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Notes on the underground: monetary policy in resource-rich economies

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Notes on the Underground:
Monetary Policy in Resource-Rich Economies*

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Abstract

How should monetary policy respond to a commodity price shock in a resource-rich economy? We study optimal monetary policy in a simple model of an oil exporting economy to provide a first answer to this question. The central bank faces a trade-off between the stabilization of domestic inflation and an appropriately defined output gap as in the reference New Keynesian model. But the welfare-relevant output gap depends on oil technology, and the weight on output stabilization is increasing in the size of the oil sector. Given substantial spillovers to the rest of the economy, optimal policy therefore calls for a reduction of the interest rate following a drop in the oil price in our model. In contrast, a central bank with a mandate to stabilize consumer price inflation may raise interest rates to limit the inflationary impact of an exchange rate depreciation.

JEL codes: E52, E58, J11

Keywords: small open economy, oil export, monetary policy.

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1 Introduction

The oil price has been very volatile over the past four and a half decades (Figure 1a). Steep price increases in the 1970s were followed by a sharp reversal in the first half of the 1980s. After two decades with oil prices below the USD 40 mark, the price swung sharply again during the first decade of the 2000s, briefly reaching its all-time high of USD 145 in July 2008. Since North Sea oil exploitation began in the early 1970s, the real price of Brent Crude has averaged 2014-USD 53 with a standard deviation of 29. More recently, the price has appeared to be relatively stable. In the three years leading up to 19 June 2014, the price moved around an average of USD 110 with a standard deviation of 6 (Figure 1b). But as we write six months later, the price has dropped from 115 to less than USD 60, corresponding to a decline of close to 50 per cent or more than eight standard deviations of the previous three years.

How should monetary policy respond to such a large oil price shock? In the literature, this question has chiefly been addressed from the perspective of countries that are net importers of oil, most notably the United States (Hamilton, 1983; Bernanke et al., 1997). This has naturally led to an emphasis on oil as a consumption good and as an input to production (Finn, 1995; Rotemberg and Woodford, 1996; Leduc and Sill, 2004). A few studies investigate monetary policy issues from the perspective of exporters of oil (Catao and Chang, 2013; Hevia and Nicolini, 2013). But also in these papers, oil is generally introduced as an intermediate and final consumption good, while supply is exogenously given as an endowment.

We depart from this approach and analyze the response of monetary policy to oil price shocks from the perspective of an economy which is dependent on oil exports for foreign currency revenue. Starting from the framework developed by Galí and Monacelli (2005) (henceforth GM), our objective is to establish a benchmark for monetary policy in resource-rich economies. This naturally means that we abstract from a number of features and frictions that may influence the appropriate stance of monetary policy in more complicated models as well as in practice. Here, we focus solely on the implications of introducing a reliance on the export of commodities into the reference model in the New Keynesian literature on monetary policy in small open economies. While for concreteness we focus on oil, our analysis and results apply more broadly to economies which produce and primarily export large quantities of a commodity whose price is determined in world markets.

In contrast to the previous literature, we abstract from domestic consumption of natural resources. Instead, we let the extraction of oil be endogenous and reliant on domestic intermediate inputs. This assumption provides a direct demand link from the oil sector to the rest of the economy. We further assume that our economy is a small player in the global market, taking the world price of

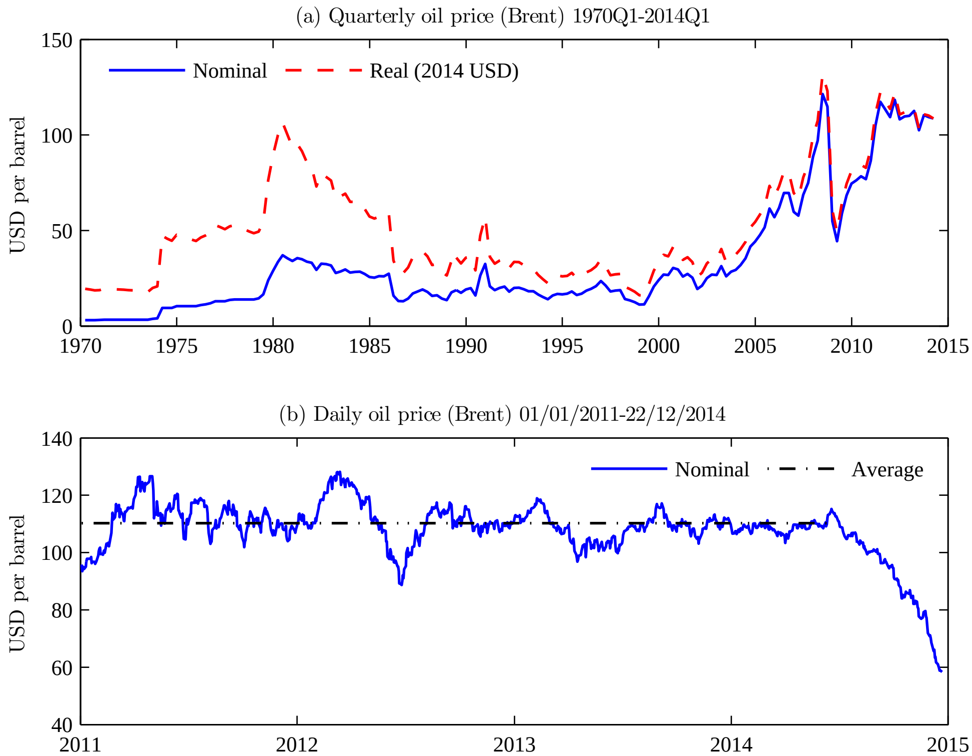

Figure 1: USD price of one barrel of oil (Brent Crude). Sources: OECD and Thomson Reuters/EIA, and own calculations.

oil as given. We allow for oil rents to be channeled into a sovereign wealth fund and spent according to a fiscal policy rule. We believe that these features are particularly important for resource-rich economies.

We evaluate optimal monetary policy for this economy in a linear quadratic framework. While the model features substantial spillovers from the oil sector to the rest of the economy, we show that, as in GM, the objective function only penalizes deviations of domestic inflation from its target (normalized to zero) and of non-oil output from its efficient level. However, the presence of the oil sector changes both the relative weight on the output gap in the loss function and the slope of the Phillips curve in addition to the efficient level of production. Ultimately, the weight on stabilization of real activity increases by the size of the oil sector, and it is higher than in GM under our baseline calibration. Given the spillovers to the rest of the economy, optimal policy therefore calls for a reduction of the interest rate following a drop in the oil price in our model. In contrast, a central bank with a mandate to stabilize consumer price inflation may raise interest rates to limit the inflationary impact of an exchange rate depreciation.

The paper is organized as follows. In Section 2, we motivate our modeling assumptions by presenting key stylized facts from Norway as an example of a resource-rich economy with an independent monetary policy. We present the model in Section 3 and its equilibrium is characterized in Section 4. Section 5 is devoted to optimal monetary policy. Section 6 compares the response of the economy to a negative oil price shock under optimal policy with several simple monetary policy rules. Section 7 concludes.

2 A Small Oil-Exporting Economy

With a total production of 214 standard cubic meters oil equivalent of petroleum in 2012, corresponding to about 3.7 million barrels per day, Norway is the world's tenth largest exporter of crude oil and third largest exporter of natural gas (Tormodsgard, 2014). Figure 2a shows how the share of value added generated by Norway's off-shore oil industry has grown to close to 25 percent of total gross domestic product (GDP) since oil extraction began in the early 1970s.[1]

Figure 2b shows how the composition of Norwegian exports have changed in the same period. Crude oil and natural gas now comprise about 50 percent of exports. A further 20 percent are exports of petroleum products and services, and other primary or basic secondary goods such as metals and chemicals. In addition, a share of remaining exports of goods and services is related to the oil and gas industry (Mellbye et al., 2012). A large share of Norway's exports are therefore either petroleum or other commodities or commodity-related products. Imports of crude oil and natural gas are negligible compared to exports.

Since 1996, the Norwegian state tax revenues and income from direct ownership of oil and gas fields have been transferred to its sovereign wealth fund.[2] Figure 2c shows that the fund's market value has grown to more than twice the size of the mainland economy. Currently, the market value of the fund exceeds USD 800 billion. Since 2001, successive governments have committed to a fiscal policy rule stipulating that a maximum of four percent of the value of the fund—corresponding to the expected average return—can be transferred to the government each year to cover the so-called structural non-oil deficits. This institutional arrangement works to save oil wealth for future generations by essentially transforming Norway's asset portfolio in the direction of less oil underground and more financial assets abroad, thus partially shielding the mainland economy from the use of oil rents.

[1] In addition to the NACE Rev. 2 sector '06 Extraction of crude petroleum and natural gas,' Statistics Norway's aggregate off-shore sector comprises '09.1 Support activities for petroleum and natural gas extraction,' '49.50 Transport via pipeline, ' '50.1 Sea and coastal passenger water transport,' and '50.2 Sea and coastal freight water transport. We follow this terminology by referring to the oil sector as the off-shore economy and to remaining sectors as the mainland economy.

[2] In addition to an ordinary corporate tax rate of 27 per cent, a special tax of 51 per cent is levied on oil companies.

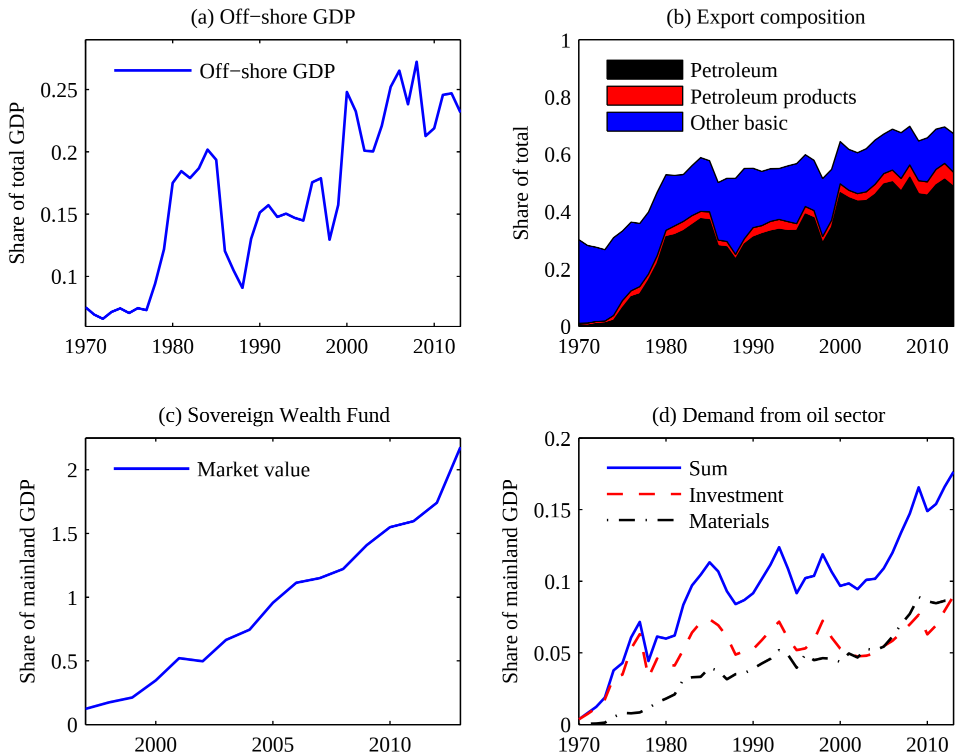

Figure 2: Stylized facts about Norway. Sources: Statistics Norway, Norges Bank Investment Management, and own calculations.

This process is not without consequences for the mainland, however. Figure 2d illustrates an important transmission channel from oil and gas exploitation to other economic activities. The dashed line shows investment in capital stock off shore as a fraction of mainland GDP. While highly volatile, this investment share has fluctuated around a stable average of about six per cent since the mid-1970s. As the capital stock has been built up and off-shore production intensified, intermediate consumption in oil and gas extraction has grown steadily and now comprises about seven percent of value added on the mainland (dashed-dotted line). Together, the oil industry's demand for investment goods and intermediate inputs have grown to constitute approximately 15 percent of mainland GDP in 2013 (solid line).[3]

Such a link from the oil industry to the rest of the economy is central to the model outlined in the next section. While materials and oil investments are equally important in the Norwegian data, we shall emphasize materials in the oil sector in what follows. This allows us to characterize the demand link from the oil sector in a particularly simple form, and we avoid going into the details of oil investment decisions, driven as they are by political and administrative processes and by long

[3]In the early part of the sample, import shares were very high. But since a domestic oil supply and services industry developed through the 1970s, direct import shares have fallen to around 20 percent (Eika et al., 2010).

swings in the oil price. Our approach therefore emphasizes a relatively high-frequency transmission of oil price shocks working through the utilization of existing oil fields, marginal investments and the terms of contracts for suppliers. However, we take the responses to be indicative of the direction if not of the timing of the transmission working through oil investments as well.

3 Model

The basic structure of the model corresponds to GM, with the exception of an oil export sector and the presence of a sovereign wealth fund. Our modeling choices are motivated by the stylized facts just described.

We let oil firms be located off shore. They operate under perfect competition and sell oil in the word market taking the oil price as given. Firms on the mainland, in contrast, sell consumption goods to domestic households and to the government, and they supply materials to off-shore firms to be used as inputs to oil extraction. Mainland firms operate under monopolistic competition and set prices on a staggered basis. A representative household consumes, supplies labor to mainland firms, and trades in a complete set of state-contingent securities in international financial markets. The consumption bundle consists of home goods produced by mainland firms and imported foreign goods.

Oil profits go into a sovereign wealth fund. The treasury receives a transfer from the fund in each period and cannot issue debt. As a consequence, for a given level of government spending, the transfer endogenously determines taxes. We let the transfer be determined by a simple rule, according to which the government can spend a fixed fraction of the fund value in each period.

3.1 Households

At time t, the representative household chooses consumption C_{t+s}, state-contingent securities D_{t+s} and hours at work N_{t+s} for periods $t+s$ where $s = 0, 1, ...$ by solving the intertemporal utility maximization problem

$$\max_{\{C_{t+s}, D_{t+s}, N_{t+s}\}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\ln C_{t+s} - \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right],$$

subject to the budget constraint

$$P_t C_t + \mathbb{E}_t (Q_{t,t+1} D_{t+1}) = W_t N_t + D_t + \Psi_t - T_t. \tag{1}$$

Here, $\mathbb{E}_t$ is the conditional expectation operator, $\beta \in (0,1)$ is the subjective discount factor, $\varphi > 0$ is the inverse Frisch elasticity of labor supply, P_t is the consumer price index (CPI), W_t is the nominal wage rate, Ψ_t represents dividends from the ownership of intermediate goods producing firms, and T_t denotes lump-sum taxes paid to the government.

The first-order condition for state-contingent securities and consumption can be combined to give

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{1}{\Pi_{t+1}}, \tag{2}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is the gross CPI inflation rate. A similar expression, adjusted for the presence of the nominal exchange rate (the price of foreign currency in units of home currency) $\mathcal{E}_t$ holds for the representative household in the rest of the world. Therefore, perfect risk-sharing implies that the ratio of consumption across countries is proportional to real exchange rate $S_t \equiv \mathcal{E}_t P_t^* / P_t$ so that

$$C_t = \vartheta C_t^* S_t, \tag{3}$$

where ϑ is a constant that depends on the initial relative net asset position.[4] By no arbitrage, the nominal net return on a one-period risk-free bond i_t denominated in domestic currency satisfies

$$(1 + i_t)^{-1} = \mathbb{E}_t Q_{t,t+1}. \tag{4}$$

A similar condition holds for a risk-free bond denominated in foreign currency, implying the uncovered interest rate parity condition

$$\mathbb{E}_t \left\{ Q_{t,t+1} \left[(1 + i_t) - (1 + i_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] \right\} = 0. \tag{5}$$

The first order condition for labor supply is

$$\frac{W_t}{P_t} = N_t^{\varphi} C_t. \tag{6}$$

The overall consumption basket C_t is a Cobb-Douglas bundle of home mainland goods C_{Ht} and imported foreign goods C_{Ft}

$$C_t \equiv \frac{C_{Ht}^{1-\alpha} C_{Ft}^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}},$$

where $\alpha \in [0,1]$ represents the import share and is a measure of the degree of openness. Expenditure

[4] In what follows, we assume symmetric initial conditions so that $\vartheta = 1$.

minimization gives rise to the downward-sloping demand functions

$$C_{Ht} = (1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-1} C_t \qquad \text{and} \qquad C_{Ft} = \alpha\left(\frac{P_{Ft}}{P_t}\right)^{-1} C_t, \tag{7}$$

where P_{Ht} and P_{Ft} are the price of domestic and foreign goods in domestic currency, respectively. The associate price consumer price index is

$$P_t = P_{Ht}^{1-\alpha} P_{Ft}^{\alpha}.$$

The terms of trade measure the price of imports in terms of the price of domestic goods. For the mainland economy, the terms of trade are defined as $\mathcal{T}_t \equiv P_{Ft}/P_{Ht}$. Consequently, the relative price of domestic mainland and foreign goods are related to the mainland terms of trade according to

$$\frac{P_{Ht}}{P_t} = \mathcal{T}_t^{-\alpha} \qquad \text{and} \qquad \frac{P_{Ft}}{P_t} = \mathcal{T}_t^{1-\alpha}.$$

We impose that the home country does not export domestic manufacturing goods ($\alpha^* = 0 \Rightarrow P_t^* = P_{Ft}^*$) and that the law of one price holds for foreign goods ($P_{Ft} = \mathcal{E}_t P_{Ft}^*$). Combining these two assumptions gives a relation between the real exchange rate and the mainland terms of trade

$$S_t = \mathcal{T}_t^{1-\alpha}.$$

3.2 Final Goods Producers

On the mainland, competitive final goods producers assemble intermediate goods $Y_{Ht}(i)$. Their problem is to minimize costs

$$\min_{Y_{Ht}(i)} \int_0^1 P_{Ht}(i) Y_{Ht}(i) di,$$

subject to an aggregation technology with constant elasticity of substitution

$$Y_{Ht} \equiv \left[\int_0^1 Y_{Ht}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $P_{Ht}(i)$ is the price charged by individual firm i and $\varepsilon > 1$ is the elasticity of substitution between varieties of mainland goods. The resulting downward-sloping demand function for firm i's product is

$$Y_{Ht}(i) = \left[\frac{P_{Ht}(i)}{P_{Ht}}\right]^{-\varepsilon} Y_{Ht}, \tag{8}$$

and the associated price index is

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}. \tag{9}$$

3.3 Intermediate Goods Producers

Intermediate goods producers set prices on a staggered basis (Calvo, 1983). Each period, a measure $(1-\theta)$ of randomly selected firms get to post a new price $\tilde{P}_{Ht}(i)$ to maximize expected discounted profits

$$\max_{\tilde{P}_{Ht}(i)} \mathbb{E}_t \left\{\sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left[(1+\varsigma)\tilde{P}_{Ht}(i) Y_{Ht,t+s}(i) - W_{t+s} N_{t+s}(i)\right]\right\},$$

where $\varsigma > 0$ is a steady-state subsidy, subject to a linear technology

$$Y_{Ht}(i) = A_{Ht} N_t(i), \tag{10}$$

where A_{Ht} is total factor productivity, and the demand for its own product (8) conditional on no further price change in the future.

In a symmetric equilibrium, all firms that can change their prices make the same choice ($\tilde{P}_{Ht}(i) = \tilde{P}_{Ht}, \ \forall i$). The first-order condition for this problem is

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \theta^s Q_{t,t+s} Y_{Ht,t+s}(i) \left(\tilde{P}_{Ht} - \frac{1}{1+\varsigma}\frac{\varepsilon}{\varepsilon-1}\frac{W_{t+s}}{A_{Ht+s}}\right)\right] = 0. \tag{11}$$

Using the demand relation (8) and the labor market clearing condition, we can also write the aggregate production function for the mainland economy as

$$Y_{Ht}\Delta_t = A_{Ht} N_t, \tag{12}$$

where labor market clearing implies

$$N_t = \int_0^1 N_t(i) di,$$

and Δ_t is an index of price dispersion defined as

$$\Delta_t \equiv \int_0^1 \left[\frac{P_{Ht}(i)}{P_{Ht}}\right]^{-\varepsilon} di.$$

From the price index (9) and the assumption of staggered price setting, the law of motion of the index of price dispersion is

$$\Delta_t = \theta \Pi_{Ht}^{\varepsilon} \Delta_{t-1} + (1-\theta)\left(\frac{1-\theta\Pi_{Ht}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{13}$$

where $\Pi_{Ht} \equiv P_{Ht}/P_{Ht-1}$ is the domestic inflation rate. Also from the definition of the price index, the optimal reset price is related to the domestic inflation rate according to

$$\frac{\tilde{P}_{Ht}}{P_{Ht}} = \left(\frac{1-\theta\Pi_{Ht}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}. \tag{14}$$

From the first order condition for firms (11), we can rewrite the optimal reset price in terms of aggregate variables only as

$$\frac{\tilde{P}_{Ht}}{P_{Ht}} = \frac{X_{1t}}{X_{2t}},$$

where X_{1t} is the present discounted value of total costs in real terms

$$X_{1t} = \frac{1}{1+\varsigma}\frac{\varepsilon}{\varepsilon-1}C_t^{-1}MC_t\frac{P_{Ht}}{P_t}Y_{Ht} + \beta\theta\mathbb{E}_t(\Pi_{Ht}^{\varepsilon}X_{1t+1}), \tag{15}$$

and X_{2t} is the present discounted value of total revenues in real terms

$$X_{2t} = C_t^{-1}\frac{P_{Ht}}{P_t}Y_{Ht} + \beta\theta\mathbb{E}_t(\Pi_{Ht}^{\varepsilon-1}X_{2t+1}), \tag{16}$$

with $MC_t \equiv W_t/(A_{Ht}P_{Ht})$.

3.4 Oil Producers

The production of oil Y_{Ot} uses mainland goods M_t as input in a diminishing-return technology

$$Y_{Ot} = A_{Ot}M_t^{\eta}, \tag{17}$$

where A_{Ot} is total factor oil extraction technology, and $\eta \in (0,1)$.

A representative producer takes the price of inputs as given and sells any quantity in the world market at the price $P_{Ot} = \mathcal{E}_t P^*_{Ot}$. We assume that the oil producer cannot affect the world price of oil P^*_{Ot}, which is instead determined in the international market. The oil firm's problem is

$$\max_{M_t} P_{Ot}Y_{Ot} - P_{Ht}M_t,$$

subject to (17). The first order condition for this problem gives rise to the demand for intermediate inputs

$$\frac{P_{Ht}}{P_{Ot}} = \eta A_{Ot} M_t^{\eta-1}, \tag{18}$$

which in turn determines oil production by (17) and oil profits as $(1-\eta)P_{Ot}Y_{Ot}$.

3.5 Government

The fiscal authority takes spending G_t as given and needs to respect the budget constraint

$$P_{Ht}G_t = T_t + R_t, \tag{19}$$

where R_t represents transfers from the sovereign wealth fund. We assume that the government follows the fiscal policy rule

$$R_t = \rho \left(1 + i^*_{t-1}\right) \mathcal{E}_t F^*_{t-1}, \tag{20}$$

where F^*_{t-1} is foreign asset fund holdings at end of the previous period, and $\rho \in (0,1)$. This rule allows the government to spend a fixed fraction ρ of the initial value of the fund each period and is similar to the "bird-in-hand rule" in Wills (2014). Since oil profits are fully taxed, the rule implies that the value of the fund evolves according to

$$\mathcal{E}_t F^*_t = (1-\rho) \left(1 + i^*_{t-1}\right) \mathcal{E}_t F^*_{t-1} + (1-\eta) P_{Ot}Y_{Ot}. \tag{21}$$

To insure that the real value of the fund is stationary, we restrict ρ to be such that

$$(1-\rho)\left(1 + i^*_{t-1}\right) < 1.$$

This restriction ensures that the government spends slightly more than the average yield on the fund each period. In this case, the value of the fund will stabilize in the long run even with a constant stream of oil revenue.

3.6 Goods Market Clearing

Domestic goods can be consumed by the household, as input in oil extraction, and for government spending. Hence, goods market clearing requires

$$Y_{Ht} = C_{Ht} + M_t + G_t. \tag{22}$$

Foreign goods are consumed at home and abroad so that

$$Y^*_{Ft} = C_{Ft} + C^*_{Ft}.$$

Finally, all oil is exported abroad

$$Y_{Ot} = C^*_{Ot}.$$

3.7 National Accounts

Nominal gross domestic product (GDP) for the mainland economy is the value of the goods produced by mainland firms at home, $GDP_{Ht} = P_{Ht}Y_{Ht}$, while nominal off-shore GDP is the value added in the oil sector, $GDP_{Ot} = P_{Ot}Y_{Ot} - P_{Ht}M_t$. Total nominal GDP is the sum of the two: $GDP_t = GDP_{Ht} + GDP_{Ot}$.

Using expenditure minimization and the resource constraint for mainland goods, we can rewrite total GDP as $GDP_t = P_tC_t - P_{Ft}C_{Ft} + P_{Ot}Y_{Ot} + P_{Ht}G_t$. Because the home country fully exports its oil production and no other manufacturing good, the previous expression can be also written in real terms as

$$Y_t \equiv \frac{GDP_t}{P_t} = C_t + \frac{P_{Ht}}{P_t}G_t + NX_t,$$

where the real trade balance is

$$NX_t \equiv \frac{P_{Ot}}{P_t}Y_{Ot} - \frac{P_{Ft}}{P_t}C_{Ft}. \tag{23}$$

Following a similar reasoning, we can also define real mainland GDP as

$$\tilde{Y}_{Ht} \equiv \frac{P_{Ht}}{P_t}Y_{Ht} = C_t + \frac{P_{Ht}}{P_t}G_t + NX_{Ht},$$

where the trade balance for the mainland economy is

$$NX_{Ht} \equiv \frac{P_{Ht}}{P_t}M_t - \frac{P_{Ft}}{P_t}C_{Ft}.$$

4 Equilibrium

All prices can be expressed in terms of the CPI and related to the terms of trade. In addition to the expressions for the prices of home and foreign goods in section 3.1, we can write the real oil price as

$$\frac{P_{Ot}}{P_t} = \frac{P^*_{Ot}}{P^*_t}\frac{\mathcal{E}_t P^*_t}{P_t} = \frac{P^*_{Ot}}{P^*_t}\mathcal{T}_t^{1-\alpha},$$

where the real foreign currency price of oil (P^*_{Ot}/P^*_t) is exogenous. The terms of trade is therefore the only relative price that matters for the characterization of the equilibrium. Hence, given monetary policy (pinning down the nominal interest rate i_t), initial conditions (Δ_{-1} and $\mathcal{T}_{-1}$) and exogenous processes for foreign output ($Y^*_t = C^*_t$), interest rates (i^*_t), inflation ($\Pi^*_{Ft} = \Pi^*_t$), productivity (A_{Ht} and A_{Ot}) and the real dollar oil price (P^*_{Ot}/P^*_t), an imperfectly competitive equilibrium is a sequence of quantities

$$\{C_t, C_{Ht}, C_{Ft}, N_t, X_{1t}, X_{2t}, Y_{Ot}, M_t, Y_{Ht}\}_{t=0}^{\infty}$$

and prices

$$\{Q_{t,t+1}, \Pi_t, \mathcal{E}_t, MC_t, \Delta_t, \Pi_{Ht}, \mathcal{T}_t\}_{t=0}^{\infty}$$

such that households and firms optimize, the government satisfies its budget constraint, and all markets clear.

4.1 Sovereign Wealth Fund Irrelevance

Inspection of equations (2) to (17), (22), and the equation that links the evolution of the terms of trade to the nominal exchange rate depreciation and the inflation differential between mainland and foreign goods ($\mathcal{T}_t/\mathcal{T}_{t-1} = (\mathcal{E}_t/\mathcal{E}_{t-1})(\Pi^*_t/\Pi_{Ht})$), reveals that the equilibrium is independent of fiscal policy decisions. In particular, the evolution of the sovereign wealth fund and the transfer rule from the sovereign wealth fund to the fiscal authority are irrelevant.

The assumptions of lump-sum taxation and complete markets are crucial for the result. Iterating the household budget constraint (1), replacing profits, and imposing the transversality condition gives

$$D_t = \mathbb{E}_t\left[\sum_{s=0}^{\infty} Q_{t,t+s}(P_{t+s}C_{t+s} - P_{Ht+s}Y_{Ht+s} + T_{t+s})\right],$$

where, without loss of generality, we have abstracted from the steady state subsidy. Further, using expenditure minimization, the resource constraint for mainland goods, the production function in the oil sector, and the government budget constraint leads to

$$D_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} (P_{Ft+s} C_{Ft+s} - \eta P_{Ot+s} Y_{Ot+s} - R_{t+s}) \right].$$

From the evolution of the sovereign wealth fund (21) and the transfer rule (20), we can then write

$$R_t = -\mathcal{E}_t [F_t^* - (1 + i_{t-1}^*) F_{t-1}^*] + (1 - \eta) P_{Ot} Y_{Ot}.$$

Replacing this relation in the expression for D_t above yields

$$D_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} Q_{t,t+s} [P_{Ft+s} C_{Ft+s} - P_{Ot+s} Y_{Ot+s} + \mathcal{E}_{t+s} F_{t+s}^* - (1 + i_{t+s-1}^*) \mathcal{E}_{t+s} F_{t+s-1}^*] \right\}.$$

Therefore, the state-contingent payment in period t compensates for any future trade imbalance (the difference between the first two terms in square brackets) and for the accumulation/decumulation of future net foreign asset positions via the sovereign wealth fund (the other two terms). In other words, state-contingent securities undo any international wealth transfer associated with the fund.

4.2 Efficient and Natural Equilibrium

The efficient allocation corresponds to the outcome of the optimization problem of a benevolent social planner who maximizes the utility of the representative agent in the absence of distortions subject only to technological, resource and international risk-sharing constraints. This problem is static and can be represented as

$$\max_{N_t, \mathcal{T}_t} \; \log(\mathcal{T}_t^{1-\alpha} C_t^*) - \frac{N_t^{1+\varphi}}{1+\varphi}$$

subject to

$$A_{Ht} N_t = (1-\alpha) \mathcal{T}_t Y_t^* + \left(\eta A_{Ot} \frac{P_{Ot}^*}{P_t^*} \mathcal{T}_t \right)^{\frac{1}{1-\eta}} + G_t,$$

where we have used the risk-sharing condition to substitute for aggregate consumption, and the demands for domestic goods as well as the production function to replace variables in the resource constraint. The first-order condition for this problem is

$$1 - \alpha = (N_t^e)^{1+\varphi} \gamma_{\tau t}, \tag{24}$$

where the superscript "e" denotes the efficient equilibrium and

$$\gamma_{\tau t} \equiv \frac{C_{Ht}}{Y_{Ht}} + \frac{1}{1-\eta} \frac{M_t}{Y_{Ht}}. \tag{25}$$

A variational argument shows that this efficiency condition equates the representative household's marginal rate of substitution of leisure for consumption with the marginal rate of transformation. From the utility function, an optimal plan must satisfy the condition

$$\frac{dC_t}{dN_t} = C_t N_t^{\varphi}. \tag{26}$$

In turn, dC_t/dN_t is determined by the constraints to the optimization problem. From the risk sharing condition we find

$$\frac{dC_t}{d\mathcal{T}_t} \frac{\mathcal{T}_t}{C_t} = 1 - \alpha,$$

and from the resource constraint

$$\frac{dY_{Ht}}{d\mathcal{T}_t} \frac{\mathcal{T}_t}{Y_{Ht}} = \gamma_{\tau t}.$$

so that γ_τ can be seen to represent the elasticity of domestic output with respect to the terms of trade. Combining these two relations with the marginal product of labor from the production function gives

$$\frac{dC_t}{dN_t} = \frac{dC_t}{d\mathcal{T}_t} \frac{d\mathcal{T}_t}{dY_{Ht}} \frac{dY_{Ht}}{dN_t} = \frac{1-\alpha}{\gamma_{\tau t}} \frac{C_t}{Y_{Ht}} A_{Ht}.$$

Inserting this in (26) gives an alternative representation of the efficiency condition

$$\frac{1-\alpha}{\gamma_{\tau t}} \frac{C_t^e}{Y_{Ht}^e} A_{Ht} = C_t^e \left(N_t^e\right)^{\varphi}, \tag{27}$$

which reduces to (24).

The flexible-price equilibrium can be characterized be the labor market equilibrium condition, which can be found by combining (6) and (11):

$$(1+\varsigma) \frac{\epsilon - 1}{\epsilon} \left(\mathcal{T}_t^n\right)^{-\alpha} A_{Ht} = C_t^n \left(N_t^n\right)^{\varphi}. \tag{28}$$

Here, the superscript "n" denotes the flexible-price or so-called natural equilibrium. It follows from a comparison with (27) that the natural equilibrium is efficient if and only if

$$(1+\varsigma)\,\frac{\epsilon-1}{\epsilon}\,(\mathcal{T}_t)^{-\alpha}=\frac{1-\alpha}{\gamma_{\tau t}}\frac{C_t}{Y_{Ht}}$$

This cannot be expected to hold in the absence of a time-varying subsidy, and the natural equilibrium is generally inefficient in our model. However, the subsidy ς can be set such that this condition holds in the steady state ensuring that $N^{1+\varphi}=(1-\alpha)/\gamma_\tau$, where we denote steady-state variables by dropping time subscripts.

We note that the term γ_τ is constant and equal to one in GM for the case with a unitary elasticity of international substitution in consumption. Consequently, the efficient level of employment is constant at $N^{1+\varphi}=1-\alpha$. Also, since $dC_t/dY_{Ht}=(1-\alpha)\mathcal{T}_t^{-\alpha}$ in that version of the model, it follows that any subsidy such that $(1+\varsigma)(\epsilon-1)/\epsilon=1-\alpha$ guarantees the efficiency of both the steady state and the flexible-price equilibrium with $(N_t^n)^{1+\varphi}=1-\alpha$ for all t. Conversely in our model, γ_τ depends on oil technology and the time-varying share of the oil sector's material demand in total demand for mainland goods. As a result, the efficient level of employment moves with the exogenous shocks out of the steady state, and the natural equilibrium is inefficient even if a subsidy ensures the efficiency of the steady state. Effectively, the oil sector introduces an externality, by which the social planner can improve on the natural equilibrium through variation in the terms of trade. This is because firms do not internalize the effect of movements in the terms of trade on consumption when setting prices.

5 Linear-Quadratic Framework

To characterize the optimal monetary policy plan away from steady state, we take a second-order approximation of the utility function of the representative agent and a first-order approximation to the equilibrium conditions. Equilibrium conditions are log-linearized around the steady state characterized in Appendix A, and the log-linearized relations are listed in Appendix B, where we use lower case letter to represent log-deviations from steady state: $z_t\equiv\ln(Z_t/Z)$ for any variable Z_t. The resulting linear-quadratic framework allows us to derive a targeting rule for the central bank that implements optimal policy. We focus on the solution under commitment from a timeless perspective (Woodford, 1999).

5.1 Efficient and Natural Output

Up to a first-order approximation, we can solve for the efficient levels of mainland output and the terms of trade using the efficiency condition and the resource and risk-sharing constraints (see Appendix C). The terms of trade become

$$\begin{aligned}&\left\{\varphi\gamma_\tau^2 + \lambda_\tau\right\}\tau_t^e = \\ &\quad (1+\varphi)\gamma_\tau a_{Ht} - s_c(1+\varphi\gamma_\tau)y_t^* - \frac{s_m}{1-\eta}\left(\varphi\gamma_\tau + \frac{1}{1-\eta}\right)(a_{Ot} + p_{Ot}^*) - \varphi\gamma_\tau(1-s_c-s_m)g_t,\end{aligned} \tag{29}$$

where $s_c \equiv C_H/Y_H$, $s_m \equiv M/Y_H$, γ_τ is the steady state value of $\gamma_{\tau t}$, and

$$\lambda_\tau \equiv s_c + \frac{s_m}{(1-\eta)^2}.$$

The efficient level of output is given as

$$\begin{aligned}&\left\{\frac{\lambda_\tau}{\gamma_\tau} + \varphi\gamma_\tau\right)y_{Ht}^e = \\ &\quad (1+\varphi)\gamma_\tau a_{Ht} + \frac{\lambda_\tau}{\gamma_\tau}\left(1-s_c-s_m\right)g_t + \left(\frac{\lambda_\tau}{\gamma_\tau} - 1\right)s_c y_t^* - \frac{s_c}{\gamma_\tau}\frac{\eta}{1-\eta}\frac{s_m}{1-\eta}\left(a_{Ot} + p_{Ot}^*\right).\end{aligned} \tag{30}$$

Note that a reduction in the oil price leads to a depreciation of the efficient terms of trade and an increase in the efficient level of mainland output. As oil production becomes less profitable, the off-shore economy demands less inputs. This reduces the relative price of home mainland goods. Under complete markets, however, the depreciation of the terms of trade corresponds to higher domestic consumption because of international risk sharing. This allows the planner to increase production of domestic goods to meet consumption demand without adverse effects on welfare.

For comparison, the approximate flexible-price equilibrium level of output is given as

$$y_{Ht}^n = \frac{1}{1+\varphi\gamma_\tau}\left[(1+\varphi)\gamma_\tau a_{Ht} + \frac{s_m}{1-\eta}(p_{Ot}^* + a_{Ot} - y_t^*) + (1-s_c-s_m)g_t\right], \tag{31}$$

while the flexible-price level of the terms of trade can be found using the relation

$$\tau_t^n = (1+\varphi)a_{Ht} - y_t^* - \varphi y_{Ht}^n. \tag{32}$$

A negative oil price shock also leads to a depreciation of the flexible-price terms of trade. But the natural level of output falls as the market does not internalize the welfare effects of the depreciation working through consumption. Notice that the natural and efficient levels of output coincide if $\gamma_\tau = \lambda_\tau = s_c$, which holds when the resource sector does not demand any resources from the mainland so that $\eta = s_m = 0$. If, in addition, government spending is zero so that $\gamma_\tau = s_c = 1$, we have $y^n_{Ht} = y^e_{Ht} = a_{Ht}$ as in GM.

5.2 Quadratic Loss Function

To derive the quadratic loss function, we use the expression for the aggregate production function to rewrite the utility function of the representative household as

$$\mathcal{W}_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} - \frac{(Y_{Ht+s}/A_{Ht+s})^{1+\varphi}}{1+\varphi} \Delta_{t+s}^{1+\varphi} \right] \right\}.$$

In Appendix E, we show that a second order approximation of this expression about a steady state with zero inflation and relative prices equal to one yields

$$\mathcal{W}_t = -\frac{\Omega}{2} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s (\pi^2_{Ht+s} + \lambda_x x^2_{Ht+s}) \right] + t.i.p. + \mathcal{O}(\|\epsilon_t\|^3), \tag{33}$$

where $t.i.p.$ stands for "terms independent of policy" (i.e. exogenous shocks) and $\mathcal{O}(\|\epsilon_t\|^3)$ collects the terms of order three or higher that we neglect by taking a second order approximation. The constants in the previous expression are functions of the structural parameters of the model

$$\begin{aligned} \Omega &\equiv \frac{(1-\alpha)\varepsilon}{\kappa\gamma_\tau}, \\ \lambda_x &\equiv \frac{\kappa}{\varepsilon}\left(\frac{\lambda_\tau}{\gamma_\tau^2} + \varphi\right), \end{aligned}$$

where

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

The welfare-relevant output gap is defined as the deviation of mainland output from its efficient level

$$x_{Ht} = y_{Ht} - y^e_{Ht}.$$

The form of the loss function in our model coincides with the one in GM. But the relative weight on the efficient output gap is different. In GM, the absence of an oil sector and government spending implies that $s_m = 0$ and $s_c = 1$ so that $\lambda_\tau = \gamma_\tau^2 = 1$. Effectively, the inefficiency gap between the marginal rate of substitution and the marginal rate of transformation is determined by the deviation of the level of employment from its efficient level in that model. A higher value of φ leads to a higher inefficiency gap for a given output gap so that the weight in the loss function is increasing in this parameter. Here, $\lambda_\tau > \gamma_\tau^2$ and the weight on the output gap is larger. This is a consequence of the terms-of-trade externality, which leads to a further opening of the inefficiency gap whenever output deviates from its efficient level.

5.3 Linear Constraints

Expressions (29) and (30) characterize the efficient equilibrium away from steady state up to the first order. We now find a representation for the Phillips curve in terms of the efficient output gap that can be used to derive the optimal policy rule in our model.

A first-order approximation of the firm price-setting condition gives the New Keynesian Phillips curve

$$\pi_{Ht} = \kappa mc_t + \beta \mathbb{E}_t \pi_{Ht+1}, \tag{34}$$

where marginal cost is given as

$$mc_t = \varphi y_{Ht} - (1+\varphi) a_{Ht} + y_t^* + \tau_t. \tag{35}$$

As we show in Appendix D, we can rewrite the previous expression in terms of the efficient output gap only as

$$\pi_{Ht} = \xi x_{Ht} + \beta \mathbb{E}_t \pi_{Ht+1} + u_t, \tag{36}$$

where

$$u_t \equiv \kappa[\varphi y_{Ht}^e + \tau_t^e - (1+\varphi) a_{Ht} + y_t^*]$$

and

$$\xi \equiv \frac{\kappa(1+\varphi\gamma_\tau)}{\gamma_\tau}.$$

The term u_t consists of a weighted sum of shocks and is generally different from zero away from the steady state. It measures the extent to which contemporaneous stabilization of inflation and the efficient output gap is impossible as a consequence of demand spillovers from the oil sector. Without the resource and government sectors when γ_τ equals one, u_t drops out of the Phillips curve since in this case $\tau_t^e = a_{Ht} - y_t^*$ and $y_{Ht}^e = a_{Ht}$. This means that the "divine coincidence" holds, and monetary policy does not face a trade-off between domestic inflation and output gap stabilization.

Note that, differently from models in which oil is an input in the production stage, oil prices have inflationary consequences only through an indirect impact on marginal costs (see equation 34 and 35). The price of oil does not enter directly in the aggregate supply relation. An increase in the price of oil leads to an appreciation of the terms of trade (τ_t falls) and higher demand of intermediate inputs from off-shore. With nominal stickiness in mainland prices, production of mainland goods increases (y_{Ht} rises). The two effects on marginal costs go in opposite directions, and their relative strength depends on the inverse Frisch elasticity of labor supply.

The Phillips curve can also be written in terms of the flexible-price output gap as

$$\pi_{Ht} = \xi(y_{Ht} - y^n_{Ht}) + \beta \mathbb{E}_t \pi_{Ht+1}.$$

Comparing this representation with the one in terms of the efficient output gap, it follows that the term that prevents contemporaneous stabilization of inflation and the efficient output gap is proportional to the difference between efficient and flexible-price level of output

$$u_t = \xi(y^e_{Ht} - y^n_{Ht}).$$

Therefore, the term u_t captures the extent of the distortions in the economy that arise because of a terms of trade externality.

Up to the first order, the consumption Euler equation expressed in terms of efficient output gap reads as

$$x_{Ht} = -\sigma_\alpha(i_t - \mathbb{E}_t \pi_{t+1} - r^e_t) + \mathbb{E}_t x_{Ht+1}, \tag{37}$$

where $\sigma_\alpha \equiv (1-\alpha)/\gamma_\tau$ and the efficient real interest rate is defined implicitly by $r^e_t = \mathbb{E}_t c^e_{t+1} - c^e_t$ and

$$y^e_{Ht} = \frac{\gamma_\tau}{1-\alpha} c^e_t + \left(s_c - \frac{\gamma_\tau}{1-\alpha} \right) y^*_t + \frac{s_m}{1-\eta}(p^*_{Ot} + a_{Ot}) + (1 - s_c - s_m) g_t.$$

For a given choice of monetary policy, the relation between CPI and domestic inflation ($\pi_t = \pi_{Ht} + \alpha(\tau_t - \tau_{t-1})$) and the relation between the efficient output gap and the terms of trade ($x_{Ht} = \gamma_\tau(\tau_t - \tau^e_t)$) complete the description of the equilibrium up to a first-order approximation.

5.4 Optimal Monetary Policy

The linear-quadratic framework then consists of maximizing the second-order approximation of the objective function $\mathcal{W}_t$ in (33). This corresponds to solving the welfare loss minimization problem

$$\min_{\{\pi_{Ht+s}, x_{Ht+s}, i_{t+s}, \pi_{t+s}, \tau_{t+s}\}} \frac{\Omega}{2}\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s (\pi_{Ht+s}^2 + \lambda_x x_{Ht+s}^2)$$

subject to the aggregate supply equation (36), the aggregate demand equation (37) and the relations between CPI and domestic inflation as well as between the output gap and the terms of trade:

$$\begin{aligned}
\pi_{Ht} &= \xi x_{Ht} + \beta \mathbb{E}_t \pi_{Ht+1} + u_t \\
x_{Ht} &= -\sigma_\alpha (i_t - \mathbb{E}_t \pi_{t+1} - r_t^e) + \mathbb{E}_t x_{Ht+1} \\
\pi_t &= \pi_{Ht} + \alpha(\tau_t - \tau_{t-1}) \\
x_{Ht} &= \gamma_\tau (\tau_t - \tau_t^e).
\end{aligned}$$

Given the representation of the loss function and the constraints, the problem is equivalent to minimizing the loss function subject to the aggregate supply equation only, thus obtaining a solution for domestic inflation and the output gap. The remaining variables (interest rate i_t, CPI inflation π_t, and terms of trade τ_t) are then the solution to the remaining three equations given the optimal values of π_{Ht} and x_{Ht}.

The first-order conditions for the simplified problem (under commitment from a timeless perspective) are

$$\pi_{Ht} - \mu_t + \mu_{t-1} = 0,$$

and

$$\lambda_x x_{Ht} + \xi \mu_t = 0,$$

where μ_t is the Lagrange multiplier on the constraint. Combining the two first-order conditions to eliminate the Lagrange multiplier yields a standard optimal targeting rule

$$\pi_{Ht} + \frac{\lambda_x}{\xi}(x_{Ht} - x_{Ht-1}) = 0. \tag{38}$$

The optimal targeting rule takes the same form as in a closed-economy model with exogenous cost-push shocks (Clarida et al., 1999; Woodford, 2003). The same result would also hold in GM. In our model, however, the term u_t is not a cost-push shock per se, but rather a linear combination

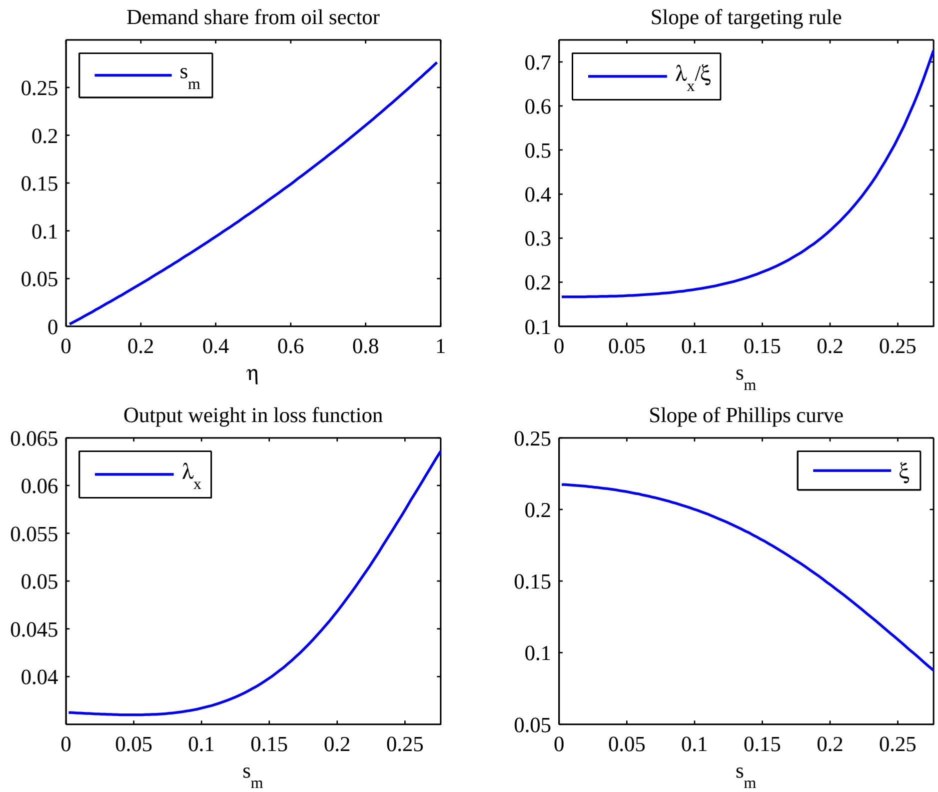

Figure 3: Sensitivity of the policy trade-off to the size of the demand impulse from the oil sector

of disturbances arising from the demand side of the economy. This convolution of demand shocks prevents contemporaneous stabilization of inflation and the output gap. Its presence in the Phillips curve depends on the reallocation of resources between the domestic and the off-shore economy due to terms of trade fluctuations that affect the marginal cost for mainland firms.

Another difference with the standard model is that the coefficient that governs the optimal policy trade-off is a function of the size of the oil sector through its effect on the composite parameters γ_τ and λ_τ:

$$\frac{\lambda_x}{\xi} = \frac{(\lambda_\tau + \varphi\gamma_\tau^2)}{(\gamma_\tau + \varphi\gamma_\tau^2)\varepsilon}.$$

In the absence of the oil sector with $\lambda_\tau = \gamma_\tau$, the weight on real activity in the optimal targeting rule equals the inverse of the elasticity of substitution among varieties. This special case encompasses both the closed and open economy counterparts (Clarida et al., 1999, and GM).

In our model, the targeting rule is steeper in that the weight on the change in the output gap is larger. This is a consequence of a larger weight on output stabilization as the Phillips curve actually becomes steeper with our benchmark calibration (outlined below), in which $\eta = 0.25$ and $s_m = 0.06$. That is, even if the sacrifice ratio (the output cost of reducing inflation) is lower in our model, the monetary policy maker penalizes output fluctuation more because of the higher welfare consequences of deviating from the efficient level of output. As we increase the size of the resource sector from our benchmark calibration, the weight on the output gap increases and the slope of the Phillips curve falls.[5] Consequently, as we illustrate in figure 3, the targeting rule becomes increasingly steeper as we increase the size of the oil sector.

6 Impulse Response Analysis

We now turn to an impulse response analysis of a shock to the dollar price of oil in a parameterized version of the model.

6.1 Parameterization

We consider a period to be one quarter and let $\beta = 0.995$. This implies that real interest rates at home and abroad are about two per cent in steady state. The inverse of the Frisch elasticity of labor supply is set to $\varphi = 1$ and the share of home goods in consumption is $\alpha = 0.4$. We let the expected duration of price contracts be about a year by setting $\theta = 0.75$. The materials share in oil production is set to $\eta = 0.25$. We let the treasury spend the fraction $\rho = 0.01$ of the value of the fund each quarter, corresponding to about four per cent on an annualized basis. The government is sizeable with $\gamma_G = 0.3$.

The values for α and η have a strong influence on the relative size of the off-shore economy, the wealth fund and the public budget financed by oil revenues. The Norwegian import share suggest a value of α no larger than 0.4, while the income distribution for the oil sector would suggest a calibration of η no smaller than 0.25. Our chosen calibration implies that the off-shore economy contributes about 14 per cent to total GDP, while six per cent of mainland GDP is exported off-shore in the form of inputs to oil extraction in line with the evidence provided in Section 2. Consumption of home mainland goods makes up about 59 per cent of mainland GDP, while the remaining 35 per cent go to government spending with this calibration. The wealth fund has grown to 29 times GDP in the steady state, which allows for a mainland trade deficit of 33 per cent of mainland GDP through the resource balance. 97 per cent of government spending is thus financed through revenue from the fund. The dynamics are not sensitive to the steady-state size of the fund, however.

[5] There is a critical point around $\eta = 0.66$ for which $\gamma_\tau = 1$ and the slope of the Phillips curve coincides with the one in GM. However, in this case λ_τ is about 2 and the weight on output stabilization is higher than in GM.

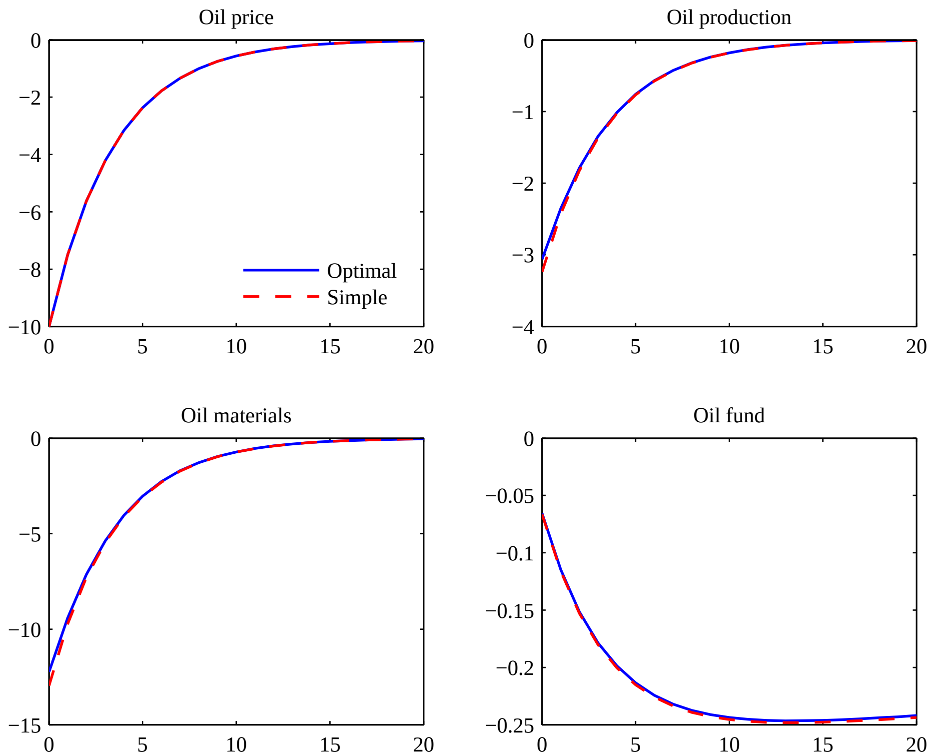

Figure 4: Impulse responses to a negative shock to the USD oil price in the off-shore economy with optimal monetary policy (solid lines) and a simple monetary policy rule (dashed line)

6.2 Oil price shock

Consider a moderately persistent fall in the dollar price of oil as in the top left panel of Figure 4. For about three years, the shock reduces the profitability of oil production. Oil producers respond by reducing the extraction of oil (see figure 4). Hence, the demand for intermediate input from the mainland declines, and both the volume of oil extracted and the profits generated in the off-shore economy decline. The value of the sovereign wealth fund drops since oil revenue falls short of spending. As the oil price shock abates, the size of the fund stabilizes at a lower level. Only slowly does the fund revert to its initial size as oil revenues recover. These responses in the off-shore sector are driven by the fall in the oil price and are only marginally affected by monetary policy.

In contrast, monetary policy will shape the propagation of the oil price shock through the mainland economy. This, in turn, will determine the optimal monetary policy response. Figure 5 shows responses of selected mainland variables under optimal policy with timeless commitment (solid lines). For comparison, the figure also shows responses for a regime in which the central bank follows the simple rule $i_t = 1.5\pi_{Ht}$ (dashed lines). In both cases, the fall in the demand for intermediates off-shore leads to an contraction of economic output on the mainland. And in both cases, monetary policy responds by reducing the interest rate. But optimal monetary policy calls

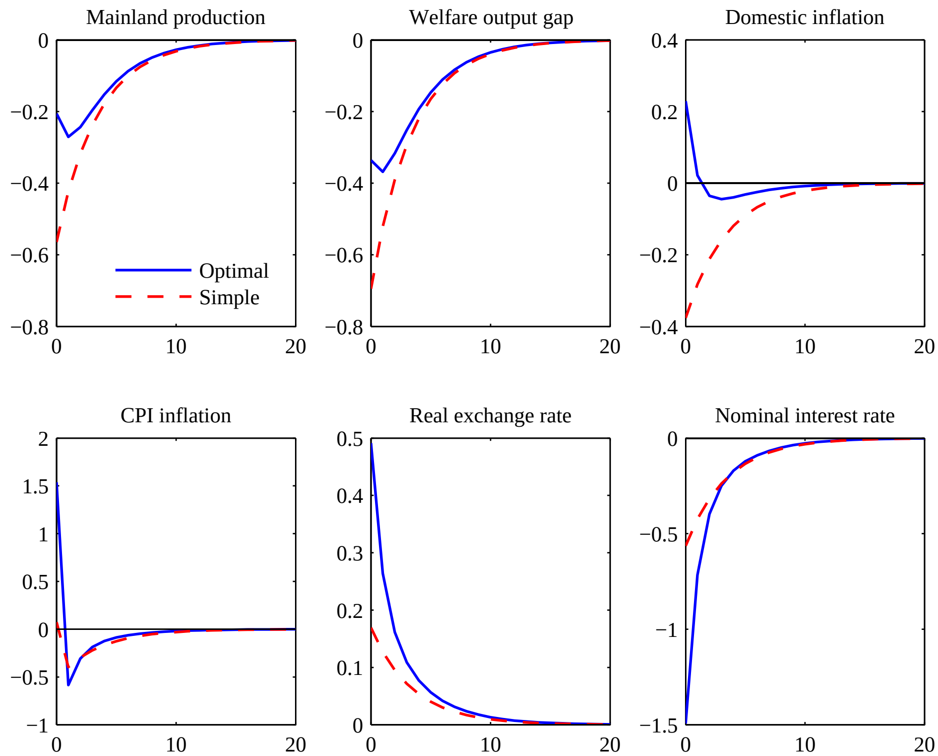

Figure 5: Impulse responses to a negative shock to the USD oil price in the mainland economy with optimal monetary policy (solid lines) and a simple monetary policy rule (dashed line)

for a stronger interest rate response than the simple rule. By reducing the interest rate by more than one percentage point, the central bank reduces the contraction in output to about 0.25 per cent.

In response to the reduction in demand from the oil industry, mainland firms cut back production and reduce their demand for labor. This works to drive down the real wage and marginal costs. Firms therefore also want to reduce prices and domestic prices tend to fall. With the simple monetary policy rule used here, the central bank simply leans against this process. A lower interest rate stimulates aggregate demand and thus reduces labor supply at given wages, and a real exchange rate appreciation reduces the relative price of home goods. But both the welfare-relevant output gap and domestic inflation fall.

With optimal monetary policy, the central bank will not allow its two target variables to move in the same direction, see the targeting rule in (38). It therefore reduces the interest rate enough to induce an increase in the real wage despite the contraction in output and employment. By stimulating private consumption, the policy contracts labor supply enough to more than offset the effect on the real wage from a fall in labor demand. In addition, a larger real exchange rate depreciation works to increase the real product wage. With higher marginal costs, firms set higher

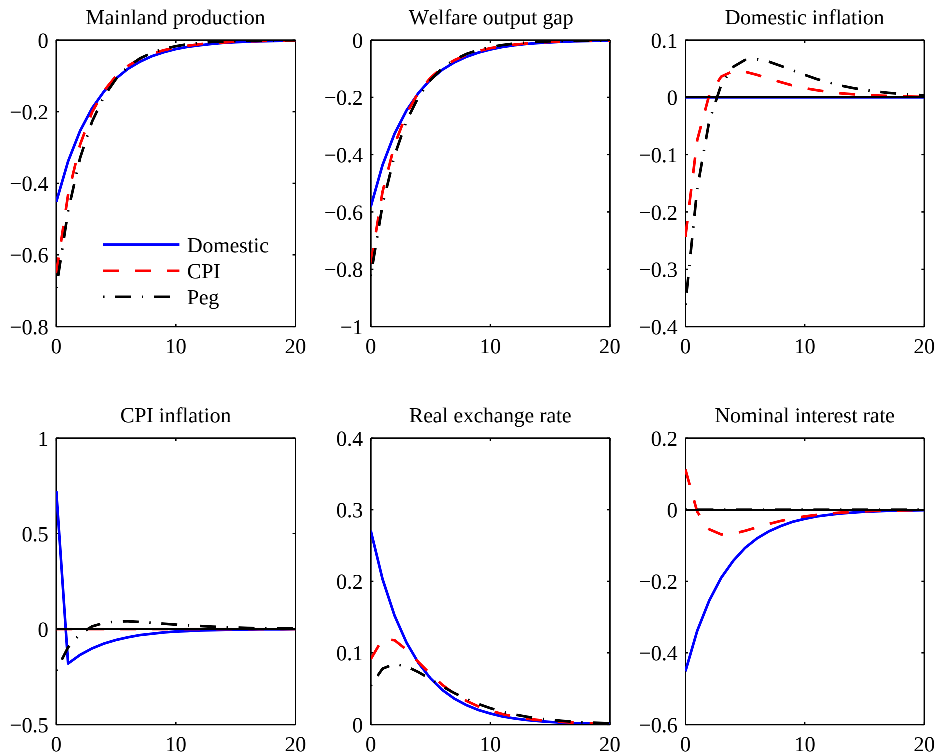

Figure 6: Impulse responses to a negative shock to the USD oil price in the mainland economy with strict simple rules: Domestic inflation targeting (solid lines), CPI inflation targeting (dashed line) and exchange rate peg (dashed-dotted lines)

prices and domestic inflation rises.

For comparison, Figure 6 shows responses under three simple but strict monetary policy regimes. Under strict domestic inflation targeting (solid lines), the central bank successfully stabilizes domestic price inflation ($\pi_{Ht} = 0$), under strict CPI inflation targeting (dashed lines) the bank stabilizes consumer price inflation ($\pi_t = 0$), and under an exchange peg (dotted lines) it stabilizes the nominal exchange rate ($e_t = 0$).

The strict domestic targeting regime is similar to the simple rule considered in Figure 5, only now the central bank leans enough against the deflationary pressure from the oil price shock to keep domestic inflation constant. In contrast, with consumer price inflation targeting, the central bank increases the nominal interest rate on impact of the shock. As the fall in the oil price reduces the relative price of domestic goods, the real exchange rate depreciates. This increases consumer price inflation through imported inflation. The central bank therefore needs to increase interest rates initially to keep CPI inflation on target. A central bank with a credible peg is restricted to keeping the interest rate in line with the foreign rate. The real exchange rate depreciation takes the form of consumer price deflation in this case, while dynamics in the real economy are similar to those under consumer price inflation targeting.

7 Conclusion

We have studied monetary policy in a simple New Keynesian model of a resource-rich economy. Given substantial spillovers from the commodity sector to the rest of the economy, optimal policy calls for a reduction of the interest rate following a drop in the commodity price. While this prescription is clear in our model, the results also illustrate that a central bank with a flexible consumer price inflation target may find itself in a dilemma after a shock to the commodity price. A fall in the price will lead to a slowdown in the domestic economy. But a sharp depreciation of the exchange rate may lead to inflationary pressure. Given its mandate, the central bank may therefore have to increase interest rates at the cost of deepening the domestic recession further.

References

Bernanke, B., M. Gertler, and M. Watson (1997). Systematic Monetary Policy and the Effects of Oil Price Shocks. *Brookings Papers on Economic Activity 1*, 91–142.

Calvo, G. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics 12*, 383–398.

Catao, L. and R. Chang (2013). Monetary Policy Rules for Commodity Traders. *IMF Economic Review 61*, 52–91.

Clarida, R., J. Galí, and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature 37*, 1661–1707.

Eika, T., J. Prestmo, and E. Tveter (2010). Ringvirkninger av petroleumsvirksomheten - hvilke naeringer leverer? Statistics Norway.

Finn, M. (1995). Variance Properties of Solow's Productivity Residual and Their Cyclical Implications. *Journal of Economic Dynamics and Control 19*, 1249–1282.

Galí, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies 72*, 707–734.

Hamilton, J. (1983). Oil and the Macroeconomy Since World War II. *Journal of Political Economy 91*, 228–248.

Hevia, C. and J. P. Nicolini (2013). Optimal Devaluations. *IMF Economic Review 61*, 22–51.

Leduc, S. and K. Sill (2004). A Quantitative Analysis of Oil-Price Shocks, Systematic Monetary Policy, and Economic Downturns. *Journal of Monetary Economics 51*, 781–808.

Mellbye, C., S. Fjose, and O. Thorseth (2012). Internasjonaliseringen av norsk oljeleverandoerindustri. Menon Business Economics.

Rotemberg, J. and M. Woodford (1996). Imperfect Competition and the Effects of Energy Price Increases. *Journal of Money, Credit, and Banking 28*, 143–148.

Tormodsgard, Y. (2014). Facts 2014. The Norwegian Petroleum Sector.

Wills, S. (2014). Optimal Monetary Responses to Oil Discoveries. OxCarre working paper no. 121, University of Oxford.

Woodford, M. (1999). Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability? In *New Challenges for Monetary Policy*, pp. 277–316. Federal Reserve Bank of Kansas City Symposium, Jackson Hole, WY.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

A Steady state

We consider a steady state in which inflation is zero and relative prices are all one. To ensure that a steady-state level exists for the sovereign wealth fund, we assume that $(1-\rho)(1+i^*) < 1$. In the steady state, the value of the fund has stabilized at its long-run value given the constant stream of revenue from oil. This means that the steady state represents a mature oil economy with a large sovereign wealth fund and low levels of taxation. Further, we set $G/Y = \gamma_G$.

A.1 Model

The household relations (1)-(7) become

$$C = \frac{W}{P}N + \frac{\Psi}{P} - \frac{T}{P} \tag{39}$$

$$\frac{1}{1+i} = \beta \tag{40}$$

$$C = C^* S \tag{41}$$

$$\frac{1}{1+i^*} = \frac{1}{1+i} \tag{42}$$

$$\frac{W}{P} = \omega N^{\varphi} C^{\sigma} \tag{43}$$

$$C = \frac{C_H^{1-\alpha} C_F^{\alpha}}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha}} \tag{44}$$

$$C_H = (1-\alpha)C \tag{45}$$

$$C_F = \alpha C \tag{46}$$

The firm relations (10), (11), (17) and (18) become

$$Y_H = A_H N \tag{47}$$

$$\frac{W_t}{P_H} = (1+\varsigma)\frac{(1-\varepsilon)}{\varepsilon}A_H \tag{48}$$

$$Y_O = A_O M^\eta \tag{49}$$

$$1 = \eta A_O M_O^{\eta-1} \tag{50}$$

The government budget relations (19), (20) and (21) become

$$G = \frac{T}{P} + \frac{R}{P} \tag{51}$$

$$\frac{R}{P} = \rho\,(1+i^*)\,\frac{F^*}{P^*} \tag{52}$$

$$\frac{F^*}{P^*} = \frac{(1-\eta)\,Y_O}{1-(1-\rho)\,(1+i^*)} \tag{53}$$

Market clearing requires

$$Y_H = C_H + G + M \tag{54}$$

while relevant definitions in the steady state are

$$NX_H = M - C_F \tag{55}$$

$$Y = \tilde{Y}_H + \tilde{Y}_O \tag{56}$$

$$\tilde{Y}_H = Y_H \tag{57}$$

and

$$\tilde{Y}_O = (1-\eta)\, Y_O \tag{58}$$

A.2 Solution

To solve for key steady-state relations, first, combine (52), (53) and (58) to get

$$\frac{R/P}{Y} = \frac{\rho\,(1+i^*)}{1-(1-\rho)\,(1+i^*)}\frac{\tilde{Y}_O}{Y} \tag{59}$$

Second, combine (49), (50) and (58) to get

$$\frac{M}{\tilde{Y}_O} = \frac{\eta}{1-\eta} \tag{60}$$

Third, combine (54), (45), national account definitions (56) and (57), and (60) to get

$$\frac{C}{Y} = \frac{1}{1-\alpha}\left(1-\gamma_G-\frac{1}{1-\eta}\frac{\tilde{Y}_O}{Y}\right) \tag{61}$$

Fourth, noting that $\Psi/P = Y_H - (W/P)N$ and substituting (54), (45), (46) and (51) into (39) gives $R/P = C_F - M$ so that

$$\frac{M}{\tilde{Y}_O}\frac{\tilde{Y}_O}{Y} = \alpha\frac{C}{Y} - \frac{R/P}{Y} \tag{62}$$

Substituting (59)-(61) in (62) and rearranging gives

$$\frac{\tilde{Y}_O}{Y} = \frac{\alpha}{1-\alpha}\,(1-\gamma_G)\left(\frac{\eta}{1-\eta}+\frac{\alpha}{1-\alpha}\frac{1}{1-\eta}+\frac{\rho\,(1+i^*)}{1-(1-\rho)\,(1+i^*)}\right)^{-1} \tag{63}$$

Remaining steady-state ratios appearing in the log-linearization can now easily be recovered.

B Linear model

B.1 Households

Using national accounts definitions, the household budget constraint gives an expression for the evolution of net foreign assets:

$$\Delta nfa_t = \frac{NX_H}{Y}\left(p_{Ht} + nx_{Ht}\right) + \frac{R/P}{Y} r_t$$

where mainland net exports are given as

$$nx_{Ht} = \frac{M}{NX_H} m_t - \frac{C_F}{NX_H}\left(p_{Ft} - p_{Ht} + c_{Ft}\right)$$

The consumption bundle can be expressed in terms of prices as

$$0 = (1 - \alpha)\, p_{Ht} + \alpha p_{Ft}$$

The Euler equation is given as

$$c_t = E_t c_{t+1} - \left(i_t - E_t \pi_{t+1}\right)$$

while international risk-sharing condition takes the form

$$c_t = y_t^* + s_t$$

Labor supply is

$$w_t = c_t + \varphi n_t$$

Demand for home goods is

$$c_{Ht} = -p_{Ht} + c_t$$

and for foreign goods

$$c_{Ft} = -p_{Ft} + c_t$$

B.2 Firms

Mainland production technology is

$$y_{Ht} = a_{Ht} + n_t$$

The price-setting first-order condition results in the New Keynesian Phillips curve

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa mc_t$$

with $\kappa \equiv= (1-\beta\theta)(1-\theta)/\theta$, marginal costs given as

$$mc_t = w_t - p_{Ht} - a_{Ht}$$

and

$$\pi_{Ht} - \pi_t = p_{Ht} - p_{Ht-1}$$

Off-shore technology is given as

$$y_{Ot} = a_{Ot} + \eta m_t$$

Input demand in the oil sector is

$$p_{Ht} - p_{Ot} = a_{Ot} + (\eta - 1)\, m_t$$

with

$$p_{Ot} = s_t + p^*_{Ot}$$

B.3 Government

The government budget constraint is

$$g_t + p_{Ht} = \frac{G - T/P}{G} r_t + \frac{G - T/P}{T/P} t_t$$

and the fiscal policy rule

$$r_t = s_t + f^*_{t-1} + i^*_{t-1} - \pi^*_t$$

The fund evolves according the process

$$f^*_t + s_t = (1-\rho)\,(1+i^*)\left(f^*_{t-1} + s_t + i^*_{t-1} - \pi^*_t\right) + \left[1 - (1-\rho)\,(1+i^*)\right](y_{Ot} + p_{Ot})$$

Monetary policy may follow a simple rule like

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_{it} + \phi_y\,(y_{it} - \tilde{y}_t)\right]$$

where $\tilde{y}_t$ is an output target, or an optimal targeting rule like (38).

B.4 Market clearing and definitions

Market clearing requires

$$y_{Ht} = \frac{C_H}{Y_H} c_{Ht} + \frac{M}{Y_H} m_t + \frac{G}{Y_H} g_t$$

By definition

$$s_t = (1-\alpha)\tau_t$$

and

$$\tau_t = p_{Ft} - p_{Ht}$$

Total GDP is

$$y_t = \frac{\tilde{Y}_H}{Y} y_{Ht} + \frac{\tilde{Y}_O}{Y} y_{Ot}$$

C Efficient and natural output

Log-linearizing the efficiency condition (24) gives

$$(1+\varphi)n_t^e = -\gamma_\tau^{-1}\left(s_c c_{Ht}^e + \frac{s_m}{1-\eta} m_t^e\right) + y_{Ht}^e$$

Using the production function, demand relations and the risk-sharing condition, this can be written as

$$\varphi\gamma_\tau y_{Ht}^e = -\lambda_\tau \tau_t^e + (1+\varphi)\gamma_\tau a_{Ht} - s_c y_t^* - \frac{1}{(1-\eta)^2} s_m \left(a_{Ot} + p_{Ot}^*\right) \tag{64}$$

Similarly, the resource constraint can be written as

$$\varphi\gamma_\tau y_{Ht}^e = \gamma_\tau \tau_t + s_c y_t^* + \frac{1}{1-\eta} s_m \left(a_{Ot} + p_{Ot}^*\right) + (1 - s_c - s_m) g_t \tag{65}$$

Equations (64) and (65) represent a system of two equations in the two unknowns y_{Ht}^e and τ_t^e. Solving it gives (29) and (30) in the text.

Independently of price setting, we can use the risk sharing and demand relations to write the resource constraint as

$$y_{Ht} = \gamma_\tau \tau_t + s_c y_t^* + \frac{s_m}{1-\eta}\left(p_{Ot} + a_{Ot}\right) + (1 - s_c - s_m) g_t \tag{66}$$

from which is follows that

$$y_{Ht} - y_{Ht}^{n} = \gamma_{\tau}\left(\tau_t - \tau_t^{n}\right)$$

The labor market equilibrium condition, which holds when prices are fully flexible, combined with the risk-sharing condition gives (32) in the text. Combining it with (66) gives (31).

D Phillips curve

Marginal costs are given as

$$mc_t + mc = w_t - p_{Ht} - a_{Ht} = \varphi\left(y_{Ht} - a_{Ht}\right) + y_t^{*} + \tau_t - a_{Ht}$$

where the second equality uses labor supply and the risk sharing condition. With flexible price, firms keep marginal costs fixed so that

$$mc = \varphi y_{Ht}^{n} + \tau_t^{n} - (1+\varphi)a_{Ht} + y_t^{*}$$

Hence

$$mc_t = \varphi\left(y_{Ht} - y_{Ht}^{n}\right) + \left(\tau_t - \tau_t^{n}\right) = \left(\gamma_{\tau}^{-1} + \varphi\right)\left(y_{Ht} - y_{Ht}^{n}\right) \tag{67}$$

Inserting (67) in (34) and rearranging gives (36) with

$$u_t = \xi(y_{Ht}^{e} - y_{Ht}^{n}) = \kappa[\varphi y_{Ht}^{e} + \tau_t^{e} - (1+\varphi)a_{Ht} + y_t^{*}]$$

where the second equality follows from manipulations using the solutions for the efficient and natural equilibria.

E Loss function

Using the expression for the aggregate production function, we can rewrite the utility function of the representative household as

$$\mathcal{W}_t = \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^{s}\left[\ln C_{t+s} - \frac{\left(Y_{Ht+s}/A_{Ht+s}\right)^{1+\varphi}}{1+\varphi}\Delta_{t+s}^{1+\varphi}\right]\right\}.$$

A second order approximation of this expression around a steady state with zero inflation and relative prices equal to one yields

$$\mathcal{W}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ c_{t+s} - \left(\frac{Y_H}{A_H}\right)^{1+\varphi} \left[y_{Ht+s} + \frac{1+\varphi}{2}(y_{Ht+s}^2 - 2a_{Ht+s}y_{Ht+s}) + \frac{1}{2}\frac{\theta\varepsilon}{(1-\theta)(1-\beta\theta)}\pi_{Ht+s}^2 \right] \right\},$$

where we have omitted terms independent of policy and of order higher than two.

In order to obtain a purely quadratic approximation of the utility function, we need to eliminate the linear terms in consumption and output. To this end, we take a second order approximation of the resource constraint

$$y_{Ht} + \frac{1}{2}y_{Ht}^2 = \frac{C_H}{Y_H}\left(c_{Ht} + \frac{1}{2}c_{Ht}^2\right) + \frac{M}{Y_H}\left(m_t + \frac{1}{2}m_t^2\right) + \frac{G}{Y_H}\left(g_t + \frac{1}{2}g_t^2\right)$$

Using this expression, the linear terms can be written as

$$c_t - N^{1+\varphi}y_{Ht} = -\frac{N^{1+\varphi}}{2}\left(\frac{C_H}{Y_H}c_{Ht}^2 + \frac{M}{Y_H}m_t^2 - y_{Ht}^2\right)$$

where we have again ignored terms independent of policy and the optimal subsidy has been imposed so that $(1-\alpha) = N^{1+\varphi}\gamma_\tau$. Inserting the exactly log-linear demand and risk sharing conditions gives

$$c_t - N^{1+\varphi}y_{Ht} = -\frac{N^{1+\varphi}}{2}\left\{\lambda_\tau\tau_t^2 + 2\left[s_c y_t^* + \frac{s_m}{(1-\eta)^2}\left(a_{Ot} + p_{Ot}^*\right)\right]\tau_t - y_{Ht}^2\right\}$$

Inserting this expression into the welfare function, using the expression

$$\varphi\gamma_\tau y_{Ht}^e + \lambda_\tau\tau_t^e - (1+\varphi)\gamma_\tau a_{Ht} = -s_c y_t^* - \frac{s_m}{(1-\eta)^2}\left(a_{Ot} + p_{Ot}^*\right)$$

from the efficient equilibrium and the implication of the resource constraint that $y_{Ht} = \gamma_\tau\tau_t + t.i.p.$, gives

$$\mathcal{W}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ -\frac{N^{1+\varphi}}{2}\left[\lambda_\tau\left(\tau_t^2 - 2\tau_t\tau_t^e\right) + \varphi\left(y_{Ht}^2 - 2y_{Ht}y_{Ht}^e\right) + \frac{\theta\varepsilon}{(1-\theta)(1-\beta\theta)}\pi_{Ht+s}^2\right]\right\},$$

where terms independent of policy have been ignored. Using the relation $y_{Ht} - y_{Ht}^e = \gamma_\tau\left(\tau_t - \tau_t^e\right)$, we can write this as

$$\mathcal{W}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ -\frac{N^{1+\varphi}}{2} \left[\left(\lambda_\tau - \gamma_\tau^2\right) \left(\tau_t - \tau_t^e\right)^2 + (1+\varphi) \left(y_{Ht} - y_{Ht}^e\right)^2 + \frac{\varepsilon}{\kappa} \pi_{Ht+s}^2 \right] \right\},$$

which is equivalent to (33).