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The Macroeconomic Forecasting Performance of Autoregressive Models with Alternative Specifications of Time-Varying Volatility *

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Abstract

This paper compares alternative models of time-varying macroeconomic volatility on the basis of the accuracy of point and density forecasts of macroeconomic variables. In this analysis, we consider both Bayesian autoregressive and Bayesian vector autoregressive models that incorporate some form of time-varying volatility, precisely stochastic volatility (both with constant and time-varying autoregressive coefficients), stochastic volatility following a stationary AR process, stochastic volatility coupled with fat tails, GARCH and mixture of innovation models. The comparison is based on the accuracy of forecasts of key macroeconomic time series for real-time post War-II data both for the United States and United Kingdom. The results show that the AR and VAR specifications with widely-used stochastic volatility dominate models with alternative volatility specifications, in terms of point forecasting to some degree and density forecasting to a greater degree.

Keywords: Stochastic volatility, GARCH, forecasting

JEL Classifications: E17, C11, C53

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1 Introduction

A growing number of studies have provided evidence of time-varying volatility in the economies of many industrialized nations. To this point, most available evidence, based on data through the early to mid-2000's, has highlighted the Great Moderation (e.g., Stock and Watson 2003, 2007, Cogley and Sargent 2005, Primiceri 2005, Koop and Potter 2007, Benati 2008, Giordani and Villani 2010, and Justiniano and Primiceri 2010). Some more recent studies have shown that, following the Great Moderation, volatility rose sharply during the sharp recession of 2007-2009 (e.g., Clark 2009, 2011, and Curdia, Del Negro, and Greenwald 2012).

Modeling the apparently significant time variation in macroeconomic volatility is important to the accuracy of a range of types of inference. In general, of course, least squares estimates of VAR coefficients remain consistent in the face of conditional heteroskedasticity, but OLS variance estimates do not. Moreover, modeling the conditional heteroskedasticity can yield more efficient (GLS) estimates of VAR coefficients; Sims and Zha (2006) have emphasized the value of volatility modeling for improving efficiency. Accordingly, in both dimensions, taking account of time variation in volatility should improve the VAR-based estimation and inference common in macroeconomic analysis. In particular, in VAR-based analysis of impulse responses, variance decompositions, and historical decompositions — used, for instance, to assess the effects of alternative monetary policies — modeling time variation in conditional volatilities is likely to be important for accurate inferences.

In addition, modeling changes in volatility should help to improve the accuracy of density forecasts from VARs. Shifts in volatility have the potential to result in forecast densities that are either far too wide or too narrow. For instance, in light of the Great Moderation, density forecasts for GDP growth in 2006 based on time series models assuming constant variances over a sample such as 1960-2005 would probably be far too wide, with inflated confidence intervals and probabilities of tail events such as recession. As another example, in late 2008, density forecasts for 2009 based on time series models assuming constant variances for 1985-2008 would have been too narrow. Results in Giordani and Villani (2010), Jore, Mitchell, and Vahey (2010), and Clark (2011) support this intuition on the gains to point and density forecasts of modeling shifts in conditional volatilities. D'Agostino, Gambetti, and Giannone (2012) show that the combination of time-varying parameters and stochastic volatility improves the accuracy of point and density forecasts. These benefits to allowing time-varying volatility could prove useful to central banks that provide density information in the form of forecast fan charts and qualitative assessments of forecast uncertainty.

In most of the recent studies providing evidence of time-varying volatility (e.g., Stock and Watson 2003, 2007, Cogley and Sargent 2005, Primiceri 2005, Benati 2008), the time variation in volatility has been captured with a single model: stochastic volatility, in which the log of volatility follows a random walk process. In Bayesian estimation algorithms, the stochastic volatility specification is computationally tractable. In addition, studies such as Clark (2011) and Carriero, Clark, and Marcellino (2012) have shown that it is effective for improving the accuracy of density forecasts from AR models and Bayesian VARs. However, there are alternatives that could also be effective for capturing changes in macroeconomic volatility. Studies such as Koop and Potter (2007), Giordani and Villani (2010), and Groen, Paap, and Ravazzolo (2012) have used models in which volatility is subject to potentially many discrete breaks; others, such as Jore, Mitchell, and Vahey (2010), have used models with a small number of discrete breaks. Yet another model of time-varying volatility would be a GARCH specification. While the pioneering development of ARCH (Engle 1982) and GARCH (Bollerslev 1986) models included applications to inflation, these models seem to have become rare in recent macroeconomic modeling, with the exception of a few studies, such as Canarella, et al. (2008) and Chung, et al. (2012). Karapanagiotidis (2012) considers yet another approach, using autoregressive Wishart processes to capture time-varying volatility in macroeconomic BVARs for forecasting. Koop and Korobilis (2012) show that a computational shortcut for allowing time-varying volatility, using a form of exponential smoothing of volatility, improves the accuracy of point and density forecasts from larger VARs.

While a number of studies in the finance literature have compared alternative models of time-varying volatility of asset returns (e.g., Amisano and Geweke 2010, Hansen and Lunde 2005, Nakajima 2012), no such broad comparison yet exists for macroeconomic variables. Accordingly, this paper compares alternative models of time-varying macroeconomic volatility, included within autoregressive and vector autoregressive specifications for key macroeconomic indicators and estimated using Bayesian inference. We base our comparison on real-time out-of-sample forecast accuracy, for both point and density forecasts of GDP growth, unemployment, inflation, and a short-term interest rate in both the United States and United Kingdom.[1]

[1]In the finance literature, some studies compare volatility models for their efficacy in modeling returns

The set of univariate AR models includes the following volatility specifications: constant volatility; stochastic volatility (with both constant AR coefficients and time-varying AR coefficients); GARCH; and a mixture of innovations model. The set of BVARs widens the specification of the stochastic volatility process, to include: constant volatility; stochastic volatility (with both constant AR coefficients and time-varying AR coefficients); stochastic volatility following a stationary AR process; stochastic volatility coupled with fat tails; and GARCH.[2] The variables modeled include GDP growth, the unemployment rate, inflation in the GDP deflator, and a short-term government bill yield. For both countries, our results indicate that the AR and VAR specifications with stochastic volatility dominate models with alternative volatility specifications, in terms of point forecasting to some degree and density forecasting to a greater degree. Therefore, at least from a macroeconomic forecasting perspective, these alternative volatility specifications seem to have no advantage over the now widely-used stochastic volatility specification.

The paper proceeds as follows. Section 2 describes the data. Section 3 presents the models and estimation methodology, with further estimation details in an appendix. Section 4 presents the results. Section 5 concludes.

2 Data

For both the U.S. and U.K., we use quarterly data to estimate models for growth of real GDP, inflation in the GDP price index or deflator (henceforth, GDP inflation), unemployment rate, and 3-month government bill rate. We compute GDP growth and as 100 times the log difference of real GDP and inflation as 100 times the log difference of the GDP price index.

In the case of the U.S., we obtained (quarterly) real time data on GDP and the GDP price index from the Federal Reserve Bank of Philadelphia's Real Time Dataset for Macroeconomists, from which the first available vintage is 1965:Q4 and the last is 2011:Q4. For simplicity, we use "GDP" and "GDP price index" to refer to the output and price series, even though the measures are based on GNP and a fixed weight deflator for much of the sample. As described in Croushore and Stark (2001), the vintages of the RTDSM are dated to reflect the information available around the middle of each quarter. Normally, in a given

(e.g., Amisano and Geweke 2010, Nakajima 2012), while others compare volatility models for their efficacy in modeling volatility (e.g., Hansen and Lunde 2005).

[2] To simplify computations, we do not consider a mixture model in the BVAR case.

vintage t, the available GDP and GDP price index data run through period $t-1$.

For the U.K., we obtained real time data on GDP and the GDP deflator from the website of the Bank of England, with vintages starting in 1990:Q1 and ending in 2011:Q2. The timing convention is similar to that for the U.S. data. While the Bank of England provides vintages for each month of the year, we selected one month's vintage per quarter. Specifically, we selected data vintages for quarters 1-4 to correspond to the series available in February, May, August, and November of each year (as in the RTDSM). In the handful of cases in which the last quarter of data that should be available was not actually available (apparently due to some publication delay), we used the vintage from the next month. For example, if the vintage from February in year t did not include GDP data through Q4 of year $t-1$, we used the vintage from March in year t in place of the February vintage.

In the case of unemployment and interest rates, for which real-time revisions are small to essentially non–existent, we simply abstract from real-time aspects of the data and use currently available time series. For the U.S., we obtained monthly data on the unemployment rate and 3-month Treasury bill rate from the FAME database of the Federal Reserve Board of Governors and formed the quarterly unemployment and interest rate as simple within-quarter averages of the monthly data. For the U.K., we obtained a quarterly unemployment rate from FAME and constructed a bill rate by merging a 1975-2011 series on the 3-month Treasury yield (sterling) obtained from Bank of England web site with a 1962-74 series on the 3-month interbank rate obtained from FAME (after merging the monthly series, we converted to the quarterly frequency by averaging within the quarter).

For the U.S., we consider a forecast evaluation period of 1985:Q1 through 2011:Q2, which involves real-time data vintages from 1985:Q1 through 2011:Q4. For each forecast origin t starting with 1985:Q1, we use the real-time data vintage t to estimate the forecast models and construct forecasts for periods t and beyond. We report results for forecast horizons of 1, 2, 4, and 8 quarters ahead. In light of the time $t-1$ information actually incorporated in the models used for forecasting at t, the 1-quarter ahead forecast is a current quarter (t) forecast, while the 2-quarter ahead forecast is a next quarter ($t+1$) forecast, etc. The starting point of the model estimation sample is always 1955:Q1 (for some models, we use data for the 1948-54 period to set the priors on some parameters, as detailed below).

For the U.K., we consider a forecast evaluation period of 1990:Q1 through 2010:Q4, which involves real-time data vintages from 1990:Q1 (first available) through 2011:Q2. For each forecast origin t starting with 1990:Q1, we use the real-time data vintage t to estimate the forecast models and construct forecasts for periods t and beyond. We report results for forecast horizons of 1, 2, 4, and 8 quarters ahead. The starting point of the model estimation sample is always 1978:Q1 (for some models, we use data for the 1971-77 period to set the priors on some parameters, as detailed in the appendix).[3]

As discussed in such sources as Romer and Romer (2000), Sims (2002), and Croushore (2006), evaluating the accuracy of real-time forecasts requires a difficult decision on what to take as the actual data in calculating forecast errors. The GDP data available today for, say, 1985, represent the best available estimates of output in 1985. However, output as defined and measured today is quite different from output as defined and measured in 1970. For example, today we have available chain-weighted GDP; in the 1980s, output in the U.S. was measured with fixed-weight GNP. Forecasters in 1985 could not have foreseen such changes and the potential impact on measured output. Accordingly, we follow studies such as Romer and Romer (2000) and Faust and Wright (2009) and use the second available estimates of GDP/GNP and the GDP/GNP deflator as actuals in evaluating forecast accuracy. In the case of h-quarter ahead forecasts made for period $t+h$ with vintage t data ending in period $t-1$, the second available estimate is normally taken from the vintage $t+h+2$ data set. In light of our abstraction from real-time revisions in unemployment and the government bill yields, for these series the real-time data correspond to the final vintage data.

3 Models

In this section we provide the specifications of our AR models and VAR models and provide an overview of the methods used for estimation. The appendix provides further detail on the estimation algorithms.

3.1 AR models

For each variable, we consider a baseline AR(p) model with constant shock variance:

$$y_t = b_0 + \sum_{i=1}^{p} b_i y_{t-i} + v_t, \quad v_t \sim N(0, \phi). \tag{1}$$

To this baseline, we compare AR models with two different formulations of time-varying volatility: GARCH and stochastic volatility. We also consider an AR model with both

[3]Our sample specification for the U.K. is constrained by the unemployment series, which doesn't start until 1971:Q1.

time-varying parameters and stochastic volatility and an AR model that takes the mixture of innovations form, developed in studies such as Koop and Potter (2007), Giordani, et al. (2007), and Groen, Paap, and Ravazzolo (2012). All of our AR models include 2 lags for GDP growth and 4 lags for inflation, unemployment rate and inflation.

The AR-GARCH model incorporates a common GARCH(1,1) process (as in, e.g., Chung, et al. (2012)):[4]

$$\begin{aligned} y_t &= b_0 + \sum_{i=1}^{p} b_i y_{t-i} + v_t \\ v_t &= h_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0,1) \\ h_t &= a_0 + a_1 v_{t-1}^2 + a_2 h_{t-1},\ a_0 > 0, a_1 + a_2 < 1. \end{aligned} \tag{2}$$

The AR-SV model, considered in such studies as Clark (2011), takes the following form:

$$\begin{aligned} y_t &= b_0 + \sum_{i=1}^{p} b_i y_{t-i} + v_t \\ v_t &= \lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0,1) \\ \log(\lambda_t) &= \log(\lambda_{t-1}) + \nu_t,\ \nu_t \sim\ N(0,\phi). \end{aligned} \tag{3}$$

The AR-TVP-SV model takes the form given in Cogley and Sargent (2005), simplified to a univariate process:

$$\begin{aligned} y_t &= b_{0,t} + \sum_{i=1}^{p} b_{i,t} y_{t-i} + v_t \\ b_t &= b_{t-1} + n_t, \quad \text{var}(n_t) = Q \\ v_t &= \lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0,1) \\ \log(\lambda_t) &= \log(\lambda_{t-1}) + \nu_t,\ \nu_t \sim\ N(0,\phi). \end{aligned} \tag{4}$$

Finally, the AR-mixture model is specified as follows:

$$\begin{aligned} y_t &= b_{0,t} + \sum_{i=1}^{p} b_{i,t} y_{t-i} + v_t \\ v_t &= \lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0,1) \\ b_{j,t} &= b_{j,t-1} + \kappa_{j,t}\ n_{j,t},\ j = 0,\ldots,p \\ \log(\lambda_t) &= \log(\lambda_{t-1}) + \kappa_{p+1,t}\ n_{p+1,t} \\ \Pr[\kappa_{j,t} = 1] &= \pi_j,\ j = 0,\ldots,p+1 \\ \text{var}\left((n_{0,t},\ldots,n_{p+1,t})'\right) &= \text{diag}(q_0, q_1, \ldots, q_{p+1}). \end{aligned} \tag{5}$$

[4]We also tried a version with Student-t residuals where the degrees of freedom τ are estimated. Results were worse than the normal case and we do not report them.

3.2 VAR models

For our set of $k = 4$ variables, we consider a baseline VAR(p) model with a constant variance-covariance matrix of shocks:

$$y_t = B(L)y_{t-1} + v_t, \quad v_t \sim N(0, \Phi). \tag{6}$$

To this baseline, we compare VAR models with various formulations of time-varying volatility.[5] Two of the models couple the VAR with GARCH or stochastic volatility. Still another model includes time-varying parameters and stochastic volatility. In other models, we consider some variations on the stochastic volatility specification that has been most common in recent macroeconomic modeling. One variation consists of a stationary, rather than random walk, stochastic volatility process. The other variation consists of adding fat tails to stochastic volatility, using the fat tails formulation of Jacquier, Polson, and Rossi (2004). All of the VAR models include 4 lags.

The VAR-GARCH model incorporates a GARCH(1,1) process for the orthogonalized error of each VAR equation:

$$\begin{aligned} y_t &= B(L)y_{t-1} + v_t \\ v_t &= A^{-1}H_t^{0.5}\epsilon_t, \ \epsilon_t \sim N(0, I_k), \ H_t = \text{diag}(h_{1,t}, \ldots, h_{k,t}) \\ h_{i,t} &= a_{0,i} + a_{1,i}v_{i,t-1}^2 + a_{2,i}h_{i,t-1}, \ a_{0,i} > 0, a_{1,i} + a_{2,i} < 1, \ \forall \ i = 1, k, \end{aligned} \tag{7}$$

where A = a lower triangular matrix with ones on the diagonal and non-zero coefficients below the diagonal.

The VAR-SV model includes the conventional formulation of a random walk process for log volatility:

$$\begin{aligned} y_t &= B(L)y_{t-1} + v_t \\ v_t &= A^{-1}\Lambda_t^{0.5}\epsilon_t, \ \epsilon_t \sim N(0, I_k), \ \Lambda_t = \text{diag}(\lambda_{1,t}, \ldots, \lambda_{k,t}) \\ \log(\lambda_{i,t}) &= \log(\lambda_{i,t-1}) + \nu_{i,t}, \ \nu_{i,t} \sim \ N(0, \phi_i) \ \forall \ i = 1, k, \end{aligned} \tag{8}$$

where A = a lower triangular matrix with ones on the diagonal and non-zero coefficients below the diagonal.

[5] In results not reported in the interest of brevity, we also considered a BVAR with fat tails but not stochastic volatility. Forecasts from this model were clearly less accurate than forecasts from the model with stochastic volatility. We also tried a version of the VAR-GARCH with Student-t residuals where the degrees of freedom τ are estimated. Results were worse than the normal case and we do not report them.

The VAR-ARSV specification treats log volatility as following an AR(1) process, which we will force to be stationary by using a tight prior:

$$\begin{aligned} y_t &= B(L)y_{t-1} + v_t \\ v_t &= A^{-1}\Lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0, I_k),\ \Lambda_t = \mathrm{diag}(\lambda_{1,t}, \ldots, \lambda_{k,t}) \\ \log(\lambda_{i,t}) &= a_{0,i} + a_{1,i}\log(\lambda_{i,t-1}) + \nu_{i,t},\ \nu_{i,t} \sim\ N(0, \phi_i)\ \forall\ i = 1, k. \end{aligned} \tag{9}$$

With macro time series fairly limited in length (especially compared to finance time series), it is likely to be difficult to reliably estimate both time-varying volatility and the parameters of the autoregressive volatility process. Accordingly, we use a tight prior to almost fix the slope coefficients $a_{1,i}, i = 1, \ldots, k$ (but not the intercept), at three different values, giving us three different VAR-ARSV specifications: 0.9, 0.8, and 0.5.

The VAR-SVt model augments the (random walk) stochastic volatility specification to include fat tails, similarly to the DSGE specification considered in Curdia, Del Negro, and Greenwald (2012):

$$\begin{aligned} y_t &= B(L)y_{t-1} + v_t \\ v_t &= A^{-1}Q_t^{0.5}\Lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0, I_k) \\ \Lambda_t &= \mathrm{diag}(\lambda_{1,t}, \ldots, \lambda_{k,t}),\ Q_t = \mathrm{diag}(q_{1,t}, \ldots, q_{k,t}) \\ \log(\lambda_{i,t}) &= \log(\lambda_{i,t-1}) + \nu_{i,t},\ \nu_{i,t} \sim\ N(0, \phi_i)\ \forall\ i = 1, k \\ d/q_{i,t} &\sim\ i.i.d.\ \chi_d^2, \end{aligned} \tag{10}$$

where d denotes the degrees of freedom of the Student-t distribution that is the marginal distribution of $q_{i,t}^{0.5}\epsilon_{i,t}$. For simplicity, in lieu of estimating the degrees of freedom, we fixed it, considering three different settings, estimating (separately) models with 5, 10, and 15 degrees of freedom.

Finally, letting X_t denote the collection of right-hand side variables of each equation of the VAR, the VAR-TVP-SV model takes the form given in Cogley and Sargent (2005):

$$\begin{aligned} y_t &= X_t' B_t + v_t \\ B_t &= B_{t-1} + n_t,\ \mathrm{var}(n_t) = Q \\ v_t &= A^{-1}\Lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim N(0, I_k),\ \Lambda_t = \mathrm{diag}(\lambda_{1,t}, \ldots, \lambda_{k,t}) \\ \log(\lambda_{i,t}) &= \log(\lambda_{i,t-1}) + \nu_{i,t},\ \nu_{i,t} \sim\ N(0, \phi_i)\ \forall\ i = 1, k, \end{aligned} \tag{11}$$

where A = a lower triangular matrix with ones on the diagonal and non-zero coefficients below the diagonal.

3.3 Estimation algorithms

We estimate all of the models described above using Bayesian Markov Chain Monte Carlo (MCMC) methods. In generating forecasts, we use a recursive estimation scheme, expanding the model estimation sample as forecasting moves forward in time. This section provides a brief overview of our methods. The appendix and the studies cited below provide additional detail on algorithms and priors.

For the AR and BVAR models with constant variances, we use the Normal-diffuse prior and posterior detailed in such sources as Kadiyala and Karlsson (1997) and estimate the models by Gibbs sampling.

For the AR-GARCH and VAR-GARCH models, we use a Metropolis-within-Gibbs MCMC algorithm, combining Gibbs sampling steps for model coefficients with a random walk Metropolis-Hastings (MH) algorithm to draw the GARCH parameters. Our MH algorithm for the GARCH parameters is similar to the ones in Vrontos, Dellaportas, and Politis (2000) and So, Chen, and Chen (2005). To speed convergence and allow optimal mixing, we employ an adaptive MH-MCMC algorithm that combines a random walk Metropolis (RW-M) and an independent kernel (IK)MH algorithm. In the case of the VAR-GARCH model, the Choleski matrix A is handled in the same way as it is in the VARs with stochastic volatility, which is the same as in Cogley and Sargent (2005).

To estimate the AR-SV, VAR-SV, AR-TVP-SV and VAR-TVP-SV models, we use Metropolis-within-Gibbs MCMC algorithms, combining Gibbs sampling steps for model coefficients with Cogley and Sargent's (2005) Metropolis algorithm (taken from Jacquier, Polson, and Rossi (1994)) for stochastic volatility. For AR models with stochastic volatility, our algorithm is the same as that used in Clark (2011). For the AR models with TVP and stochastic volatility, our algorithm takes the form described in Cogley and Sargent (2005). For the VAR-ARSV specification, the algorithm is the same, but for the addition of a step to draw the coefficients of the AR processes of each variable's volatility. As noted above, we nearly fix the slope coefficient at particular values, by setting the prior mean to either 0.9, 0.8, or 0.5, with a prior standard deviation of 0.05.

To estimate the VAR-SVt model, we extend the algorithm used for the VAR-SV specification to accommodate fat tails, following the approach of Jacquier, Polson, and Rossi (2004). The key extension is the addition of a step to draw, for each variable, the time series of $q_{i,t}$ from an inverse Gamma distribution. The other steps are the same as those of the VAR-SV algorithm, but for a few small normalizations of data or innovations to reflect the $q_{i,t}$ terms.

Finally, our approach to estimating the AR-mixture model is taken from Groen, Paap, and Ravazzolo (2012). The steps in their Gibbs sampler include: using the algorithm of Gerlach, et al. (2000) to sample the latent states $\kappa_{j,t}$ that indicate the timing of breaks in the coefficients and variance; using the simulation smoother of Carter and Kohn (1994) to sample the regression parameters; and using the algorithm of Kim, Shepard, and Chib (1998) to draw the time-varying volatility and the variance of innovations to volatility.

All of our reported results are based on samples of 5000 posterior draws, retained from larger samples of draws. However, we use different burn periods and thinning intervals for different models, depending on the mixing properties of the algorithms (drawing on our own results on mixing properties and others in the literature, such as those in Primiceri (2005), Clark and Davig (2011), and Carriero, Clark, and Marcellino (2012)). Details are given in the appendix.

4 Results

To evaluate the models, we compare their accuracy in real-time out-of-sample forecasting, first for the U.S. and then for the U.K. To focus on the efficacy of alternative models of time-varying volatility, we separate our comparisons of AR models from our comparisons of VAR models. Among AR models, we compare the AR models with different volatility specifications to a baseline AR with constant volatility. Among VAR models, we compare the VARs with different volatility models to a baseline VAR with constant volatility. As noted above, we use a recursive estimation scheme in generating forecasts, expanding the model estimation sample as forecasting moves forward in time. In all cases, we provide results for our full sample and for a sample ending in 2007:Q4, to strip out possible effects of the severe recession.

For each country, we first consider the accuracy of point forecasts, using root mean square errors (RMSEs). We then consider density forecasts, using both the average log predictive score and the average continuous ranked probability score (CRPS). The predictive score, motivated and described in such recent sources as Geweke and Amisano (2010), is commonly viewed as the broadest measure of density accuracy. At each forecast origin, we compute the log predictive score using the quadratic approximation of Adolfson, Linde, and

Villani (2007):

$$s_t(y^o_{t+h}) = -0.5 \left(n \log(2\pi) + \log |V_{t+h|t}| + \left(y^o_{t+h} - \bar{y}_{t+h|t}\right)' V^{-1}_{t+h|t} \left(y^o_{t+h} - \bar{y}_{t+h|t}\right)\right), \quad (12)$$

where y^o_{t+h} denotes the observed outcome, $\bar{y}_{t+h|t}$ denotes the posterior mean of the forecast distribution, and $V_{t+h|t}$ denotes the posterior variance of the forecast distribution.

As indicated in Gneiting and Raftery (2007) and Gneiting and Ranjan (2011), some researchers view the continuous ranked probability score as having advantages over the log score. In particular, the CRPS does a better job of rewarding values from the predictive density that are close to but not equal to the outcome and is less sensitive to outlier outcomes. The CRPS, defined such that a lower number is a better score, is given by

$$\begin{aligned} CRPS_t(y^o_{t+h}) &= \int_{-\infty}^{\infty} \left(F(z) - 1\{y^o_{t+h} \leq z\}\right)^2 dz \\ &= E_f|Y_{t+h} - y^o_{t+h}| - 0.5 E_f|Y_{t+h} - Y'_{t+h}|, \end{aligned} \quad (13)$$

where F denotes the cumulative distribution function associated with the predictive density f, $1\{y^o_{t+h} \leq z\}$ denotes an indicator function taking value 1 if $y^o_{t+h} \leq z$ and 0 otherwise, and Y_{t+h} and Y'_{t+h} are independent random draws from the posterior predictive density. See Ravazzolo and Vahey (2010) for an application to disaggregate inflation.

4.1 U.S. results

Table 1 provides RMSEs for real-time forecasts obtained with U.S. data. For the baseline AR and VAR models with constant volatilities, we report the actual RMSEs. For the other AR models, we report ratios of each model's RMSE to the baseline AR model with constant volatility. Similarly, for the BVAR models with time-varying volatility, we report ratios of each model's RMSE to the baseline VAR with constant volatility. Entries less than 1 indicate that the given model yields forecasts more accurate than those from the baseline. In summarizing the results, we will focus on the full sample for most of the discussion and then conclude with a review of differences in the results for the shorter sample.

The results in Table 1 indicate that, for AR models, allowing time varying volatility tends to slightly to modestly improve forecast accuracy, for all variables except GDP growth. Taking the stochastic volatility specification as the baseline for time-varying volatility, none of the other volatility formulations yield any consistent, sizable advantage over stochastic volatility. The same is true for the model with both TVP and stochastic volatility. Sometimes a GARCH, mixture, or TVP-SV model can be better, but other times these models can be worse, with the performance of the mixture model tending to be the most variable. Consider, for example, inflation forecasts over the 1985:Q1-2011:Q2 sample. At the 1-quarter horizon, the AR-SV model has a lower RMSE ratio than the AR-GARCH and AR-mixture models. At the 8-quarter horizon, the AR-mixture has a lower RMSE than the AR-SV and AR-GARCH specifications. At both horizons, adding TVP further improves (slightly to modestly) on the RMSE of the AR-SV model.

Within the set of VAR models, the standard stochastic volatility specification (with log volatility following a random walk) consistently yields small to modest gains in point forecast accuracy. The VAR-SV model almost always has a lower RMSE than the VAR with GARCH. The modifications of stochastic volatility that make volatility stationary, allow fat tails, or allow TVP don't offer any notable gains over the standard VAR-SV specification. Consider, for example, forecasts of GDP growth. At horizons between 1 and 4 quarters, the VAR-SV model improves on the RMSE of the VAR by about 7 to 12 percent, depending on the horizon. The VAR-ARSV model with an AR(1) coefficient of 0.9 in the volatility processes improves on the RMSE of the VAR by about 6 to 10 percent of horizons between 1 and 4 quarters. The VAR-SVt model with fat tails based on 5 degrees of freedom improves on the baseline RMSE by 8 to 12 percent at the same horizons. In contrast, the VAR-GARCH specification yields RMSEs that exceed the baseline RMSEs by 3 to 9 percent between horizons of 1 and 8 quarters.

Table 2 provides average log predictive scores for real-time forecasts obtained with U.S. data. For the baseline AR and VAR models with constant volatilities, we report the actual scores (defined so that a higher score is a better result). For the other AR models, we report differences in score relative to the baseline AR model with constant volatility, such that a positive number indicates a model beats the baseline. Similarly, for the BVAR models with time-varying volatility, we report differences in score relative to the baseline VAR with constant volatility.

The results in Table 2 indicate that, within the AR class of models, allowing time varying volatility generally improves the accuracy of density forecasts, more so at shorter horizons than longer horizons. At shorter horizons, the gains in average scores are bigger than the gains in RMSEs associated with time-varying volatility models. As with the point forecasts, taking the stochastic volatility specification as the baseline for time-varying volatility, none of the other volatility formulations or the TVP-SV model yield any consistent, sizable advantage over the baseline model with stochastic volatility. Sometimes a GARCH or mixture model or the AR-TVP-SV model can be better, but other times these models can be worse, with the mixture model generally performing poorly.[6] Consider forecasts of GDP growth. At the 1-quarter horizon, the AR-SV model has a score about 20 percent better (higher) than the score of the AR with constant volatility, while the AR-GARCH model has a score about 17 percent above the baseline, and the AR-mixture model has a score about 7 percent below the baseline. At the 8-quarter horizon, the AR-SV model's score is essentially the same as the baseline model's, while the scores of the GARCH and mixture models are lower than the score of the AR model with constant volatility.

Within the set of VAR models, the standard stochastic volatility specification (with log volatility following a random walk) yields healthy gains in average log predictive scores for most variables and horizons, with the exception of unemployment and interest rates at longer horizons. The VAR-SV model dominates the VAR with GARCH, again with the exception of unemployment and interest rates at longer horizons. The modifications of stochastic volatility that make volatility stationary or allow fat tails don't offer any consistent gains over the standard VAR-SV specification, but stationarity does help longer-horizon forecasts of unemployment and the interest rate. Similarly, allowing TVP helps in some cases and hurts in others.

Consider, for example, forecasts of GDP growth. At horizons between 1 and 8 quarters, the VAR-SV model improves on the average log score of the VAR by about 6 to 19 percent, depending on the horizon. At the 1-quarter horizon, the VAR-GARCH specification improves on the baseline score by 8 percent (compared to 19 percent for the VAR-SV model); at other horizons, GARCH lowers the score by about 3 to 19 percent. The VAR-ARSV specification with an AR(1) coefficient of 0.9 improves on the average log score of the VAR by about 0 to 19 percent, depending on the horizon. The VAR-SVt model using 10 degrees of freedom raises scores (relative to the constant volatility benchmark) by 8 to 19 percent.

However, in the case of unemployment and interest rates, making the stochastic volatility process stationary improves on the VAR-SV model's accuracy for longer-horizon density forecasts. As we detail below, this pattern seems to be associated with the extreme outcomes of the recent sharp recession. Consider forecasts of unemployment. The relative score measure for the VAR-SV model declines from about 16 percent at the 1-quarter horizon to

[6] GARCH seems to work best for the interest rate. For example, at the 1-quarter ahead horizon, the score difference of the AR-GARCH model is 37.3 percent, compared to 9.6 percent for the AR-SV specification.

-71 percent at the 8-quarter horizon. Making the volatility process stationary mitigates the sharp decline in performance at longer horizons. For example, the relative score measure for the VAR-ARSV model using a coefficient of 0.9 declines from about 16 percent at the 1-quarter horizon to -12 percent at the 8-quarter horizon, while the score of the model using a coefficient of 0.8 declines from about 12 percent to -5 percent.

Table 3 provides average CRPS results for real-time U.S. forecasts. For the baseline AR and VAR models with constant volatilities, we report the levels of the average CRPS. For the other AR models, we report ratios of each model's average CRPS to the baseline AR model with constant volatility. Similarly, for the BVAR models with time-varying volatility, we report ratios of each model's average CRPS to the baseline VAR with constant volatility. Entries less than 1 indicate that the given model performs better, by the CRPS metric, than the baseline model.

The results in Table 3 indicate that, within the AR class of models, allowing time varying volatility consistently yields increases in density accuracy as measured by the CRPS. Taking the stochastic volatility specification as the baseline for time-varying volatility, none of the other volatility formulations yield any consistent, sizable advantage over stochastic volatility. Sometimes a GARCH model, mixture model, or TVP-SV specification can be very similar to or better than the baseline stochastic volatility model, but other times these models can be worse. Consider, for example, inflation forecasts over the 1985:Q1-2011:Q2 sample. Relative to the baseline AR model with constant volatility, the AR-SV specification improves the CRPS by 10 to 13 percent, depending on horizon. Adding time-varying parameters further improves (slightly at most horizons) the accuracy of density forecasts of inflation. The gains in CRPS are smaller for both the AR-GARCH and AR-mixture models, ranging from 1 to 2 percent for the former and 7 to 12 percent for the latter. In the case of unemployment forecasts, the AR-SV model improves the CRPS by 1 to 4 percent, while the AR-GARCH does a bit better at some horizons, improving the CRPS by 1 to 5 percent, and the mixture model worsens the average CRPS by 2 to 41 percent, depending on horizon.

Within the set of VAR models, the standard stochastic volatility specification yields healthy gains in average CRPS for most variables and horizons, with the exception of unemployment and interest rates at longer horizons. The VAR-SV almost uniformly dominates the VAR with GARCH, which is generally inferior to the baseline VAR with constant volatility. The modifications of stochastic volatility that make volatility stationary or allow fat tails don't offer any consistent, notable gains over the standard VAR-SV specification. The same applies to the VAR-TVP-SV specification. Consider, for example, forecasts of GDP growth. At horizons between 1 and 8 quarters, the VAR-SV model improves on the average CRPS of the VAR by about 7 to 14 percent. The VAR-GARCH's CRPS are worse than the baseline VAR's, by an amount ranging from a little more than 0 to as much as 13 percent. The VAR-ARSV specification with an AR(1) coefficient of 0.9 improves on the average CRPS of the VAR by about 6 to 13 percent, while the VAR-SVt model using 10 degrees of freedom improves the CRPS by 10 to 14 percent.

In light of the unprecedented developments of the 2007-2009 recession, it is possible that some of the findings described above are distorted by the recession. To assess that possibility, we consider a shorter sample of 1985:Q1-2007:Q4, which omits the severe outcomes of the recession. For the most part, the findings we just described for the 1985:Q1-2011:Q2 sample also apply to the 1985:Q1-2007:Q4 sample. However, there are some differences across results for the samples, likely due to some very large forecast errors during the recession (essentially, the severity of the recession was a very small tail event, based on the post-war history).[7] The most notable difference is in average log predictive scores: in the pre-crisis sample compared to the full sample, there is less of a tendency for scores from models with random walk stochastic volatility to decline as the horizon increases. Accordingly, there are fewer cases in which the stochastic volatility specification that treats volatility as stationary has some advantage (at longer horizons) over the specification that treats volatility as a random walk. For example, for forecasts of GDP growth from the AR and VAR models, while the gains to stochastic volatility decline as the horizon increases in the full sample results (e.g., for the VAR-SV, the relative score declines from 0.193 at $h = 1$ to 0.060 at $h = 8$), the gains do not decline in the shorter, pre-crisis sample (for the VAR-SV, the relative score is 0.222 at $h = 1$ and 0.214 at $h = 8$). A similar pattern applies to unemployment rate forecasts.

These patterns reflect some broad influences of the crisis on density forecast performance. To understand these, for the BVAR-SV and the constant volatility BVAR model we have taken a closer look at 1-step ahead predictive scores over the 2006-2010 period. This analysis indicates the performance of the stochastic volatility specification briefly deteriorates relative to the performance of the constant volatility BVAR for the following reasons.

[7] Large shocks are evident in the baseline AR and VAR RMSEs, log predictive scores, and CRPS, which are worse in the full sample than in the pre-crisis sample.

Before the crisis, the BVAR-SV specification generally scores better than the BVAR because the BVAR-SV model better picks up the effects of the Great Moderation on volatility. Some of the extreme outcomes of the crisis period are more unusual by the standards of the BVAR-SV-estimated (narrower) predictive densities than by the standards of the BVAR-estimated (wider) predictive densities. Consequently, for a few quarters, the BVAR tends to score better than the BVAR-SV models. But after a few quarters, the BVAR-SV model has picked up enough of a rise in volatility that it resumes yielding predictive scores better than the scores from the BVAR.

4.2 U.K. results

Tables 4-6 provide, respectively, RMSE, average log score, and average CRPS results for real-time U.K. forecasts. As in the U.S. results, the tables provide the levels of RMSEs, scores, and CRPS for the baseline AR and VAR models and relative RMSEs, scores, and CRPS for all other models. We deliberately provide a discussion of the U.K. results that is briefer than the discussion of the U.S. results.

The RMSE results in Table 4 indicate that, for AR models, allowing time varying volatility is somewhat less helpful in U.K. data for the full sample than in U.S. data. In the U.K. case, allowing time-varying volatility consistently improves point forecast accuracy for only the interest rate; time-varying volatility makes GDP growth forecasts consistently less accurate and has mixed effects on the accuracy of inflation and unemployment forecasts. Neither GARCH nor the mixture model has any consistent advantage over stochastic volatility, and the performance of the AR-mixture model seems to be most variable (across variables and horizons). Adding TVP also fails to yield systematic gains over the baseline model with stochastic volatility. Consider, for example, unemployment rate forecasts over the 1985:Q1-2010:Q4 sample. At the 1-quarter horizon, the AR-SV model has the lowest RMSE, by a small margin, with a RMSE ratio of 0.980, while the AR-GARCH, AR-mixture, and AR-TVP-SV models yield RMSE ratios of 1.030, 1.234, and 1.025, respectively. At the 8-quarter horizon, the RMSE ratios are 0.942 for the AR-SV model and 0.958, 1.549, and 0.869 for the AR-GARCH, AR-mixture, and AR-TVP-SV specifications.

Within the set of VAR models, the standard stochastic volatility specification (with log volatility following a random walk) consistently yields gains in point forecast accuracy, except in the case of GDP growth at multi-step horizons. As in the U.S. results, the modifications of stochastic volatility that make volatility stationary or allow fat tails or allow TVP don't offer any notable gains over the standard VAR-SV specification. Similarly, the VAR-GARCH sometimes yields forecasts more accurate than those from the VAR-SV, but often yields forecasts considerably less accurate. Consider, for example, forecasts of inflation. At horizons between 1 and 8 quarters, the VAR-SV model improves on the RMSE of the VAR by about 10 to 26 percent, depending on the horizon. The VAR-ARSV model with an AR(1) coefficient of 0.9 improves on the baseline RMSE by 11 to 28 percent, while the VAR-SVt model using 5 degrees of freedom improves on the baseline by 9 to 23 percent. For inflation, the VAR-GARCH yields gains in the accuracy that are comparable to, but not as large, as those from the VAR-SV.

The average log score results in Table 5 indicate that, for the full sample of 1985-2010, allowing time varying volatility is somewhat less helpful in U.K. data than in U.S. data, particularly with AR models. With the set of AR models, including stochastic volatility improves average log scores for inflation and unemployment rate forecasts at all horizons, but not for interest rates at any horizons or GDP growth at horizons greater than 1 quarter. The models with time-varying volatility fare much better in the 1985-2007 sample than the full sample, suggesting very large effects of the sharp recession. Among AR models with time-varying volatility, it is once again the case that none of the alternatives offer any consistent advantage over the standard stochastic volatility specification (in which volatility follows a random walk). Consider, for example, forecasts of GDP growth. In the full sample, the score differential for the AR-SV model declines from 0.240 at the 1-quarter horizon to -2.871 at the 8-quarter horizon, while the score of the AR-mixture model falls from -1.982 at the 1-quarter horizon to -2.445 at the 8-quarter horizon. In the 1985-2007 sample, the score differential of the AR-SV model is better, at 0.403 and 0.272 at the 1-quarter and 8-quarter horizons. The score differentials of the AR-mixture model are also better in the shorter sample than the full, at -0.025 at the 1-quarter horizon and 0.500 at the 8-quarter horizon.

Within the set of VAR models, it is also the case that time-varying volatility is less helpful to scores over the full sample than over the shorter sample, although the scores of VARs with time-varying volatility in the full sample look a little better than do the scores of AR models with time-varying volatility (we included in the U.S. results presentation an explanation of the effects of the crisis on predictive scores). In most cases, the VAR-SV model dominates the VAR with GARCH. The modifications of stochastic volatility that make volatility stationary or allow fat tails or allow TVP don't offer any consistent gains over the standard VAR-SV specification, except that, in the full sample, stationarity of volatility often improves longer-horizon forecast scores. Consider forecasts of GDP growth. In the full sample, the VAR-SV improves on the score of the baseline VAR at the 1-step horizon but not the 8-step horizon, yielding score differentials of 0.362 and -1.147 at these horizons. For the same sample, the VAR-ARSV with an AR(1) coefficient of 0.8 yields corresponding score differentials of 0.367 and -0.087. In the pre-crisis period, the VAR-SV yields score differentials of 0.483 at the 1-quarter horizon and 0.222 at the 8-quarter horizon, compared to differentials of 0.345 and 0.018 for the VAR-ARSV with an AR(1) coefficient of 0.8.

Finally, the CRPS results for the U.K. in Table 6 are more similar to the results for the U.S. than are the results for the other forecast metrics. This greater similarity in CRPS results seems to reflect much less sensitivity of the CRPS to the large shocks of the 2007-2009 recession. Within the AR class of models, allowing time varying volatility often, although not always, increases density accuracy as measured by the CRPS. Taking the stochastic volatility specification as the baseline for time-varying volatility, none of the other volatility formulations yields any consistent, sizable advantage over stochastic volatility. Consider, for example, inflation forecasts over the 1985:Q1-2010:Q4 sample. Relative to the baseline AR model with constant volatility, the AR-SV specification improves the CRPS by 6 to 13 percent, depending on horizon. The gains in CRPS are smaller for both the AR-GARCH and AR-mixture models, ranging from a loss of 5 percent to a gain of 7 percent for the former and a gain of 2 to 30 percent for the latter.

Within the set of VAR models, the standard stochastic volatility specification (with log volatility following a random walk) yields healthy gains in average CRPS for most variables and horizons, with the exception of unemployment forecasts at longer horizons. The VAR-SV often (but not always) dominates the VAR with GARCH. The modifications of stochastic volatility that make volatility stationary or allow fat tails or allow TVP don't offer any consistent, notable gains over the standard VAR-SV specification. Consider, for example, forecasts of inflation. At horizons between 1 and 8 quarters, the VAR-SV model improves on the average CRPS of the VAR by about 14 to 30 percent. The VAR-ARSV specification with an AR(1) coefficient of 0.9 improves on the average CRPS of the VAR by about 14 to 32 percent, while the VAR-SVt model using 10 degrees of freedom improves the CRPS by 14 to 29 percent. For the same models, the CRPS ratios are very similar in the 1985-2007 sample.

5 Conclusions

This paper compares, from a forecasting perspective, alternative models of time-varying macroeconomic volatility, included within autoregressive and vector autoregressive specifications for key macroeconomic indicators. The set of models includes constant volatility; stochastic volatility (with both constant AR coefficients and time-varying AR coefficients); stochastic volatility following a stationary AR process; stochastic volatility coupled with fat tails; GARCH; and a mixture of innovations model. Real-time forecasts of U.S. and U.K. GDP growth, the unemployment rate, inflation in the GDP deflator, and a short-term government bill yield over the last three decades are produced. For both countries, our results indicate that the AR and VAR specifications with stochastic volatility dominate models with alternative volatility specifications, in terms of point forecasting to some degree and density forecasting to a greater degree, in particular when using proper scoring rules such as the CRPS. Results are robust to the inclusion of the recent Great Recession period. We conclude that, from a macroeconomic forecasting perspective, these alternative volatility specifications seem to have no advantage over the now widely-used stochastic volatility specification.

6 Appendix

This appendix details the MCMC algorithms used to estimate and forecast with the models considered in the paper. In the interest, in most cases we provide details for the VAR specifications and omit details on AR specifications, which differ in that they do not involve the step necessary to estimate the Choleski matrix A or use A to orthogonalize innovations. For generality and simplicity in presentation, in this exposition we use y_t to refer to the endogenous variable or vector of variables, and we use X_t to denote the vector of variables on the right-hand side of each equation.

6.1 VAR with constant volatility

Under the Normal-diffuse prior, we estimate the VAR model with a two-step Gibbs sampler, detailed in such studies as Kadiyala and Karlsson (1997).

Step 1: Draw the vector of VAR coefficients B conditional on the error variance-covariance matrix Φ.

We draw the VAR coefficients from a conditional posterior distribution that is multivariate normal, as in equation (16a) of Kadiyala and Karlsson (1997).

Step 2: Draw the error covariance matrix Φ conditional on the VAR coefficients B.

We draw the error variance matrix from a conditional posterior distribution that is inverse Wishart, as in equation (16b) of Kadiyala and Karlsson (1997).

Forecast density:

To generate draws of forecasts, for each draw of the VAR coefficients and error covariance matrix, we generate shocks from $t+1$ through $t+H$, where H denotes the maximum forecast horizon considered, using the given draw of Φ. We use the shocks, the autoregressive structure of the VAR, and the draw of coefficients to compute the draw of y_{T+h}, $h = 1, \ldots, H$. The resulting set of draws of y_{T+h} are used to compute the forecast statistics of interest (RMSE based on the posterior median, etc.).

Priors:

For the VAR model, we use a conventional Minnesota prior, without cross-variable shrinkage:

$$\underline{\mu}_B \text{ such that } E[B_l^{(ij)}] = 0 \; \forall \; i, j, l \tag{14}$$

$$\underline{\Omega}_B \text{ such that } V[B_l^{(ij)}] = \begin{cases} \frac{\theta^2}{l^2} \frac{\sigma_i^2}{\sigma_j^2} & for\ l > 0 \\ \varepsilon^2 \sigma_i^2 & for\ l = 0 \end{cases} \tag{15}$$

Following common settings, we use $\theta = 0.2$ and $\varepsilon = 1000$, and we set the scale parameters σ_i^2 at estimates of residual variances from AR(4) models from the estimation sample. With all of the variables of our VAR models transformed for stationarity (in particular, we use growth rates of GDP, the price level, etc.), we set the prior mean of all the VAR coefficients to 0.

6.2 VAR with TVP and Stochastic Volatility

We estimate the VAR-TVP-SV model with a five-step Metropolis-within-Gibbs MCMC algorithm, following studies such as Cogley and Sargent (2005) and Primiceri (2005). The Metropolis step is used for the estimation of stochastic volatility, following Cogley and Sargent (2005) in their use of the Jacquier, Polson, and Rossi (1994) algorithm.

Step 1: Draw the time series of the vector of VAR coefficients B_t conditional on the history of Λ_t, Q, A, and Φ, where Φ denotes a diagonal matrix with elements $\phi_i, i = 1, \ldots, k$.

As detailed in Primiceri (2005), drawing the VAR coefficients involves using the Kalman filter to move forward in time, a backward smoother to obtain posterior means and variances of the coefficients at each point in time, and then drawing coefficients from the posterior normal distribution. For the backward smoothing, we use the algorithm of Durbin and Koopman (2002) instead of the Carter and Kohn (1994) algorithm used by Primiceri. Koop and Korobilis (2009) note that either algorithm can be used for VARs with time-varying parameters; the software we use makes the Durbin and Koopman (2002) algorithm faster. The mean and variance of the period 0 coefficient vector used in the smoother is fixed at the prior mean and variance described below.

Step 2: Draw the elements of A conditional on the history of B_t, the history of Λ_t, Q, and Φ.

Following Cogley and Sargent (2005), rewrite the VAR as

$$A(y_t - X_t' B_t) = A\hat{y}_t \equiv \tilde{y}_t = \Lambda_t^{0.5} \epsilon_t, \tag{16}$$

where, conditional on B_t, $\hat{y}_t$ is observable. This system simplifies to a set of $i = 2, \ldots, k$ equations, with equation i having as dependent variable $\hat{y}_{i,t}$ and as independent variables $-1 \cdot \hat{y}_{j,t}, j = 1, \ldots, i-1$, with coefficients a_{ij}. Multiplying equation i by $\lambda_{i,t}^{-0.5}$ eliminates the heteroskedasticity associated with stochastic volatility. Then, proceeding separately for each transformed equation i, draw the i'th equation's vector of coefficients a_i (a vector containing a_{ij} for $j = 1, \ldots, i-1$) from a normal posterior distribution with the mean and variance implied by the posterior mean and variance computed in the usual (OLS) way. See Cogley and Sargent (2005) for details.

Step 3: Draw the elements of the variance matrix Λ_t conditional on the history of B_t, A, Q, and Φ.

Following Cogley and Sargent (2005) and Primiceri (2005), the VAR can be rewritten as

$$A(y_t - X_t' B_t) \equiv \tilde{y}_t = \Lambda_t^{0.5} \epsilon_t,$$

where $\epsilon_t \sim N(0, I_k)$. Taking logs of the squares yields

$$\log \tilde{y}_{i,t}^2 = \log \lambda_{i,t}^2 + \log \epsilon_{i,t}^2, \ i = 1, \ldots, k.$$

The conditional volatility process is

$$\log(\lambda_{i,t}^2) = \log(\lambda_{i,t-1}^2) + \nu_{i,t}, \ \nu_{i,t} \sim \text{iid } N(0, \phi_i), \ i = 1, \ldots, k.$$

The estimation of the time series of $\lambda_{i,t}^2$ proceeds equation by equation, using the measured $\log \tilde{y}_{i,t}^2$ and Cogley and Sargent's (2005) version of the Metropolis algorithm of Jacquier, Polson, and Rossi (1994); see Cogley and Sargent for further detail.

Step 4: Draw the variance matrix Q conditional on the history of B_t, the history of Λ_t, A, and Φ.

Following Cogley and Sargent (2005) and Primiceri (2005), the sampling of Q, the variance-covariance matrix of innovations to the VAR coefficients, is based on inverse Wishart priors and posteriors. The scale matrix of the posterior distribution is the sum of the prior mean $\times$ the prior degrees of freedom and $\sum_{t=1}^{T} \hat{n}_t \hat{n}_t'$, where $\hat{n}_t$ denotes the innovations to the posterior draws of coefficients obtained in step 1.

Step 5: Draw the variances $\phi_i, i = 1, \ldots, k$, conditional on the history of B_t, the history of Λ_t, A, and Q.

Following Cogley and Sargent (2005), the sampling of ϕ_i, the variance of innovations to log variance associated with VAR equation i, is based on inverse Gamma priors and posteriors. Each equation's volatility is treated independently. The scale factor of the posterior distribution is the sum of the prior mean $\times$ the prior degrees of freedom and $\sum_{t=1}^{T} \hat{\nu}_{i,t}^2$, where $\hat{\nu}_{i,t}$ denotes the innovations to the posterior draw of the volatility for variable i obtained in step 3.

Forecast density:

To generate draws of forecasts, we follow Cogley, Morozov, and Sargent's (2005) approach to simulating the predictive density. Let H denote the maximum forecast horizon considered. From a forecast origin of period T, for each retained draw of the time series of B_t up through T, Λ_t up through T, A, Q, and Φ, we: (1) draw innovations to coefficients for periods $T+1$ through $T+H$ from a normal distribution with variance-covariance matrix Q and use the random walk structure to compute $B_{T+1}, \ldots, B_{T+H}$; (2) draw innovations to log volatility for each variable i for periods $T+1$ through $T+H$ from a normal distribution with variance ϕ_i and use the random walk model of $\log \lambda_{i,t+h}$ to compute $\lambda_{i,T+1}, \ldots, \lambda_{i,T+H}$; (3) draw innovations to y_{T+h}, $h = 1, \ldots, H$, from a normal distribution with variance $\Sigma_{T+h} = A^{-1} \Lambda_{T+h} A^{-1\prime}$, and use the vector autoregressive structure of the model along with the time series of coefficients B_{T+h} to obtain draws of y_{T+h}, $h = 1, \ldots, H$. The resulting draws of y_{T+h} are used to compute the forecast statistics of interest (RMSE based on the posterior mean, etc.).

Priors:

The prior for the initial values of the parameters B_t, B_0 is normally distributed with zero mean and unit variance. The prior for Q follows an inverted Wishart distribution:

$$Q \sim IW(\underline{Q}, \mu_Q), \tag{17}$$

where $\underline{Q}$ is diagonal matrix with diagonal elements equal to 0.035, and μ_Q is set to 1.

In the prior for the volatility-related components of the model, we use an approach to setting them similar to that of such studies as Clark (2011), Cogley and Sargent (2005), and Primiceri (2005). The prior for A is uninformative:

$$\underline{\mu}_{a,i} = 0, \ \underline{\Omega}_{a,i} = 1000^2 \cdot I_{i-1}. \tag{18}$$

In line with other studies such as Cogley and Sargent (2005), we make the priors on the volatility-related parameters loosely informative. As for the B_t parameters, the prior on each ϕ_i use a mean of 0.035 and 1 degrees of freedom. For the initial value of the volatility of each equation i, we use

$$\underline{\mu}_{\lambda,i} = \log \hat{\lambda}_{i,0,OLS}, \ \underline{\Omega}_{\lambda} = 4. \tag{19}$$

To obtain $\log \hat{\lambda}_{i,0,OLS}$, we use a training sample of observations preceding the estimation sample to fit AR(4) models for each variable and, for each $j = 2, \ldots, n$, we regress the residual from the AR model for j on the residuals associated with variables 1 through $j-1$ and compute the error variance (this step serves to filter out covariance as reflected in the A matrix). Letting $\hat{\sigma}^2_{i,0}$ denote these error variances, we set the prior mean of log volatility in period 0 at $\log \hat{\lambda}_{i,0,OLS} = \log \hat{\sigma}^2_{i,0}$. For simplicity, since some of the data vintages do not start until 1959, we use the same prior mean on initial volatility for all vintages (forecast origins). We compute that volatility value using the last available vintage of data, with a training sample of 36 observations for the U.S. and 24 for the U.K.

6.3 VAR with Stochastic Volatility

We estimate the model with a four-step Metropolis-within-Gibbs MCMC algorithm.

Step 1: Draw the vector of VAR coefficients B conditional on the history of Λ_t, A, and Φ.

The vector of coefficients is sampled from a multivariate normal posterior distribution with mean $\bar{\mu}_B$ and variance $\bar{\Omega}_B$, based on prior mean $\underline{\mu}_B$ and variance $\underline{\Omega}_B$. Letting $\Sigma_t =$

$A^{-1}\Lambda_t A^{-1\prime}$, the posterior mean and variance are:

$$\text{vec}(\bar{\mu}_B) = \bar{\Omega}_B \left\{ \text{vec}\left(\sum_{t=1}^{T} X_t y_t' \Sigma_t^{-1} \right) + \underline{\Omega}_B^{-1} \text{vec}(\underline{\mu}_B) \right\} \quad (20)$$

$$\bar{\Omega}_B^{-1} = \underline{\Omega}_B^{-1} + \sum_{t=1}^{T} (\Sigma_t^{-1} \otimes X_t X_t'). \quad (21)$$

Step 2: Draw the elements of A conditional on B, the history of Λ_t, and Φ.

This step proceeds as with step 2 of the AR-TVP-SV algorithm, except that the VAR coefficients are constant.

Step 3: Draw the elements of the variance matrix Λ_t conditional on B, A, and Φ.

This step proceeds as with step 3 of the AR-TVP-SV algorithm, except that the VAR coefficients are constant.

Step 4: Draw the variances $\phi_i, i = 1, \ldots, k$, conditional on B, the history of Λ_t, A, and Q.

This step proceeds as with step 5 of the AR-TVP-SV algorithm, except that the VAR coefficients are constant.

Forecast density:

The simulation of the predictive density follows the steps described above for the VAR-TVP-SV model, except that the steps for simulating time series of the VAR coefficients are eliminated.

Variant with stationary volatility

To estimate the VAR-ARSV model, we modify the VAR-SV algorithm in two ways. First, we need to make some small adjustments to the Metropolis step for sampling the volatilities $\lambda_{i,t}$, to reflect a volatility process that is an AR(1) process rather than a random walk. These adjustments affect the mean and standard deviation of the conditional distributions used to sample volatility. The general equations are given in Jacquier, Polson, and Rossi (1994). Second, we need to add a step to draw the AR coefficients of the volatility process:

$$\log(\lambda_{i,t}) = a_{0,i} + a_{1,i} \log(\lambda_{i,t-1}) + \nu_{i,t}, \;\; \nu_{i,t} \sim \;\; N(0, \phi_i) \; \forall \; i = 1, k.$$

This step uses a multivariate normal prior and posterior for the vector of coefficients for each equation i, treating each equation independently.

Variant with fat tails

To estimate the VAR-SVt model, we modify some steps of the VAR-SV algorithm to normalize innovations by $q_{i,t}^{-0.5}$ and add a step to draw the fat tails term. Step 1 (VAR coefficients) above is modified to use a reduced form variance matrix that takes the form $\Sigma_t = A^{-1} Q_t \Lambda_t A^{-1\prime}$. Step 2 ($A$ coefficients) is modified to multiply equation i by $q_{i,t}^{-0.5} \lambda_{i,t}^{-0.5}$. Step 3 ($\Lambda_t$) is modified to use $q_{i,t}^{-0.5} \tilde{y}_{i,t}$ in lieu of $\tilde{y}_{i,t}$. Finally, following Jacquier, Polson, and Rossi (2004), we add a Step 5 to independently sample $q_{i,t}$ from an inverse Gamma distribution using d degrees of freedom and a posterior scale term of $(\tilde{y}_{i,t}^2/\lambda_{i,t} + d)$ [which is exactly the same as using $(\tilde{y}_{i,t}^2/\lambda_{i,t} + d)/\chi^2(d)$ for the sampling].

Priors

For the VAR-SV model, we use a conventional Minnesota prior, without cross-variable shrinkage:

$$\underline{\mu}_B \text{ such that } E[B_l^{(ij)}] = 0 \; \forall \; i, j, l \tag{22}$$

$$\underline{\Omega}_B \text{ such that } V[B_l^{(ij)}] = \begin{cases} \frac{\theta^2}{l^2} \frac{\sigma_i^2}{\sigma_j^2} & for \; l > 0 \\ \varepsilon^2 \sigma_i^2 & for \; l = 0 \end{cases} \tag{23}$$

Following common settings, we use $\theta = 0.2$ and $\varepsilon = 1000$, and we set the scale parameters σ_i^2 at estimates of residual variances from AR(4) models from the estimation sample. With all of the variables of our VAR models transformed for stationarity (in particular, we use growth rates of GDP, the price level, etc.), we set the prior mean of all the VAR coefficients to 0.

For the priors on the volatility-related components of the model, we use the approach detailed above for the VAR-TVP-SV. In the case of the model with stationary stochastic volatility, we set the prior mean and standard deviation of each volatility equation's intercept at 0 and 0.4, respectively, and we set the prior mean of the AR(1) coefficient at either 0.5, 0.8, or 0.9, with a standard deviation of 0.05.

6.4 AR mixture of innovation models

The AR(p) mixture of innovations model, developed in studies such as Koop and Potter (2007), Giordani, et al. (2007), and Groen, Paap, and Ravazzolo (2012), is specified as follows:

$$
\begin{aligned}
y_t &= b_{0,t} + \sum_{i=1}^{p} b_{i,t} y_{t-i} + u_t \\
u_t &= \lambda_t^{0.5} \epsilon_t, \ \epsilon_t \sim N(0,1) \\
b_{j,t} &= b_{j,t-1} + \kappa_{j,t} \ n_{j,t}, \ j = 0, \ldots, p \\
\log(\lambda_t) &= \log(\lambda_{t-1}) + \kappa_{p+1,t} \ n_{p+1,t} \\
\Pr[\kappa_{j,t} = 1] &= \pi_j, \ j = 0, \ldots, p+1 \\
\text{var}\left((n_{0,t}, \ldots, n_{p+1,t})'\right) &= \text{diag}(q_0, q_1, \ldots, q_{p+1}).
\end{aligned} \tag{24}
$$

For posterior simulation we run the Gibbs sampler in combination with the data augmentation technique by Tanner and Wong (1987). Defining the latent variables $B = \{b_{0,t}, ..., b_{p,t}\}_{t=1}^{T}$, $R = \{\lambda_t\}_{t=1}^{T}$,, $\mathcal{K} = (\mathcal{K}_b, \mathcal{K}_\lambda)$, $\mathcal{K}_b = \{\kappa_{0,t}, ..., \kappa_{p,t}\}_{t=1}^{T}$ and $\mathcal{K}_\lambda = \{\kappa_{p+1,t}\}_{t=1}^{T}$, alongside the model parameters, $\theta \equiv (\pi_0, ..., \pi_{p+1}, q_0, ..., q_{p+1})'$, the sampling scheme consists of the following iterative steps:

1. Draw $\mathcal{K}_b$ conditional on $R, \mathcal{K}_\lambda, \theta$, and r .

2. Draw B conditional on $R, \mathcal{K}, \theta$ and r.

3. Draw $\mathcal{K}_\lambda$ conditional on $B, \mathcal{K}_b, \theta$, and r.

4. Draw R conditional on $B, \mathcal{K}, \theta$ and r.

5. Draw θ conditional on $B, \mathcal{K}$ and r.

The first and third step applies the efficient sampling algorithm of Gerlach, Carter and Kohn (2000). The second and fourth steps follows the forward-backward algorithm of Carter and Kohn (1994).

The vector of parameters θ is easily sampled as we use conjugate priors. For the structural break probability parameters we take Beta distributions

$$
\pi_j \sim \text{Beta}(a_j, b_j) \qquad \text{for } j = 0, \ldots, p+1. \tag{25}
$$

The parameters a_j and b_j can be set according to our prior belief about the occurrence of structural breaks. The expected prior probability of a break is $a_j/(a_j + b_j)$. We set $a_j = 0.8$ and $b_j = 30$ for $j = 0, \ldots, p$ and $a_{p+1} = 0.5$ and $b_{p+1} = 2$. For the variance parameters which reflect our prior beliefs about the size of the structural breaks we take an inverted

Gamma-2 prior that depends on scale parameter $\bar{\omega}_j$ and degrees of freedom parameter ν_j, that is,

$$q_j^2 \sim \text{IG-2}(\bar{\omega}_j, \nu_j) \qquad \text{for } j = 0, \ldots, k+1, \tag{26}$$

with $\bar{\omega}_j = \omega_j \nu_j$. The expected prior break size equals therefore the square root of $(\omega_j \nu_j)/(\nu_j - 2)$ for $\nu_j > 2$. We set ω_j equal to OLS estimates of the variance of the autoregressive parameters and residual variance divided by 100.

6.5 GARCH models

We estimate the VAR model in equation (7) using a Metropolis-Hastings algorithm. Define the vector $\alpha_i = (B_0, ..., B_L, a_{0,i}, a_{1,i}, a_{2,i})'$, with $B(L) = (B_0, ..., B_L)$, $i = 1, .., k$, and α_j the j-th element of it. The sampling scheme consists of the following iterative steps.

Step 1: At iteration s, generate a point α_j^* from the random walk kernel

$$\alpha_j^* = \alpha_j^{i-1} + \epsilon_j, \;\; \epsilon \sim N(0, Q), \tag{27}$$

where Q is a diagonal matrix and σ_j^2 is its j-th diagonal element, and α_j^{s-1} is the $(s-1)$th iterate of α_j. Therefore, we draw row elements of $B_0, ..., B_L$ and $a_{0,i}, a_{1,i}, a_{2,i}$ independently for each $i = 1, .., k$. Then accept α_j^* as α_j^s with probability $p = min\left[1, f(\alpha_j^*)/f(\alpha_j^{s-1})\right]$, where $f()$ is the likelihood of model (7) times priors. Otherwise, set $\alpha_j^* = \alpha_j^{s-1}$. The elements of Q are tuned by monitoring the acceptance rate to lie between 25% and 50%.

Step 2: After M iterations, we apply the following independent kernel MH algorithm. Generate α_j^* from

$$\alpha_j^* = \mu_{\alpha_j}^{i-1} + \epsilon_j, \quad \epsilon \sim N(0, Q_{\alpha_j}), \tag{28}$$

where μ_{α_j} and Q_{α_j} are, respectively, the sample mean and the sample covariance of the first M iterates for α_j. Then accept α_j^* as α_j^i with probability

$$p = min\left[1, \frac{f(\alpha_j^*) g(\alpha_j^{s-1})}{f(\alpha_j^{s-1}) g(\alpha_j^*)}\right], \tag{29}$$

where $g()$ is a Gaussian proposal density (28). The non-diagonal elements of the variance covariance matrix grouped in the matrix A are simulated as in Cogley and Sargent (2005), as described in step 2 in section 6.3.

Priors

We set normal priors for $B(L)$ with mean and variance equal to frequentist estimates. The priors for $a_{0,i}, a_{1,i}, a_{2,i}$ are uniform distributed and satisfy the restrictions $a_0 > 0,\ a_1 + a_2 < 1$.

6.6 Details of burn samples and thinning invervals

As noted in section 3, our results are based on samples of 5000 draws retained from a larger number of draws, with the larger number of draws reflecting settings on burn samples and thinning intervals meant to yield reasonable mixing of the MCMC chains associated with each model.

Burn samples and thinning intervals

model	burn	thin interval	total draws
AR	5000	5	30,000
AR-GARCH	2000	1	7,000
AR-SV	1000	20	101,000
AR-TVP-SV	1000	20	101,000
AR-mixture	2000	2	12,000
VAR	5000	5	30,000
VAR-GARCH	2000	1	7,000
VAR-SV	5000	8	45,000
VAR-ARSV	5000	5	30,000
VAR-SVt	5000	8	45,000
VAR-TVP-SV	1000	20	101,000

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Table 1. Real-Time Forecast RMSEs, U.S. Forecasts

(RMSEs for AR and BVAR benchmarks, RMSE ratios in all others)

	GDP growth, 1985:Q1-2007:Q4				**GDP growth, 1985:Q1-2011:Q2**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.440	0.454	0.475	0.475	0.518	0.558	0.583	0.587
VAR	0.517	0.550	0.511	0.456	0.675	0.747	0.711	0.593
AR-GARCH	1.007	1.011	1.013	1.042	0.998	1.011	1.010	1.032
AR-SV	1.016	1.013	1.017	1.029	1.010	1.002	1.007	1.020
AR-mixture	1.039	1.046	1.093	1.175	1.014	1.018	1.050	1.089
AR-TVP-SV	1.016	1.009	1.027	1.048	1.004	0.995	1.007	1.017
VAR-GARCH	1.054	1.022	1.094	1.055	1.049	1.040	1.094	1.027
VAR-SV	0.955	0.949	1.008	1.041	0.896	0.878	0.932	1.007
VAR-ARSV, a_1=0.9	0.967	0.962	1.016	1.033	0.909	0.896	0.941	0.998
VAR-ARSV, a_1=0.8	0.978	0.971	1.014	1.022	0.928	0.915	0.950	0.992
VAR-ARSV, a_1=0.5	0.984	0.985	1.000	1.009	0.957	0.948	0.961	0.987
VAR-SVt, d=5	0.947	0.943	0.986	1.015	0.891	0.877	0.915	0.978
VAR-SVt, d=10	0.945	0.945	0.996	1.022	0.891	0.880	0.924	0.985
VAR-SVt, d=15	0.949	0.947	0.996	1.017	0.893	0.880	0.925	0.987
VAR-TVP-SV	0.977	0.953	1.084	1.048	0.996	0.926	1.038	0.987
	Inflation, 1985:Q1-2007:Q4				**Inflation, 1985:Q1-2011:Q2**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.252	0.265	0.281	0.355	0.266	0.272	0.298	0.367
VAR	0.254	0.281	0.302	0.386	0.269	0.284	0.318	0.400
AR-GARCH	0.996	0.992	0.986	0.972	0.992	0.989	0.983	0.967
AR-SV	0.972	0.981	0.940	0.910	0.977	0.978	0.940	0.907
AR-mixture	1.004	0.958	0.936	0.885	0.985	0.974	0.943	0.877
AR-TVP-SV	0.972	0.962	0.907	0.825	0.962	0.963	0.903	0.826
VAR-GARCH	1.102	0.957	0.980	0.909	1.074	0.968	0.981	0.920
VAR-SV	0.972	0.956	0.920	0.887	0.966	0.949	0.915	0.883
VAR-ARSV, a_1=0.9	0.968	0.960	0.934	0.916	0.962	0.949	0.925	0.911
VAR-ARSV, a_1=0.8	0.972	0.967	0.951	0.937	0.966	0.957	0.941	0.931
VAR-ARSV, a_1=0.5	0.980	0.985	0.979	0.979	0.977	0.975	0.964	0.972
VAR-SVt, d=5	0.972	0.971	0.944	0.937	0.970	0.960	0.938	0.929
VAR-SVt, d=10	0.972	0.967	0.944	0.926	0.966	0.957	0.935	0.919
VAR-SVt, d=15	0.968	0.971	0.944	0.924	0.962	0.957	0.935	0.914
VAR-TVP-SV	1.083	1.046	0.868	0.824	1.071	1.077	0.984	0.882

Table 1, continued. Real-Time Forecast RMSEs, U.S. Forecasts

(RMSEs for AR and BVAR benchmarks, RMSE ratios in all others)

	Unemployment, 1985:Q1-2007:Q4				**Unemployment, 1985:Q1-2011:Q2**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.164	0.292	0.548	0.914	0.222	0.403	0.814	1.354
VAR	0.161	0.291	0.541	0.747	0.240	0.483	1.003	1.503
AR-GARCH	1.018	1.010	0.987	0.952	1.005	0.968	0.986	0.995
AR-SV	0.982	0.986	0.971	0.982	0.995	0.995	0.978	0.984
AR-mixture	1.110	1.171	1.250	1.388	1.032	1.069	1.221	1.618
AR-TVP-SV	1.000	0.997	0.978	0.996	1.023	0.998	1.034	1.055
VAR-GARCH	1.037	1.058	1.039	1.043	1.004	1.019	1.041	1.031
VAR-SV	0.969	0.944	0.933	1.018	0.967	0.930	0.914	0.971
VAR-ARSV, a_1=0.9	0.981	0.958	0.949	1.038	1.000	0.969	0.946	0.992
VAR-ARSV, a_1=0.8	0.988	0.969	0.959	1.033	1.017	0.986	0.962	0.999
VAR-ARSV, a_1=0.5	0.994	0.990	0.976	1.016	1.038	1.014	0.988	1.008
VAR-SVt, d=5	0.969	0.955	0.951	1.033	1.008	0.969	0.941	0.977
VAR-SVt, d=10	0.969	0.955	0.948	1.033	1.000	0.963	0.936	0.979
VAR-SVt, d=15	0.969	0.951	0.946	1.033	1.000	0.961	0.936	0.981
VAR-TVP-SV	1.081	1.052	1.044	1.313	1.108	1.039	0.967	1.068
	Interest rate, 1985:Q1-2007:Q4				**Interest rate, 1985:Q1-2011:Q2**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.475	0.869	1.373	2.036	0.469	0.873	1.435	2.227
VAR	0.452	0.795	1.258	1.839	0.454	0.810	1.330	2.012
AR-GARCH	0.823	0.867	0.887	0.907	0.832	0.860	0.889	0.919
AR-SV	0.977	0.967	0.995	0.992	0.977	0.964	0.990	0.980
AR-mixture	0.815	0.900	1.070	1.255	0.851	0.911	1.040	1.164
AR-TVP-SV	0.760	0.826	0.950	1.077	0.780	0.806	0.885	0.981
VAR-GARCH	0.909	0.990	0.955	0.990	0.923	0.993	0.963	0.985
VAR-SV	0.946	0.962	0.999	1.016	0.933	0.944	0.974	0.990
VAR-ARSV, a_1=0.9	0.960	0.971	1.004	1.031	0.944	0.954	0.979	1.005
VAR-ARSV, a_1=0.8	0.962	0.972	1.007	1.034	0.948	0.957	0.982	1.009
VAR-ARSV, a_1=0.5	0.969	0.978	1.007	1.031	0.955	0.963	0.987	1.014
VAR-SVt, d=5	0.955	0.966	1.007	1.036	0.937	0.948	0.984	1.013
VAR-SVt, d=10	0.953	0.962	1.005	1.031	0.935	0.944	0.980	1.006
VAR-SVt, d=15	0.944	0.961	1.002	1.028	0.930	0.943	0.978	1.004
VAR-TVP-SV	0.810	0.960	1.156	1.476	0.815	0.916	1.050	1.322

Notes: 1. In each quarter t from 1985:Q1 through 2011:Q2, vintage t data (which end in $t-1$) are used to form forecasts for periods t through $t+7$, corresponding to horizons of 1 through 8 quarters ahead. The forecast errors are calculated using the second-available (real–time) estimates of growth and inflation and currently available measures of unemployment and the short-term interest rate as the actuals. All variables are defined in annualized percentage points.

2. The models are detailed in section 3. The notation VAR-ARSV refers to a VAR model in which log volatility follows an AR(1) process, with a tight prior on the slope coefficient a_1. The notation VAR-SVt refers to a VAR model with stochastic volatility and fat tails, in which d denotes the degrees of freedom of the Student-t distribution that is the marginal distribution of innovations to the model. The forecasts are produced with recursive estimation of the models.

3. For the baseline AR and VAR models with constant volatilities, the table reports the RMSEs (first two rows of each panel). For the AR models with time-varying volatilities, the table reports the ratio of each model relative to the RMSE of the AR baseline. For the VAR models with time-varying volatilities, the table reports the ratio of each model relative to the RMSE of the VAR baseline. Entries less than 1 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

Table 2. Average log predictive scores, U.S. Forecasts

(Scores for AR and BVAR benchmarks, score differences in all others)

	GDP growth, 1985:Q1-2007:Q4				**GDP growth, 1985:Q1-2011:Q2**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	-1.005	-1.038	-1.063	-1.060	-1.044	-1.093	-1.120	-1.119
VAR	-0.996	-1.006	-1.012	-1.044	-1.118	-1.176	-1.163	-1.119
AR-GARCH	0.235	0.196	0.188	0.149	0.172	0.077	0.014	-0.068
AR-SV	0.268	0.251	0.252	0.225	0.200	0.142	0.096	0.004
AR-mixture	0.205	0.184	0.110	0.085	-0.066	-0.221	-0.314	-0.343
AR-TVP-SV	0.272	0.244	0.218	0.189	0.211	0.150	0.101	0.018
VAR-GARCH	0.088	0.005	-0.150	-0.207	0.080	-0.028	-0.149	-0.187
VAR-SV	0.222	0.187	0.202	0.214	0.193	0.133	0.090	0.060
VAR-ARSV, a_1=0.9	0.171	0.135	0.090	0.020	0.187	0.150	0.092	0.004
VAR-ARSV, a_1=0.8	0.112	0.081	0.034	-0.009	0.132	0.106	0.053	-0.010
VAR-ARSV, a_1=0.5	0.034	0.011	-0.009	-0.009	0.057	0.036	0.018	0.000
VAR-SVt, d=5	0.182	0.157	0.166	0.165	0.175	0.147	0.127	0.058
VAR-SVt, d=10	0.213	0.187	0.194	0.204	0.190	0.146	0.118	0.077
VAR-SVt, d=15	0.218	0.183	0.207	0.215	0.199	0.137	0.124	0.070
VAR-TVP-SV	0.121	0.036	-0.116	-0.412	0.121	0.047	-0.094	-0.350
	Inflation, 1985:Q1-2007:Q4				**Inflation, 1985:Q1-2011:Q2**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	-0.192	-0.267	-0.353	-0.521	-0.224	-0.267	-0.374	-0.531
VAR	-0.200	-0.306	-0.411	-0.599	-0.235	-0.299	-0.428	-0.612
AR-GARCH	0.012	0.051	0.051	0.069	0.014	0.045	0.044	0.052
AR-SV	0.177	0.157	0.176	0.145	0.156	0.133	0.154	0.127
AR-mixture	-0.154	-0.113	0.010	-0.296	-0.171	-0.176	-0.058	-0.268
AR-TVP-SV	0.192	0.195	0.223	0.172	0.177	0.165	0.199	0.163
VAR-GARCH	-0.327	-0.300	-0.309	-0.314	-0.306	-0.311	-0.284	-0.294
VAR-SV	0.077	0.102	0.173	0.152	0.075	0.083	0.155	0.138
VAR-ARSV, a_1=0.9	0.040	0.055	0.063	0.042	0.047	0.043	0.063	0.044
VAR-ARSV, a_1=0.8	0.019	0.018	0.021	0.012	0.029	0.015	0.027	0.017
VAR-ARSV, a_1=0.5	-0.029	-0.038	-0.045	-0.041	-0.012	-0.032	-0.031	-0.031
VAR-SVt, d=5	0.046	0.056	0.086	0.057	0.051	0.037	0.075	0.048
VAR-SVt, d=10	0.072	0.080	0.126	0.093	0.073	0.064	0.113	0.083
VAR-SVt, d=15	0.079	0.089	0.135	0.104	0.078	0.071	0.121	0.092
VAR-TVP-SV	0.059	0.019	-0.097	-0.412	0.056	-0.010	-0.124	-0.396

Table 2, continued. Average log predictive scores, U.S. Forecasts

(Scores for AR and BVAR benchmarks, score differences in all others)

	Unemployment, 1985:Q1-2007:Q4				**Unemployment, 1985:Q1-2011:Q2**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.122	-0.423	-0.971	-1.388	-0.014	-0.581	-1.223	-1.707
VAR	0.177	-0.358	-0.876	-1.157	-0.034	-0.707	-1.535	-2.208
AR-GARCH	0.160	0.074	-0.032	-0.083	0.155	0.087	-0.031	-0.062
AR-SV	0.081	0.090	0.089	0.049	0.046	0.038	-0.008	-0.083
AR-mixture	0.121	0.084	-0.168	-0.704	-0.002	0.082	-0.065	-0.355
AR-TVP-SV	-1.176	-1.340	-1.426	-1.508	-1.066	-1.192	-1.139	-1.023
VAR-GARCH	-0.548	-0.510	-0.354	-0.247	-0.403	-0.257	0.002	-0.016
VAR-SV	0.187	0.189	0.104	-0.065	0.162	0.098	-0.182	-0.707
VAR-ARSV, a_1=0.9	0.139	0.154	0.100	-0.041	0.157	0.160	0.034	-0.120
VAR-ARSV, a_1=0.8	0.097	0.113	0.080	-0.029	0.119	0.128	0.056	-0.054
VAR-ARSV, a_1=0.5	0.028	0.047	0.035	-0.023	0.052	0.058	-0.006	-0.080
VAR-SVt, d=5	0.104	0.134	0.075	-0.074	0.121	0.111	-0.092	-0.424
VAR-SVt, d=10	0.161	0.177	0.083	-0.091	0.172	0.143	-0.161	-0.656
VAR-SVt, d=15	0.172	0.184	0.088	-0.094	0.176	0.131	-0.189	-0.680
VAR-TVP-SV	0.035	-0.011	-0.177	-0.531	-0.003	-0.045	0.032	0.163
	Interest rate, 1985:Q1-2007:Q4				**Interest rate, 1985:Q1-2011:Q2**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	-0.867	-1.348	-1.750	-2.144	-0.853	-1.339	-1.758	-2.151
VAR	-0.837	-1.272	-1.680	-2.041	-0.832	-1.274	-1.692	-2.034
AR-GARCH	0.391	0.188	0.063	0.077	0.373	0.187	0.047	0.038
AR-SV	0.095	0.031	-0.049	-0.048	0.096	0.034	-0.075	-0.149
AR-mixture	0.278	0.042	-0.139	-0.323	0.161	-0.002	-0.104	-0.193
AR-TVP-SV	0.056	-0.146	-0.283	-0.324	0.029	-0.159	-0.238	-0.191
VAR-GARCH	-0.018	-0.061	-0.074	-0.115	-0.042	-0.071	-0.059	-0.062
VAR-SV	0.316	0.088	-0.215	-0.261	0.284	0.043	-0.304	-0.403
VAR-ARSV, a_1=0.9	0.313	0.157	-0.026	-0.077	0.287	0.128	-0.066	-0.109
VAR-ARSV, a_1=0.8	0.287	0.156	-0.004	-0.062	0.267	0.135	-0.024	-0.069
VAR-ARSV, a_1=0.5	0.234	0.135	0.014	-0.035	0.219	0.124	0.006	-0.036
VAR-SVt, d=5	0.217	0.062	-0.126	-0.156	0.199	0.035	-0.216	-0.249
VAR-SVt, d=10	0.256	0.068	-0.161	-0.186	0.235	0.034	-0.280	-0.327
VAR-SVt, d=15	0.261	0.064	-0.173	-0.190	0.239	0.026	-0.294	-0.312
VAR-TVP-SV	0.358	0.069	-0.201	-0.423	0.351	0.109	-0.114	-0.272

Notes: 1. In each quarter t from 1985:Q1 through 2011:Q2, vintage t data (which end in $t-1$) are used to form forecasts for periods t through $t+7$, corresponding to horizons of 1 through 8 quarters ahead. The forecast errors are calculated using the second-available (real–time) estimates of growth and inflation and currently available measures of unemployment and the short-term interest rate as the actuals. All variables are defined in annualized percentage points.

2. The models are detailed in section 3. The notation VAR-ARSV refers to a VAR model in which log volatility follows an AR(1) process, with a tight prior on the slope coefficient a_1. The notation VAR-SVt refers to a VAR model with stochastic volatility and fat tails, in which d denotes the degrees of freedom of the Student-t distribution that is the marginal distribution of innovations to the model. The forecasts are produced with recursive estimation of the models.

3. For the baseline AR and VAR models with constant volatilities, the table reports (in the first two rows of each panel) the average values of log predictive density scores, computed with the Gaussian (quadratic) approximation given in equation (12), defined so that a higher score implies a better model. For the AR models with time-varying volatilities, the table reports the average score of each model less the average score of the AR baseline. For the VAR models with time-varying volatilities, the table reports the average score of each model less the average score of the VAR baseline. Entries greater than 0 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

Table 3. Average CRPS, U.S. Forecasts

(CRPS for AR and BVAR benchmarks, CRPS ratios in all others)

	GDP growth, 1985:Q1-2007:Q4				**GDP growth, 1985:Q1-2011:Q2**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.308	0.318	0.327	0.328	0.329	0.347	0.358	0.360
VAR	0.325	0.335	0.325	0.319	0.384	0.414	0.401	0.361
AR-GARCH	0.877	0.890	0.902	0.927	0.903	0.922	0.939	0.956
AR-SV	0.864	0.862	0.875	0.875	0.891	0.893	0.911	0.911
AR-mixture	0.867	0.862	0.924	0.982	0.906	0.902	0.964	0.989
AR-TVP-SV	0.864	0.865	0.890	0.899	0.888	0.893	0.919	0.919
VAR-GARCH	0.988	1.015	1.123	1.169	1.003	1.034	1.122	1.130
VAR-SV	0.886	0.885	0.901	0.906	0.864	0.856	0.890	0.927
VAR-ARSV, a_1=0.9	0.898	0.901	0.929	0.935	0.875	0.873	0.908	0.941
VAR-ARSV, a_1=0.8	0.921	0.920	0.949	0.955	0.899	0.898	0.928	0.955
VAR-ARSV, a_1=0.5	0.952	0.950	0.965	0.965	0.939	0.935	0.951	0.960
VAR-SVt, d=5	0.883	0.882	0.888	0.894	0.859	0.859	0.874	0.907
VAR-SVt, d=10	0.883	0.882	0.891	0.887	0.862	0.856	0.882	0.904
VAR-SVt, d=15	0.879	0.882	0.888	0.884	0.859	0.859	0.879	0.904
VAR-TVP-SV	0.938	0.961	1.086	1.188	0.935	0.915	1.045	1.125
	Inflation, 1985:Q1-2007:Q4				**Inflation, 1985:Q1-2011:Q2**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.161	0.170	0.181	0.216	0.166	0.172	0.188	0.221
VAR	0.164	0.177	0.190	0.233	0.170	0.177	0.197	0.240
AR-GARCH	0.988	0.976	0.978	0.972	0.994	0.983	0.979	0.977
AR-SV	0.845	0.876	0.851	0.894	0.873	0.890	0.867	0.900
AR-mixture	0.913	0.876	0.851	0.903	0.928	0.913	0.883	0.905
AR-TVP-SV	0.845	0.841	0.807	0.806	0.861	0.860	0.824	0.814
VAR-GARCH	1.183	1.186	1.211	1.189	1.165	1.192	1.178	1.158
VAR-SV	0.959	0.937	0.884	0.884	0.954	0.944	0.895	0.887
VAR-ARSV, a_1=0.9	0.959	0.950	0.919	0.920	0.961	0.950	0.917	0.918
VAR-ARSV, a_1=0.8	0.959	0.962	0.948	0.938	0.961	0.963	0.945	0.939
VAR-ARSV, a_1=0.5	0.979	1.000	1.000	0.991	0.980	0.988	0.989	0.987
VAR-SVt, d=5	0.959	0.956	0.919	0.938	0.954	0.957	0.923	0.939
VAR-SVt, d=10	0.959	0.956	0.908	0.924	0.954	0.950	0.912	0.926
VAR-SVt, d=15	0.945	0.950	0.908	0.920	0.948	0.950	0.912	0.922
VAR-TVP-SV	0.939	0.972	0.958	1.017	0.953	1.000	1.020	1.033

Table 3, continued. Average CRPS, U.S. Forecasts
(CRPS for AR and BVAR benchmarks, CRPS ratios in all others)

	Unemployment, 1985:Q1-2007:Q4				**Unemployment, 1985:Q1-2011:Q2**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.104	0.184	0.331	0.538	0.126	0.219	0.418	0.702
VAR	0.101	0.177	0.319	0.435	0.130	0.243	0.490	0.726
AR-GARCH	0.942	0.951	0.985	0.980	0.952	0.959	0.998	0.994
AR-SV	0.952	0.940	0.946	0.976	0.976	0.963	0.962	0.986
AR-mixture	1.019	1.038	1.163	1.422	1.016	1.055	1.153	1.406
AR-TVP-SV	1.442	1.571	1.628	1.494	1.365	1.479	1.486	1.343
VAR-GARCH	1.505	1.401	1.216	1.124	1.323	1.230	1.118	1.070
VAR-SV	0.900	0.880	0.917	1.042	0.922	0.897	0.906	1.001
VAR-ARSV, a_1=0.9	0.910	0.891	0.920	1.035	0.938	0.922	0.926	1.008
VAR-ARSV, a_1=0.8	0.930	0.914	0.933	1.030	0.961	0.942	0.944	1.010
VAR-ARSV, a_1=0.5	0.970	0.954	0.955	1.016	1.000	0.984	0.978	1.017
VAR-SVt, d=5	0.910	0.903	0.933	1.051	0.946	0.930	0.928	1.005
VAR-SVt, d=10	0.900	0.886	0.927	1.056	0.930	0.918	0.922	1.010
VAR-SVt, d=15	0.900	0.886	0.923	1.056	0.938	0.914	0.920	1.012
VAR-TVP-SV	1.040	1.045	1.113	1.430	1.038	1.025	1.004	1.185
	Interest rate, 1985:Q1-2007:Q4				**Interest rate, 1985:Q1-2011:Q2**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.288	0.498	0.775	1.162	0.283	0.497	0.800	1.245
VAR	0.276	0.459	0.718	1.059	0.277	0.465	0.748	1.127
AR-GARCH	0.747	0.839	0.885	0.916	0.756	0.839	0.894	0.929
AR-SV	0.934	0.960	1.000	1.006	0.936	0.960	1.002	1.005
AR-mixture	0.750	0.898	1.099	1.309	0.788	0.907	1.074	1.202
AR-TVP-SV	0.733	0.857	1.015	1.159	0.753	0.847	0.963	1.047
VAR-GARCH	0.986	1.013	0.997	1.033	1.000	1.019	0.999	1.013
VAR-SV	0.813	0.919	1.023	1.075	0.822	0.915	1.010	1.059
VAR-ARSV, a_1=0.9	0.825	0.915	1.004	1.060	0.829	0.910	0.992	1.041
VAR-ARSV, a_1=0.8	0.828	0.915	1.003	1.053	0.833	0.910	0.991	1.033
VAR-ARSV, a_1=0.5	0.840	0.919	1.000	1.041	0.840	0.917	0.988	1.030
VAR-SVt, d=5	0.847	0.928	1.023	1.073	0.847	0.923	1.016	1.061
VAR-SVt, d=10	0.836	0.922	1.024	1.079	0.836	0.919	1.016	1.067
VAR-SVt, d=15	0.828	0.922	1.020	1.075	0.833	0.917	1.013	1.062
VAR-TVP-SV	0.746	0.937	1.150	1.450	0.751	0.892	1.047	1.300

Notes: 1. In each quarter t from 1985:Q1 through 2011:Q2, vintage t data (which end in $t-1$) are used to form forecasts for periods t through $t+7$, corresponding to horizons of 1 through 8 quarters ahead. The forecast errors are calculated using the second-available (real–time) estimates of growth and inflation and currently available measures of unemployment and the short-term interest rate as the actuals. All variables are defined in annualized percentage points.

2. The models are detailed in section 3. The notation VAR-ARSV refers to a VAR model in which log volatility follows an AR(1) process, with a tight prior on the slope coefficient a_1. The notation VAR-SVt refers to a VAR model with stochastic volatility and fat tails, in which d denotes the degrees of freedom of the Student-t distribution that is the marginal distribution of innovations to the model. The forecasts are produced with recursive estimation of the models.

3. For the baseline AR and VAR models with constant volatilities, the table reports (in the first two rows of each panel) the average cumulative ranked probability score (CRPS), computed with formula given in equation (13), defined so that a lower CPRS implies a better model. For the AR models with time-varying volatilities, the table reports the ratio of the average CRPS of each model to the average CRPS of the AR baseline. For the VAR models with time-varying volatilities, the table reports the ratio of the average CRPS of each model to the average CRPS of the VAR baseline. Entries less than 1 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

Table 4. Real-Time Forecast RMSEs, U.K. Forecasts

(RMSEs for AR and BVAR benchmarks, RMSE ratios in all others)

	GDP growth, 1985:Q1-2007:Q4				**GDP growth, 1985:Q1-2010:Q4**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.461	0.454	0.421	0.357	0.591	0.605	0.629	0.601
VAR	0.483	0.424	0.361	0.373	0.685	0.624	0.617	0.609
AR-GARCH	1.028	1.035	1.002	0.964	1.078	1.074	1.022	1.005
AR-SV	1.011	1.095	1.074	1.022	0.992	1.073	1.051	1.043
AR-mixture	1.104	1.150	1.181	1.182	1.118	1.180	1.196	1.083
AR-TVP-SV	1.074	1.161	1.121	1.031	1.078	1.144	1.092	1.038
VAR-GARCH	1.050	1.222	1.377	1.206	0.953	1.064	1.139	1.171
VAR-SV	0.849	1.044	1.099	1.094	0.891	1.034	1.039	1.052
VAR-ARSV, a_1=0.9	0.859	1.047	1.110	1.138	0.903	1.034	1.041	1.061
VAR-ARSV, a_1=0.8	0.863	1.040	1.096	1.130	0.900	1.027	1.037	1.057
VAR-ARSV, a_1=0.5	0.884	1.026	1.074	1.125	0.906	1.021	1.031	1.050
VAR-SVt, d=5	0.865	1.028	1.085	1.132	0.893	1.021	1.034	1.065
VAR-SVt, d=10	0.861	1.037	1.088	1.122	0.891	1.027	1.036	1.061
VAR-SVt, d=15	0.853	1.035	1.094	1.114	0.890	1.027	1.036	1.058
VAR-TVP-SV	1.021	1.210	1.277	1.147	0.889	1.122	1.104	1.064
	Inflation, 1985:Q1-2007:Q4				**Inflation, 1985:Q1-2010:Q4**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	0.530	0.545	0.630	0.747	0.552	0.566	0.630	0.745
VAR	0.630	0.748	0.954	1.278	0.665	0.777	0.973	1.224
AR-GARCH	1.002	1.022	1.030	1.091	1.005	1.018	1.027	1.075
AR-SV	1.028	1.028	0.998	0.916	1.011	1.011	1.000	0.914
AR-mixture	1.064	1.015	0.979	0.759	1.072	1.000	0.986	0.774
AR-TVP-SV	0.983	0.956	0.940	0.841	0.984	0.954	0.938	0.839
VAR-GARCH	0.948	1.048	0.865	0.660	0.896	0.960	0.841	0.778
VAR-SV	0.903	0.833	0.780	0.729	0.897	0.848	0.785	0.737
VAR-ARSV, a_1=0.9	0.897	0.831	0.771	0.713	0.894	0.849	0.780	0.722
VAR-ARSV, a_1=0.8	0.911	0.857	0.804	0.758	0.909	0.876	0.817	0.765
VAR-ARSV, a_1=0.5	0.941	0.895	0.862	0.836	0.942	0.918	0.878	0.840
VAR-SVt, d=5	0.910	0.850	0.798	0.765	0.909	0.869	0.811	0.771
VAR-SVt, d=10	0.908	0.841	0.785	0.740	0.901	0.857	0.793	0.746
VAR-SVt, d=15	0.907	0.840	0.783	0.730	0.898	0.854	0.790	0.737
VAR-TVP-SV	1.041	0.941	0.895	0.765	1.029	0.983	0.991	0.830

Table 4, continued. Real-Time Forecast RMSEs, U.K. Forecasts

(RMSEs for AR and BVAR benchmarks, RMSE ratios in all others)

	Unemployment, 1985:Q1-2007:Q4				**Unemployment, 1985:Q1-2010:Q4**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.183	0.321	0.676	1.459	0.201	0.359	0.745	1.400
VAR	0.158	0.250	0.431	0.623	0.181	0.310	0.579	0.900
AR-GARCH	1.060	1.047	0.975	0.936	1.030	1.025	0.977	0.958
AR-SV	0.978	0.975	0.964	0.922	0.980	0.983	0.970	0.942
AR-mixture	1.295	1.458	1.565	1.577	1.234	1.362	1.458	1.549
AR-TVP-SV	1.033	1.037	0.956	0.805	1.025	1.022	0.970	0.869
VAR-GARCH	1.342	1.700	2.246	3.072	1.227	1.361	1.368	1.408
VAR-SV	1.000	0.980	0.979	1.127	0.983	0.971	0.978	1.052
VAR-ARSV, a_1=0.9	1.000	0.980	0.979	1.127	0.983	0.974	0.981	1.052
VAR-ARSV, a_1=0.8	1.000	0.984	0.989	1.108	0.983	0.977	0.985	1.043
VAR-ARSV, a_1=0.5	1.006	0.996	0.991	1.077	0.994	0.984	0.988	1.033
VAR-SVt, d=5	0.994	0.988	0.982	1.082	0.989	0.984	0.983	1.030
VAR-SVt, d=10	0.994	0.980	0.975	1.088	0.983	0.977	0.981	1.036
VAR-SVt, d=15	0.994	0.980	0.972	1.086	0.989	0.977	0.978	1.036
VAR-TVP-SV	1.044	1.124	1.290	1.703	1.061	1.113	1.223	1.376
	Interest rate, 1985:Q1-2007:Q4				**Interest rate, 1985:Q1-2010:Q4**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.759	1.337	2.385	3.708	0.794	1.393	2.442	3.766
VAR	0.679	1.225	2.192	3.750	0.706	1.259	2.200	3.670
AR-GARCH	0.768	0.844	0.849	0.834	0.793	0.863	0.858	0.836
AR-SV	0.839	0.835	0.770	0.713	0.854	0.851	0.790	0.737
AR-mixture	0.651	0.717	0.751	0.744	0.704	0.769	0.783	0.754
AR-TVP-SV	0.747	0.823	0.853	0.830	0.766	0.835	0.853	0.833
VAR-GARCH	0.941	1.028	0.983	0.873	1.154	0.992	0.954	1.079
VAR-SV	0.885	0.849	0.817	0.784	0.906	0.875	0.834	0.800
VAR-ARSV, a_1=0.9	0.905	0.868	0.838	0.799	0.918	0.889	0.854	0.817
VAR-ARSV, a_1=0.8	0.942	0.911	0.879	0.836	0.945	0.921	0.890	0.848
VAR-ARSV, a_1=0.5	0.984	0.957	0.932	0.895	0.977	0.956	0.933	0.898
VAR-SVt, d=5	0.934	0.898	0.864	0.828	0.945	0.914	0.875	0.836
VAR-SVt, d=10	0.936	0.898	0.857	0.810	0.946	0.913	0.869	0.821
VAR-SVt, d=15	0.937	0.898	0.856	0.806	0.948	0.913	0.867	0.818
VAR-TVP-SV	0.853	0.831	0.785	0.754	0.931	0.952	0.912	0.785

Notes: 1. See the notes to Table 1.

Table 5. Average log predictive scores, U.K. Forecasts

(Scores for AR and BVAR benchmarks, score differences in all others)

	GDP growth, 1985:Q1-2007:Q4				**GDP growth, 1985:Q1-2010:Q4**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	-0.880	-0.885	-0.912	-0.911	-1.002	-1.030	-1.092	-1.090
VAR	-0.837	-0.825	-0.823	-0.876	-1.077	-1.041	-1.076	-1.083
AR-GARCH	0.369	0.333	0.275	0.167	-0.366	-1.193	-1.996	-1.793
AR-SV	0.403	0.299	0.360	0.272	0.240	-0.242	-1.883	-2.871
AR-mixture	-0.025	0.018	0.203	0.500	-1.982	-3.174	-3.973	-2.445
AR-TVP-SV	0.229	0.227	0.328	0.261	-0.349	-1.013	-2.440	-2.001
VAR-GARCH	0.160	0.197	-0.029	-0.068	-0.384	-0.740	-1.106	-1.193
VAR-SV	0.483	0.345	0.347	0.222	0.362	-0.135	-1.187	-1.147
VAR-ARSV, a_1=0.9	0.423	0.302	0.210	0.049	0.401	0.111	-0.304	-0.199
VAR-ARSV, a_1=0.8	0.345	0.223	0.133	0.018	0.367	0.134	-0.158	-0.087
VAR-ARSV, a_1=0.5	0.203	0.105	0.036	-0.007	0.261	0.077	-0.041	-0.048
VAR-SVt, d=5	0.460	0.341	0.330	0.219	0.325	-0.086	-0.759	-0.813
VAR-SVt, d=10	0.475	0.358	0.347	0.229	0.340	-0.149	-1.082	-1.186
VAR-SVt, d=15	0.470	0.348	0.349	0.255	0.337	-0.152	-1.131	-1.076
VAR-TVP-SV	0.363	0.235	0.161	-0.074	0.375	-0.102	-0.579	-0.461
	Inflation, 1985:Q1-2007:Q4				**Inflation, 1985:Q1-2010:Q4**			
	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$	$h=1Q$	$h=2Q$	$h=4Q$	$h=8Q$
AR	-0.953	-0.991	-1.112	-1.263	-0.968	-1.003	-1.108	-1.254
VAR	-0.993	-1.146	-1.360	-1.727	-1.022	-1.170	-1.377	-1.664
AR-GARCH	0.179	0.129	0.050	-0.095	0.151	0.112	0.040	-0.077
AR-SV	0.216	0.172	0.209	0.232	0.169	0.154	0.176	0.225
AR-mixture	0.163	0.279	0.355	0.331	0.062	0.195	0.297	0.288
AR-TVP-SV	0.234	0.258	0.210	0.187	0.199	0.216	0.199	0.196
VAR-GARCH	0.242	0.298	0.307	0.210	0.178	0.160	0.368	0.282
VAR-SV	0.179	0.263	0.324	0.483	0.187	0.247	0.296	0.439
VAR-ARSV, a_1=0.9	0.149	0.220	0.285	0.446	0.154	0.207	0.265	0.407
VAR-ARSV, a_1=0.8	0.099	0.154	0.217	0.366	0.105	0.145	0.202	0.333
VAR-ARSV, a_1=0.5	0.017	0.071	0.128	0.254	0.023	0.062	0.116	0.227
VAR-SVt, d=5	0.131	0.200	0.268	0.411	0.138	0.178	0.231	0.381
VAR-SVt, d=10	0.169	0.232	0.301	0.452	0.173	0.214	0.267	0.416
VAR-SVt, d=15	0.175	0.241	0.311	0.465	0.181	0.225	0.281	0.426
VAR-TVP-SV	0.095	0.160	0.170	0.207	0.090	0.138	0.123	0.170

Table 5, continued. Average log predictive scores, U.K. Forecasts

(Scores for AR and BVAR benchmarks, score differences in all others)

	Unemployment, 1985:Q1-2007:Q4				**Unemployment, 1985:Q1-2010:Q4**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.273	-0.304	-1.061	-1.871	0.166	-0.445	-1.180	-1.717
VAR	0.403	-0.056	-0.595	-1.042	0.268	-0.290	-0.960	-1.374
AR-GARCH	-0.675	-0.971	-1.235	-1.472	-0.567	-0.787	-0.987	-1.249
AR-SV	0.015	0.029	0.035	0.093	0.027	0.023	0.019	0.048
AR-mixture	-0.299	-0.248	-0.379	-0.906	-0.301	-0.238	-0.331	-0.792
AR-TVP-SV	-0.054	-0.035	0.016	0.153	-0.047	-0.011	0.038	0.088
VAR-GARCH	-0.916	-0.669	-1.806	-2.075	-0.674	-1.327	-1.294	-1.375
VAR-SV	-0.016	0.004	-0.009	-0.088	-0.005	-0.033	-0.208	-0.360
VAR-ARSV, a_1=0.9	-0.047	-0.037	-0.050	-0.137	-0.012	-0.023	-0.134	-0.191
VAR-ARSV, a_1=0.8	-0.044	-0.038	-0.049	-0.124	-0.005	0.002	-0.047	-0.085
VAR-ARSV, a_1=0.5	-0.061	-0.064	-0.074	-0.134	-0.012	0.008	0.013	-0.024
VAR-SVt, d=5	-0.029	-0.021	-0.043	-0.114	0.001	-0.024	-0.192	-0.358
VAR-SVt, d=10	-0.012	0.001	-0.013	-0.089	0.011	-0.028	-0.236	-0.401
VAR-SVt, d=15	-0.019	-0.002	-0.013	-0.092	0.004	-0.026	-0.230	-0.417
VAR-TVP-SV	-0.036	-0.089	-0.255	-0.518	-0.051	-0.099	-0.346	-0.596
	Interest rate, 1985:Q1-2007:Q4				**Interest rate, 1985:Q1-2010:Q4**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	-1.204	-1.706	-2.349	-3.192	-1.236	-1.747	-2.346	-3.016
VAR	-1.122	-1.616	-2.265	-3.187	-1.147	-1.639	-2.224	-2.939
AR-GARCH	0.388	0.258	0.333	0.607	-0.261	-0.362	-0.107	0.239
AR-SV	0.449	0.301	0.387	0.813	-0.102	-0.570	-0.582	-0.659
AR-mixture	0.661	0.502	0.479	0.761	-0.909	-0.868	-0.234	0.366
AR-TVP-SV	0.532	0.316	0.402	0.671	-0.095	-0.293	-0.143	-0.229
VAR-GARCH	0.258	0.106	0.172	0.231	-0.795	-0.803	-0.237	0.419
VAR-SV	0.367	0.267	0.278	0.545	0.158	-0.034	-0.079	0.002
VAR-ARSV, a_1=0.9	0.246	0.176	0.271	0.735	0.178	0.068	0.126	0.497
VAR-ARSV, a_1=0.8	0.182	0.145	0.233	0.600	0.148	0.082	0.164	0.478
VAR-ARSV, a_1=0.5	0.068	0.070	0.151	0.429	0.065	0.055	0.135	0.388
VAR-SVt, d=5	0.304	0.262	0.306	0.614	0.098	-0.035	0.018	0.206
VAR-SVt, d=10	0.345	0.270	0.281	0.570	0.107	-0.070	-0.084	0.068
VAR-SVt, d=15	0.345	0.268	0.279	0.547	0.099	-0.078	-0.088	0.017
VAR-TVP-SV	0.297	0.230	0.335	0.703	0.132	0.069	0.163	0.512

Notes: 1. See the notes to Table 2.

Table 6. Average CRPS, U.K. Forecasts

(CRPS for AR and BVAR benchmarks, CRPS ratios in all others)

	GDP growth, 1985:Q1-2007:Q4				GDP growth, 1985:Q1-2010:Q4			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.278	0.279	0.279	0.265	0.326	0.332	0.347	0.337
VAR	0.281	0.262	0.253	0.264	0.356	0.333	0.337	0.339
AR-GARCH	0.853	0.864	0.853	0.857	0.969	0.988	0.965	0.973
AR-SV	0.770	0.860	0.842	0.762	0.844	0.952	0.960	0.923
AR-mixture	0.942	0.961	0.975	0.826	1.077	1.139	1.133	0.970
AR-TVP-SV	0.874	0.925	0.889	0.770	0.972	1.039	1.014	0.923
VAR-GARCH	1.078	1.164	1.364	1.163	1.025	1.210	1.412	1.301
VAR-SV	0.726	0.823	0.855	0.880	0.806	0.923	0.950	0.971
VAR-ARSV, a_1=0.9	0.740	0.853	0.887	0.931	0.808	0.929	0.962	0.986
VAR-ARSV, a_1=0.8	0.757	0.865	0.895	0.942	0.814	0.932	0.959	0.986
VAR-ARSV, a_1=0.5	0.806	0.891	0.918	0.964	0.842	0.943	0.971	0.991
VAR-SVt, d=5	0.740	0.827	0.848	0.909	0.811	0.914	0.944	0.991
VAR-SVt, d=10	0.733	0.827	0.844	0.894	0.808	0.920	0.944	0.986
VAR-SVt, d=15	0.726	0.823	0.855	0.898	0.803	0.920	0.947	0.986
VAR-TVP-SV	0.843	0.996	0.996	1.030	0.840	1.045	1.036	1.050
	Inflation, 1985:Q1-2007:Q4				**Inflation, 1985:Q1-2010:Q4**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.313	0.322	0.370	0.441	0.323	0.331	0.370	0.439
VAR	0.358	0.425	0.552	0.773	0.374	0.438	0.560	0.731
AR-GARCH	0.914	0.938	0.976	1.059	0.932	0.952	0.981	1.050
AR-SV	0.917	0.932	0.930	0.871	0.926	0.937	0.938	0.872
AR-mixture	0.955	0.882	0.816	0.669	0.978	0.894	0.846	0.695
AR-TVP-SV	0.875	0.863	0.854	0.794	0.892	0.879	0.862	0.795
VAR-GARCH	0.832	0.852	0.705	0.542	1.016	0.826	0.713	0.669
VAR-SV	0.860	0.791	0.748	0.686	0.856	0.808	0.759	0.699
VAR-ARSV, a_1=0.9	0.860	0.795	0.742	0.663	0.859	0.812	0.756	0.677
VAR-ARSV, a_1=0.8	0.887	0.828	0.780	0.716	0.883	0.846	0.793	0.727
VAR-ARSV, a_1=0.5	0.939	0.880	0.851	0.804	0.936	0.898	0.865	0.812
VAR-SVt, d=5	0.871	0.814	0.774	0.723	0.872	0.835	0.791	0.734
VAR-SVt, d=10	0.862	0.800	0.758	0.698	0.859	0.817	0.772	0.709
VAR-SVt, d=15	0.862	0.798	0.757	0.686	0.859	0.814	0.770	0.697
VAR-TVP-SV	0.953	0.864	0.824	0.745	0.955	0.902	0.896	0.792

Table 6, continued. Average CRPS, U.K. Forecasts

(CRPS for AR and BVAR benchmarks, CRPS ratios in all others)

	Unemployment, 1985:Q1-2007:Q4				Unemployment, 1985:Q1-2010:Q4			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.103	0.183	0.392	0.882	0.115	0.203	0.423	0.810
VAR	0.088	0.142	0.247	0.368	0.101	0.171	0.306	0.475
AR-GARCH	1.252	1.448	1.630	1.137	1.183	1.345	1.499	2.164
AR-SV	0.981	0.973	0.954	0.916	0.974	0.980	0.960	0.937
AR-mixture	1.291	1.317	1.347	1.364	1.226	1.261	1.307	1.378
AR-TVP-SV	1.029	1.022	0.934	0.768	1.017	1.015	0.946	0.827
VAR-GARCH	2.045	1.894	2.591	4.712	1.970	1.205	2.232	3.469
VAR-SV	1.000	0.979	0.984	1.121	0.990	0.977	0.984	1.089
VAR-ARSV, a_1=0.9	1.000	0.986	0.980	1.139	0.990	0.983	0.981	1.089
VAR-ARSV, a_1=0.8	1.011	0.993	0.988	1.118	0.990	0.977	0.981	1.069
VAR-ARSV, a_1=0.5	1.011	1.000	1.000	1.102	1.000	0.988	0.988	1.056
VAR-SVt, d=5	0.989	0.979	0.984	1.102	0.990	0.977	0.984	1.073
VAR-SVt, d=10	0.989	0.972	0.976	1.099	0.980	0.971	0.981	1.077
VAR-SVt, d=15	0.989	0.979	0.976	1.097	0.990	0.977	0.981	1.075
VAR-TVP-SV	1.045	1.106	1.263	1.704	1.059	1.105	1.252	1.482
	Interest rate, 1985:Q1-2007:Q4				**Interest rate, 1985:Q1-2010:Q4**			
	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$	$h = 1Q$	$h = 2Q$	$h = 4Q$	$h = 8Q$
AR	0.419	0.728	1.349	2.329	0.429	0.745	1.370	2.292
VAR	0.381	0.669	1.258	2.387	0.393	0.684	1.247	2.244
AR-GARCH	0.728	0.804	0.799	0.766	0.751	0.836	0.833	0.804
AR-SV	0.740	0.762	0.718	0.630	0.760	0.796	0.765	0.688
AR-mixture	0.554	0.643	0.685	0.615	0.597	0.702	0.740	0.668
AR-TVP-SV	0.649	0.740	0.775	0.735	0.669	0.770	0.812	0.774
VAR-GARCH	0.976	0.868	0.828	0.796	1.176	1.104	1.135	0.820
VAR-SV	0.775	0.791	0.773	0.721	0.810	0.834	0.813	0.765
VAR-ARSV, a_1=0.9	0.808	0.820	0.802	0.726	0.833	0.852	0.835	0.765
VAR-ARSV, a_1=0.8	0.850	0.864	0.840	0.771	0.866	0.886	0.864	0.797
VAR-ARSV, a_1=0.5	0.915	0.930	0.903	0.846	0.914	0.936	0.912	0.856
VAR-SVt, d=5	0.819	0.832	0.820	0.766	0.846	0.862	0.849	0.798
VAR-SVt, d=10	0.811	0.830	0.815	0.754	0.841	0.862	0.848	0.790
VAR-SVt, d=15	0.816	0.829	0.812	0.748	0.843	0.863	0.845	0.787
VAR-TVP-SV	0.819	0.815	0.743	0.664	0.878	0.917	0.851	0.705

Notes: 1. See the notes to Table 3.