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The risk components of liquidity

by

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# The Risk Components of Liquidity

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March 4, 2008

## Abstract

Does liquidity risk differ depending on our choice of liquidity proxy? Unlike literature that considers common liquidity variation, we focus on identifying different components of liquidity, statistically and economically, using more than a decade of US transaction data. We identify three main statistical liquidity factors which are utilized in a linear asset pricing framework. We motivate a correspondence of the statistical factors to traditional dimensions of liquidity as well as the notion of order and trade based liquidity measures. We find evidence of multiple liquidity risk premia, but only a subset of the financial liquidity factors are associated with significant risk premia. These are the factors that we relate to the dimensions of immediacy and resiliency, while the depth dimension does not command a risk premium in any of the models. Our results suggests caution when choosing liquidity variables in asset pricing applications, since liquidity premia may be reflected in only some dimensions of liquidity.

**JEL Codes:** G12; G14

**Keywords:** Liquidity Risk; Liquidity Factors; Asset Pricing; Market Microstructure

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\*The views expressed in this paper are those of the authors and do not necessarily represent those of Norges Bank (Central Bank of Norway).

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# 1 Introduction and Motivation

How should we measure liquidity? What type of liquidity risks are non-diversifiable and command a risk premium? These questions are important for individuals and institutional investors as well as regulatory authorities. There is evidence that various liquidity measures are persistent and tend to move together both across securities within the same liquidity measure and across different liquidity measures.<sup>1</sup> However, the results are mixed, and there is still an open question what dimensions of liquidity are important for asset prices.<sup>2</sup> If different liquidity measures yield different predictions about the investment climate and costs of transacting, it is crucial to distinguish which type of liquidity measures one should look at. Moreover, in view of the large number of liquidity measures examined in the literature, we require more insight into what types and dimensions of liquidity are likely candidates for capturing systematic liquidity risk. This paper adds to the growing literature on liquidity along three important dimensions: we decompose liquidity into multiple orthogonal factors, provide an economic interpretation of the different liquidity factors and evaluate whether some dimensions of liquidity are more important for expected returns than others.

More specifically, the paper first attempts to statistically identify different components of liquidity. In particular, we use a factor analysis, in conjunction with an economic interpretation of the factors. Chen (2005) and Korajczyk and Sadka (2007) use a similar methodology to identify the common variation among and across various liquidity measures. While Chen (2005) uses the largest (across measures) factor as her candidate for liquidity risk, Korajczyk and Sadka (2007) examine both the across-measure liquidity factors and within-measure factors, but use only the first-measure principal component as their risk candidate. Neither of these studies examine the composition of the additional factors or provide an economic interpretation of the factors they extract. This paper extends the factor analysis approach for measuring liquidity by also studying the additional liquidity composites extracted from the factor analysis. This is important for several reasons. First, the additional factors may contain information about liquidity risks that are not measured by the largest factor. Also, there is still no consensus liquidity measure: existing research uses more than a dozen distinct liquidity measures, all with different properties (see Aitken and Comerton-Forde (2003), Holl and Winn (1995), and Roll (2005).) Thus, by examining how different liquidity variables relate to the set of common factors, we might be able to understand better how and why different types of liquidity co-move.

The second point of the paper, is that we try to attribute an economic interpretation to the various common liquidity factors we estimate. In addition to linking these factors to the dimensions of liquidity proposed by Kyle (1985) and Harris (1990), we also explore a natural

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<sup>1</sup>See, for example, Chordia et al. (2000), Huberman and Halka (2001) and Sadka (2006).

<sup>2</sup>See e.g. Hasbrouck and Seppi (2001), Aitken and Comerton-Forde (2003), Roll (2005), Acharya and Pedersen (2005), Korajczyk and Sadka (2007). An excellent overview of the current status of research on liquidity and liquidity risk is given by Amihud et al. (2005).

distinction between trade and order based liquidity proposed by Aitken and Comerton-Forde (2003). Trade based measures reflect consummated liquidity, and include such variables as realized transaction volume and variables that are calculated based on actual transaction prices. By contrast, order based measures reflect real time availability and cost of liquidity before the transaction, and include bid-ask spreads and related variables.

The third contribution of the paper is that we examine whether the different market-wide liquidity factors contain differential information about expected returns. The main question asked is whether there are some types and dimensions of liquidity that are more important risk factors than others. For this purpose, we use a stochastic discount factor approach (see e.g. Cochrane, 2001) to test and compare different liquidity augmented asset pricing models. As shown by Aitken and Comerton-Forde (2003) different liquidity measures capture different aspects of the trading climate. Hence different liquidity measures may yield mixed signals about actual liquidity. For example, during financial crises one may observe signals of both low liquidity along one dimension (high spreads and transaction costs) and simultaneously observe high liquidity along another dimension (high trading volume). Furthermore, as shown by Sadka (2006), shocks to liquidity variables are persistent. If shocks to some liquidity measures are un-diversifiable, while shocks to other liquidity variables can be more easily diversified away or are less important to investors, it is important to determine what types of liquidity measures are priced.

## 1.1 Overview of the paper

Our paper proceeds in three basic steps. First, we pool a set of 12 distinct marketwide liquidity measures that cover different liquidity dimensions, and extract and interpret the three most significant common factors. The liquidity variables are calculated using the NYSE Trades and Automated Quotes (TAQ) dataset for the period 1993 through 2005. In the language of Korajczyk and Sadka (2007) we restrict our analysis to across-measure liquidity factors.<sup>3</sup> We interpret the three factors we obtain to be related to price-impact and transaction costs (resiliency), activity (immediacy) and depth (thickness). The factors can also be interpreted in the framework of Aitken and Comerton-Forde (2003) where one factor has high loadings on order based variables, one on trade based variables and one mixed factor with high loadings on both types of variables. The mixed factor has high loadings on transaction cost variables and price impact variables. We also estimate the factor model for sub-samples by splitting the sample period in two. The sub-sample results show a surprising stability in the estimated factor structure. Second, we provide an economic interpretation of our liquidity factors and find supporting evidence for the conjecture that different liquidity measures point in conflicting directions during important events. Third, using our liquidity factors, we perform asset pricing

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<sup>3</sup>We do not claim that our factors capture all possible aspects of liquidity. We do, however, claim that our factors seem to capture the most important aspects of liquidity, as evidenced by the robust subsample results.

tests in a stochastic discount factor framework and compare different liquidity augmented factor models in the Fama-French tradition. The main result in that part of the paper is that in a cross section of industry portfolios and stocks sorted on size and book to market, the CAPM augmented with liquidity factors prices out the SMB and HML factors. Moreover, we find evidence of multiple liquidity risk premia, SINCE the tests attach a significant premium to several, but not all, liquidity dimensions.

The remainder of the paper is organized as follows. Section 2 describes the liquidity measures used in our study and provides a description of the data with sample statistics. Section 3 explains the common factor methodology employed and documents the results of the common factor analysis. We then investigate the relationship between the common liquidity factors and asset returns in section 4. Section 5 concludes.

## 2 Data and Liquidity variables

### 2.1 Data

The analysis in this paper use mainly two datasets. For the purpose of calculating a set of market-wide liquidity variables we use the New York Stock Exchange Trades and Automated Quotes (TAQ) database. The part of the TAQ database used in this paper contains intraday data for NYSE, AMEX and NASDAQ listed stocks from February 1993 through December 2005. For the asset pricing tests in section 4, we also use portfolios and risk factors provided by Kenneth French from his homepage.<sup>4</sup>

Before we calculate the liquidity measures for the TAQ data, we filter the data. First, we restrict the analysis to only NYSE and AMEX listed securities. The main reasons for excluding NASDAQ listed securities is the difference in trading system and an issue of double counting of reported volume by NASDAQ market makers for certain periods during the sample period. We also apply various filters. First, we remove firms with less than 500 daily observations over the entire sample period, less than an average of 10 trades per day, a stock price higher than \$500 and lower than \$1. For the remaining securities we also filter away observations with negative spreads, with missing bid or ask quotes and quoted spreads relative to the midpoint price that are greater than 10%.<sup>5</sup> The filtering leaves us with a sample of 4955 NYSE and AMEX listed stocks for the period February 1993 through December 2005.

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<sup>4</sup>The 25 Fama-French portfolio returns, SIC industry portfolios, size (SMB) value (HML) factors aswell as the market return and risk free rate are downloaded from Kenneth French's homepage at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>

<sup>5</sup>We also apply various other filters on the intraday data. We purge trades out of sequence, trades recorded before the open or after the close and trades with special settlement conditions.

## 2.2 Liquidity measures

For the filtered securities, we calculate 12 liquidity proxies using the intraday TAQ data. To calculate the market-wide versions of these liquidity variables, we first calculate each measure for each firm on each date. Then we aggregate to a monthly frequency and calculate equally weighted monthly averages to obtain the market-wide versions of the variables.

The activity variables that we calculate is the monthly number of trades, monthly trading volume in shares and the average trade-size in shares. The price impact measures we calculate is Amihud's Illiquidity ratio (Amihud (2002)), intraday volatility (the absolute trade to trade return) and the average daily volatility (the average open to close return). The Illiquidity ratio is calculated as the daily absolute return divided by the dollar-volume traded on the respective day. The cost measures calculated are the average quoted spread (difference between the best ask and bid prices), the average relative spread (the quoted spread divided by the midpoint between the bid and ask quotes) and the average effective spread (measured as the difference between the trade price and the prevailing bid-ask midpoint just before the trade was executed divided by the bid-ask midpoint). The quantity measures used are the total number of quotes and the average bid and ask volume (in lots).

For simplicity, and to guide us in the interpretation of the various liquidity factors, we divide the measures into *trade based* and *order based* measures, motivated by the study of Aitken and Comerton-Forde (2003). The measures classified as trade based measures are the measures that use executed trades (transaction volume) and transaction prices in their calculations. The trade based measures are thus: total trades, trading volume, trade size, intraday- and daily volatility, relative effective spread and the Amihud Illiquidity ratio. The order based measures are measures that assess available liquidity and cost of liquidity (before the trade) at any point in time. The order based measures are: the quoted spread, relative quoted spread, total quotes and the bid- and ask- volume.

## 2.3 Descriptive statistics of the liquidity variables

The data that we use is summarized in Tables 1 and 2. Table 1 presents descriptive statistics. It is evident from this table that there is a large degree of diversity in the range and levels of the various liquidity variables across measures and over time within the same measure. For example, the median trading volume is about 700 million shares, while the median quoted spread is only 0.18. For the purpose of the factor analysis, the difference in levels across measures is handled by standardizing and normalizing the variables. There is obviously some non-stationarity in some of the variables, this is handled by detrending the respective variables. From Table 2, the main finding is that there is also a large variation in the correlation of various liquidity measures. Among trade based measures, the correlation ranges from -0.23 to 0.93. For order based measures, the range is similarly large, ranging from -0.18 to 0.93. In the table correlations

Table 1: Summary Statistics

The table presents some summary statistics for the 12 marketwide liquidity variables in our sample from February 1993 to December 2005. The statistic reflect the time-series mean, median, standard deviation (STD) and maximum and minimum value of the respective variables.

	Mean	Median	STD	Max	Min
<b>Trade based measures</b>					
Total trades (1000)	557	334	504	3670	37
Trading volume (mill. shares)	745	700	415	2687	79
Trade size (shares)	1424	1622	488	2509	507
Intraday volatility (%)	0.35	0.39	0.13	0.61	0.13
Relative eff. spread (%)	0.61	0.67	0.21	1.02	0.23
Daily volatility (%)	1.25	1.14	0.37	5.26	0.58
Illiq. ratio (%)	0.0028	0.0027	0.0008	0.0080	0.0011
<b>Order based measures</b>					
Quoted spread	0.16	0.18	0.06	0.28	0.07
Rel. quoted spread (%)	1.72	1.94	0.60	2.84	0.63
Total quotes (1000)	2138	551	2930	22171	44
Bid volume (lots)	60	56	29	127	16
Ask volume (lots)	60	52	28	124	16

greater than 0.3 in absolute value are in dark grey cells, while correlations between 0.2 and 0.3 in absolute value are in lighter grey cells. White cells indicate an absolute correlation less than 0.2. Taken together, these two tables illustrate part of the challenge of empirical work on liquidity: the various measures of liquidity are tremendously diverse, both in terms of their ranges and their correlations. While some liquidity proxies are highly correlated, others that we also think of being important for capturing market liquidity have a low correlation. This dispersion in correlations may be seen as one empirical motivation for exploring the multi-dimensionality of liquidity. In the subsequent section, we shall explore whether such diversity is also part of liquidity’s pricing behavior.

### 3 Factor decomposition

#### 3.1 Methodology and design issues

Factor analysis comprises a family of statistical techniques concerned with the reduction of a set of observable variables in terms of a small number of latent factors.<sup>6</sup> In this section, we first discuss a few details on the research design and choice of methodology. We then present the main results from the factor analysis for the whole sample period and for sub-periods.

The two main factor methods are common factors and principal components. In a *common factor* analysis it is assumed that the variance can be decomposed in two parts: common

<sup>6</sup>For a detailed discussion of factor analysis, see for example Hair et al. (1998).



Table 2: Correlation of Liquidity Measures

The table presents the correlations between the 12 liquidity variables in our sample from February 26, 1993 to December 31, 2005. We distinguish between trade-based measures and order-based measures. Correlations greater than 0.3 in absolute value are in dark grey cells, while correlations between 0.2 and 0.3 in absolute value are in lighter grey cells. White cells indicate an absolute correlation less than 0.2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
<b>Trade Based</b>											
(1) Total trades	1.00										
(2) Trading volume	0.86	1.00									
(3) Trade size	0.27	0.62	1.00								
(4) Intraday volatil.	0.23	0.34	0.26	1.00							
(5) Effective spread	0.18	0.28	0.22	0.93	1.00						
(6) Daily volatil.	0.50	0.56	0.29	0.57	0.54	1.00					
(7) Illiq.ratio (ILR)	-0.23	-0.19	-0.14	0.25	0.31	0.23	1.00				
<b>Order based</b>											
(8) Quoted spread	0.07	0.17	0.13	0.74	0.73	0.41	0.26	1.00			
(9) Relative spread	0.11	0.21	0.18	0.93	0.93	0.50	0.39	0.77	1.00		
(10) Total quotes	0.90	0.71	0.17	0.11	0.08	0.37	-0.17	-0.03	0.03	1.00	
(11) Bid volume	-0.02	0.06	0.23	0.05	0.02	-0.07	-0.15	-0.03	-0.01	-0.02	1.00
(12) Ask volume	-0.02	0.09	0.30	-0.06	-0.09	-0.13	-0.18	-0.13	-0.12	-0.04	0.72

variance that is shared by other variables in the model, and unique variance that is unique to a particular variable, including an error component. As the name suggests, a common factor analysis focus on the common variance of the observed variables. Specific variation and the error term are excluded from the analysis. In a *principal component* analysis however, no distinction is made between common and unique (idiosyncratic) variance, and the objective is to account for the maximum portion of variance present in the original set of variables with a minimum number of composite components. Thus, while both techniques are widely used for the same purpose (data reduction), they are quite different in terms of the underlying assumptions.

The focus of this paper is to identify the latent dimensions that explain why different types of liquidity variables are correlated with each other. Thus, we want to extract a small number of factors to account for the inter-correlations among our observed liquidity variables. For that purpose a common factor approach may be more suitable rather than a principal component analysis. Moreover, the scarce amount of prior knowledge we have about the composition of the variance of different liquidity measures also points towards using the common factor method. It should also be noted that in large samples, the difference between the two methods becomes small.<sup>7</sup>

A second design issue concerns how many factors to extract. While there exist several criteria

<sup>7</sup>We have estimated both common factor and principal component models for our our sample, and the results are very similar. However, we have chosen to focus on the common factor results in the paper.

or empirical guidelines for determining this number, the decision is ultimately subjective. We consider several criteria before we decide on the number of common liquidity factors. First, we use a version of the latent root criterion which says that the latent root (or eigenvalue) of the factors should exceed the average of the initial communality estimates. Second, we check that the number of factors lies close to the elbow of the scree plot.

To make the factors interpretable, factor rotation is a way to simplify the rows and columns of the factor matrix. In an orthogonal rotation, the axes are maintained at 90 degrees, while in an oblique rotation, there is no such restriction, meaning that the factors can be correlated with each other. The orthogonal methods are most widely used, although the oblique rotation is more flexible and also more realistic, since important underlying dimensions are not necessarily uncorrelated.<sup>8</sup> Since we want to use the factors as risk factors in the section 4, it is most convenient for us to work with uncorrelated factors constructed using an orthogonal factor rotation.

An important criterion for performing factor analysis in the first place, is that enough variables are sufficiently correlated. As mentioned above, we use the rule of thumb that a substantial number of the correlation coefficients should exceed 0.30. In Table 2 correlations greater than 0.30 are in dark grey cells. Visual inspection of the matrix indicates that a factor analysis is appropriate. We also see from Table 2 that there are many variables that have a low correlation with some variables and a high correlation with others. This further motivates the notion that there are different classes of liquidity measures (e.g. order- and trade based) that are more closely related with measures of the same type than with measures of the other type as proposed by Aitken and Comerton-Forde (2003).

### 3.2 Factor results

We now turn to the results of our factor analysis, which reduces our set of liquidity variables into a smaller number of latent liquidity factors. For the purpose of the factor analysis, each of the marketwide variables are detrended, standardized and normalized to have a mean zero.

Table 3 shows the results of our factor analysis based on the full sample. We find that there are three common statistical factors, which we denote Factor 1, Factor 2 and Factor 3. For simplicity, we consider a factor to have a large or dominant influence from trade (or order) based measures if the factor has significant loadings from two or more liquidity measures that are trade (or order) based. The table shows rotated factor loadings for three extracted factors as well as the final estimates of shared variance among the variables. A rule of thumb frequently used is that factor loadings greater than 0.30 in absolute value are considered significant. These loadings are marked gray in the table. We also report Kaisers Measure of Sampling Adequacy (MSA), both overall and for individual variables. In general, values of MSA greater than 0.8

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<sup>8</sup>On the other hand, the analytical procedures for performing orthogonal rotation are better developed than the procedures for oblique rotations.

are considered good, while values less than 0.5 are unacceptable. We find an overall MSA of 0.74, and all individual variables have a MSA greater than 0.5. Hence, the set of liquidity variables seems well suited for factor analysis. Furthermore, Factor 1 explains 51% of total shared variance among the variables, while Factors 2 and 3 explain 30% and 19%, respectively. The factor loadings reflect the correlations between the original liquidity variable and the latent factors. To give the reader a visual impression of how the latent factors vary over time, figure 1 plots the factor score series for each of the three factors as well as the cross sectional average volatility for each month and the VIX S&P500 index<sup>9</sup>. The correlation between the VIX index and our cross sectional volatility measure is 0.81. We see that the first factor has the closest relationship with market volatility and expected volatility (VIX). This is expected since Factor 1 has a high loading on the Illiquidity ratio and volatility in Table 3.

To assess whether we find evidence of different types and dimensions of liquidity, we examine whether the variables have a high loading on only one factor. The main result is that most variables tend to have a major loading on one factor and a smaller loading (in absolute terms) on the two other factors. This suggests that there is differential information in the different liquidity variables.

With respect to the order- and trade dimensions, the results are ambiguous. Factor 1 is a hybrid of both trade- and order-based variables. On the other hand, Factor 2 seem to be largely trade based, and Factor 3 is largely order based. This finding is also robust to the sample period used, as shown in Table 4. Consequently, we refer to Factor 1 as a hybrid factor, Factor 2 as a trade based factor, and factor 3 as an order based factor.<sup>10</sup>

An alternative way of attaching an economic interpretation to the factors is by utilizing the concepts of liquidity dimensions in Kyle (1985) and Harris (1990). Note that the three latent factors are uncorrelated as a consequence of the orthogonal factor rotation. We see that there is some relation between the factors and liquidity dimensions. Factor 1 has high loadings on mainly price impact (Illiquidity ratio, volatility) and cost measures (spread). The positive loadings means that when the market becomes more illiquid, Factor 1 increases. Factor 1 seem to be consistent with the notion of *resiliency*, since an increase in this factor reflects a larger price impact from trades. The variables that have a significant loadings on Factor 2 are mainly variables related to trading activity. Thus, this factor can be thought of as being related to *immediacy*. An increase in factor 2 reflect an increase in trading activity. Thus, when this factor is high traders are likely to be able to get their orders executed quickly. We also see that the daily volatility has a split loading across factor 1 and 2. This split loading is intuitive in the sense that one would expect volatility to have a high loading on the same factor as the

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<sup>9</sup>VIX is the ticker symbol for the Chicago Board Options Exchange (CBOE) Volatility Index. The index measure of the implied volatility of S&P 500 index options and represents a popular measure of the market's expectation of volatility over the next 30 day period.

<sup>10</sup>As mentioned before, these descriptions of the factors are a simple way to assess the dominant economic influence on our various factors, which will be helpful for analyzing our test results. While we find these descriptions appealing, alternative descriptions are possible.

Table 3: Results from the estimation of a common factor model on the liquidity measures  
The table presents the main results from a common factor model estimated over the full sample period on 12 marketwide liquidity measures. We group the measures into 7 trade based liquidity variables and 5 order based liquidity variables. MSA is Kaiser’s measure of sampling adequacy. The factors are rotated orthogonally using the Varimax method. Grey cells indicate a factor loading above 0.30.

<b>Model A</b>		<b>Shared</b>	<b>Rotated factor loadings</b>		
<b>Full sample</b>	<b>MSA</b>	<b>variance</b>	<b>Factor 1</b>	<b>Factor 2</b>	<b>Factor 3</b>
<b>Trade based measures</b>					
Total trades	0.59	0.96	0.04	0.98	-0.03
Trading volume, shares	0.66	0.91	0.17	0.92	0.17
Trade size, shares	0.51	0.32	0.18	0.38	0.37
Intraday volatility	0.82	0.93	0.94	0.20	0.07
Relative effective spread	0.90	0.91	0.94	0.14	0.03
Daily volatility	0.89	0.51	0.52	0.49	-0.07
Illiquidity ratio (ILR)	0.59	0.23	0.38	-0.22	-0.20
<b>Order based measures</b>					
Quoted spread	0.94	0.60	0.77	0.03	-0.03
Relative quoted spread	0.80	0.96	0.98	0.06	-0.00
Total quotes	0.70	0.69	-0.05	0.83	-0.06
Bid volume, lots	0.55	0.58	-0.01	-0.02	0.76
Ask volume, lots	0.58	0.84	-0.12	-0.01	0.91
Overall MSA	0.74				
Shared variance explained			4.30	2.56	1.58
Percent of total shared variance			51	30	19

illiquidity ratio since the illiquidity ratio is closely related to market volatility. A significant loading of volatility on factor 2 is also expected due to the volume-volatility relation proposed by Clark (1973) and examined in e.g. Jones et al. (1994). Factor 3 has high loadings on the depth variables (bid- and ask volume), and may be argued to be an order-based factor. This factor is related to the notion of market *tickness* as in Kyle (1985) or the *depth* dimension of Harris (1990). The factor also have a high positive loading on trade size. One interpretation of this is that in periods when the market is deep (large volume at the quotes) traders also trade larger sizes since the market can absorb greater volumes without moving the price. In line with this interpretation, we also see that there is a negative, but small, factor loading of -0.2 on the illiquidity ratio for factor 3. Thus, when factor 3 is high (the market is deep) and more volume is needed to move the price.

To examine the robustness of the whole sample factor structure, we also split the sample into two sub-periods, and estimate the factor structure for each sub-sample separately. The results from this analysis is shown in Table 4. From the table we see that there is a remarkable stable factor structure among the variables. The overall MSA of 0.70 for the first period and 0.74 for the second period suggest that both the sub-sample factor models capture a large fraction of the common variation in the liquidity variables. We also see that the factor loadings are very similar across the two periods both with respect to the sign and size. The only major difference between the two periods is that the illiquidity ratio has an absolute loading greater than 0.3 in all three factors for the first sub-period. The loading on the illiquidity ratio for the second sub-sample and for the whole sample results in Table 3 is much smaller, although the illiquidity ratio still have a negative loading on factor 2 and 3. Thus, in the first sub-period the significant negative loading on the illiquidity ratio in factor 2 and 3 is intuitive in the sense that it reflects that the illiquidity ratio is low when liquidity is high along the other two dimensions (trading activity and depth).

## 4 Asset Pricing Models

We now estimate factor models in the style of Fama and French (1993).<sup>11</sup> This approach enables us to ask whether the cross section of stock returns can be explained by one or more of our liquidity factors in *addition* to the CAPM beta and other standard risk factors (SMB and HML). We use the score series from our three orthogonal statistical factors in the previous section as our liquidity risk factors. This is similar in spirit to the approach of Pastor and Stambaugh (2003).<sup>12</sup> We then conduct standard asset pricing tests, first by using a set of 48 value weighted

<sup>11</sup>In this context, the term 'factor' refers to a financial risk factor, which is not the same as the *statistical* risk factors that we have discussed thus far.

<sup>12</sup>Since our factors are nontraded, the value of the premia are not easy to interpret, although the significance is similar qualitatively to what it would be in the case of traded factors. Since our focus is on pricing significance for our factors, we adopt this approach.

Figure 1: Time series plots of monthly factor scores

The figures shows the factor score series for each of the latent liquidity factors (left axis) two measures of volatility; the cross sectional average realized volatility for each month and the VIX S&P-500 index.

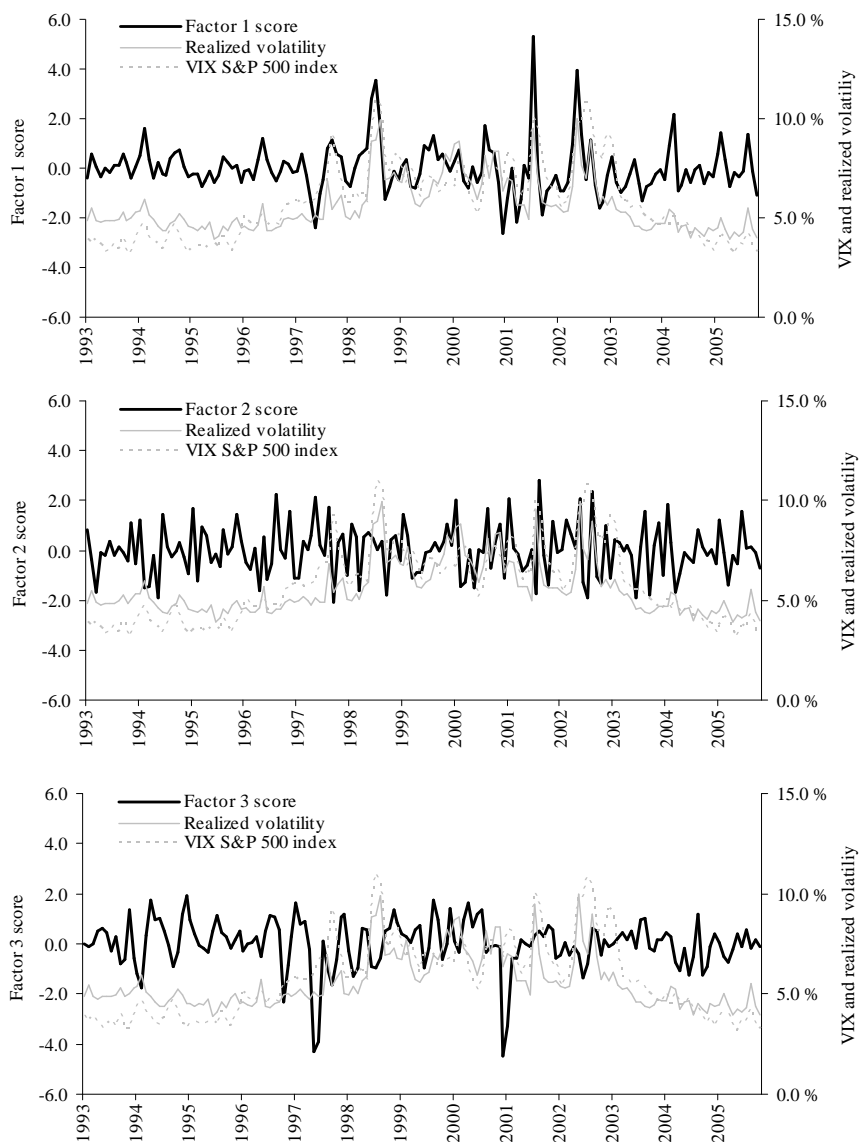


Table 4: Robustness of the factor analysis - sub period estimation

The table presents the main results from estimating the common factor model in Table 3 over two sub-samples. The liquidity variables include 7 trade based liquidity variables and 5 order based liquidity variables. MSA is Kaiser's measure of sampling adequacy. The factors are rotated orthogonally using the Varimax method. Grey cells indicate a factor loading above 0.30.

<b>Panel (a): February 1993 to June 1999</b>					
	MSA	Shared variance	Rotated factor loadings		
			Factor 1	Factor 2	Factor 3
<b>Trade based measures</b>					
Total trades	0.56	0.95	0.02	0.97	-0.04
Trading volume, shares	0.61	0.95	0.08	0.95	0.17
Trade size, shares	0.36	0.29	0.05	0.35	0.40
Intraday volatility	0.75	0.91	0.94	0.13	0.02
Relative effective spread	0.90	0.86	0.92	0.10	0.00
Daily volatility	0.90	0.50	0.46	0.49	-0.20
Illiquidity ratio (ILR)	0.65	0.34	0.38	-0.32	-0.31
<b>Order based measures</b>					
Quoted spread	0.95	0.42	0.62	0.01	-0.19
Relative quoted spread	0.77	0.95	0.97	0.02	-0.08
Total quotes	0.74	0.76	0.05	0.87	-0.05
Bid volume, lots	0.63	0.70	-0.14	-0.07	0.82
Ask volume, lots	0.63	0.89	-0.12	-0.04	0.94
Overall MSA	0.70		3.93	2.91	1.66
Shared variance explained			46	34	20
Percent of total shared variance					
<b>Panel (b): July 1999 to December 2005</b>					
	MSA	Shared variance	Rotated factor loadings		
			Factor 1	Factor 2	Factor 3
<b>Trade based measures</b>					
Total trades	0.61	0.97	0.05	0.99	-0.02
Trading volume, shares	0.71	0.89	0.23	0.90	0.15
Trade size, shares	0.65	0.33	0.24	0.42	0.32
Intraday volatility	0.86	0.94	0.93	0.25	0.09
Relative effective spread	0.83	0.94	0.95	0.18	0.04
Daily volatility	0.87	0.54	0.55	0.49	0.03
Illiquidity ratio (ILR)	0.53	0.15	0.36	-0.12	-0.09
<b>Order based measures</b>					
Quoted spread	0.91	0.70	0.83	0.05	0.03
Relative quoted spread	0.78	0.95	0.97	0.10	0.04
Total quotes	0.60	0.63	-0.11	0.79	-0.05
Bid volume, lots	0.53	0.50	0.05	0.03	0.71
Ask volume, lots	0.52	0.85	-0.12	0.02	0.91
Overall MSA	0.74				
Shared variance explained			4.65	2.33	1.42
Percent of total shared variance			55	28	17

industry portfolios as our test assets, and second using the 25 value weighted Fama-French portfolios, constructed based on size and book-to-market, as our test assets. The frequency is monthly, and the sample period is from March 1993 to December 2005.

To estimate and evaluate the various linear multifactor models we use the stochastic discount factor approach (see e.g. Cochrane (2001)). To estimate the models we use the Generalized Method of Moments (GMM) estimator. The stochastic discount factor approach utilizes that if the law of one price holds, there exists a random variable that prices which prices all stocks. This variable is called the stochastic discount factor (SDF), and we denote it by  $m_t$ . For any *excess return*,  $r_{i,t}$ , there exist a stochastic discount factor that makes the following expression hold,

$$E[r_{i,t}m_t] = 0 \quad (1)$$

In a multifactor framework, the stochastic discount factor can be expressed as,

$$m_t = 1 - b'f_t \quad (2)$$

where the constant term is normalized to 1 since we look at excess returns, and  $f_t$  are the risk factors with loadings  $b$ . Since in all our model tests we have more test assets than risk factors, we have an over-identified system. The set of sample moment conditions are given by,

$$w_T = r_T - m_T b \quad (3)$$

where  $r_T$  is the vector of test portfolios sample mean excess returns,  $r_T = (1/T) \sum_{t=1}^T r_t$  and  $m_T$  is the vector  $m_T = (1/T) \sum_{t=1}^T r_T f_t'$ . For a given weighting matrix  $\Omega$ , the estimates of  $b$  (the factor loadings) are those that minimize  $J(b)$ ,

$$J(b) = w_T' \Omega^{-1} w_T \quad (4)$$

which gives the set of factor loadings ( $b_T$ ),

$$b_T = (m_T' \Omega^{-1} m_T)^{-1} m_T' \Omega^{-1} r_T \quad (5)$$

In this study we use the optimal weighting matrix proposed by Hansen (1982) where we start out with the identity matrix for  $\Omega$  and iterate until there is only a marginal change in the objective function. The risk premiums are calculated as  $E[ff']b$ . Obviously, the size of the risk premiums are only economically meaningful when the factors are returns. Note that our liquidity factors are not returns. However, the significance and relative size of the estimated risk premiums for the liquidity factors are still meaningful.



Table 5: Factor correlations and descriptive statistics

The table shows the correlations between the three liquidity factors and the value weighted market return ( $r_m^{vw}$ ), the size (SMB) factor and the value (HML) factor. Correlations greater than 0.30 in absolute value are in bold.

	Factor 1	Factor 2	Factor 3	$r_m^{vw}$	SMB	HML
<i>(a) Correlations:</i>						
Factor 2	-0.02					
Factor 3	0.02	-0.01				
$r_m^{vw}$	-0.24	0.10	0.25			
SMB	<b>-0.34</b>	-0.04	0.05	0.20		
HML	-0.09	-0.15	-0.28	<b>-0.53</b>	<b>-0.50</b>	
<i>(b) Descriptive statistics:</i>						
Min.	-2.63	-2.06	-4.47	-16.20 %	-16.70 %	-12.80 %
Mean	0.00	0.00	0.00	0.62 %	0.23 %	0.39 %
Median	-0.12	0.00	0.04	1.33 %	-0.06 %	0.41 %
Max.	5.33	2.82	1.93	8.18 %	22.18 %	13.80 %
Std.dev.	1.01	1.02	0.99	4.27 %	3.97 %	3.61 %

### How does liquidity compare to standard suspects?

Before we present the results from the asset pricing tests, it is useful to examine the correlation between the risk factor candidates. Table 5 shows the correlations between all the variables used in the different asset pricing tests. The table also shows some descriptive statistics for each variable. We see that the three latent liquidity factors are uncorrelated. This is due to the orthogonal factor rotation. Furthermore, we see that Factor 1 has a relatively large negative correlation with the size (SMB) factor of -0.34. Factor 2 is not correlated with any of the other variables, while factor 3 is positively correlated with the market and negatively correlated with the value (HML) factor. Even though the Fama-French factors are constructed in a fashion that tries to reduce their correlation we see that they have a correlation of -0.50 with each other.<sup>13</sup> However, since we are *not* evaluating the Fama-French model, the important point is that our liquidity factors have a modest correlation with all other variables and with each other.

**Industry portfolios as test assets** We start by estimating different models trying to minimize pricing errors of a set of 48 industry portfolios as our test assets. The estimation period is from February 1993 though December 2005, and the frequency is monthly. The industry

<sup>13</sup>The Fama-French factors are constructed using 6 value-weight portfolios formed on size and book-to-market. SMB is the average return on the three small portfolios minus the average return on the three big portfolios. HML is the average return on the two value portfolios minus the average return on the two growth portfolios.

portfolios are constructed in the end of June every year based on the securities most recent four digit SIC (Security Industry Code) number.<sup>14</sup> A benefit of examining this set of test assets is that they are constructed based on a characteristic exogenous to the market characteristics (e.g. the Fama-French factors).

The estimated risk premia and model tests are summarized in Table 6. We estimate 10 different model specifications: a standard one-factor CAPM model, the Fama-French model, three models which are essentially CAPM models augmented with each of our three liquidity factor separately, two models with combinations of two of our liquidity variables, one model that is a CAPM augmented with all three liquidity variables and a final model where we augment the Fama-French model with all of the three liquidity factors.

The first thing to note in Table 6 is that there is a significant, and positive, risk premium associated with holding market risk in all models. When estimating the Fama-French model, we also get a significant positive risk premium for the SMB factor, and a negative premium for the HML factor. With respect to our liquidity factors, the result for Factor 1 is strong and stable across all models. We see from the table that Factor 1 receives a significant negative risk premium regardless of model specification, while Factor 2 and 3 do not receive a significant premium in any of the estimated models. This result is striking as there does not seem to be any role for Factor 2 when we try to price the industry portfolios. The interpretation of the sign of the Factor 1 risk premium is that investors value assets that give a high payoff in illiquid states when their marginal utility of wealth is high and it is costly to liquidate assets. Note however, that since the factors are not traded, we cannot interpret the premium as an expected return per unit risk. However, we can interpret the sign for the significant premiums. Also, Factor 1 is similar to the liquidity measure used in Acharya and Pedersen (2005) since it has a high loading on the illiquidity ratio. Thus, the negative loading on this factor is consistent with the results in their paper. Interpreting Factor 1 as proxying for resiliency, this result suggests that price impact and transaction costs are the important liquidity risks for investors, while trading activity (immediacy) and depth (market thickness) is less so. One reason for this finding may be that once we take into account the effect of Factor 1, there is no cross-sectional variation left to be explained by Factor 2 and 3. On the other hand, looking at Model 2 and Model 3 where we only include Factor 1 and Factor 2 in the models, these factors does not receive a significant risk premium.

We now turn to the model tests. The HJ distance of Hansen and Jagannathan (1997) measures the maximum annualized pricing error for each model. The Wald test assesses whether the coefficients on all the factors are equal to zero. The delta-J test of Newey and West (1987) examines whether SMB and HML have additional ability to explain asset prices, relative to each

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<sup>14</sup>Kenneth French describes the construction of these portfolios in the following way: “We assign each NYSE, AMEX, and NASDAQ stock to an industry portfolio at the end of June of year  $t$  based on its four-digit SIC code at that time. (We use Compustat SIC codes for the fiscal year ending in calendar year  $t-1$ . Whenever Compustat SIC codes are not available, we use CRSP SIC codes for June of year  $t$ .)”

Table 6: Risk premia estimates - 48 industry portfolios

The table presents the estimated risk premia (with the associated t-value in square brackets below each estimate) using our statistical liquidity factors as risk factors. The test assets are 48 industry portfolios based on the four-digit SIC code at June every year. For each model, the asset pricing tests are reported in the last four columns, with the associated p-values in round brackets below each test statistic. These are the HJ-distance metric of Hansen and Jagannathan (1997), the Wald-p test of the factor coefficients being zero in the linear pricing kernel. The delta-J test of Newey and West (1987) is reported, which tests whether including the Fama-French factors improves the model.

	Estimated risk premia						Tests		
	$r_m^{vw}$	SMB	HML	F1	F2	F3	HJ-dist.	Wald-p	delta J
CAPM	<b>0.021</b> [6.99]						0.610 (0.171)	0.000 (0.000)	12.751 (0.002)
FF	<b>0.022</b> [7.49]	<b>0.005</b> [2.35]	<b>-0.005</b> [-2.42]				0.595 (0.190)	0.000 (0.000)	- -
Model 1	<b>0.024</b> [8.02]			<b>-0.355</b> [-2.91]			0.610 (0.143)	0.000 (0.000)	3.841 (0.147)
Model 2	<b>0.019</b> [6.49]				-0.086 [-0.96]		0.609 (0.159)	0.000 (0.000)	3.128 (0.209)
Model 3	<b>0.022</b> [6.99]					0.206 [1.77]	0.610 (0.142)	0.000 (0.000)	8.990 (0.011)
Model 4	<b>0.023</b> [7.58]			<b>-0.325</b> [-2.70]	0.014 [0.16]		0.609 (0.133)	0.000 (0.000)	2.168 (0.338)
Model 5	<b>0.024</b> [7.92]			<b>-0.356</b> [-2.85]		0.167 [1.43]	0.610 (0.118)	0.000 (0.000)	5.405 (0.067)
Model 6	<b>0.019</b> [6.03]				-0.116 [-1.24]	0.188 [1.58]	0.609 (0.131)	0.000 (0.000)	16.733 (0.000)
Model 7	<b>0.023</b> [7.39]			<b>-0.323</b> [-2.61]	0.014 [0.16]	0.159 [1.34]	0.609 (0.108)	0.000 (0.000)	3.194 (0.203)
All variables	<b>0.023</b> [7.47]	<b>0.006</b> [2.54]	<b>-0.007</b> [-2.96]	<b>-0.302</b> [-2.50]	-0.023 [-0.24]	0.178 [1.45]	0.590 (0.168)	0.000 (0.000)	- -

alternative model. In all cases, a low p-value indicates evidence against the model.<sup>15</sup> When we look at the model tests in the four last columns of Table 6, we see that whenever Factor 1 is included in the model we cannot reject the null hypothesis (delta-J test) that the Fama-French factors do not improve the pricing of the test assets. This is interesting especially since both the SMB and HML factors have significant premia. In all other cases, when we exclude Factor 1, the delta-J test rejects the null in favor of including the Fama-French factors. This makes a strong case for Factor 1 as an additional risk factor. When looking at the HJ-distance tests, we also see that none of the models are rejected. This is probably a result of a very significant risk premium associated with the market risk across all models.

<sup>15</sup>These tests are standard in asset pricing literature. For more details on these tests, see Cochrane (2001).

Table 7: Average sample returns for the test assets

The table shows average monthly returns for the 25 Fama-French portfolios that we use as test assets in the asset pricing tests for our sample period from February 1993 through December 2005. The 25 portfolios are constructed based on securities rank along two dimensions; size and B/M. The portfolios are reconstructed July every year and kept fixed through June the next year. The last column shows the average return across all sizes for the respective B/M categories, and the last row shows the average return across all B/M categories for each size group.

	Small firms	2	3	4	Big firms	Mean ret. B/M groups
Low B/M	0.51 %	0.83 %	0.80 %	1.07 %	0.90 %	0.82 %
2	1.43 %	1.06 %	1.08 %	1.20 %	1.08 %	1.17 %
3	1.54 %	1.32 %	1.18 %	1.32 %	1.06 %	1.28 %
4	1.75 %	1.35 %	1.13 %	1.30 %	1.02 %	1.31 %
High B/M	1.75 %	1.34 %	1.49 %	1.14 %	0.89 %	1.32 %
Mean ret. size groups	1.39 %	1.18 %	1.14 %	1.21 %	0.99 %	

**Fama-French portfolios as test assets** It is useful to examine whether the different models are able to price an alternative set of test assets. For this purpose, we have chosen the 25 Fama-French portfolios. The Fama-French portfolios are constructed in July every year based on firms size and B/M value ranking. Table 7 shows the average monthly returns for these portfolios during our sample period from February 1993 through December 2005. We see that the portfolios containing the smallest firms with the highest B/M values have had the highest realized returns over the period. To see how the portfolios relate to our liquidity factors, Table 8 shows the exposures for each portfolio to our three liquidity factors. These exposures are estimated by simply regressing the respective portfolio return on the the respective marketwide liquidity factor with a constant term. Numbers in bold denote significant exposures at the 1% level. Interestingly, all the portfolios, except one, have a significant negative exposure to liquidity Factor 1, while none of the portfolios have a significant exposure to Factor 2. For Factor 3 we find a positive and significant exposure for all the portfolios (regardless of size) containing firms with the lowest B/M value. The negative exposures to Factor 1 means that when the overall market becomes more illiquid (recall that an increase in Factor 1 indicate lower liquidity), the return is low. Note that this exposure is similar to the third beta in the model of Acharya and Pedersen (2005) that captures that a premium related to the risk of holding assets that give a low payoff when the overall market becomes illiquid.

We now compare the pricing performance of our liquidity factors to that of other factors using the 25 Fama-French portfolios constructed based on size and B/M-value as our test assets<sup>16</sup>. Table 9 shows estimated risk premia for the same set of model specifications that we estimated

<sup>16</sup>A detailed description of the construction of the Fama-French portfolios is provided on Kenneth French's homepage. It is also described in detail in Fama and French (1993).

Table 8: Liquidity exposures for the 25 size and B/M portfolios

The table shows the exposures for the 25 Fama-French portfolios. The exposures are estimated by regressing the respective portfolio return on the the respective liquidity factor with a constant term. Exposure estimates in bold are significant at the 1% level.

	Small firms	2	3	4	Big firms
<i>(a) Factor 1 exposures</i>					
Low B/M	<b>-3.00</b>	<b>-2.42</b>	<b>-1.85</b>	<b>-1.66</b>	-0.38
2	<b>-2.93</b>	<b>-2.26</b>	<b>-1.70</b>	<b>-1.62</b>	<b>-1.14</b>
3	<b>-2.36</b>	<b>-2.01</b>	<b>-1.66</b>	<b>-1.49</b>	<b>-1.33</b>
4	<b>-2.51</b>	<b>-2.28</b>	<b>-1.78</b>	<b>-1.61</b>	<b>-0.97</b>
High B/M	<b>-2.78</b>	<b>-2.76</b>	<b>-2.14</b>	<b>-1.45</b>	<b>-0.98</b>
<i>(b) Factor 2 exposures</i>					
Low B/M	0.34	0.34	0.38	0.38	0.65
2	0.25	0.03	0.11	0.33	0.35
3	0.26	-0.10	-0.07	0.18	0.40
4	0.14	-0.10	-0.26	-0.11	0.14
High B/M	0.08	-0.25	0.14	-0.02	-0.30
<i>(c) Factor 3 exposures</i>					
Low B/M	<b>1.91</b>	<b>1.60</b>	<b>1.76</b>	<b>1.67</b>	<b>1.27</b>
2	<b>1.31</b>	0.64	0.66	0.49	<b>0.76</b>
3	0.84	0.47	0.37	0.39	0.60
4	0.76	0.43	0.33	0.49	0.29
High B/M	0.59	0.36	0.32	0.14	0.04

Table 9: Risk premia estimates - 25 size and B/M portfolios

The table presents the estimated risk premia (with the associated t-value in square brackets below each estimate) using our statistical liquidity factors as risk factors. For each model, the asset pricing tests are reported in the last four columns, with the associated p-values in round brackets below each test statistic. These are the HJ-distance metric of Hansen and Jagannathan (1997), the Wald-p test of the factor coefficients being zero in the linear pricing kernel. The delta-J test of Newey and West (1987) is reported, which tests whether including the Fama-French factors improves the model.

	Estimated risk premia					Tests			
	$r_m^{vw}$	SMB	HML	F1	F2	F3	HJ-dist.	Wald-p	delta J
CAPM	<b>0.007</b> [ <b>2.09</b> ]						0.780 (0.000)	0.000 (0.037)	6.659 (0.036)
FF	0.007 [1.80]	-0.002 [-0.73]	0.003 [1.11]				0.733 (0.000)	0.000 (0.034)	- -
Model 1	<b>0.009</b> [ <b>2.29</b> ]			-0.115 [-0.87]			0.726 (0.001)	0.000 (0.001)	10.430 (0.005)
Model 2	0.002 [0.80]				<b>0.772</b> [ <b>4.89</b> ]		0.76 (0.000)	0.000 (0.000)	30.603 (0.000)
Model 3	0.006 [1.55]					0.402 [1.80]	0.779 (0.000)	0.000 (0.084)	6.495 (0.039)
Model 4	0.005 [1.19]			<b>-0.763</b> [ <b>-4.99</b> ]	<b>0.736</b> [ <b>4.68</b> ]		0.683 (0.019)	0.000 (0.000)	9.319 (0.010)
Model 5	0.006 [1.54]			0.002 [0.02]		0.403 [1.80]	0.725 (0.001)	0.000 (0.179)	6.365 (0.041)
Model 6	0.005 [1.47]				<b>0.336</b> [ <b>2.72</b> ]	0.395 [1.84]	0.754 (0.001)	0.000 (0.008)	10.814 (0.004)
Model 7	0.005 [1.09]			<b>-0.747</b> [ <b>-4.84</b> ]	<b>0.730</b> [ <b>4.65</b> ]	-0.099 [-0.38]	0.683 (0.016)	0.000 (0.000)	8.424 (0.015)
All variables	0.006 [1.27]	0.000 [0.05]	<b>0.008</b> [ <b>2.38</b> ]	<b>-0.692</b> [ <b>-3.47</b> ]	<b>1.032</b> [ <b>5.40</b> ]	0.104 [0.32]	0.656 (0.013)	0.000 (0.000)	- -

for the industry portfolios.

The most significant finding, relative to the previous case of pricing the industry portfolios, is that there are evidence of both Factor 1 and 2 being priced. However, we do not find such a strong and stable risk premium associated with Factor 1 as before, although it still has a negative sign across all models. Even though Factor 1 and 2 are uncorrelated, there seems to be a relationship between the factors in the sense that the risk premia for Factor 1 is not significant unless we also include Factor 2 in the model. On the other hand, Factor 2 receives a positive risk premium regardless of whether Factor 1 is included or not. A surprising difference from the result for the industry portfolios is that neither the market nor the Fama-French factors receives a significant risk premia in most models. So what does the sign of the premiums for Factor 1 and Factor 2 mean? If we look at model 7, we see that Factor 1 has a significant negative premium and Factor 2 has a significant positive premium. Since Factor 1 has a negative sign, it has the

same interpretation as before. Recall that Factor 1 is high when liquidity is low (price impact and spread is high). Thus, the risk premium reflects that assets that have a high payoff in an illiquid market are particularly valuable to investors, such that a negative premium reflect that investors are willing to accept a lower return on assets that give a high payoff when the market as a whole becomes illiquid. Since the stochastic discount factor measures investors' marginal utility of wealth, it is exactly in illiquid states that investors value wealth the most and it is more costly to liquidate assets. Thus, assets that provide a hedge against wealth fluctuations are valuable. Similarly, the positive premium attached to Factor 2 reflects that investors require a premium to hold assets that offer a high payoff when activity (immediacy) is high and it is easy to liquidate assets and offers a low payoff in states when trading is slower and immediacy is potentially lower. Factor 3 does not have a significant premium in any of the models. However, if we look across models, there might be a positive premium attached to that factor, at least at the 10% significance level. Similarly as for Factor 2, investors require a premium for holding stocks that yield a high payoff when the market is deep (liquid) and consequently a low payoff when the market is thin.

With respect to the asset pricing tests we see that in all cases except for the comprehensive liquidity model (Model 7), the HJ distance has minute p-values, providing evidence against the null hypothesis of correct pricing. Therefore, the only model that could plausibly price assets correctly at the 1% level is Model 7, which includes all three liquidity factors. The delta J test has p-values beneath 0.01 for Models 1, 2, 4 and 6, indicating evidence against the null hypothesis of correct pricing without SMB and HML. This indicates that SMB and HML potentially add information, for models with Factor 2. The Wald test has small p-values only in Models 1 and 4, the models including Factor 1 and/or Factor 2. Therefore we can say that only in models with these factors included is there evidence that the coefficients are nonzero.<sup>17</sup>

To summarize the results of our asset pricing tests when we try to price the 25 Fama-French portfolios, we find it interesting that only the trade based factor, Factor 2 can always price returns (alone or combined with other factors), while Factor 1 only receive a premium when Factor 2 is also included in the model. Since the factors are orthogonal by construction, there seems to be important information in Factor 2 that, once controlled for, gives rise to a significant premium for Factor 1. Moreover, despite the fact that neither the SMB or HML factor receives a significant risk premium, the Fama-French factors seem to add information, even in the presence of a model of the CAPM augmented with all three liquidity factors.

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<sup>17</sup>We also estimated SupLM tests of parameter stability of Andrews (1993). We do not report these since in all models the parameters were always stable at 10% to 1% levels. Results are available from the authors on request.

## 5 Conclusion

This paper addresses the question of whether the effect of liquidity risk on asset prices is sensitive to our choice of liquidity proxies. There is currently little research on how best to define liquidity. This makes it difficult to examine asset pricing implications, since the observed correlations between many liquidity measures used in the literature have very different correlations with each other, especially across different types of liquidity variables. With this background in mind, our paper proceeds to address our leading question, in three steps.

First, we construct a set of 12 diverse liquidity measures. We utilize more than a decade of high frequency data from the NYSE Trades and Automated Quotes (TAQ) database in order to build both trade and order based liquidity measures. Second, we apply a factor decomposition on this set of liquidity variables and, unlike previous studies, offer an economic interpretation of the three liquidity factors we extract. The main result is that the three latent liquidity factors may be associated with the concept of different liquidity dimensions. Third, we estimate and test various asset pricing models both using 48 industry portfolios as well as the 25 Fama-French portfolios as test assets. When estimating the asset pricing models on the set of 48 industry portfolios, we find that only liquidity Factor 1 has a significant risk premium. This risk premium is very strong and stable in the sense that in all of the models where we augment the CAPM with liquidity Factor 1, the delta-J test suggest that there is no additional gain by including the Fama-French factors in the model. It should also be noted that both the market and the Fama-French factors have significant risk premia and are pricing the industry portfolios. With this in mind, the result that liquidity factor 1 “prices” out the Fama-French factors is very interesting.

When trying to price the 25 Fama-French portfolios the results are a bit weaker with respect to a risk premium for Factor 1. In this case we document that only the liquidity factor that consistently receives a risk premium is Factor 2, which reflects trading activity (immediacy). However, we also find evidence in this set of test assets for a negative risk premium associated with the first liquidity factor. Thus, the factor with loadings on price impact and implicit transaction costs is still an important risk factor. Overall, the signs for the risk premiums for Factor 1 and 2 are reasonable in the sense that a negative risk premium for the first factor reflects that investors value securities that payoff well in states when the market is illiquid and the liquidation of assets is costly. Similarly for Factor 2, the positive premium reflect that investors require a premium for holding stocks that yield a low payoff when the general market activity (and immediacy) is low.

In summary, to the best of our knowledge, we are the first to construct and systematically interpret a comprehensive set of liquidity factors. Using the natural distinction of liquidity dimensions and a natural dichotomy between trade and order based factors, we document that the effect of liquidity on asset prices may be sensitive to the choice of liquidity proxy. We



discover evidence that a factor which can be interpreted as capturing resiliency and transaction costs is the most important candidate for liquidity risk. However, we also find that trading activity is important when pricing portfolios constructed on size and B/M. In that setting we document some support for multiple liquidity risk premia. Interestingly, in both sets of asset pricing tests we do not find any premium associated with depth or market tickness.

Thus, our results suggest a potentially complex aspect to liquidity based asset pricing, due to diversity in both sign and significance of liquidity premia. Uncovering this complexity may be an important task for future research.

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