, we present the alternative models we consider, i.e., the MF-VAR cast in state-space form and the MF-VAR in stacked form. In Section 4, we discuss the Monte Carlo experiments. In Section 5, we present the empirical applications that forecast monthly economic indicators using the quarterly SPF. In Section 6, we summarize the main results and conclude.

[1]We focus on the trade-weighted exchange rate since it is a relevant indicator of the competitiveness of a country. Moreover, for bilateral exchange rates, macroeconomic differentials (or expectations of macroeconomic differentials) would likely be more relevant than expectations related to the U.S. economy only. However, survey-based expectations for other countries are not available over a long enough sample or not fully comparable across countries, so that we refrained from considering the case of forecasting bilateral exchange rates with survey data.

2 RU-MIDAS

In this section, we derive the Reverse Unrestricted MIDAS (RU-MIDAS) regression approach from a general dynamic linear model, discuss its estimation, and consider the use of RU-MIDAS as a forecasting device.

Let us denote the high frequency (HF) time index as $t = 0, \frac{1}{k}, \frac{2}{k}, ..., \frac{k-1}{k}, 1, ...,$ where k denotes the frequency mismatch, and the low frequency (LF) time index as $t = 0, 1, 2, ...$ The variable x can be observed in HF (for each t) while the variable y^* can be only observed in LF (every k periods). More generally, if we denote by L the HF lag operator so that $L^j x_t = x_{t-j/k}$, then we can introduce the operator $\omega(L)$:

$$\omega(L) = \omega_0 + \omega_1 L + ... + \omega_{k-1} L^{k-1}, \tag{1}$$

which characterizes the temporal aggregation scheme applied to the y^* variable. For example, $\omega(L) = 1 + L + ... + L^{k-1}$ in the case of flow variables and $\omega(L) = 1$ for stock variables. What we observe in LF is $y_t = \omega(L) y_t^*$.

We assume that the variable x is generated by an AR(p) process with the variable y^* as an exogenous regressor:

$$c(L)x_t = d(L)y_t^* + e_{xt}, \tag{2}$$

where $d(L) = d_1 L + ... + d_p L^p$, $c(L) = I - c_1 L - ... - c_p L^p$, and the errors are white noise. For simplicity, we suppose that the starting values $y^*_{-p/k}, ..., y^*_{-1/k}$ and $x_{-p/k}, ..., x_{-1/k}$ are all fixed and equal to zero.[2]

Let us now introduce the LF lag operator, Z, with $Z = L^k$ so that $Z^j y_t = y_{t-j}$, and define a polynomial in the HF lag operator, $\gamma_0(L)$, such that the product $g_0(L) = \gamma_0(L)d(L)$ only contains powers of $L^k = Z$, so that $g_0(L) = g_0(L^k) = g_0(Z)$. Multiplying both sides of (2) by $\gamma_0(L)$ and $\omega(L)$, we get:

$$\begin{aligned} \gamma_0(L)\, c(L)\, \omega(L)\, x_t &= \gamma_0(L)\, d(L)\, \omega(L)\, y_t^* + \gamma_0(L)\, \omega(L)\, e_{xt}, \\ t &= 0, 1, 2, ... \end{aligned} \tag{3}$$

[2]The multivariate version where y_t or x_t or both are vectors, different lag lengths of the polynomials in $d(L)$ and $c(L)$, and an MA component, can be easily handled, but at the cost of an additional complication in the notation.

or

$$\begin{aligned}\widetilde{c}_0\left(L\right)x_t &= g_0(Z)y_t + \widetilde{\gamma}_0\left(L\right)e_{xt}, \\ t &= 0,1,2,...\end{aligned} \tag{4}$$

The specification in (4) is an example of an *exact reverse unrestricted MIDAS model.* A few comments are in order. First, in (4) the HF variable x depends on its own HF lags, on LF lags of the (observable) LF variable y, and on an error term that in general has an MA structure.

Second, the model specification depends on the particular HF period we are in. Specifically, for each $i = 0, ..., k-1$, we can define a polynomial in the HF lag operator, $\gamma_i(L)$, such that the product $g_i(L) = \gamma_i(L)d(L)$ only contains powers of L^{k-i}, so that multiplying both sides of (2) by $\gamma_i(L)$ and $\omega\left(L\right)$, we get:

$$\begin{aligned}\widetilde{c}_i\left(L\right)x_t &= g_i(L^{k-i})y_t + \widetilde{\gamma}_i\left(L\right)e_{xt}, \\ t &= 0+\frac{i}{k}, 1+\frac{i}{k}, 2+\frac{i}{k}, ... \\ i &= 0,...,k-1.\end{aligned} \tag{5}$$

Therefore, RU-MIDAS models have a *periodic* structure.

Third, it can be shown that the polynomials $\gamma_i\left(L\right)$ exist and can be analytically determined (see, e.g., Marcellino (1999) and the references therein).

Fourth, in practice the parameters of (2) are unknown so that $\gamma_i\left(L\right)$ cannot be exactly determined. Hence, empirically, we will use *approximate reverse unrestricted MIDAS* (RU-MIDAS) models based on linear lag polynomials, such as

$$\begin{aligned}\widetilde{a}_i\left(L\right)x_t &= b_i(L^{k-i})y_t + \xi_{it}, \\ t &= 0+\frac{i}{k}, 1+\frac{i}{k}, 2+\frac{i}{k}, ... \\ i &= 0,...,k-1.\end{aligned} \tag{6}$$

where the orders of $\widetilde{a}_i\left(L\right)$ and $b_i(L^{k-i})$, a_i and b_i respectively, are large enough to make ξ_{it} white noise. Moreover, the RU-MIDAS model in (6) can be easily extended to allow for additional HF or LF explanatory variables.

The estimation of the model is straightforward, since it is a linear model. Hence, for each value of i the parameters of the model (6) can be estimated by OLS, and the lag orders a_i and b_i can be selected with standard information criteria.

Alternatively, given that the error terms ξ_{it} are in general correlated across i, the RU-MIDAS equations for different values of i can be jointly estimated by means of a system estimation method. This can be also achieved by grouping the equations in (6) into a single equation with a proper set of dummy variables. As an example, consider the case where $k = 3$ (e.g., monthly and quarterly variables), and $a_i = 3$, $b_i = 1$. The single equation version of (6) is then

$$\begin{aligned} x_t &= \alpha_1 (1 - D_2 - D_3)\, y_{t-\frac{1}{3}} + \alpha_2 D_2 y_{t-\frac{2}{3}} + \alpha_3 D_3 y_{t-1} + \\ &\quad \beta_{11} (1 - D_2 - D_3)\, x_{t-\frac{1}{3}} + \beta_{12} D_2 x_{t-\frac{1}{3}} + \beta_{13} D_3 x_{t-\frac{1}{3}} + \\ &\quad \beta_{21} (1 - D_2 - D_3)\, x_{t-\frac{2}{3}} + \beta_{22} D_2 x_{t-\frac{2}{3}} + \beta_{23} D_3 x_{t-\frac{2}{3}} + \\ &\quad \beta_{31} (1 - D_2 - D_3)\, x_{t-1} + \beta_{32} D_2 x_{t-1} + \beta_{33} D_3 x_{t-1} + \\ &\quad + v_t, \\ t &= 0, \frac{1}{3}, \frac{2}{3}, 1, ..., \end{aligned} \tag{7}$$

where D_2 and D_3 are dummy variables taking the value one in, respectively, each third and first month of a quarter (so, respectively, for $t = \frac{2}{3}, 1 + \frac{2}{3}, 2 + \frac{2}{3}, ...$ and $t = 0, 1, 2, ...$). The model in (7) should be then estimated by GLS to allow for possible serial correlation and heteroskedasticity.[3]

To conclude, a major advantage of the RU-MIDAS specifications in (6) is that they are linear, so that standard techniques can be used for forecasting future values of the x variable using information from their past values and the past values of the LF variable y. The direct forecasting method (see, e.g., Marcellino et al. (2006)) is particularly useful in this context, since it does not require to forecast future values of the y variable, although it requires to change the model specification for each forecasting horizon.

3 Competing Models

An alternative approach to modeling the dependence of high-frequency on low-frequency variables is based on a mixed frequency VAR (MF-VAR), using its two most popular formulations. First, the typical MF-VAR, as in Mariano and Murasawa (2010), is cast in

[3]We do not expect any major efficiency gains from the use of the system estimation approach with respect to the univariate method, while model specification becomes more complex in the case of higher frequency mismatch or several explanatory variables. Hence, in the Monte Carlo and empirical applications we will focus on the univariate estimation approach.

state-space form, and the low-frequency series is treated as high-frequency series with missing observations. Second, the MF-VAR as described in Ghysels (2014), who decomposes each high-frequency variable into a set of k low-frequency variables and models them in low-frequency jointly with the low-frequency variable.

In the rest of this section, we provide details for both approaches and compare them to the RU-MIDAS introduced in the previous section. We anticipate that a key advantage of the RU-MIDAS is to provide an analytical relationship between high-frequency and low-frequency variables, and a model that directly operates in high-frequency, emphasizing the changing impact of the low-frequency variable on the high-frequency variable.

3.1 Mixed-Frequency VAR in state-space form

The mixed-frequency VAR cast in state-space form originates from the work of Mariano and Murasawa (2010), and was subsequently used in a forecasting exercise by Kuzin et al. (2011), see also Foroni et al. (2013a). Schorfheide and Song (2015) show how to estimate this model in a Bayesian framework, thereby making the estimation of relatively large scale models computationally feasible.

Focusing on the case studied by Mariano and Murasawa (2010), where y_t is quarterly GDP growth (a flow variable) and x_t is a monthly variable, so that $k = 3$, y_t is disaggregated at the monthly frequency via the geometric mean:

$$y_t = \frac{1}{3}y_t^* + \frac{2}{3}y_{t-\frac{1}{3}}^* + y_{t-\frac{2}{3}}^* + \frac{2}{3}y_{t-1}^* + \frac{1}{3}y_{t-\frac{4}{3}}^*, \tag{8}$$

where, as in the case of the RU-MIDAS model, y_t^* is the corresponding unobserved monthly variable.

The latent variable y_t^* and the monthly indicator x_t follow a bivariate VAR(p) process:

$$\mathbf{\Phi}(L)\,\mathbf{z}_t = \mathbf{u}_t, \qquad \mathbf{u}_t \sim i.i.d.N\left(\mathbf{0}, \mathbf{\Sigma}\right), \tag{9}$$

where $\mathbf{\Phi}(L) = I - \mathbf{\Phi}_1 L - ... - \mathbf{\Phi}_p L^p$ and

$$\mathbf{z}_t = \begin{bmatrix} y_t^* \\ x_t \end{bmatrix}.$$

Assuming that $p \leq 5$, defining the state vector of unobserved variables $\mathbf{s_t}$ as

$$\mathbf{s}_t = \begin{bmatrix} \mathbf{z}_t & \mathbf{z}_{t-\frac{1}{3}} & \mathbf{z}_{t-\frac{2}{3}} & \mathbf{z}_{t-1} & \mathbf{z}_{t-\frac{4}{3}} \end{bmatrix}',$$

and given the relationship between y_t and y_t^* in (8), the state-space representation of the MF-VAR(p) is given by the following transition and measurement equations:[4]

$$\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-\frac{1}{3}} + \mathbf{B}\mathbf{v}_t \tag{10}$$

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \mathbf{C}\mathbf{s}_t, \tag{11}$$

where $\mathbf{v}_t \sim i.i.d.N(\mathbf{0}, \mathbf{I}_2)$ and the system matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \end{bmatrix}, \quad \mathbf{A_1} = [\mathbf{\Phi_1} ... \mathbf{\Phi_p} \;\; \mathbf{0_{2*2(5-p)}}], \quad \mathbf{A_2} = [\mathbf{I_8} \;\; \mathbf{0_{8*2}}],$$

$$\mathbf{B} = \begin{bmatrix} \Sigma^{1/2} \\ 0_{8*2} \end{bmatrix}, \quad \mathbf{C} = [\mathbf{H_0} ... \mathbf{H_4}],$$

where the matrix $\mathbf{C}$ contains the coefficient matrices in the lag polynomial $\mathbf{H}(L) = \sum_{i=0}^{4} \mathbf{H}_i L^i$, which is defined according to the aggregation constraint defined in equation (8), and with $L^j x_t = x_{t-\frac{j}{k}}$ as in Section 2:

$$\mathbf{H}(L) = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{pmatrix} L + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} L^2 + \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{pmatrix} L^3 + \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 0 \end{pmatrix} L^4.$$

We refer to this model as the MF-VAR-KF.

In terms of parameter estimation for the MF-VAR-KF, the optimal procedure in this context is Maximum Likelihood, which can be easily implemented. As in Mariano and Murasawa (2010), we replace missing observations in the low-frequency variable with zeros and rewrite the measurement equation accordingly, as if the missing values were random draws from a Normal distribution $N(0, 1)$, independent of model parameters:

$$\begin{bmatrix} y_t^+ \\ x_t \end{bmatrix} = \begin{bmatrix} C_{1t} \\ C_2 \end{bmatrix} \mathbf{s}_t + \begin{bmatrix} D_{1t} \\ 0 \end{bmatrix} \mathbf{v}_t, \tag{12}$$

[4]For brevity, we do not report here the representation for $p > 5$. Details for this specification can be found in Mariano and Murasawa (2010). The state-space form can be easily modified to model stock rather than flow LF variables, or to consider alternative aggregation schemes.

where $\mathbf{v}_t$ is the random draw, and

$$C_{1t} = \begin{cases} C_1 & \text{if } y_t \text{ is observable} \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

$$D_{1t} = \begin{cases} 0 & \text{if } y_t \text{ is observable} \\ I & \text{otherwise} \end{cases}. \tag{14}$$

For further details on the estimation, we refer to Mariano and Murasawa (2010). For the VAR lag length choice, we use standard information criteria.

Moving to forecasting, from equation (10), the h-period-ahead forecast for the state vector is given by:

$$\hat{\mathbf{s}}_{t+h|t} = E[\mathbf{s}_{t+h}|(y_t, y_{t-1}, ..., y_1, x_t, x_{t-\frac{1}{3}}, ..., x_1)] = \mathbf{A}^h \hat{\mathbf{s}}_{t|t}, \tag{15}$$

and the associated forecast error is

$$(\mathbf{s}_{t+h} - \hat{\mathbf{s}}_{t+h|t}) = \mathbf{A}^h(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t}) + \mathbf{A}^{h-1}\mathbf{v}_{t+\frac{1}{3}} + \mathbf{A}^{h-2}\mathbf{v}_{t+\frac{2}{3}} + ... + \mathbf{A}\mathbf{v}_{t+h-\frac{1}{3}} + \mathbf{v}_{t+h}, \tag{16}$$

so that the forecast error variance for the state vector is:

$$\mathbf{P}_{t+h|t} = E[(\mathbf{s}_{t+h} - \hat{\mathbf{s}}_{t+h|t})(\mathbf{s}_{t+h} - \hat{\mathbf{s}}_{t+h|t})] = \mathbf{A}^h \mathbf{P}_{t|t}(\mathbf{A}^h)' + \mathbf{A}^{h-1}\mathbf{B}(\mathbf{A}^{h-1})' + ... + \mathbf{B}.$$

From equation (12), the h-period ahead forecast for the observable HF x variable, $\hat{x}_{t+h|t}$, can then be obtained from the following equation

$$\hat{\mathbf{z}}_{t+h|t} = \begin{bmatrix} \hat{y}_{t+h|t} \\ \hat{x}_{t+h|t} \end{bmatrix} = \mathbf{C}_{t+h}\hat{\mathbf{s}}_{t+h|t}.$$

Hence, the forecast error is the second element of the vector $\mathbf{C}_{t+h}(\mathbf{s}_{t+h} - \hat{\mathbf{s}}_{t+h|t})$, while the forecast error variance is the element (2,2) of the matrix $\mathbf{C}_{t+h}\mathbf{P}_{t+h|t}\mathbf{C}'_{t+h}$.

Finally, to compare the MF-VAR-KF and the RU-MIDAS forecasts, we note that in practice with the MF-VAR-KF we obtain the best estimates and forecasts for the HF variable y_t^* and then we iterate forward the VAR in (9) and take expectations conditional on the available information set to produce forecasts for x_t. This is similar to using equation (2), replacing y^* with interpolated values of y. Instead, in the (approximate) RU-MIDAS model, we use the observable LF y as explanatory variable, which generates the periodic structure of the model, and adopt a direct forecasting approach, which does not require the specification of a model for y (and y^*). Since the Kalman Filter produces the best linear forecasts in the MSE sense, the MF-VAR-KF should dominate the RU-MIDAS if we know the joint HF data generating process (DGP), that is, if the variables are generated by (9). However, in practice the DGP is unknown, so that the comparison between MF-VAR-KF and the RU-MIDAS is more complex and depends on the extent of the mis-specification of the assumed joint HF DGP, which prevents the derivation of general theoretical results. From the Monte Carlo simulations and empirical analyses reported in the following sections, there is indeed no clear winner.

3.2 Mixed-Frequency VAR in stacked form

The mixed-frequency VAR in stacked form (MF-VAR-SF), developed by Ghysels (2014), deals with the different data frequencies by stacking the HF variables in a LF vector of dependent variables, depending on the timing of their releases. Continuing the case of the previous subsection where there is only one quarterly variable y and one monthly variable x, the stacked vector is $\mathbf{q}_t = \left(x_{t-\frac{2}{3}}, x_{t-\frac{1}{3}}, x_t, y_t\right)'$, where $t = 1, 2, 3....$

Let us assume that $\mathbf{q}_t$ follows a VAR(p) process:

$$\mathbf{q}_t = \mathbf{A}_1\mathbf{q}_{t-1} + ... + \mathbf{A}_p\mathbf{q}_{t-p} + \mathbf{e}_t, \tag{17}$$

where $\mathbf{e}_t$ is white noise with $E\left(\mathbf{e}_t\mathbf{e}_t'\right) = \mathbf{V}$. The matrices $\mathbf{A}_i$, $i = 1, ..., p$, and $\mathbf{V}$ are subject to a set of restrictions when (17) is explicitly derived from a HF VAR such as the one described by equation (9), see Ghysels (2014) for additional details. However, typically these restrictions are not imposed to avoid dependence of the MF-VAR-SF from an unknown HF DGP, as in the case of the RU-MIDAS. Given that there are no missing observations in the system, this approach circumvents the estimation via the Kalman filter, and standard least squares methods can be used to estimate the MF-VAR-SF. Lag length selection can be based on information criteria.

Forecasting from this model seems simple, since it is a standard VAR. However, the timing in (17) is in LF, $t = 1, 2, 3...$, while we may also want to forecast in HF, e.g., update the forecast for x in each month of the quarter. This creates additional complications, and the formula for the optimal forecast changes periodically. We illustrate this issue, assuming that $p = 1$ and $h = \{1, 2, 3\}$ to simplify the notation.

The first possibility is that we want to forecast x_{t+1} without knowing $x_{t+\frac{1}{3}}$, $x_{t+\frac{2}{3}}$ and y_{t+1}, i.e., we are interested in $\widehat{x}_{t+1|t}$, which is a 3-step ahead forecast (or a 1-step ahead forecast in LF). This is the standard case where one just uses the MF-VAR-SF model in (17) and $\widehat{x}_{t+1|t}$ coincides with the third element of the vector $\widehat{\mathbf{q}}_{t+1|t}$, with

$$\widehat{\mathbf{q}}_{t+1|t} = \mathbf{A}_1\mathbf{q}_t. \tag{18}$$

Let us now assume that we have the same target, x_{t+1}, but we are in period $t+\frac{1}{3}$, so that we want to derive $\widehat{x}_{t+1|t+\frac{1}{3}}$, which is a 2-step ahead forecast. In this case we need to use the structural representation of the MF-VAR-SF, which describes the contemporaneous correlations among the variables. Therefore, let us define $\mathbf{P}$ as the matrix of contemporaneous correlations that is obtained via a Choleski decomposition

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ p_{21} & 1 & 0 & 0 \\ p_{31} & p_{32} & 1 & 0 \\ p_{41} & p_{42} & p_{43} & 1 \end{bmatrix}, \tag{19}$$

with $\mathbf{V} = \mathbf{P}^{-1}\mathbf{D}\left(\mathbf{P}^{-1}\right)'$, where $\mathbf{D}$ is a diagonal matrix. Premultiplying (17) by $\mathbf{P}$, we obtain the structural MF-VAR-SF representation:

$$\mathbf{P}\mathbf{q}_t = \mathbf{P}\mathbf{A}\mathbf{q}_{t-1} + \mathbf{u}_t, \tag{20}$$

or, equivalently,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ p_{21} & 1 & 0 & 0 \\ p_{31} & p_{32} & 1 & 0 \\ p_{41} & p_{42} & p_{43} & 1 \end{bmatrix} \begin{bmatrix} x_{t-\frac{2}{3}} \\ x_{t-\frac{1}{3}} \\ x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix} \begin{bmatrix} x_{t-\frac{5}{3}} \\ x_{t-\frac{4}{3}} \\ x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^{x1} \\ u_t^{x2} \\ u_t^{x3} \\ u_t^{y} \end{bmatrix}, \tag{21}$$

where $[\alpha_{ij}] = [\mathbf{PA}]_{ij}$ is the matrix of structural coefficients, and $\mathbf{u}_t = \mathbf{P}\mathbf{e}_t$ is the vector of structural errors.

If we focus on the dynamics of the x variable, we can see how it changes depending on the month of the quarter. In the first month of the quarter ($x_{t-\frac{2}{3}}$ or $x_{t-1+\frac{1}{3}}$), the dynamics of x_t is:

$$x_{t-\frac{2}{3}} = \alpha_{11}x_{t-\frac{5}{3}} + \alpha_{12}x_{t-\frac{4}{3}} + \alpha_{13}x_{t-1} + \alpha_{14}y_{t-1} + u_t^{x1}. \tag{22}$$

In the second month of the quarter ($x_{t-\frac{1}{3}}$ or $x_{t-1+\frac{2}{3}}$), the dynamics of x_t becomes:

$$x_{t-\frac{1}{3}} = \alpha_{21}x_{t-\frac{5}{3}} + \alpha_{22}x_{t-\frac{4}{3}} + \alpha_{23}x_{t-1} - p_{21}x_{t-\frac{2}{3}} + \alpha_{24}y_{t-1} + u_t^{x2}. \tag{23}$$

In the third month (x_t), the dynamics of x_t is:

$$x_t = \alpha_{31}x_{t-\frac{5}{3}} + \alpha_{32}x_{t-\frac{4}{3}} + \alpha_{33}x_{t-1} - p_{31}x_{t-\frac{2}{3}} - p_{32}x_{t-\frac{1}{3}} + \alpha_{34}y_{t-1} + u_t^{x3}. \tag{24}$$

Therefore, our 2-step ahead forecast of interest $\widehat{x}_{t+1|t+\frac{1}{3}}$ can be obtained as:

$$\widehat{x}_{t+1|t+\frac{1}{3}} = \alpha_{31}x_{t-\frac{2}{3}} + \alpha_{32}x_{t-\frac{1}{3}} + \alpha_{33}x_t + \alpha_{34}y_t - p_{31}x_{t+\frac{1}{3}} - p_{32}\widehat{x}_{t+\frac{2}{3}|t+\frac{1}{3}}, \tag{25}$$

$$\widehat{x}_{t+\frac{2}{3}|t+\frac{1}{3}} = \alpha_{21}x_{t-\frac{2}{3}} + \alpha_{22}x_{t-\frac{1}{3}} + \alpha_{23}x_t + \alpha_{24}y_t - p_{21}x_{t+\frac{1}{3}}. \tag{26}$$

If instead we want $\widehat{x}_{t+1|t+\frac{2}{3}}$ (a 1-step ahead forecast), we should use the expression:

$$\widehat{x}_{t+1|t+\frac{2}{3}} = \alpha_{31}x_{t-\frac{2}{3}} + \alpha_{32}x_{t-\frac{1}{3}} + \alpha_{33}x_t + \alpha_{34}y_t - p_{31}x_{t+\frac{1}{3}} - p_{32}x_{t+\frac{2}{3}}. \tag{27}$$

All these expressions can be easily generalized to longer forecast horizons.

Ghysels (2014) compares in details the MF-VAR-SF and the MF-VAR-KF, which are basically equivalent when the restrictions imposed by the HF VAR underlying the MF-VAR-KF are imposed on the MF-VAR-SF. When instead the restrictions are not imposed, the relative performance depends on the mis-specification of the HF DGP assumed by the MF-VAR-KF and the goodness of fit for the MF-VAR-SF specification.

To conclude, at first sight, the MF-VAR-SF is rather different from the RU-MIDAS approach. However, the detailed derivation of the optimal forecasts from the MF-VAR-SF reveals that the two approaches are very similar, in the sense that also in the MF-VAR-SF we need to change the specification of the HF variable model depending on the specific month of the quarter, and the HF variable is regressed (in LF) on its own HF lags and on LF lags of the LF variable. However, while the MF-VAR-SF is specified as a system, the RU-MIDAS equations are specified one by one, which can give more flexibility. Moreover, the forecasting approach is based on the direct method in the case of the RU-MIDAS and on the iterated method for the MF-VAR-SF.

4 Monte Carlo Experiments

4.1 High-frequency DGP

In this section, we evaluate in a controlled experiment the forecasting ability of the three different models outlined in the previous sections (RU-MIDAS, MF-VAR-KF, and MF-VAR-SF models). The DGP we use is a bivariate model in the HF temporal unit described as

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \rho & \delta_l \\ \delta_h & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^y \\ \epsilon_t^x \end{pmatrix}, \tag{28}$$

where $\epsilon_t^y \sim N(0, \sigma^y)$ and $\epsilon_t^x \sim N(0, \sigma^x)$.

The design of the Monte Carlo experiment is as follows. First, time series with sample size $T = 600$ are generated after discarding the first 100 observations to account for start-up effects. The size of the evaluation sample is set to 150, and the estimation sample is recursively expanded as we progress in the forecasting exercise, with the first estimation sample extending over the first 450 observations.[5] Second, for all models, we assume that the y_t variable is only observed once every three periods, so as to mimic the case of an empirically relevant situation of a mix between quarterly and monthly variables. Third, for all models we calculate forecasts for the HF variable with forecast horizons $h = \{1, 2, 3, 6, 9, 12\}$ using an AR model as a benchmark. Fourth, the MF-VAR-SF model is estimated without imposing any restrictions on the parameters of the autoregressive matrices to provide a straightforward comparison with the RU-MIDAS model. The unrestricted version of the MF-VAR-SF model is also easier to estimate, since standard least squares estimation method can be used. In addition, results from Foroni et al. (2015) suggest that the forecasting performance of the restricted and unrestricted versions are relatively similar. Finally, for all models, the lag length is set so as to include one LF temporal unit of information. In practice, this implies that the lag length for the MF-VAR-KF, RU-MIDAS and AR models is set to 3, whereas the lag length in the MF-VAR-SF model is set to 1.

In addition, we use different values for the parameters when generating time series. First, we assume a recursive DGP, setting $\delta_l = 0$. The persistence parameter ρ is set to

[5] Note that for computational reasons, the MF-VAR-KF model is estimated only once for each replication (at the beginning of the forecasting exercise). Limited evaluation of a fully recursive forecasting exercise produced qualitatively similar results.

0.8, a value that is relevant given the high persistence typically observed in macroeconomic variables, and the parameter δ_h is set to 0.5. Second, we assume a non-recursive DGP, using the following set of parameters $(\rho, \delta_h, \delta_l) = (0.5, 0.5, 0.4)$. These values are chosen so as to make sure that the VAR satisfies (weak) stationarity conditions. Finally, the shocks ϵ_t^y and ϵ_t^x are drawn independently from a normal distribution using the following parameter values for the variance $(\{\sigma^y, \sigma^x\} = \{2, 1\})$.

We also calculate forecasts obtained with an Autoregressive Distributed Lag (ARDL) model. This model is written as follows

$$x_t = \alpha + \sum_{i=1}^{p} \beta_i x_{t-\frac{i}{k}} + \sum_{j=1}^{q} \gamma_j y^*_{t-\frac{j}{k}} + u_t, \tag{29}$$

where y_t^* is the high-frequency estimate of the low-frequency variable obtained by interpolation. We use a linear interpolator, where the interpolated values for the LF variable are obtained as follows

$$y^*_{t+\lambda} = (1-\lambda)y_t + \lambda y_{t+1}, \tag{30}$$

where $\lambda = \frac{i}{k}$, $i = \{1, 2\}$, $k = 3$, y_{t+1} is the next non-missing value, and y_t is the previous non-missing value. This interpolation method requires the use of future values for the LF variable in equation (30) (i.e., y_{t+1}), which is problematic in a forecasting context. As a result, in the forecasting exercise, the predicted values for the LF variable in equation (30) are obtained from an AR model. Moreover, the lag length in equation (29) is set to $p = 3$ and $q = 3$, so as to include one LF temporal unit of information to ensure a similar information set across all models. Forecasts from ARDL models are calculated with the direct method.

To evaluate the forecasts, we use the mean square prediction errors (MSPE) relative to the MSPE calculated from the forecasts derived from an AR model. We report the median of the estimates over 1000 replications. In doing so, we distinguish four different cases: (i) MSPE calculated from the 150 observations of the evaluation sample, (ii) MSPE calculated from the 50 observations where the forecast with horizon $h = 1$ corresponds to the first month of the quarter (or first HF temporal unit of the LF temporal unit), (iii) MSPE calculated from the 50 observations where the forecast with horizon $h = 1$ corresponds to the second month of the quarter, and (iv) MSPE calculated from the 50 observations where the forecast with horizon $h = 1$ corresponds to the third month of the quarter. This is relevant given that the LF variable is observed periodically in our simulation set-up (i.e., in the third month of each quarter). Hence, in the third month of the quarter, the low-frequency variable has just been released, and it provides a valuable information when forecasting the HF variable in the first month of the next quarter. As a result, it could well be that for a given forecast horizon, the forecasting performance of the model varies depending on the temporal distance to the release of the LF variable. Table 1 presents the results when all observations of the evaluation sample are pooled when calculating MSPEs, whereas Tables 2, 3 and 4 present the results when using only observations corresponding to a specific month of the quarter when calculating the MSPEs.

The results for the high-frequency DGPs are presented in Panels A of these tables. First, looking at Panel A of Table 1, the MF-VAR-SF model obtains the best forecasting results for forecast horizons $h = \{1, 2\}$. Second, the RU-MIDAS model obtains the best forecasting performance for $h = \{3, 6\}$ in the case of the recursive DGP, and for $h = \{3\}$ in the case of the non-recursive DGP. Third, the MF-VAR-KF model typically ranks closely to the MF-VAR-SF model. In fact, it obtains the best forecasting performance for horizon $h = \{6, 9, 12\}$ when the data are generated from a non-recursive DGP.[6] Finally, the ARDL model forecasting performance is somewhat inferior compared with the other models, especially for fairly long forecast horizons (i.e., for $h > 2$).

Table 2 presents the results when the forecasts are evaluated in the first month of each quarter, that is, right after the release of the LF variable. Unsurprisingly, the forecasting performance of all models drastically improves. However, the ranking of the models is little changed in that the MF-VAR-SF model continues to obtain the best forecasting performance in the case of the recursive DGP, and it is closely followed by the MF-VAR-KF and RU-MIDAS models. Table 3 shows the results when the forecasts are evaluated in the second month of each quarter, that is, the forecasts are calculated one month after the release of the LF variable. The forecasting performance of the models deteriorates compared with Table 2. Interestingly, the MF-VAR-SF model continues to rank as the best performing model for most forecast horizons with both the recursive and non-recursive DGPs. Table 4 shows that the forecasting performance of all models further deteriorates when forecasts are calculated two months after the release of the LF variable. In fact, in nearly all cases, the forecasting performance of the AR model is better or comparable to that of the mixed-frequency models.

[6] In the appendix, we also consider a different aggregation rule for the MF-VAR-KF model than the one described by equation (8). We find that, conditional on our simulation setup, there is no substantial forecasting gain to expect from the use of a different aggregation rule.

4.2 Mixed-frequency DGP

One caveat on the simulations presented so far is that data are generated from a high-frequency VAR, which may not be realistic given that macroeconomic time series are typically sampled at different frequencies. As a result, we evaluate here the forecasting ability of the different models using mixed-frequency DGPs, generating data from a MF-VAR-SF model.

In line with simulations presented in Götz and Hecq (2014), data generated from a MF-VAR-SF are obtained from the following dynamic structural equations:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.4 & 1 & 0 & 0 \\ 0.2 & -0.4 & 1 & 0 \\ 0 & 0 & \delta & 1 \end{bmatrix} \begin{bmatrix} x_{t-\frac{2}{3}} \\ x_{t-\frac{1}{3}} \\ x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.6 & -0.2 & 0.4 & 0.3 \\ 0 & 0.6 & -0.2 & 0.3 \\ 0 & 0 & 0.6 & 0.3 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_{t-\frac{5}{3}} \\ x_{t-\frac{4}{3}} \\ x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^{x1} \\ u_t^{x2} \\ u_t^{x3} \\ u_t^{y} \end{bmatrix}.$$

We choose two different values for the parameter δ that determines the contemporaneous correlation between the LF and HF variable ($\delta = 0$ and $\delta = 0.2$).[7] Note that while data are not directly generated from a RU-MIDAS model, setting $\delta = 0$ roughly corresponds to the case of a RU-MIDAS DGP. Apart from generating time series from different DGPs, the design of the Monte Carlo exercise is identical to the one presented in the previous section.

Panels B of Tables 1 to 4 present the results. As expected, the MF-VAR-SF model (i.e., the model used to generate the data) obtains in nearly all cases the best forecasting performance. It is also interesting to note that the RU-MIDAS model exhibits a very similar forecasting performance to the MF-VAR-SF model for forecast horizons $h = \{3, 6, 9, 12\}$, a pattern that could be also observed when data were generated from a high-frequency DGP. While the ranking of the models is somewhat similar across the different types of DGP (high-frequency or mixed-frequency DGPs), when data are generated from the mixed-frequency DGP, we no longer observe a clear trend towards a better forecasting performance depending on the distance to the release of the LF variable. This suggests that the relevance of calculating the forecasting performance in a periodic way varies depending on the data generating process. Hence, it remains ultimately an empirical issue that we investigate in the next section.

[7]Note that when the LF and HF variables are heavily correlated, a multicollinearity problem may arise, which heavily deteriorates the forecasting performance of the RU-MIDAS model at selected forecast horizons. In such cases, appropriate transformation of the HF variable permits to alleviate this issue.

Overall, the Monte-Carlo results suggest that, conditional on our simulation set-up, the forecasting performance may vary considerably depending on the temporal distance to the release of the LF variable. In terms of ranking of the models, our simulation exercise suggests that there is no single outperforming model. However, the MF-VAR-SF model tends to outperform the competing models, and the RU-MIDAS model performs largely in line with the MF-VAR-SF model at longer forecasting horizons.

5 Empirical Applications

It is well documented in the literature that surveys are helpful indicators to predict key macroeconomic variables (see, for example, Chun (2011), Chernov and Mueller (2012) and Stark (2010)). For the U.S. economy, it is typically difficult to improve upon the forecasting performance of well established surveys such as the Survey of Professional Forecasters (see, e.g., Faust and Wright (2013)). In a similar vein, survey data can also be used in standard macroeconomic forecasting models. For example, Wright (2013) shows that using survey data to calibrate prior distributions in Bayesian VAR models yields a substantial improvement in real-time macroeconomic predictions compared with models that exclude such information.

To illustrate the forecasting ability of the different mixed-frequency data models presented in the previous sections, we exploit the natural frequency mismatch that exists between some widely used survey data (typically available on a quarterly frequency) and the target variable (typically sampled at a monthly frequency). In doing so, we use the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia, which is widely recognized as a very accurate survey (see, e.g., Stark (2010), who performs an extensive evaluation of the SPF forecasting performance).

We run two different empirical exercises. First, we illustrate the forecasting performance of the different models using the quarterly SPF for three key monthly variables representative of the U.S. economy: CPI inflation, industrial production growth, and personal consumption growth. Our goal is to check whether using the information from quarterly SPF data is helpful to improve the forecasts of the corresponding monthly series. Second, we forecast the monthly nominal U.S. dollar trade-weighted exchange rate, which is a challenging task given that the random walk model is typically found to be the best performing model when forecasting exchange rates, see e.g. Meese and Rogoff (1983a) for early evidence and Engel and West (2005) for a theoretical explanation of this finding. Again, the objective is to evaluate whether the information conveyed by the quarterly SPF helps to forecast the monthly U.S. trade-weighted exchange rate.

5.1 Design of the forecasting experiments

We apply the following transformation to the monthly variables. Industrial production growth, consumption growth and inflation rate are the annualized three-month percentage change (i.e., 400 times the three-month log change in the level of the industrial production index, real personal consumption expenditures index and the CPI index). Moreover, the exchange rate measure we use is the nominal trade-weighted U.S. exchange rate (i.e., a weighted average of the foreign exchange value of the U.S. dollar against major currencies), and it is taken in logs. All the data are downloaded from the ALFRED real-time database of the Saint Louis Federal Reserve Bank. The full sample size for all time series extends from July 1981 to November 2013 and we use real-time data, whenever available.

For the quarterly variables, we consider the median forecasts for the CPI inflation rate, industrial production and personal consumption. The SPF survey for inflation and consumption growth starts in the third quarter of 1981, which explains the start date of our sample. For ease of comparison across models and variables, the estimation sample for industrial production also starts in 1981Q3 (even though the SPF for this variable is available from 1968Q4).

Figure 1 reports the monthly series together with the expectations derived from the SPF referring to the current quarter and one-quarter ahead. This shows that the SPF tracks relatively well the actual realizations of the monthly macroeconomic variables, albeit the SPF is less volatile than the actual underlying series.

The first estimation sample extends from July 1981 to December 1999, and it is recursively expanded until September 2011. For each month of the evaluation sample, we calculate forecasts from 1 to 24 months ahead, and the forecasts are evaluated against the first release, calculating the Mean Square Prediction Error (MSPE). We consider an AR model as a benchmark with lag length selected according to the BIC criterion (and a maximum lag equal to 12). The forecasting performance of a given model is then presented as a ratio with respect to the MSPE obtained from the AR benchmark. When forecasting the monthly trade-weighted exchange rate, we use the random walk as benchmark model, since it is the traditional benchmark model in this literature (see Rossi (2013)). Note also that in line with the Monte-Carlo experiments, in the forecasting exercise, both the MF-VAR-KF and MF-VAR-SF models are estimated once a quarter, but the (iterated) forecasts are updated on a monthly basis so as to take into account the latest information. However, both ARDL model and RU-MIDAS models are estimated on a monthly basis, since the forecasts are calculated with the direct method so that the parameter estimates change with the forecast horizon.

In total, for each forecast horizon and model, we obtain 144 forecasts. In line with the Monte Carlo experiments, we report the MSPE calculated when pooling all forecasts from the evaluation sample, but also report the MSPE calculated in the first, second and third month of each quarter. This is relevant since we are looking at periodic models, in which the interaction between the monthly and quarterly variables changes for every month of the quarter.

Moreover, the SPF provides forecasts for different quarterly horizons. For each of the variables, we can consider five series of forecasts: the series of forecasts related to the current quarter, and the series of forecasts from 1 to 4-quarter ahead. However, when reporting the relative MSPE, we use the SPF related to the specific forecast horizon. In detail, for forecast horizon $h = \{1, 2, 3\}$, we use the SPF related to 1-quarter ahead forecast, for forecast horizon $h = \{6\}$, we use the SPF related to 2-quarter ahead forecast, for forecast horizon $h = \{9\}$, we use the SPF related to 3-quarter ahead forecast, and for forecast horizons $h = \{12, 15, 18, 21, 24\}$, we use the SPF related to 4-quarter ahead forecasts.

5.2 Predicting monthly macroeconomic variables

In this subsection, we assess the relevance of the SPF to forecast the corresponding monthly variables, based on competing low to high frequency models. In Tables 5 to 7 we present the MSPE for each of our models relative to the AR benchmark. The tables report the ratio of the MSPE computed on the entire evaluation sample, but also separate results for the first, second and third month of each quarter. The numbers in bold highlight the cases in which the mixed-frequency models outperform the benchmark (i.e., when the ratio of the MSPE of the mixed-frequency model relative to the MSPE of the AR benchmark is smaller than 1). In addition, we implement the Diebold and Mariano test to formally test for significant differences in the forecasting performance between a given model and the AR benchmark model.

Table 5 shows that SPF expectations on inflation contain useful information for predicting monthly inflation. Using information included in the SPF allows us to improve upon the AR benchmark at almost every horizon (with the exception of the very short ones) for all the models we analyse. In a substantial number of cases, the improvement is also statistically significant according to the Diebold and Mariano test. In detail, for short forecast horizons (h={1,2,3}), the ARDL model based on interpolated survey data performs particularly well. At longer horizons (i.e., for $h > 3$), the more sophisticated models have an advantage, with the MF-VAR-SF often ranking as best and the RU-MIDAS as second best. Results are generally somewhat better in the third month of the quarter. This hints at the fact that useful information is contained in the new release of the SPF, which are available in the middle of the second month of each quarter, and consequently enter the forecast in the third month in a timely way. The gains are overall rather stable across the three months of the quarter, in the range of 10 to 20 per cent, and are often statistically significant.

We find similar results when forecasting monthly industrial production growth (see Table 6). The gains from the use of the corresponding quarterly surveys are evident, for any forecast horizon. In particular, good results are obtained in the third month of the quarter, when the new releases of the SPF become available. In line with the results for forecasting inflation, at very short horizons ($h = \{1, 2\}$), the ARDL provides the most accurate forecasts. The other models are typically better for $h > 2$, with the differences across models generally small and gains with respect to the AR benchmark between 5% and 10% and often significant.

The information contained in the SPF appears to be less useful for predicting personal consumption growth (see Table 7). Looking at Figure 1, it is clear that personal consumption growth is more volatile than the other two series we consider, and than the corresponding SPF. Therefore, in this case, survey data do not convey useful information especially at short horizons. However, the SPF seems to have some predictive content for the longer horizons, with some gains for $h \geq 6$. The best models in this case are the MF-VARs, which obtain gains close to 5%.

5.3 Predicting the monthly U.S. trade-weighted exchange rate

As a second forecasting exercise, we now focus on predicting exchange rates. In particular, we investigate the predictive power of the expectations on future inflation and interest rates for the monthly nominal U.S. trade-weighted exchange rate.[8] The rationale for this choice of predictors is that using expected inflation relates to the purchasing power parity (PPP) theory, whereas interest rates have predictive power for exchange rate according to the uncovered interest rate parity (UIP) relation. While both UIP and PPP refer to contemporaneous values for inflation and interest rate, using expectations for both variables is likely to be more relevant in a forecasting context. Note also that, according to both UIP and PPP, differentials between U.S. and foreign variables should be the relevant metrics to look at for exchange rate fluctuations. However, expectations data for other countries are limited in that they do not cover a long enough sample. As a result, we only use U.S. data as regressors.[9]

Following the literature (see Rossi (2013) for an extensive review), in estimating our models we focus on $E_t(s_{t+h} - s_t)$, where s_t indicates the log of the exchange rate, with t expressed in months and h the monthly forecast horizon. All models are evaluated relative to a benchmark random walk without drift. The predictive ability of our models is evaluated based on the mean squared prediction error.

The results are then presented in the same way as in the previous subsection. Tables 8 and 9 present the results when using the expectations on CPI inflation and on T-Bill yields, respectively. Note also we use the SPF referring to the current quarter for all the forecasting horizons, since, contrary to the previous subsection, there is no economic rationale for doing otherwise. Interestingly, mixed-frequency data models outperform the benchmark random walk model.

In particular, Table 8 shows that SPF expectations on inflation contain useful information for predicting the monthly nominal trade-weighted U.S. exchange rate. Improvements

[8] In detail, this is a weighted average of the foreign exchange value of the U.S. dollar against a subset of the broad index currencies that circulate widely outside the country of issue. Major currencies index includes the Euro Area, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden.

[9] For the same reason, we focus on the trade-weighted exchange rate and not on bilateral exchange rates, in which expectations for both countries would be relevant. Note that results based on out-of-sample forecasting of selected currency pairs (GBP/USD, JPY/USD, and CAD/USD) only produced marginal improvements in forecast accuracy for selected horizons. For the use of survey data for explaining bilateral exchange rate fluctuations, see also Fratzscher et al. (2015).

are evident with every model we consider, and at every horizon. The gains are even in the range of 10 to more than 15 percent, which are substantial in the context of exchange rate forecasting, in which it is difficult to beat the no-change (or random walk) benchmark. Admittedly, results are not always statistically significant, except for the results obtained with the MF-VAR estimated via the Kalman filter. This method provides better and significant results in almost every case.[10]

Table 9 shows that expectations on the T-Bill seem to be a useful predictor only when used in the MF-VAR setup, and especially with the MF-VAR estimated with the Kalman filter. RU-MIDAS and interpolation methods do not provide good results.

All in all, we obtain encouraging results by using surveys of professional forecasters to forecast the U.S. trade-weighted exchange rate in a mixed-frequency setup. In many cases, mixed-frequency models obtain better forecasting results than the random walk benchmark, and the forecasting gains are often statistically significant.

6 Conclusions

In this paper, we analyze how to incorporate low-frequency information in models that forecast high-frequency variables. While the literature has mostly concentrated on the use of high-frequency variables for predicting a lower frequency variable (e.g., forecasting quarterly GDP growth with monthly indicators), little work has been done regarding the use of low-frequency variables to predict higher frequency variables.

First, we introduce a new model, denoted as RU-MIDAS, in which the high-frequency variable has a dynamic relation with the low-frequency variable. We then compare this model with two different versions of the mixed-frequency VAR model (the Kalman filter and stacked-form versions), so far only used to exploit high frequency information to predict low frequency variables.

Second, we evaluate the forecasting ability of these competing mixed-frequency models, by means of a number of Monte Carlo experiments. Our simulation results suggest that the predictive performance of the low-frequency variable for a related high-frequency variable increases soon after the release of the low-frequency variable, suggesting that the timing

[10] This is consistent with the results from Dal Bianco et al. (2012), who use the MF-VAR-KF model to forecast the euro-dollar exchange rate at the weekly frequency.

of the release of the low-frequency variable matters when predicting the high-frequency variable. Moreover, across the different simulations we perform, no clear-cut ranking of the competing mixed-frequency models emerges.

Finally, we illustrate the empirical relevance of the different mixed-frequency models using the quarterly Survey of Professional Forecasters for forecasting three key US monthly macroeconomic indicators (industrial production, consumption and inflation) as well as the monthly nominal U.S. trade-weighted exchange rate. In line with the Monte Carlo experiments, we do not find a clear-cut ranking in the forecasting performance of the models. However, our results clearly indicate that there is additional insight to gain from the quarterly SPF right after its publication when one is interested in forecasting the corresponding monthly indicator. Hence, low frequency information can indeed be useful when predicting high frequency variables.

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7 Appendix

Comparison of different aggregation rules for the MF-VAR-KF model

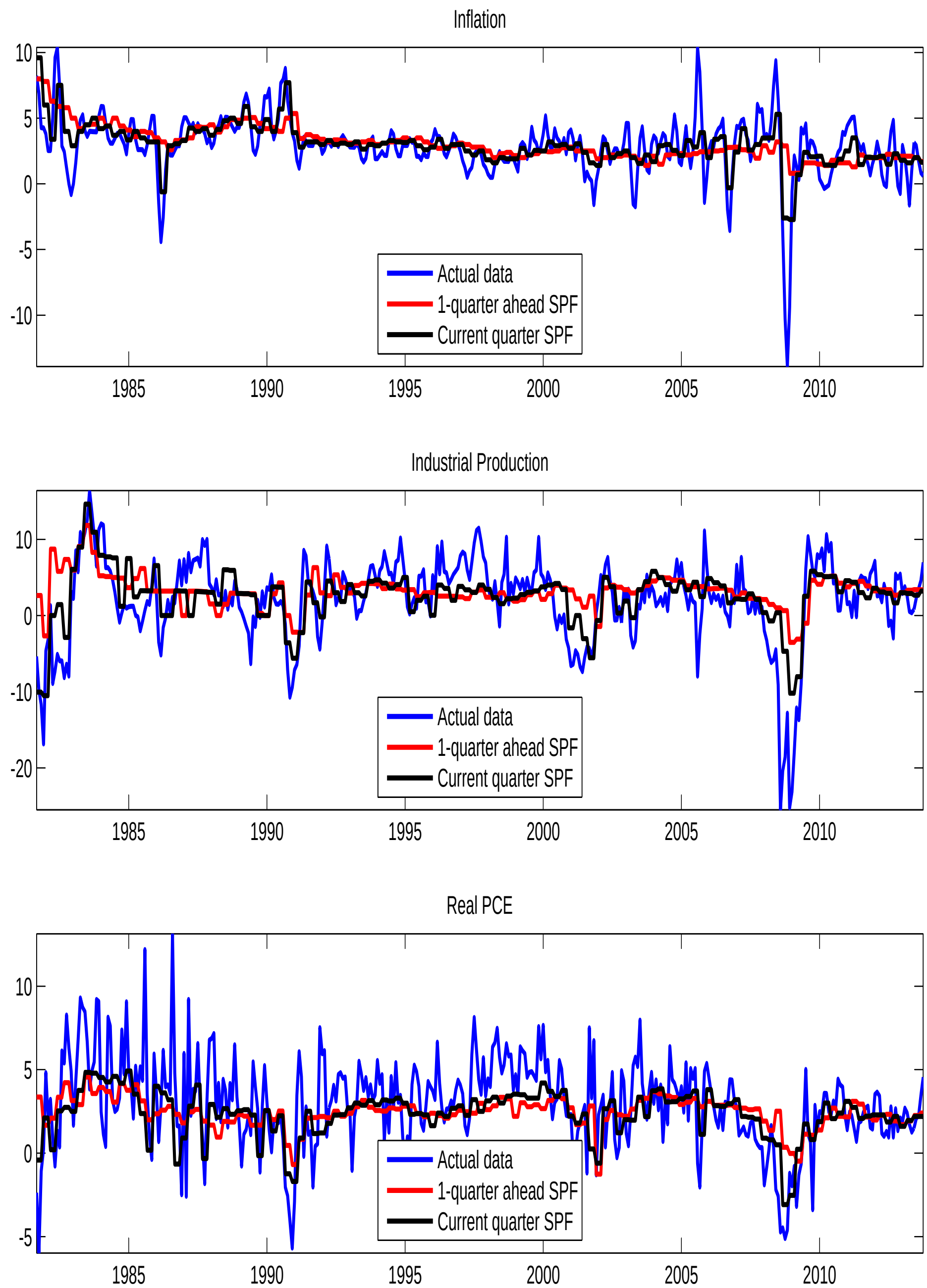
In this section, we perform additional simulations to check the relevance of the aggregation constraint from Mariano and Murasawa (2003) when estimating the MF-VAR-KF model (i.e., equation (8) in Section 3.1). This aggregation constraint is derived from a geometric mean, assuming that the (flow) data on a given quarter is three times its monthly values pertaining to that quarter. However, this type of constraint is not appropriate in the context of a stock variable where the aggregation rule is straightforward since stock variables are just a particular quantity at a specific time. Also, the MF-VAR-KF model is not directly comparable with the MF-VAR-SF and RU-MIDAS models to the extent that there is no such disaggregation of the LF variable. Instead, in both RU-MIDAS and MF-VAR-SF models, the LF variable is assumed to be observed every k periods. Hence, the comparison of the MF-VAR-KF model with the RU-MIDAS and MF-VAR-SF models may be distorted by the different aggregation rules adopted in these different models. As a result, we consider a stock variable type of aggregation so that equation (8) becomes:

$$\mathbf{H}(L) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} L + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} L^2 + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} L^3 + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} L^4 \quad (31)$$

if the LF variable is observed (otherwise, the upper row of H(L) is set to 0).

Table 10 shows the results. First, in the case of data generated from a high-frequency VAR, the forecasting performance of the MF-VAR-KF models is very close, regardless of the aggregation rule adopted. However, in the case of data generated from a mixed-frequency VAR DGP, the results suggest that the MF-VAR-KF model with an aggregation rule for stock variable outperforms the MF-VAR-KF model with an aggregation rule à la Mariano and Murasawa (2003) for forecast horizon h={1}. However, at longer forecast horizons, both models exhibit a similar forecasting performance. Overall, conditional on this simulation exercise, this suggests that there is no substantial forecasting gain to expect from using a different aggregation rule for the MF-VAR-KF model.

Figure 1: Macroeconomic variables and SPF expectations



Note: All macroeconomic variables are taken as 400 times the three-month change in the logarithm of the underlying index.

Table 1: Simulation Results - All months

Panel A: High-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP:	$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.8, 0.5, 0, 2, 1)$						$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.5, 0.5, 0.4, 2, 1)$					
RU-MIDAS	0.859	0.914	**0.804**	**0.915**	0.983	1.005	0.946	0.979	**0.934**	0.987	1.006	1.007
MF-VAR-SF	**0.771**	**0.771**	0.809	0.915	**0.968**	**0.990**	**0.827**	**0.898**	0.935	0.978	0.986	0.989
MF-VAR-KF	0.794	0.792	0.832	0.932	0.989	1.006	0.881	0.917	0.941	**0.971**	**0.974**	**0.971**
ARDL	0.794	0.823	0.876	0.965	1.005	1.011	0.860	0.943	0.994	1.013	1.012	1.009

Panel B: Mixed-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP:	MF-VAR-SF DGP ($\delta = 0$)						MF-VAR-SF DGP ($\delta = 0.2$)					
RU-MIDAS	0.794	0.792	0.799	0.830	0.874	0.929	0.801	0.816	0.805	0.847	0.895	0.946
MF-VAR-SF	**0.685**	**0.672**	**0.797**	**0.819**	**0.858**	**0.907**	**0.684**	**0.675**	**0.801**	**0.834**	**0.878**	**0.922**
MF-VAR-KF	0.927	0.877	0.826	0.874	0.936	0.990	0.931	0.897	0.831	0.896	0.950	0.998
ARDL	0.724	0.678	0.819	0.832	0.871	0.915	0.722	0.683	0.835	0.856	0.898	0.935

Note: This table reports the median of the relative Mean Square Prediction Error for the RU-MIDAS, MF-VAR-KF, MF-VAR-SF and ARDL models averaged over 1000 replications. The benchmark model is an AR model. Boldface indicates the model with the lowest relative MSPE for a given horizon and DGP. Additional details on the DGPs are provided in the text.

Table 2: Simulation Results - First month of the quarter

Panel A: High-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP	$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.8, 0.5, 0, 2, 1)$						$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.5, 0.5, 0.4, 2, 1)$					
RU-MIDAS	0.552	0.548	0.528	0.792	0.923	0.992	0.661	0.714	**0.788**	0.910	0.961	0.981
MF-VAR-SF	**0.432**	**0.432**	**0.528**	**0.787**	**0.914**	**0.969**	**0.483**	0.683	0.788	**0.898**	0.947	0.967
MF-VAR-KF	0.440	0.445	0.543	0.797	0.926	0.974	0.503	**0.679**	0.794	0.904	**0.934**	**0.957**
ARDL	0.436	0.453	0.584	0.838	0.953	0.998	0.489	0.710	0.843	0.932	0.965	0.985

Panel B: Mixed-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP:	MF-VAR-SF DGP ($\delta = 0$)						MF-VAR-SF DGP ($\delta = 0.2$)					
RU-MIDAS	0.912	0.811	0.564	0.628	0.758	0.863	0.943	0.847	**0.568**	0.656	0.803	0.898
MF-VAR-SF	**0.716**	0.559	**0.563**	**0.625**	**0.751**	**0.842**	**0.718**	0.556	0.569	**0.652**	**0.790**	**0.875**
MF-VAR-KF	0.741	0.568	0.590	0.651	0.780	0.889	0.757	0.563	0.596	0.685	0.827	0.923
ARDL	0.768	**0.551**	0.575	0.639	0.761	0.853	0.775	**0.549**	0.582	0.667	0.802	0.888

Note: This table reports the median of the relative Mean Square Prediction Error for the RU-MIDAS, MF-VAR-KF, MF-VAR-SF and ARDL models averaged over 1000 replications. This table reports the results when the MSPEs are calculated only from the months where the forecast with horizon $h = 1$ refers to the first month of the quarter, that is one month after the LF variable has been released. The benchmark model is an AR model. Boldface indicates the model with the lowest relative MSPE for a given horizon and DGP. Additional details on the DGPs are provided in the text.

Table 3: Simulation Results - Second month of the quarter

Panel A: High-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP	$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.8, 0.5, 0, 2, 1)$						$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.5, 0.5, 0.4, 2, 1)$					
RU-MIDAS	0.893	0.936	**0.875**	0.953	0.994	1.007	1.068	1.085	0.986	1.015	1.018	1.011
MF-VAR-SF	**0.864**	**0.859**	0.882	**0.949**	**0.983**	**0.988**	**0.962**	**0.988**	**0.986**	**0.996**	1.002	1.001
MF-VAR-KF	0.940	0.918	0.914	0.972	1.007	1.006	1.046	1.015	1.000	1.001	1.004	**0.979**
ARDL	0.864	0.863	0.888	0.951	0.986	0.993	0.972	0.999	1.004	1.017	1.017	1.006

Panel B: Mixed-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP:	MF-VAR-SF DGP ($\delta = 0$)						MF-VAR-SF DGP ($\delta = 0.2$)					
RU-MIDAS	0.750	0.620	0.881	0.883	0.895	0.923	0.747	0.623	0.886	0.908	0.935	0.949
MF-VAR-SF	**0.641**	**0.600**	**0.877**	0.871	0.886	**0.907**	**0.639**	**0.600**	**0.885**	0.896	0.911	**0.928**
MF-VAR-KF	0.972	0.817	0.894	0.931	0.967	0.997	0.985	0.830	0.908	0.957	0.994	1.028
ARDL	0.683	0.605	0.886	**0.863**	**0.875**	0.912	0.681	0.603	0.905	**0.895**	**0.909**	0.935

Note: This table reports the median of the relative Mean Square Prediction Error for the RU-MIDAS, MF-VAR-KF, MF-VAR-SF and ARDL models averaged over 1000 replications. This table reports the results when the MSPEs are calculated only from the months where the forecast with horizon $h = 1$ refers to the second month of the quarter, that is two months after the LF variable has been released. The benchmark model is an AR model. Boldface indicates the model with the lowest relative MSPE for a given horizon and DGP. Additional details on the DGPs are provided in the text.

Table 4: Simulation Results - Third month of the quarter

Panel A: High-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP	$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.8, 0.5, 0, 2, 1)$						$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.5, 0.5, 0.4, 2, 1)$					
RU-MIDAS	1.135	1.248	0.997	1.011	1.020	1.029	1.095	1.155	1.012	1.020	1.019	1.026
MF-VAR-SF	1.011	1.008	1.006	1.011	1.005	1.012	1.030	1.020	1.011	1.010	1.008	1.010
MF-VAR-KF	1.018	1.020	1.015	1.031	1.016	1.008	1.066	1.046	1.007	1.005	**0.989**	**0.989**
ARDL	1.096	1.152	1.154	1.100	1.052	1.039	1.120	1.140	1.131	1.078	1.043	1.038

Panel B: Mixed-frequency DGP

Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
DGP:	MF-VAR-SF DGP ($\delta = 0$)						MF-VAR-SF DGP ($\delta = 0.2$)					
RU-MIDAS	0.732	0.944	**0.939**	0.954	0.971	0.976	0.995	1.019	1.007	1.015	1.022	1.019
MF-VAR-SF	**0.691**	**0.859**	0.942	**0.950**	**0.954**	**0.951**	**0.944**	0.942	0.997	**0.992**	**0.997**	**0.996**
MF-VAR-KF	1.050	1.259	0.980	1.034	1.062	1.064	1.076	1.281	**0.975**	1.032	1.038	1.039
ARDL	0.732	0.878	0.972	0.975	0.975	0.974	0.960	**0.905**	1.179	1.104	1.050	1.039

Note: This table reports the median of the relative Mean Square Prediction Error for the RU-MIDAS, MF-VAR-KF, MF-VAR-SF and ARDL models averaged over 1000 replications. This table reports the results when the MSPEs are calculated only from the months where the forecast with horizon $h = 1$ refers to the third month of the quarter, that is three months after the LF variable has been released. The benchmark model is an AR model. Boldface indicates the model with the lowest relative MSPE for a given horizon and DGP. Additional details on the DGPs are provided in the text.

Table 5: Forecasting monthly inflation with the quarterly SPF

All months pooled

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.175	1.001	**0.915**	**0.865***	**0.919**	**0.946**	**0.865***	**0.921**	**0.953**	**0.951**
MF-VAR-SF	1.218	1.079	**0.908**	**0.819****	**0.811****	**0.871***	**0.857****	**0.890****	**0.904****	**0.904****
MF-VAR-KF	1.783	1.299	1.015	**0.890**	**0.868***	**0.929**	**0.918***	**0.958**	**0.973**	**0.981**
ARDL	**0.906***	**0.874***	**0.845***	**0.882****	**0.891***	**0.966**	**0.958**	**0.938**	**0.949**	**0.961**

First month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.374	**0.939**	**0.944***	**0.950**	**0.955**	1.063	**0.898****	**0.940****	**0.908****	**0.979**
MF-VAR-SF	1.323	1.046	**0.967**	**0.872***	**0.857***	**0.924**	**0.886***	**0.917***	**0.908****	**0.924****
MF-VAR-KF	1.394	1.308	1.170	**0.956**	**0.902**	**0.981**	**0.940**	**0.974**	**0.970**	**0.995**
ARDL	**0.901***	**0.915**	**0.923**	**0.916**	**0.919**	**0.985**	**0.955**	**0.958**	**0.931**	**0.896***

Second month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.101	1.004	**0.897***	**0.863***	**0.852****	**0.851****	**0.874****	**0.912****	1.041	**0.965****
MF-VAR-SF	1.243	1.166	**0.900**	**0.826**	**0.812***	**0.857**	**0.877***	**0.886***	**0.921***	**0.939***
MF-VAR-KF	2.084	1.472	**0.942**	**0.897**	**0.875**	**0.911**	**0.945**	**0.955**	**0.990**	1.022
ARDL	**0.862**	**0.852**	**0.814**	**0.917**	**0.917**	**0.991**	1.031	**0.975**	1.021	1.096

Third month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.060	1.072	**0.889****	**0.775****	**0.944**	**0.911***	**0.816****	**0.908****	**0.918****	**0.905****
MF-VAR-SF	1.092	1.031	**0.853**	**0.755***	**0.757**	**0.823**	**0.802****	**0.860***	**0.879***	**0.846****
MF-VAR-KF	1.852	1.078	**0.921**	**0.811**	**0.823**	**0.887**	**0.864***	**0.942**	**0.960**	**0.923**
ARDL	**0.954**	**0.850***	**0.792****	**0.814****	**0.833***	**0.918**	**0.890**	**0.876**	**0.896**	**0.908**

Note: Boldface indicates improvements over the benchmark AR model. Lag selection is done with the SIC. Statistically significant reductions in MSPE according to the Diebold-Mariano test are marked using **(5% significance level) and *(10% significance level). The first estimation sample extends from July 1981 to December 1999, and the evaluation sample extends from January 2000 to November 2013.

Table 6: Forecasting monthly industrial production with the quarterly SPF

All months pooled

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.035	**0.974**	**0.942***	**0.952****	**0.935***	**0.913**	**0.928**	**0.969**	**0.959**	**0.966**
MF-VAR-SF	1.430	1.141	1.006	**0.954**	**0.912****	**0.919****	**0.928****	**0.939****	**0.947****	**0.950****
MF-VAR-KF	1.281	1.038	**0.905****	**0.859****	**0.913****	**0.940***	**0.937****	**0.943****	**0.949***	**0.949***
ARDL	**0.931****	**0.924***	**0.926***	**0.928**	**0.929**	**0.922**	**0.928**	**0.947***	**0.953***	**0.981**

First month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.138	1.087	1.085	1.073	**0.931****	**0.925****	**0.944***	**0.975**	**0.953***	**0.974****
MF-VAR-SF	1.973	1.314	1.064	1.115	**0.961**	**0.945****	**0.951***	**0.950***	**0.954****	**0.966**
MF-VAR-KF	1.855	1.128	1.042	**0.924**	**0.925***	**0.983**	**0.972**	**0.955**	**0.960**	**0.965**
ARDL	**0.932**	**0.965**	1.012	**0.990**	**0.935**	**0.936**	**0.943**	**0.943**	**0.949***	1.003

Second month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.006	**0.875****	**0.950**	**0.894****	**0.943****	**0.922****	**0.936***	**0.952***	**0.964***	**0.991**
MF-VAR-SF	1.412	1.017	1.066	**0.911****	**0.914****	**0.916****	**0.927****	**0.939****	**0.947****	**0.954****
MF-VAR-KF	1.257	1.044	**0.892**	**0.841****	**0.924****	**0.936****	**0.934****	**0.940***	**0.949**	**0.947***
ARDL	**0.896****	**0.899****	**0.867***	**0.921**	**0.948**	**0.936**	**0.941**	**0.951**	**0.960**	**0.994**

Third month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.996**	**0.991**	**0.803****	**0.913****	**0.933****	**0.895****	**0.907***	**0.981**	**0.960**	**0.932****
MF-VAR-SF	1.151	1.134	**0.895***	**0.869****	**0.868****	**0.899****	**0.909****	**0.927****	**0.940****	**0.933****
MF-VAR-KF	**0.990**	**0.953**	**0.793****	**0.824****	**0.892****	**0.907****	**0.909****	**0.935***	**0.938***	**0.936***
ARDL	**0.949****	**0.916***	**0.904***	**0.887**	**0.907**	**0.897**	**0.903**	**0.950**	**0.949**	**0.947**

Note: Boldface indicates improvements over the benchmark AR model. Lag selection is done with the SIC. Statistically significant reductions in MSPE according to the Diebold-Mariano test are marked using **(5% significance level) and *(10% significance level). The first estimation sample extends from July 1981 to December 1999, and the evaluation sample extends from January 2000 to November 2013.

Table 7: Forecasting monthly consumption with the quarterly SPF

All months pooled

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.138	1.053	1.196	**0.935***	**0.977**	1.029	1.019	1.092	1.077	1.014
MF-VAR-SF	1.212	1.214	1.129	**0.991**	**0.979**	**0.938****	**0.951**	**0.950****	**0.945****	**0.946****
MF-VAR-KF	1.387	1.291	1.072	1.001	**0.956**	**0.931***	**0.947***	**0.942****	**0.941****	**0.945****
ARDL	1.109	1.215	1.226	1.010	1.074	1.080	1.080	1.056	1.069	1.027

First month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.087	1.112	1.069	1.027	**0.982**	1.006	**0.993**	1.101	1.042	**0.999**
MF-VAR-SF	**0.976**	1.213	1.230	**0.955**	1.007	**0.940**	**0.957***	**0.964****	**0.961****	**0.960****
MF-VAR-KF	1.260	1.141	1.081	**0.956**	**0.987**	**0.932**	**0.945**	**0.958****	**0.955****	**0.956****
ARDL	**0.969**	1.480	1.290	1.012	1.051	1.132	1.333	1.068	1.068	1.058

Second month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.038	1.161	1.253	**0.845****	**0.960****	1.061	**0.994**	1.180	1.166	1.023
MF-VAR-SF	1.212	1.291	1.107	**0.970**	**0.925**	**0.914****	**0.931****	**0.959****	**0.944****	**0.947****
MF-VAR-KF	1.336	1.299	1.135	1.031	**0.916**	**0.914****	**0.930****	**0.950****	**0.941****	**0.947****
ARDL	1.217	1.192	1.124	**0.981**	1.124	1.032	1.052	1.032	1.086	**0.998**

Third month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.241	**0.880****	1.343	**0.914**	**0.997**	1.020	1.098	**0.958***	1.008	1.022
MF-VAR-SF	1.391	1.121	1.001	1.088	1.015	**0.971**	**0.971**	**0.916****	**0.920****	**0.925****
MF-VAR-KF	1.510	1.391	**0.976**	1.040	**0.967**	**0.958**	**0.974**	**0.909****	**0.917****	**0.928****
ARDL	1.160	1.051	1.261	1.048	1.035	1.063	1.033	1.003	1.048	1.023

Note: Boldface indicates improvements over the benchmark AR model. Lag selection is done with the SIC. Statistically significant reductions in MSPE according to the Diebold-Mariano test are marked using **(5% significance level) and *(10% significance level). The first estimation sample extends from July 1981 to December 1999, and the evaluation sample extends from January 2000 to November 2013.

Table 8: Forecasting monthly exchange rate with the CPI quarterly SPF

All months pooled

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.910***	1.037	**0.966**	**0.884**	**0.940**	**0.955**	**0.942**	**0.979**	**0.896**	**0.924**
MF-VAR-SF	**0.915***	**0.964**	**0.934**	**0.877***	**0.895**	**0.912**	**0.887**	**0.874***	**0.878***	**0.88***
MF-VAR-KF	1.087	**0.964**	**0.772****	**0.859****	**0.889****	**0.913****	**0.919****	**0.915****	**0.905****	**0.902****
ARDL	**0.904****	**0.944**	**0.923**	**0.856***	**0.854***	**0.908**	**0.918**	**0.914**	**0.924**	**0.949**

First month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.829****	1.047	1.043	**0.884****	**0.954**	**0.919***	**0.915***	**0.968**	**0.903***	**0.893****
MF-VAR-SF	**0.881**	**0.985**	**0.991**	**0.883**	**0.887**	**0.904**	**0.875**	**0.878**	**0.862**	**0.875**
MF-VAR-KF	1.135	1.026	**0.858****	**0.912****	**0.909****	**0.921****	**0.921****	**0.907****	**0.909****	**0.908****
ARDL	**0.835****	**0.950**	**0.946**	**0.840***	**0.854**	**0.901**	**0.912**	**0.947**	**0.919**	**0.950**

Second month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.925****	1.010	**0.927**	**0.854****	**0.953**	1.054	1.003	1.034	**0.930**	**0.905****
MF-VAR-SF	**0.884***	**0.961**	**0.920**	**0.853****	**0.900**	**0.924**	**0.916**	**0.907**	**0.910**	**0.906**
MF-VAR-KF	**0.999**	**0.904**	**0.738****	**0.827****	**0.872****	**0.903****	**0.916****	**0.924****	**0.920****	**0.916****
ARDL	**0.916**	**0.971**	**0.936**	**0.859***	**0.858***	**0.902**	**0.924**	**0.930**	**0.964**	**0.966**

Third month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.997**	1.049	**0.935****	**0.911****	**0.915***	**0.891****	**0.904****	**0.930**	**0.852****	**0.973**
MF-VAR-SF	1.015	**0.940**	**0.897****	**0.892***	**0.897**	**0.906**	**0.870**	**0.832**	**0.859**	**0.859***
MF-VAR-KF	1.169	**0.943**	**0.727****	**0.839****	**0.888****	**0.914****	**0.920****	**0.913****	**0.885****	**0.882****
ARDL	**0.981**	**0.910**	**0.891***	**0.87***	**0.849***	**0.922**	**0.919**	**0.861**	**0.885**	**0.931**

Note: Boldface indicates improvements over the benchmark RW model. Lag selection is done with the SIC. Statistically significant reductions in MSPE according to the Diebold-Mariano test are marked using **(5% significance level) and *(10% significance level). The first estimation sample extends from July 1981 to December 1999, and the evaluation sample extends from January 2000 to November 2013.

Table 9: Forecasting monthly exchange rate with the T-bill quarterly SPF

All months pooled

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.901****	1.075	1.023	1.022	1.112	1.131	1.197	1.405	1.535	1.728
MF-VAR-SF	**0.907***	**0.991**	**0.992**	**0.982**	**0.991**	**0.966**	**0.930**	**0.912**	**0.926**	**0.947**
MF-VAR-KF	1.097	**0.931***	**0.686****	**0.799****	**0.846****	**0.880****	**0.892****	**0.888****	**0.874****	**0.873****
ARDL	**0.911****	**0.983**	1.001	1.038	1.099	1.151	1.164	1.211	1.318	1.417

First month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.837****	1.141	1.087	1.031	1.125	1.086	1.184	1.363	1.596	1.718
MF-VAR-SF	**0.881**	**0.997**	1.030	**0.999**	**1.008**	**0.990**	**0.936**	**0.922**	**0.929**	**0.949**
MF-VAR-KF	1.072	**0.965**	**0.773****	**0.855****	**0.862****	**0.878****	**0.877****	**0.860****	**0.861****	**0.864****
ARDL	**0.841****	**0.978**	1.022	1.024	1.092	1.128	1.125	1.194	1.304	1.409

Second month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	**0.889****	1.087	0.998	1.018	1.135	1.213	1.233	1.491	1.551	1.691
MF-VAR-SF	**0.884***	**0.986**	**0.978**	**0.961**	**1.004**	**0.978**	**0.963**	**0.949**	**0.960**	**0.975**
MF-VAR-KF	1.043	**0.889**	**0.655****	**0.763****	**0.825****	**0.870****	**0.893***	**0.905***	**0.902****	**0.898****
ARDL	**0.914***	1.014	**0.996**	1.046	1.092	1.147	1.178	1.241	1.355	1.429

Third month

Forec. hor.	1	2	3	6	9	12	15	18	21	24
RU-MIDAS	1.010	**0.981**	**0.989**	1.016	1.076	1.092	1.171	1.354	1.457	1.775
MF-VAR-SF	**0.983**	**0.990**	**0.972**	**0.984**	**0.963**	**0.931**	**0.888**	**0.861**	**0.887**	**0.914**
MF-VAR-KF	1.224	**0.929**	**0.6374****	**0.780****	**0.852****	**0.891****	**0.905****	**0.898****	**0.858****	**0.855****
ARDL	1.001	**0.960**	**0.985**	1.045	1.114	1.177	1.187	1.195	1.293	1.414

Note: Boldface indicates improvements over the benchmark RW model. Lag selection is done with the SIC. Statistically significant reductions in MSPE according to the Diebold-Mariano test are marked using **(5% significance level) and *(10% significance level). The first estimation sample extends from July 1981 to December 1999, and the evaluation sample extends from January 2000 to November 2013.

Table 10: Different aggregation rules for the MF-VAR-KF model

Panel A: High-frequency DGP
$(\rho, \delta_h, \delta_l, \sigma^x, \sigma^y) = (0.5, 0.5, 0.4, 2, 1)$

	All months						*First month*					
Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
MF-VAR-KF (MM)	0.881	0.917	0.941	0.971	0.974	0.971	0.503	0.679	0.794	0.904	0.934	0.957
MF-VAR-KF (stock)	0.886	0.943	0.972	1.003	1.008	1.003	0.478	0.680	0.797	0.910	0.945	0.969
	Second month						*Third month*					
Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
MF-VAR-KF (MM)	1.046	1.015	1.000	1.001	1.004	0.979	1.066	1.046	1.007	1.005	0.989	0.989
MF-VAR-KF (stock)	1.005	1.024	1.008	1.011	1.009	1.004	1.162	1.142	1.095	1.069	1.041	1.036

Panel B: Mixed-frequency DGP ($\delta = 0.2$)

	All months						*First month*					
Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
MF-VAR-KF (MM)	0.684	0.675	0.801	0.834	0.878	0.922	0.718	0.556	0.569	0.652	0.790	0.875
MF-VAR-KF (stock)	0.733	0.698	0.814	0.850	0.897	0.929	0.740	0.563	0.600	0.679	0.806	0.884
	Second month						*Third month*					
Forec. hor.	1	2	3	6	9	12	1	2	3	6	9	12
MF-VAR-KF (MM)	0.639	0.600	0.885	0.896	0.911	0.928	1.076	1.281	0.975	1.032	1.038	1.039
MF-VAR-KF (stock)	0.682	0.631	0.884	0.902	0.914	0.938	0.766	0.895	0.944	0.969	0.965	0.977

Note: This table reports the median of the relative Mean Square Prediction Error for the MF-VAR-KF models averaged over 1000 replications. The benchmark model is an AR model. The MF-VAR-KF model is estimated using two different aggregation constraints: (i) Mariano and Murasawa (2003) constraint (denoted as MF-VAR-KF (MM) model), and (ii) an aggregation rule designed for stock variable (see equation (31) (denoted as MF-VAR-KF (stock)). Additional details on the DGPs are provided in the text.