

2014 | 02

# Working Paper

Monetary Policy

## Forecasting recessions in real time

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# Forecasting recessions in real time\*

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February 7, 2014

## Abstract

We review several methods to define and forecast classical business cycle turning points in Norway. In the paper we compare the Bry - Boschan rule (BB) with a Markov Switching model (MS), using alternative vintages of Norwegian Gross Domestic Product (GDP) as the business cycle indicator. The timing of business cycles depends on the vintage and the method used. BB provides the most reasonable definition of business cycles. The forecasting exercise, where the models are augmented with surveys or financial indicators, respectively, leads to the conclusion that the BB rule applied to density forecasts of GDP augmented with either the consumer confidence index or a financial conditions index provides the most timely predictions of peaks. For troughs, augmenting with surveys or financial indicators does not increase forecastability.

**JEL-codes:** C32, C52, C53, E37, E52

**Keywords:** Forecast densities; Turning Points; Real-time data

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\*This Working Paper should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. We thank Hilde Bjørnland, Marcelle Chauvet, Nicholas Fawcett, Laurant Ferrara, Domenico Giannone, Christian Schumacher and Shaun Vahey for helpful comments. We also thank seminar and conference participants at the workshop on “New Methods for Forecasting Macroeconomic Data” at the University of Nottingham, the 33rd Annual International Symposium on Forecasting, the 7th International Conference on Computational and Financial Econometrics, the 9th Annual CIRANO-CIREQ Workshop on “Data Revision in Macroeconomic Forecasting and Policy”, Deutsche Bundesbank and Norges Bank.

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# 1 Introduction

Short-term analysis in central banks and other policy institutions is intended to provide policy makers, and possibly a larger audience, with assessments of the recent past and current business cycle. Point and density forecasts of variables of interest are often provided. However, the analysis of current economic conditions does not rely just on this information, and there is a long tradition in business cycle analysis and related research of separating periods of contraction from periods of expansion, see [Schumpeter \(1954\)](#). Policy decisions vary depending on whether the economy is in an expansionary or a recessionary period. Most of the research has focused on US data, where the NBER cycle is regarded as the official reference cycle. Most other countries have not defined such an “official” dating of classical business cycles.

In this paper we review some alternative methods to define classical business cycle turning points for the Norwegian economy. The Norwegian business cycle is neither fully synchronized with the cycles of other Scandinavian countries, nor with the European or the US cycles. One reason for this asymmetry is Norway’s position as a small open economy with large exports of energy (gas and oil) goods. We compare the non-parametric Bry - Boschan rule (BB) as in [Harding and Pagan \(2002\)](#) with the parametric autoregressive Markov Switching model (MS) as in [Hamilton \(1989\)](#). Both methods are applied to quarterly mainland Norwegian Gross Domestic Product (GDP).<sup>1</sup>

Macroeconomic data, and in particular GDP, are subject to revisions over time. Benchmark revisions, but also revisions due to new information may change GDP substantially after several years. Economic decisions are not immune to such changes (see [Orphanides and van Norden \(2002\)](#)). [Hamilton \(2011\)](#) also shows that business cycle dating exhibits important differences when alternative vintages of data are considered. We estimate business cycle turning points using the methods listed above, comparing results using ex-post revised data and data available in real time.

Finally, data are released with substantial delays. Hence economic decisions must

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<sup>1</sup>Since the volume of exports of oil and gas are disconnected from the domestic business cycle, we exclude this from GDP.

rely on forecasts of the missing recent information.

While there is a large body of literature on point forecasts (see [Timmermann \(2006\)](#) for a recent review), and a growing literature on density forecasting, research on classical turning point prediction, in particular for countries other than the US, has been more limited. See [Anas et al. \(2008\)](#) and [Billio et al. \(2012\)](#) for applications to the Euro Area.

Markov switching models can produce point, density and turning point forecasts for GDP, whereas forecasts of recent and future values for GDP are produced in order to use the BB rule to predict turning points. Both methods can be augmented with additional leading indicators. A leading indicator is interpreted as a variable that summarizes the common cyclical movements of some coincident macroeconomic variables in a timely manner. We focus our analysis on two classes of data: financial indices and survey data.

[Harvey \(1989\)](#) was one of the first authors to document the relationship between financial variables and macroeconomic aggregates, focusing on the links between the term structure and consumption growth. More recently, [Cochrane and Piazzesi \(2005\)](#) and [Ludvigson and Ng \(2009\)](#) have extended such analysis. There are several studies documenting that the information obtained from surveys has high forecasting power for macroeconomic variables. See, for example, [Thomas \(1999\)](#), [Mehra \(2002\)](#), [Fama and Gibbons \(1984\)](#), and [Ang et al. \(2007\)](#) for the use of quantitative surveys, and [Hansson et al. \(2005\)](#), [Abberger \(2007\)](#), [Claveria et al. \(2007\)](#) and [Lui et al. \(2010a,b\)](#) for the application of qualitative surveys. Specifically, for Norwegian data [Næs et al. \(2011\)](#) and [Aastveit and Trovik \(2012\)](#) document the role of financial indicators, and [Martinsen et al. \(2014\)](#) apply survey data to forecast Norwegian economic aggregates.

We find that the BB provides reasonable estimates of business cycles over the last three to four decades, while business cycle downturns estimated by the MS tend to be too short and might not be detected at all. When predicting business cycle peaks in real time, financial and survey data seem to add substantial predictability. The BB applied to density forecasts augmented with either a financial condition index, consumer confidence or Norges Bank's regional network survey provide superior predictions compared to the ones from Markov Switching models. Troughs are predicted in a less timely manner.

In contrast to forecasting peaks, augmenting the models with financial and survey data does not improve trough predictability.

The rest of the paper is organized as follows: the next section describes the modeling framework and discusses how business cycle turning points are defined. The third section presents data and dating of business cycles in Norway over the past four decades. The fourth section focuses on the prediction of turning points in real time, describes the recursive forecasting exercise and presents the results. Section 5 concludes.

## 2 Business cycle dating approaches

Following [Burns and Mitchell \(1946\)](#), we define business cycles as fluctuations in aggregate economic activity. This is the classical business cycle characterized by peaks and troughs, describing developments in the level of economic activity across many sectors. An alternative concept is the growth cycle. Economic fluctuations are characterized by “high” or “low” growth, most commonly relative to trend growth. An attractive feature of the classical business cycle is that it is not necessary to calculate unobserved trend growth. This is particularly important when it comes to forecasting turning points, since the uncertainty in the measurement of trend growth is at its highest at the end of the time series for commonly used two-sided filters.

Classical business cycles in the US are defined by the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER). The committee decides when a turning point occurs, i.e. in which months a recession respectively starts and ends. Decisions are made by deliberation based on available data, hence announcements of turning points are not very timely. The December 2007 peak was announced December 1, 2008 and the following June 2009 trough was announced September 20, 2010. The dating of the turning points is normally not revised.

A number of methods have been suggested in order to develop mechanical algorithms for calculating the start and end of recessions, in particular for US data where recessions defined by the NBER serve as benchmarks. See [Hamilton \(2011\)](#) for a survey. Here, we

concentrate on two different methods.

## 2.1 Bry - Broschan

Bry and Boschan (1971) describe a method that was able to (almost) replicate the business cycles in the US as measured by the dating committee of the NBER. Harding and Pagan (2002) build on the work by Bry and Boschan and develop an algorithm for detecting turning points in quarterly data. The procedure picks potential turning points and subjects them to conditions that ensure that relevant criteria for business cycles are met.

In the first step, the BB procedure identifies a *potential peak* in a quarter if the value is a local maximum. Correspondingly, a *potential trough* is identified if the value is a local minimum. Searching for maxima and minima over a window of 5 quarters seems to produce reasonable results. After potential turning points are identified, the choice of final turning points depends on several rules to ensure alternating peaks and troughs and minimum duration of phases and cycles. Formally, definitions of peaks can be written

$$\wedge_t = 1\{(y_{t-2}, y_{t-1}) < y_t > (y_{t+1}, y_{t+2})\} \quad (1)$$

Correspondingly for troughs:

$$\vee_t = 1\{(y_{t-2}, y_{t-1}) > y_t < (y_{t+1}, y_{t+2})\} \quad (2)$$

When forecasting peaks and troughs, the values on the right-hand side of the equations are replaced by forecasts. Formally:

$$\wedge_t = 1\{(y_{t-2}, y_{t-1}) < y_t, Prob(y_{t+1}, y_{t+2}) < y_t) > 0.5\} \quad (3)$$

and

$$\vee_t = 1\{(y_{t-2}, y_{t-1}) > y_t, Prob(y_{t+1}, y_{t+2}) > y_t) > 0.5\} \quad (4)$$

The business cycle can be interpreted as a state  $S_t$ , which takes the value 1 in expansions and 0 in recessions. Turning points occur when the state changes. The

relationship between the business cycle and the local peaks and troughs can be written as

$$S_t = S_{t-1}(1 - \wedge_{t-1}) + (1 - S_{t-1})\vee_{t-1} \quad (5)$$

If the economy is in an expansion,  $S_{t-1} = 1$ . If no peak occurred in (t-1), then  $\wedge_{t-1} = 0$  and it follows that the state  $S_t = 1$ . On the other hand, if there is a peak in (t-1) then  $\wedge_{t-1} = 1$  and the state changes to  $S_t = 0$ . The state will remain at 0 until a trough is detected.

## 2.2 Markov Switching

There is a long tradition of using nonlinear models to capture the asymmetry and the turning points in business cycle dynamics. Among such classes of models, Markov Switching (MS) models (see for example [Goldfeld and Quandt \(1973\)](#), [Hamilton \(1989\)](#), [Clements and Krolzig \(1998\)](#), [Kim and Murray \(2002\)](#), [Kim and Piger \(2002\)](#), and [Krolzig \(2000\)](#) for further extensions) are dominant. We consider an autoregressive MS model for GDP growth similar to [Hamilton \(1989\)](#), where only the intercept is allowed to switch between regimes:

$$y_t = \nu_{s_t} + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + u_t, \quad u_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2) \quad (6)$$

$t = 1, \dots, T$ , where  $\nu_{s_t}$  is the MS intercept,  $\phi_l$ , with  $l = 1, \dots, p$ , are the autoregressive coefficients; and  $\{s_t\}_t$  is the regime-switching process, that is an  $m$ -states ergodic and aperiodic Markov-chain process. This process is unobservable (latent) and  $s_t$  represents the current phase, at time  $t$ , of the business cycle (e.g. contraction or expansion). Therefore, the MS model does not require knowledge of  $y_{t+1}$  and  $y_{t+2}$ , as the BB rule does, to define the cycle at time  $t$ . The latent process takes integer values, say  $s_t \in \{1, \dots, m\}$ , and has transition probabilities  $\mathbb{P}(s_t = j | s_{t-1} = i) = p_{ij}$ , with  $i, j \in \{1, \dots, m\}$ . The transition matrix  $P$  of the chain is

$$P = \begin{pmatrix} p_{11} & \dots & p_{1m} \\ \vdots & & \vdots \\ p_{m1} & \dots & p_{mm} \end{pmatrix}$$



and has, as a special case, the one-forever-shift model that is widely used in structural-break analysis (e.g., see [Jochmann et al. \(2010\)](#) and references therein). In our applications we assume that the initial values,  $(y_{-p+1}, \dots, y_0)$ , and  $s_0$ , of the processes  $\{y_t\}_t$  and  $\{s_t\}_t$  respectively, are known. A suitable modification of the procedure in [Vermaak et al. \(2004\)](#) can be applied for estimating the initial values of both the observable and the latent variables.<sup>2</sup>

The choice of the number of regimes is often crucial. Following previous literature we investigate specification from two regimes (as for example in [Hamilton \(1989\)](#)) to four regimes (such as in [Billio et al. \(2012\)](#)). Evidence of more than two regimes, even in forecasting applications, is rather common in finance and has suggestive economic meanings. See [Guidolin \(2011\)](#) for an up-to-date literature review with an in-depth discussion of this aspect. However, in our empirical application we only identify two regimes.

We apply a Bayesian inference approach. There are at least two reasons for this choice. First, inference for latent variable models calls for simulation based methods, which can be naturally included in a Bayesian framework. Second, predictive densities essential for density forecasting are natural output in a Bayesian framework, overcoming difficulties of the frequentist approach in dealing with parameter uncertainty either by ignoring it or by implementing a time-consuming bootstrapping approach.

Our proposed Bayesian inference framework relies on data augmentation (see [Tanner and Wong \(1987\)](#)) and on a Monte Carlo approximation of the posterior distributions as in [Billio et al. \(2012\)](#). We follow [Frühwirth-Schnatter \(2006\)](#) and define the vector of regressors,  $\mathbf{x}_{0t} = (y_{t-1}, \dots, y_{t-p}, \sigma)'$ , with regime invariant coefficients,  $\boldsymbol{\phi} = (\phi_1, \dots, \phi_p)'$ , and the vector,  $\boldsymbol{\nu} = (\nu_1, \dots, \nu_m)'$  of regime-specific parameters. In this notation the regression model in equation (6) can be written as

$$y_t = \boldsymbol{\xi}_t' \boldsymbol{\nu} + \mathbf{x}_{0t}' \boldsymbol{\phi} + u_t, \quad u_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \gamma)$$

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<sup>2</sup>Following [Krolzig \(2000\)](#) and [Anas et al. \(2008\)](#), we also investigate an MS model which assumes that both the intercept and the volatility are driven by a regime-switching variable. The results are qualitatively similar and available upon request.

The data-augmentation procedure (see also [Frühwirth-Schnatter \(2006\)](#)) yields the complete likelihood function of model (6)

$$L(\mathbf{y}_{1:T}, \boldsymbol{\xi}_{1:T} | \boldsymbol{\theta}) = \prod_{t=1}^T \prod_{k=1}^m \prod_{j=1}^m p_{jk}^{\xi_{jt-1}\xi_{kt}} (2\pi\sigma_k^2)^{-\frac{\xi_{kt}}{2}} \exp \left\{ -\frac{\xi_{kt}}{2\sigma_k^2} (y_t - \nu_k - \mathbf{x}'_{0t}\boldsymbol{\phi})^2 \right\} \quad (7)$$

where  $\boldsymbol{\theta} = (\boldsymbol{\nu}', \boldsymbol{\phi}', \boldsymbol{\sigma}', \mathbf{p})'$  is the parameter vector, with  $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_m)'$ ,  $\mathbf{p}_k = (p_{k1}, \dots, p_{km})$  the  $k$ -th row of the transition matrix, and  $\mathbf{z}_{s:t} = (\mathbf{z}_s, \dots, \mathbf{z}_t)'$ ,  $1 \leq s \leq t \leq T$ , denotes a subsequence of a given sequence of variables,  $\mathbf{z}_t$ ,  $t = 1, \dots, T$ .

In a Bayesian framework we need to complete the description of the model by specifying the prior distributions of the parameters. Again following [Billio et al. \(2012\)](#) we apply the data-dependent prior approach suggested by [Diebolt and Robert \(1994\)](#) and consider a partially improper conjugate prior. Improper conjugate priors are numerically close to the Jeffreys prior, provide similar inferences and yield easier posterior simulations. We assume uniform prior distributions for all the autoregressive coefficients, the intercept and the precision parameters

$$\begin{aligned} (\phi_1, \dots, \phi_p) &\propto \mathbb{I}_{\mathbb{R}^p}(\phi_1, \dots, \phi_p) \\ \nu_k &\propto \mathbb{I}_{\mathbb{R}}(\nu_k), \quad k = 1, \dots, m \\ \sigma_k^2 &\propto \frac{1}{\sigma_k^2} \mathbb{I}_{\mathbb{R}_+}(\sigma_k^2), \quad k = 1, \dots, m \end{aligned}$$

and do not impose stationarity constraints for the autoregressive coefficients. We assume standard conjugate prior distributions for the transition probabilities. These distributions are independent and identical Dirichlet distributions, one for each row of the transition matrix

$$(p_{k1}, \dots, p_{km})' \sim \mathcal{D}(\delta_1, \dots, \delta_m)$$

with  $k = 1, \dots, m$ .

When estimating an MS model, which is a dynamic mixture model, one needs to deal with the identification issue arising from the invariance of the likelihood function and of the posterior distribution (which follows from the assumption of symmetric prior distributions) to permutations of the allocation variables. Many different ways to solve

this problem are discussed, see for example [Frühwirth-Schnatter \(2006\)](#). We identify the regimes by imposing some constraints on the parameters, a standard procedure in business cycle analysis. We consider the following identification constraints on the intercept:  $\nu_1 < 0$  and  $\nu_1 < \nu_2 < \dots < \nu_m$ , which allow us to interpret the first regime as the one associated with the recession phase.

Samples from the joint posterior distribution of the parameters and the allocation variables are obtained by iterating a Gibbs sampling algorithm. We refer to [Billio et al. \(2012\)](#), section 3.3, for specific details of the sampling procedure for the posterior of the allocation variables (see also [Krolzig \(1997\)](#)). The methodology produces predictive densities for  $y_{t+h}$ ,  $p(y_{t+h}|y_t)$  as final output. We apply iterative forecasting when  $h > 1$ .

The MS specification in the previous paragraph can be augmented with leading indicators. We investigate several indicators, based on financial indices or surveys. The idea is that a leading indicator is a variable that summarizes the common cyclical movements of some coincident macroeconomic variables. Hence, our approach is to model the idea of business cycles as the simultaneous movement of economic activity in various sectors by using a simple indicator. In addition, the asymmetric nature of expansions and contractions is captured by assuming again a Markov process for GDP growth:

$$y_t = \nu_{s_t} + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \beta z_t + u_t, \quad u_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2) \quad (8)$$

$t = 1, \dots, T$ , where  $\nu_{s_t}$  is the MS intercept;  $\phi_l$ , with  $l = 1, \dots, p$ , are the autoregressive coefficients;  $\{s_t\}_t$  is the regime-switching process, and  $z_t$  is an exogenous indicator. Priors and posteriors for the model are similar to a standard MS model, where the vector of regime invariant coefficients is extended with the  $\beta$  coefficient,  $\mathbf{x}_{0t} = (y_{t-1}, \dots, y_{t-p}, \beta, \sigma)'$ . Equation (8) is again estimated using Bayesian inference. We employ 5 alternative exogenous indices, see section 4 for details.

[Chauvet \(1998\)](#) extracts  $z_t$  from a Markov Switching factor model in the observation equations for  $y_t$ . [Chauvet and Piger \(2008\)](#) use this model to detect US business cycles in real time.

### 3 Norwegian business cycle dating

There is no official dating of business cycles in Norway. Several studies of the Norwegian business cycle exist, see for instance [Eika and Lindquist \(1997\)](#), [Bjørnland \(2000\)](#), and [Bjørnland et al. \(2008\)](#). All these analyze the growth cycle. In a study by [Christoffersen \(2000\)](#), classical business cycles in the Nordic countries are defined by using the Bry and Boschan algorithm on the monthly index of manufacturing production. As far as we know, this is the only study so far aiming to date classical turning points in the Norwegian economy. However, manufacturing production is small compared with other sectors of the economy, and comprises merely 10 per cent of mainland Norway activity in 2012. Developments in manufacturing production might still be a good indicator of overall economic developments, but the broader measure of GDP in mainland Norway captures all sectors of the economy.<sup>3</sup>

National accounts data are revised. Revisions may lead to changes in the dating of business cycles, regardless of the method used to extract the cycles. Until November 2011, seasonally adjusted growth rates were substantially revised as far back as the early 1980s, even if the unadjusted data were not revised. This led to changes in the dating of peaks and troughs, and made it difficult to define business cycles historically. In November 2011, Statistics Norway changed their seasonal adjustment method. They also took steps to ensure that seasonally adjusted growth in historical data (prior to the base year, which changes every year) are kept unchanged. See description of methods for seasonal adjustment on Statistic Norway's website.<sup>4</sup> Seasonally adjusted historical growth rates will now only change when the unadjusted data are revised. The revision in unadjusted data related to the annual change of the base year is minor, but there will be larger main revisions from time to time.

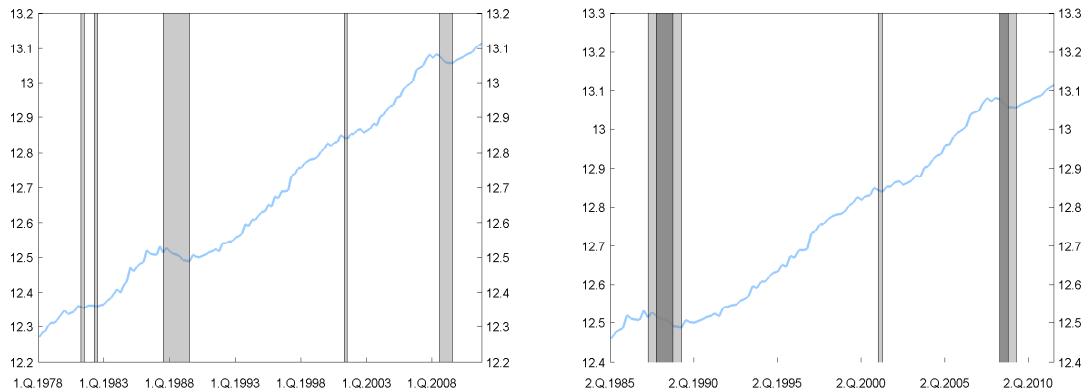
With more stable and consistent historical data, it is interesting to search for a method to define classical business cycles in Norway. Since no official dating of business

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<sup>3</sup>Since April 2012, OECD uses GDP as the reference for identification of turning points in the growth cycle.

<sup>4</sup>See [http://www.ssb.no/a/english/kortnavn/knr\\_en/sesongjustering\\_en.html](http://www.ssb.no/a/english/kortnavn/knr_en/sesongjustering_en.html)

Figure 1. *Business cycle dating*



(a) BB

(b) BB and MS

*The logarithmic transformation of GDP published in February 2012. The shaded areas indicate recessions dated by the Bry - Boschan (BB) rule (light grey) and the Markov Switching (MS) model (dark grey).*

cycles exists, we will base the choice of method on how “reasonable” the turning points are and compare with the general idea of developments in the Norwegian economy. We use the two methods presented in section 2 applied to GDP Mainland Norway as alternatives.

Quarterly national accounts data exist from 1978. We will search for turning points for the whole sample using the BB method. As the Markov Switching method (MS) entails having a training period, we can only define turning points from 1985 with this method.

The two panels in Figure 1 illustrate the alternative methods for defining turning points. According to the BB method, there is a double dip recession starting in the second quarter of 1981 and ending in the third quarter of 1982. Christoffersen (2000) finds, using the monthly seasonally adjusted manufacturing production index, that the peak occurred in September 1981 while the trough was pinpointed to October 1982. Hence, the recession lasted around 4 quarters. Our result is in line with the result in Christoffersen (2000), taking into account the different data and frequencies and data

revisions. The main message is that the recession was mild.

The recession in the late 1980s was deep and long-lasting. The BB method defines a 9-quarter long recession, starting in the third quarter of 1987 and ending in the third quarter of 1989. Using the MS algorithm, the recession starts in the first quarter of 1988 and ends in the first quarter of 1989, i.e. lasting 5 quarters.

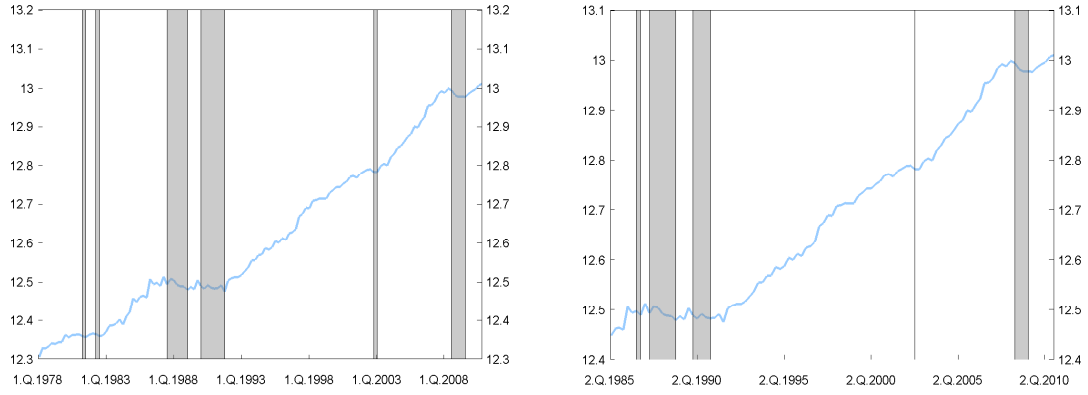
The characteristics of this period depend on the data vintage. In order to illustrate the challenges associated with data revisions, we have calculated turning points using national account vintages published in February 2011 (figure 2) and February 2010 (figure 3). For both vintages, the recession in the late 1980s becomes a double-dip recession according to the BB method and a triple-dip recession when employing the MS method. With the 2011 and the 2010 vintages, the BB recession starts in the third quarter of 1987 and does not end until the fourth quarter of 1991, a length of 18 quarters. The recession defined by the Markov switching method starts in the fourth quarter of 1986 and ends in the first quarter of 1991 (2011 vintage) or the third quarter of 1991 (2010 vintage).

Results using data published after Statistics Norway changed their seasonal adjustment of historical data favor the use of the BB method. The MS method produces a recession that seems to start too late and end too early, based on a discretionary assessment of developments in the Norwegian economy in that period. It also seems more reasonable when visually inspecting the log level of GDP.

We can compare the results from this period with further findings in [Christoffersen \(2000\)](#). He finds a peak in April 1989 and a trough in July 1990, ie around 5 quarters. Compared with analyzing quarterly GDP, using monthly manufacturing production points to a shorter recession, starting later and ending sooner. Again, this result seems unreasonable. In particular the peak occurred several quarters earlier than indicated in the Christoffersen study.

The next recession in the early 2000s is defined by the BB, while the MS method does not pick any turning points in this period. This mild recession is associated with the bursting of the “dot-com” bubble. The peak quarter is estimated to 2001Q1. The next

Figure 2. *Business cycle dating using vintage published February 2011*

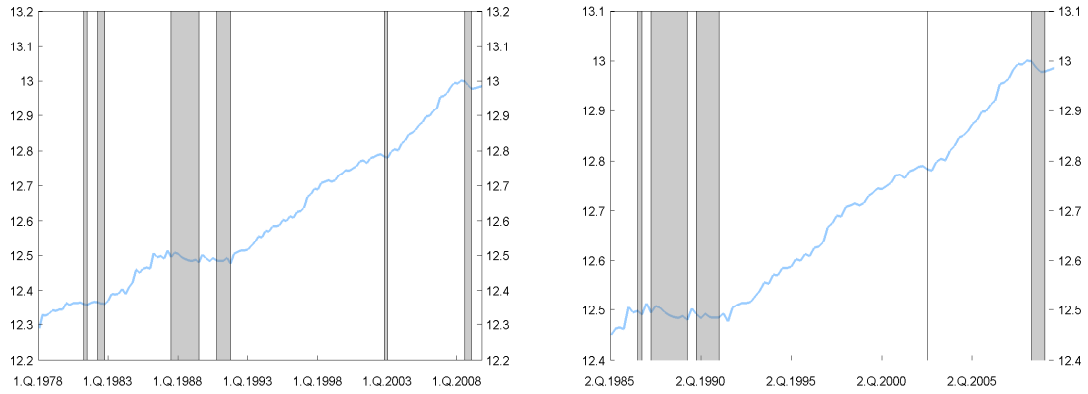


(a) BB

(b) MS

*The logarithmic transformation of GDP published in February 2011. The shaded areas indicate recessions dated by the Bry - Boschan (BB) rule (the left hand side) and the Markov Switching (MS) model (the right hand side).*

Figure 3. *Business cycle dating using Vintage published February 2010*

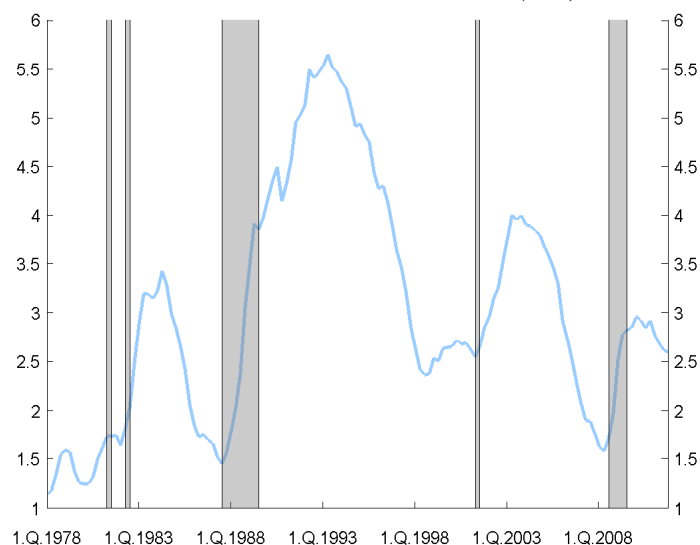


(a) BB

(b) MS

*The logarithmic transformation of GDP published in February 2010. The shaded areas indicate recessions dated by the Bry - Boschan (BB) rule (the left hand side) and the Markov Switching (MS) model (the right hand side).*

Figure 4. *Unemployment rate and business cycle dating (BB) based on GDP*



*Unemployment rate (labor force survey). The shaded areas indicate recessions dated by the BB rule.*

recession is defined by both methods. The MS method defines a very short recession, 3 quarters, with a peak in 2008Q2. The BB method defines the peak in the same quarter, but the trough does not occur until the second quarter of 2009 - a recession lasting 5 quarters.

With the two older vintages, the MS method defines a one-quarter long recession in 2002Q4, undetected in the vintage published in February 2012. Furthermore, the recession in 2008-2009 defined by the MS method is one quarter longer when using the two oldest vintages, while it is two quarters shorter defined by the BBQ method when using the oldest vintage.

An alternative way of assessing how reasonable the turning points are, is to compare their timing with developments in the unemployment rate, which has the advantage of not being revised, see figure 4. Except for the mild recession in the early 1980, developments in the unemployment rate supports the timing of the three peaks defined by the BB method. We would not expect the same congruence for the timing of the troughs, since production normally picks up before unemployment improves. The trough in unemployment occurs in the third quarter of 1987, supporting the timing of a peak in 1987Q2 as defined by the BB method. In the next recession, the unemployment rate has a



trough in the second quarter of 2001, again supporting the timing of the peak in 2001Q1. In contrast to the two downturns discussed above, the trough in unemployment in the second quarter of 2008 occurs in the same quarter as the peak defined for this recession. The labor market normally contains more frictions than the goods- and service markets, leading to a delayed reaction to shocks in the labor market. The reason for the more speedy reaction in the labor market in 2008 could be a feature of the data frequency. With monthly variables, labor market reactions could still be delayed compared with goods- and service market reactions. But it is also possible that this can be explained by the nature of the shocks that occurred during the third quarter of 2008.<sup>5</sup>

## 4 Forecasting Norwegian turning points in real time

If we interpret the turning points defined by the quarterly version of the Bry-Boschan method as describing turning points in the “true” Norwegian classical business cycles, one interesting question is then: Is it possible to predict the turning points in real time?

### 4.1 Forecasting exercise

In this exercise we will concentrate on the latest recession. Prior to November 2011, revisions of seasonally adjusted national accounts altered data as far back as in the early 1980s. Hence, data vintages from the period prior to November 2011 are increasingly different from the latest published vintage as we move backwards in time. We would not expect, using real-time data, to be able to predict turning points far back in time defined by the latest available vintage. This is also to some extent true for the latest recession, since data are final only after three years of revisions. However, these are revisions based on new information and we cannot avoid taking this into account.

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<sup>5</sup>Labor market variables are commonly used for detecting turning points. From figure 4, we would expect the unemployment rate to be useful for detecting peaks, but not for detecting troughs. We augmented the Markov Switching model with several labor market measures; hours worked, employment and two alternative unemployment rates. These did not improve the timeliness of turning point detection compared to financial market variables.

We will apply the two methods described in section 2 using real-time data and compare their ability to forecast turning points. The Markov switching techniques already compute predicted probabilities of being in one regime or the other (i.e. in recession or expansion). The quarterly Bry-Boschan requires predictions of GDP to be able to forecast a turning point in real time. We produce predictive densities for GDP from an autoregressive model  $AR(p)$ , where we fix  $p = 4$  as in the Markov switching example.

Both the BB and the MS can be augmented with leading indicators. A leading indicator is interpreted as a variable that summarizes the common cyclical movements of some coincident macroeconomic variables in a timely manner.

For augmenting the BB, we follow [Aastveit et al. \(2011\)](#) and produce predictive densities from different bivariate vector autoregressive models using GDP and one single leading indicator:

$$Y_t = \alpha + \sum_{i=1}^p \beta_i Y_{t-i} + \epsilon_t, \quad \epsilon_t \sim N(0, \Sigma_\epsilon), \quad (9)$$

where  $Y_t = (y_{1,t}, y_{2,t})$  and  $y_{1,t}$  and  $y_{2,t}$  denotes GDP growth and growth in the leading indicator, respectively.

Several papers, such as [Evans \(2005\)](#), [Giannone et al. \(2008\)](#) and [Aastveit et al. \(2014\)](#), show that accounting for the timeliness of data is crucial for nowcasting and short-term forecasting. The exploitation of timely information leads to improvement in the forecast accuracy. It is therefore essential to take into account that leading indicators are available prior to the GDP release.<sup>6</sup> The models are put into a state space form and Kalman filter techniques can easily be applied to deal with missing data, i.e. the unbalanced data problem.

The forecast uncertainties are obtained through simulations, where the final densities are derived using kernel smoothing techniques. By applying the Kalman filter we can

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<sup>6</sup>Some of the leading indicators included here are of monthly frequency, and we need to bridge the monthly indicators with quarterly GDP. This is done by constructing quarterly averages of the monthly series. If a monthly series only contains one or two months of a quarter, we simply construct the average of the one or two observations from the quarter of interest, see [Baffigi et al. \(2004\)](#) and [Angelini et al. \(2011\)](#) for a more detailed discussion of alternative bridge equations.

obtain conditional forecasts when the data set is unbalanced. More precisely, we use the smoothed covariance matrix of the predictors, which will resemble the mean squared error (MSE) matrix of the system, and draw from the normal distribution to obtain simulated forecasts for each horizon.<sup>7</sup> The median value of the predictive densities is then used to extend the real-time GDP level with the forecasts.<sup>8</sup> The median forecasts can be directly compared with defining recessions using the MS when regime 1 has 50% (or higher) probability. Since quarterly GDP is released with a lag of approximately 7 weeks, this means that if we add forecasts for 2 quarters (i.e. a nowcast and a forecast) to the latest available vintage, we may at the earliest predict a turning point 7 weeks after it occurred. If we add a three-steps ahead forecast, it would in theory be possible to forecast a turning point 5 weeks before it occurs. As uncertainty increases with the horizon we will, however, confine ourselves to predictive densities one and two steps ahead.

We employ leading indicators transformed into differences as exogenous variables in the augmented MS autoregressive models. As discussed in the section 2.2, the MS does not require predictions for  $y_{t+h}$ ,  $h = 1, 2$  to produce an indicator at time  $t$ .

We focus our analysis on two classes of data: financial indices and survey data. Indicator models based on financial data and survey data are likely candidates for detecting turning points early. Publication is timely compared to GDP, and the nature of the statistics ensures that a wide range of information and considerations are taken into account by financial market participants, see [Næs et al. \(2011\)](#), and by the respondents in the surveys, see [Martinsen et al. \(2014\)](#). For financial data we use monthly averages of daily observations. All the surveys are quarterly. There are no monthly surveys in Norway that have been published long enough to be useful for model-based forecasting. They are, however, released earlier than GDP, hence indicators are in general available for quarter  $t$ , while GDP is only available for quarter  $t - 1$ . We have chosen two financial

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<sup>7</sup>See for example [Lütkepohl \(2005\)](#).

<sup>8</sup>Generally, complete probability distributions over outcomes provide information helpful for making economic decisions; see, for example, the discussions in [Granger and Pesaran \(2000\)](#), [Timmermann \(2006\)](#) and [Gneiting \(2011\)](#).

variables and three alternative surveys. The financial indices are:

- Financial conditions index (FCI). The index is constructed using a monthly dynamic factor model with financial variables, including interest rates, money and credit and interest rate spreads.
- Relative spread (RS). The RS is a measure of liquidity and is calculated as the quoted spread as a fraction of the midpoint price (monthly average) and measures the implicit cost of trading (a small number of) shares at the Oslo stock exchange.

The surveys are:

- The overall business confidence indicator from the business tendency survey for manufacturing, mining and quarrying (BTS). The survey is conducted by Statistics Norway in the last three weeks of the quarter and is published at the end of the first month in the next quarter.
- The overall consumer confidence index (CC). The survey is conducted by TNS Gallup in the fifth week of the quarter and is published around 4 weeks before the end of the quarter.
- Expected growth in 6 months (all industries), from Norges Bank's regional network survey (RN). The survey is conducted in the first half of the quarter and published around three weeks before the end of the quarter.

## 4.2 Results

Results are presented in tables 1 to 3 and summarized in figure 5. From table 1, we observe that all the augmented BB models predict 2008Q3 as the peak quarter in real time. The first model to predict the downturn is the model with the financial conditions index (FCI) as the augmenting variable. As soon as data for the second month of the fourth quarter is known, this model pinpoints a peak in the third quarter. All three survey models predict the peak as soon as the survey for the fourth quarter of 2008 is

Table 1. *Forecasting turning points in real time - peaks*

Model	date of detection	Peak quarter
BB with AR(4)	2009 19 May	2008Q3
BB with Consumer confidence (CC)	2008 2 December	2008Q3
BB with Business Tendency Survey (BTS)	2009 28 January	2008Q3
BB with Regional Network Survey (RN)	2008 17 December	2008Q3
BB with Financial conditions index (FCI)	2008 1 December	2008Q3
BB with RS	2009 1 March	2008Q3
Markov switching with AR(4)	2009 19 May	2008Q2
Markov Switch with change in CC	2009 1 June	2008Q1
Markov Switch with change in BTS	2009 27 April	2008Q3
Markov Switch with change in RN	2008 17 December	2007Q4
Markov Switch with change in FCI	2009 1 April	2008Q1
Markov Switch with change in RS	2009 1 July	2008Q2

*The table reports the real-time predicted peak quarter and the exact date of detection using several alternative methods defined in section 2.*

published. The consumer confidence indicator is the first survey to be published on 2 December, while the business tendency survey is not published until 17 January 2009. Using the RS as the augmenting factor does not produce a very timely forecast of the peak, since in this case the turning point in 2008Q3 is not forecasted until 1 March 2009.

With one exception, the Markov Switching models do not predict a peak until well into 2009. The MS model augmented with the Norges Bank’s regional network survey predicts the peak when the survey is published 17 December. The predicted peak quarter is as early as 2007Q4, which seems too early for Norway.

Our results differ from [Chauvet and Piger \(2008\)](#) who find that the Markov Switching dynamic-factor model provides more timely information than the BB rule in US real-time dating. The different data set, the alternative way of augmenting the MS model and the fact that we augment the BB rule with forecast densities, meaning we can predict up to time  $t + 2$ , are among the possible explanations for the different results.

Table 2 summarizes the peaks and troughs identified in the previous section using

Table 2. *Defining turning points ex post*

Method	Peak quarter	Length of downturn	Trough quarter
BB	2008Q2	5 quarters	2009Q3
Markov Switching	2008Q2	3 quarters	2009Q1

*The table reports peak and trough quarter and length of the downturn using the vintage published in February 2012 with two alternative methods.*

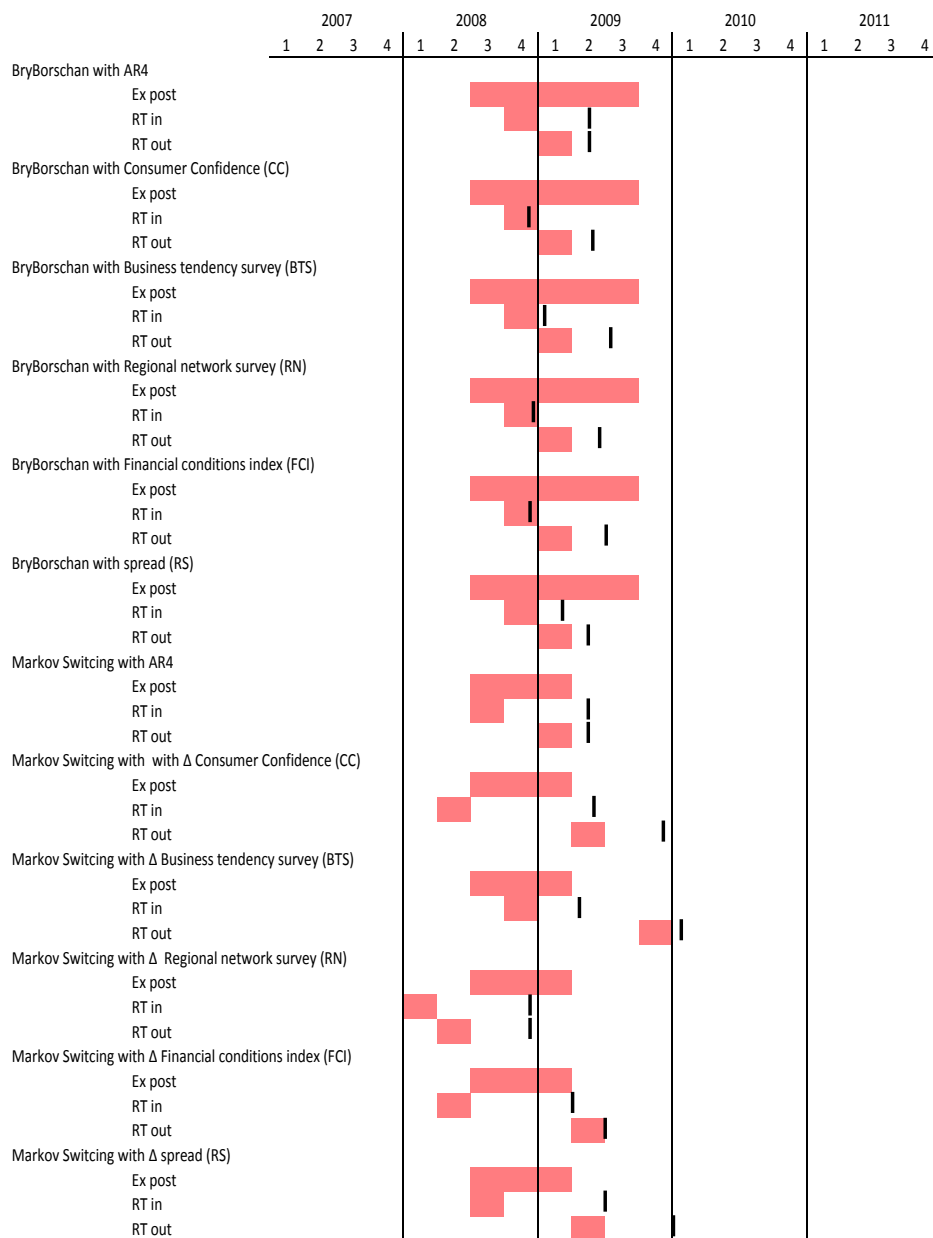
ex-post data, more precisely the vintage published in February 2012. In real time the peaks forecasted by BB occur 1 quarter later than the peaks defined ex post using the latest data vintage. Substantial data revisions are the main reason for the difference. In figure 6 we compare the log levels of GDP vintages published in November 2008 and February 2012, respectively. In the 2008 vintage growth continues through 2008, although the growth rate is slightly negative in the first quarter of 2008. In the February 2012 vintage, on the other hand, growth is on average negative the first three quarters of 2008.

Table 3 contains an overview of the models' ability to forecast troughs in real time. All the BB models forecast a trough in 2009Q1 and most of them forecast the trough during the following quarter. The first models to predict a trough are the BB augmented with AR(4) forecasts and the BB augmented with relative spread. These models predict the turning point as soon as national accounts for 2009Q1 were published. The MS models provide more mixed evidence, often with substantial delays, with troughs ranging from 2008Q2 to 2009Q2.

Figure 5 provides a graphical summary of all results. Since we do not have any benchmark to compare the results with, we discuss results in the light of what seems reasonable given what we know of economic developments. The aim is to forecast turning points in real time as accurately as possible compared to an ex post benchmark, and as timely as possible.

The BB method used on the GDP level appended with forecasts given by survey data, the RS or an AR(4) gives consistent results, dating the peak to 2008Q3 and the trough

Figure 5. *Turning point forecasts*



*For each class of models, the figure illustrates the length of the recession in lines “Ex-post” using ex-post data. Red denotes quarters where the economy is in recession. The lines denoted “RT in” show which quarter is the first in the recession in real time, i.e the peak quarter is the quarter before the quarter marked in red. In the lines denoted “RT out”, the red quarter is the trough quarter in real time. Black vertical lines with real-time data illustrate on which date the turning point was detected.*

Table 3. *Forecasting turning points in real time - troughs*

Model	date of detection	Trough quarter
BB with AR(4)	2009 19 May	2009Q1
BB with Consumer confidence (CC)	2009 1 June	2009Q1
BB with Business Tendency Survey (BTS)	2009 28 July	2009Q1
BB with Regional Network Survey (RN)	2009 10 June	2009Q1
BB with Financial conditions index (FCI)	2009 1 July	2009Q1
BB with RS	2009 19 May	2009Q1
Markov switching with AR(4)	2009 19 May	2009Q1
Markov Switch with change in CC	2009 7 December	2009Q2
Markov Switch with change in BTS	2010 27 January	2009Q4
Markov Switch with change in RN	2008 17 December	2008Q2
Markov Switch with change in FCI	2009 1 July	2009Q2
Markov Switch with change in RS	2010 1 January	2009Q2

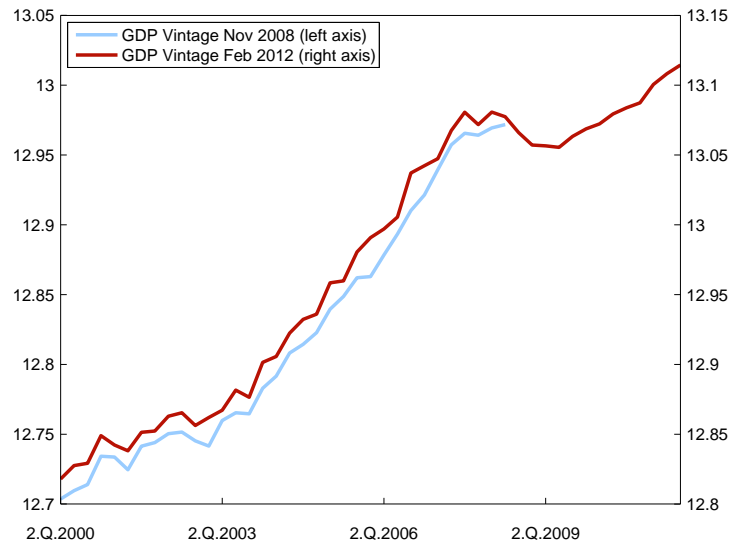
*The table reports the real-time predicted trough quarter and the exact date of detection using several methods.*

to 2009Q1. Compared to using AR(4) forecasts, the peak is predicted more timely, in particular if using either Consumer Confidence (CC) or a Financial Conditions Index (FCI) as an augmenting variable. The trough is in general forecasted with a much longer delay than is the case when forecasting the peak. In contrast to forecasting the peak, AR(4) and RS are now the most timely alternative, i.e. forecastability of the trough does not increase by augmenting the BB method by either surveys or financial variables.

The pure Markov switching model pinpoints 2008Q2 as the peak quarter, both in real time and ex post. But the real-time forecast is not produced until 19 May 2009. By then national accounts for 2009Q1 are available and the accounts for 2008 have been revised several times. On the other hand, the MS method forecasts the trough in 2009Q1 at the same time, which is as timely as the most timely BB model. Augmenting the MS with any of the surveys, the financial condition index or the RS, respectively, does not seem very helpful. These models are not able to forecast the turning points in a timely manner in real time. Peaks are not forecasted until well into 2009. Forecasted



Figure 6. *Log level of GDP*



*The logarithm transformation of GDP published in November 2008 and in February 2012.*

peak quarter varies from as early as 2007Q4 to 2008Q4 and the trough quarter ranges from 2008Q2 to 2009Q4.

We have used real-time data vintages and augmented the BB method and the MS model with surveys and financial data in order to forecast the turning points in the 2008-2009 recession. The BB (and survey and financial data in the augmented version) requires predictions for  $y_{t+h}$ ,  $h = 1, 2$ , to produce an indicator at time  $t$ , while the MS model does not. Data revisions entail that ex-post data that we use for defining the turning points are different from real-time data existing at the time. Hence, it is not surprising that the turning points are forecasted in different quarters than what we define ex post. Taking account of the data revisions, using the BB method with real-time data augmented with forecasts gives more timely predictions of turning points than the MS models do. When predicting the peak, forecastability using BB increases when augmenting with surveys or financial variables. This is not the case when predicting the trough.

## 5 Conclusion

We compare several methods to define and forecast (in real time) classical business cycle turning points in Norway, a country which does not have an official business cycle indicator. We apply the Bry - Boschan rule (BB), an autoregressive Markov Switching model (MS), and the two methodologies augmented with financial indicators and survey data, using several vintages of Norwegian mainland GDP as the business cycle indicator. We find that the BB yields the most reliable definition of the business cycle. When augmented with density forecasts from survey models or financial indicators, BB provides more timely predictions for the peak in 2008 than augmented MS models do. Results for predicting the trough is mixed. All BB models predict the trough to occur in the same quarter, while the MS models predict the trough to occur in a range of alternative quarters. Except for one of the MS models, the trough is predicted in a considerably less timely manner than the peak is. Augmenting with financial indicators or survey data do not add forecastability.

Our analysis focuses on a single country and just one recession. We believe, however, the analysis provides a useful and simple methodology for forecasting classical business cycle turning points using timely information.

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