

ANO 2007 / 14

Oslo

December 22, 2007

Working Paper

Research Department

Robust-satisficing monetary policy under parameter uncertainty

by

Q. Farooq Akram, Yakov Ben-Haim and Øyvind Eitrheim

Working papers fra Norges Bank kan bestilles over e-post:
tjenestetorget@norges-bank.no
eller ved henvendelse til:
Norges Bank, Abonnementservice
Postboks 1179 Sentrum
0107 Oslo
Telefon 22 31 63 83, Telefaks 22 41 31 05

Fra 1999 og senere er publikasjonene tilgjengelige som pdf-filer
på www.norges-bank.no, under "Publikasjoner".

Working papers inneholder forskningsarbeider
og utredninger som vanligvis
ikke har fått sin endelige form.
Hensikten er blant annet at forfatteren
kan motta kommentarer fra kolleger
og andre interesserte.

Synspunkter og konklusjoner i arbeidene
står for forfatterens regning.

*Working papers from Norges Bank can be ordered by e-mail:
tjenestetorget@norges-bank.no
or from Norges Bank, Subscription service,
P.O.Box. 1179 Sentrum
N-0107Oslo, Norway.
Tel. +47 22 31 63 83, Fax. +47 22 41 31 05*

*Working papers from 1999 onwards are available as pdf-files on the bank's
web site: www.norges-bank.no, under "Publications".*

*Norges Bank's working papers present
research projects and reports
(not usually in their final form)
and are intended inter alia to enable
the author to benefit from the comments
of colleagues and other interested parties.*

*Views and conclusions expressed in working papers are
the responsibility of the authors alone.*

ISSN 0801-2504 (printed) 1502-8143 (online)

ISBN 978-82-7553-415-4 (printed), 978-82-7553-416-1 (online)

Robust-satisficing monetary policy under parameter uncertainty*

Q. Farooq Akram,[†] Yakov Ben-Haim[‡] and Øyvind Eitrheim[§]

December 22, 2007

Abstract

We employ the robust-satisficing approach to derive robust monetary policy when parameters of a macro model are uncertain. There is a trade-off between robustness of policies and their performance. Hence, under uncertainty, the policy maker is assumed to be content with policy performance at some satisfactory level rather than a level thought to be optimal based on available information. Our empirical analysis illustrates key properties of robust-satisficing policies and compares them with min-max policies implied by the robust-control approach. Intuitively, our empirical results suggest that higher robustness can be achieved by overstating challenges to the economy and understating the abilities to meet them. How much to overstate the challenges or understate the abilities depends on the robustness sought. Robustness is achieved by lowering one's aspirations regarding the performance of policies and is therefore costly. Moreover, costs of robustness increase with the level of robustness, making robustness to apparently extreme parameter values particularly costly. We also find that robust-satisficing policies are generally less aggressive than min-max policies.

Keywords: Robust monetary policy, Knightian uncertainty, parameter uncertainty, info-gap decision theory.

JEL Codes: D81, E52, E58.

*The views expressed in this paper are those of the authors and do not necessarily represent those of Norges Bank (the Central Bank of Norway). We have received useful comments from participants at the International Conference on Computing in Economics and Finance 2007, the European Economic Association meeting 2007 and seminar participants at Norges Bank.

[†]Corresponding author: farooq.akram@norges-bank.no. Research Department, Norges Bank, Box 1179 Sentrum, N-0107 Oslo, Norway. Phone: +4722316692; Fax: +4722424062.

[‡]yakov@technion.ac.il. Yitzhak Moda'i Chair in Technology and Economics, Technion – Israel Institute of Technology, Haifa 32000 Israel.

[§]oyvind.eitrheim@norges-bank.no. Research Department, Norges Bank.

1 Introduction

Studies of monetary policy decisions under uncertainty are mainly based on the Bayesian and the robust control approaches; see e.g. Hansen and Sargent (2001), Giannoni (2002), Onatski and Williams (2003), Levin and Williams (2003), Tetlow and von zur Muehlen (2001), Coenen (2007) and Leitemo and Söderström (2004). The Bayesian approach requires that one assigns a probability distribution on the uncertain aspect of a decision problem, e.g. model parameters. This enables one to choose an expected-loss-minimizing policy. The robust control theory suggests designing policies to perform relatively well in worst-case scenarios, i.e. when the underlying premises turn out to be false in the most unfortunate way. Thereby, this approach enables one to limit the potential loss if the underlying premises turn out to be false.¹

However, both of the approaches require some assumption(s) about the probability distribution of the uncertain entity. Within the Bayesian approach, such an assumption is made explicit while it is made implicit under the robust control approach, by limiting the outcome space of the uncertain entity. This may not be innocuous since the outcome space assumed will generally affect the policy decision. One can also argue that by invoking some probability distribution, explicitly or implicitly, one would not be deriving policy response to Knightian uncertainty in the strict sense, since it implies lack of probabilistic information; see Knight (1921).

It has also been argued that robust policies can be quite costly in terms of forsaken performance in the normal course of events if the policy is geared towards limiting potential losses under extreme events; cf. Tetlow and von zur Muehlen (2001) and Cogley and Sargent (2005). In particular, the min-max policy implied by the robust control approach will be sub-optimal under all cases but the worst case. A similar objection can also be raised against the Bayesian approach where the “worst case scenario” can have undue influence on the policy decision; cf. Cogley and Sargent (2005). Relatively high potential costs of robust policies may therefore discourage one from adopting such policies.

In this paper, we employ the robust-satisficing approach to derive monetary policy response under parameter uncertainty. This approach does not require any assumption about the probability distribution of the uncertain entity. And second, potential costs of robustness play a key role in defining robust policies; see e.g. Ben-Haim (2006). The robust-satisficing approach is quite general and can be easily employed to derive decisions under various kinds of uncertainties individually or jointly.²

¹Accordingly, a fictitious malevolent agent who represents a policy maker’s worst fears concerning misspecification is introduced into the optimization problem and motivates her to minimize the loss function under the worst-case scenario. The level of uncertainty facing the decision maker can be regulated by adjusting the resources available to the malevolent agent.

²The robust-satisficing approach has been previously applied to a wide variety of decision problems with Knightian uncertainty, including financial risk assessment (Ben-Haim 2005); environmental regulation (Stranlund and Ben-Haim 2007); search behavior in animal foraging (Carmel and Ben-Haim 2005); policy decisions in marine reserve design (Halpern *et al* 2006); natural resource conservation decisions (Moilanen and Wintle 2006); inspection decisions by port authorities to detect terrorist weapons (Moffitt *et al* 2005) and to detect invasive species (Moffitt

The robust-satisficing approach bases decision making on two main premises. The first premise is that the decision maker faces uncertainty of the Knightian kind. Hence, it does not require one to specify either a probability distribution or bounds on the outcome space of the uncertain entity. The second premise is that the decision maker aims for performance at some satisfactory level rather than at a level which is deemed to be optimal based for instance on an estimated model; cf. Simon (1959) and (1979) and the references therein. The policy maker may still use such an optimal level as a reference, but is assumed willing to accept deviations from it to control her potential loss in case of faulty assumptions.

The robust-satisficing policy maximizes robustness at a given level of acceptable performance. Robustness is measured as the extent of deviation from a decision's underlying premises at which the performance will not deteriorate beyond some acceptable level. The robust-satisficing approach offers a trade-off between the robustness and the level of acceptable performance. Robustness of a policy can be raised by lowering one's aspirations regarding its performance and accepting a higher level of loss; see e.g. Ben-Haim (2006). Common with the min-max approach, the robust-satisficing approach also allows one to cap one's potential losses when the outcome space is given. Ben-Haim *et al* (2007) show that when the level of uncertainty is given, there would exist a robust-satisficing policy that is observationally equivalent to the min-max policy. Nonetheless, there are important differences between the two approaches, as shown in Ben-Haim *et al* (2007) and in this paper.

The robust-satisficing approach is attractive when a decision maker would be content with performing relatively close to the optimal level derived under a specific set of scenarios, and with relatively poor performance under an alternative set of scenarios. For example, when a policy maker's credibility depends on performing satisfactorily under ordinary events, but not under (apparently) extreme events, despite heavy losses. This could be the case for instance when private agents agree on the "ordinary-extreme classification" of events and are conscious of the potential costs of highly robust policies.

We employ the robust-satisficing approach to derive monetary policy response when there is uncertainty about key parameters of an aggregate model of the US economy, estimated by Rudebusch and Svensson (1999).³ These parameters represent degrees of persistence in the demand and supply shocks, the slope of the Phillip's curve and the response of the output gap to interest rates. Uncertainty regarding persistence in shocks can be interpreted broadly as it can proxy uncertainty regarding omission of relevant variables as well as functional form misspecification, beside representing uncertainty regarding genuine shock persistence. Monetary policy is characterized by a simple Taylor-type interest rate rule, where the decision parameters are the response coefficients associated with inflation and output gaps as well as degree of interest rate smoothing; see Taylor *et al* 2007); technological fault diagnosis (Pierce *et al* 2006) and testing (Vinot *et al* 2005); and project management (Regev *et al* 2006).

³Results based on an alternative model with hybrid New Keynesian Phillips curve and IS curve are available upon request to the authors.

(1999). In addition to illustrating key properties of robust-satisficing policies, we also point out differences and possible observational equivalence with min-max policies. This paper draws on but goes beyond Ben-Haim *et al* (2007) who considered the case of uncertainty in the persistence of supply shock to contrast robust-satisficing policies with the min-max policy.

We find that higher robustness can be achieved by basing policy on relatively high degrees of persistence in the shocks and relatively weak effects of the output gap on inflation and of interest rates on the output gap. How much to raise the degree of persistence and lower the effects of the output gap and the interest rate depends on the level of robustness sought. Robustness is achieved by lowering one's aspirations regarding the policy performance and is therefore costly. Such costs are found to increase with the level of robustness, making robustness to a wide set of parameter values as well as apparently extreme events particularly costly. This also implies relatively high costs of adopting min-max policies.

We also show that a policy decision based on the robust satisficing approach offers a higher degree of robustness than min-max policies if one aims to perform relatively well under a subset of all possible parameter values conjectured rather than limiting the loss under the worst case values of the parameters. The policy implications of the two approaches may differ substantially in such cases. However, both approaches can suggest the same policy if the acceptable level of loss is equal or higher than the maximum level under the min-max policy. The robust-satisficing policies are found to be generally less aggressive than min-max policies. They may therefore be easier to reconcile with observed interest rate setting than min-max policies; see e.g. Giannoni (2002), Leitemo and Söderström (2004), Tetlow and von zur Muehlen (2001) and the references therein.

The paper is organized as follows. The next section briefly presents the robust-satisficing approach. Section 3 presents the empirical model and characterizes monetary policy. Section 4 employs the robust-satisficing approach to deal with uncertainty in parameters individually and jointly. Section 5 presents the main conclusions followed by an appendix.

2 Robust-Satisficing Decisions

This section presents the basic concepts related to the robust-satisficing approach, definitions of different decision strategies and their properties.

2.1 Uncertainty and Robustness

We denote a policy maker's decisions by the parameter vector Ω , which for instance may consist of parameters of a simple Taylor-type interest rate rule. The policy maker's decisions are based on models and data. However, these models and data, including the probabilistic elements and

parameters, may be incomplete or erroneous in various unknown ways. There may be e.g. relevant variables missing from the models, the appropriate model specification could be unknown, estimates of key parameters could be unavailable because of lack of data or one may lack confidence in them because of measurement errors in the data and so on.

We denote uncertain elements by θ which can be e.g. specific parameters, functions, missing variables and/or probability distributions. $\tilde{\theta}$ symbolizes some specific value of θ which can be an estimate or one's choice.

We represent the uncertainty associated with θ by the family of sets $\mathcal{U}(\ell, \tilde{\theta})$. Each of these sets contains possible realizations of θ in the ‘‘vicinity’’ ℓ of $\tilde{\theta}$. $\mathcal{U}(\ell, \tilde{\theta})$ is referred to as an information gap (info-gap) model of uncertainty. Info-gap models entail no probabilistic information and thus are one possible quantification of Knightian uncertainty; see Ben-Haim (2006) for details. An info-gap model obeys two axioms:

$$\text{Contraction:} \quad \mathcal{U}(0, \tilde{\theta}) = \{\tilde{\theta}\} \quad (1)$$

$$\text{Nesting:} \quad \ell < \ell' \implies \mathcal{U}(\ell, \tilde{\theta}) \subseteq \mathcal{U}(\ell', \tilde{\theta}) \quad (2)$$

The contraction axiom asserts that $\tilde{\theta}$ is the only possibility when there is no uncertainty ($\ell = 0$). Here, we consider ℓ as an unbounded unidimensional indicator of parameter uncertainty. The nesting axiom asserts that the range of possible realizations increases as the level of uncertainty increases, *ceteris paribus*. That is, the set $\mathcal{U}(\ell, \tilde{\theta})$ becomes more inclusive as ℓ gets larger, implying that the range of possible realizations of θ in the vicinity $\tilde{\theta}$ increases with ℓ . It can therefore be referred to as the level of uncertainty and is related to the level of robustness as explained later.

The loss resulting from decision Ω when the uncertain elements take the values θ is $L(\Omega, \theta)$. The loss may be a statistical expectation or a deterministic value.⁴ The satisficing policy maker desires low loss, and would prefer loss no greater than some satisfactory level L_s :

$$L(\Omega, \theta) \leq L_s \quad (3)$$

We treat L_s as a parameter which can be chosen small or large, so the satisficing requirement in eq.(3) includes minimizing the loss as a special case. The policy maker is satisficing if she does not aim to minimize the loss but would be content with a loss no larger than L_s , recognizing that the loss may exceed L_s for some $\theta \in \mathcal{U}(\ell, \tilde{\theta})$.

⁴Our discussion can be readily extended to multiple loss functions.

2.2 Decision strategies: Robust-Satisficing, Conditional Estimation, and Min-Maxing

We consider three types of decision strategies for choosing a decision or policy Ω from a set R of feasible policies.

The robustness of decision Ω , with the satisficing requirement L_s of eq.(3), is the greatest level of uncertainty ℓ up to which all realizations θ would result in a loss no greater than L_s :

$$\widehat{\ell}(\Omega, L_s) = \max \left\{ \ell : \left(\max_{\theta \in \mathcal{U}(\ell, \tilde{\theta})} L(\Omega, \theta) \right) \leq L_s \right\} \quad (4)$$

$\widehat{\ell}(\Omega, L_s)$ is a robustness function indicating the robustness of a specific policy Ω at some acceptable loss level L_s .

The *robust-satisficing* decision maximizes the robustness (4) while satisficing the loss at the value L_s :

$$\Omega_s(L_s) = \arg \max_{\Omega \in R} \widehat{\ell}(\Omega, L_s) \quad (5)$$

Maximization of $\widehat{\ell}(\Omega, L_s)$ conditional on some L_s amounts to maximizing $\mathcal{U}(\ell, \tilde{\theta})$, by the nesting axiom.

Conditional optimization is the decision, $\Omega_{\tilde{\theta}}$, which minimizes the loss based on a specific value of the uncertain entities, $\tilde{\theta}$:

$$\Omega_{\tilde{\theta}} = \arg \min_{\Omega \in R} L(\Omega, \tilde{\theta}) \quad (6)$$

A special case of *conditional optimization* is optimization conditional on a value of θ implying the highest level of loss which defines a min-max policy. A min-max policy may be defined as follows.

The *min-max decision* minimizes the maximum loss based on a conjecture of the greatest level of uncertainty, ℓ_m :

$$\Omega_m(\ell_m) = \arg \min_{\Omega \in R} \max_{\theta \in \mathcal{U}(\ell_m, \tilde{\theta})} L(\Omega, \theta) \quad (7)$$

The min-max policy $\Omega_m(\ell_m)$ would not lead to a loss higher than some specific level for any value of θ from the parameter space defined by $\ell_m, \mathcal{U}(\ell_m, \tilde{\theta})$.

2.3 Basic properties of the decision strategies

Here, we note several basic properties of the three decision strategies: robust-satisficing $\Omega_s(L_s)$, conditional optimization $\Omega_{\tilde{\theta}}$, and min-maxing $\Omega_m(\ell_m)$. These properties, presented as propositions 1–3, characterize the relationship between robustness $\widehat{\ell}(\Omega, L_s)$ and acceptable loss L_s .

Proposition 1 *Performance trades-off against robustness, both at any fixed decision, Ω , and at the robust-satisficing decision $\Omega_s(L_s)$, if $L(\Omega, \theta)$ is uniformly continuous in θ .*

At a fixed decision Ω :

$$L_s < L'_s \implies \widehat{\ell}(\Omega, L_s) \leq \widehat{\ell}(\Omega, L'_s) \quad (8)$$

At the robust-satisficing decision $\Omega_s(L_s)$:

$$L_s < L'_s \implies \widehat{\ell}[\Omega_s(L_s), L_s] \leq \widehat{\ell}[\Omega_s(L'_s), L'_s] \quad (9)$$

Better performance (lower loss L_s), entails lower robustness $\widehat{\ell}(\Omega, L_s)$. Relation (8) asserts that this holds at any fixed decision such as the optimal (conditional) decision $\Omega_{\tilde{\theta}}$ or the min-max decision $\Omega_m(\ell_m)$ where ℓ_m , and hence $\Omega_m(\ell_m)$, is fixed. Relation (9) asserts that this trade-off also holds for the robust-satisficing decision $\Omega_s(L_s)$, which may vary as L_s varies. The proof of proposition 1 appears in Ben-Haim (2000, thm. 1 and cor. 1/1).

Proposition 2 *Conditional-optimization aspirations have no robustness. For any decision Ω for which $L(\Omega, \theta)$ is not a local maximum at $\tilde{\theta}$,*

$$L_s = L(\Omega_{\tilde{\theta}}, \tilde{\theta}) \implies \widehat{\ell}(\Omega, L_s) = 0 \quad (10)$$

Proposition 2 asserts that, for any choice of $\tilde{\theta}$, aspiring to a loss level as low as $L(\Omega_{\tilde{\theta}}, \tilde{\theta})$ has no robustness to errors or deviations from $\tilde{\theta}$. $L(\Omega_{\tilde{\theta}}, \tilde{\theta})$ refers to the optimal loss level when θ turns out to be $\tilde{\theta}$. This loss level need not be attained if θ differs from $\tilde{\theta}$. When θ is $\tilde{\theta}$, any decision Ω other than $\Omega_{\tilde{\theta}}$ would be suboptimal. Since this is true for *any* Ω , it is also true for each of the decision-strategies in eqs.(5), (6) and (7).⁵ The proof of proposition 2 derives immediately from the contraction axiom and will not be elaborated.

Proposition 3 *The robust-satisficing and min-maxing policies, $\Omega_s(L_s)$ and $\Omega_m(\ell_m)$, are identical for appropriate choices of the parameters L_s and ℓ_m .*

- For any ℓ_m for which $\Omega_m(\ell_m)$ exists, there is an L_s such that:

$$\Omega_s(L_s) = \Omega_m(\ell_m) \quad (11)$$

- For any L_s for which $\Omega_s(L_s)$ exists, there is an ℓ_m such that:

$$\Omega_m(\ell_m) = \Omega_s(L_s) \quad (12)$$

See Ben-Haim *et al* (2007) for the proof of proposition 3.

⁵The condition that “ $L(\Omega, \theta)$ is not a local maximum at $\tilde{\theta}$ ” means that outcomes could be worse than the outcome conditional on $\tilde{\theta}$, $L(\Omega, \tilde{\theta})$. If the outcomes cannot be worse than $L(\Omega, \tilde{\theta})$, then uncertainty is strictly favorable and entails only the possibility of better-than-anticipated outcomes.

Proposition 3 states that both the min-max policy and the robust-satisficing policy can equal each other for specific values of key parameters: ℓ_m and L_s . A modeler can therefore describe a min-max policy as a robust-satisficing policy and the converse. To equate the robust-satisficing policy with the min-max policy, one has to upward adjust L_s to, say, L_m such that $\widehat{\ell}(\Omega, L_m) = \ell_m$. To equate the min-max policy with the robust-satisficing policy, one has to downward adjust ℓ_m to the level $\widehat{\ell}(\Omega, L_s)$, if $L_s < L_m$. This amounts to assuming away uncertainty, which may not be reasonable, for example when ℓ_m is unbounded in principle.

Even though one can arrive at the same decision from the two different perspectives to decision making under uncertainty, robust satisficing policies will generally differ from the min-max policy. This will become evident in the examples considered later, but for now suppose there is a largest possible level of uncertainty, ℓ_m , e.g. when the parameter space is bounded by definition. Let L_m denote the min-max loss at uncertainty ℓ_m . In this case, $\ell_m > \widehat{\ell}(\Omega, L_s)$ for any Ω , except when L_s is sufficiently large and equals L_m , as $\widehat{\ell}(\Omega, L_s)$ would then equal ℓ_m . However, the robust satisficing policies $\Omega_s(L_s)$ will lead to a lower loss than L_m for parameter values defined by the range $\widehat{\ell}_s$ ($\equiv \widehat{\ell}(\Omega_s(L_s), L_s)$). On the other hand, a robust satisficing policy would not necessarily offer a lower loss than the acceptable level, i.e. $L(\Omega_s(L_s), \theta) \geq L_s$, for the parameter space corresponding to the range of uncertainty exceeding $\widehat{\ell}_s$, since set $\mathcal{U}(\widehat{\ell}_s, \tilde{\theta})$ is contained in set $\mathcal{U}(\ell_m, \tilde{\theta})$. Still, a robust-satisficing policy maker who aspires to a lower level of loss, $L_0 < L_m$, under a subset of scenarios (θ values), could prefer a robust-satisficing policy $\Omega_s(L_0)$ to the min-max policy (under which the loss will not exceed L_m). The reason for this preference is that the robustness of $\Omega_s(L_0)$ for satisficing the loss at the value L_0 is no less (and usually greater) than the robustness of the min-max policy.

3 Model and monetary policy

This section presents the estimated model of the USA by Rudebusch and Svensson (1999) and characterizes monetary policy.

3.1 Model

Following is the well known aggregate model of the US economy developed by Svensson and Rudebusch (1999):

$$\pi_t = 0.7\pi_{t-1} - 0.1\pi_{t-2} + 0.28\pi_{t-3} + 0.12\pi_{t-4} + 0.13y_{t-1} + u_{\pi,t}, \quad (13)$$

$$y_t = 1.16y_{t-1} - 0.25y_{t-2} - 0.1(\bar{r}_{t-1} - \bar{\pi}_{t-1}) + u_{y,t}. \quad (14)$$

Here, π_t is the quarterly inflation rate, y is the output gap, while $\bar{\pi}$ and \bar{r} are smoothed values of quarterly inflation rate and the nominal interest rate, respectively. Precisely, $\bar{\pi}_t = \frac{1}{4} \sum_{i=0}^3 \pi_{t-i}$, while $\bar{r}_t = \frac{1}{4} \sum_{i=0}^3 r_{t-i}$. Finally, $u_{\pi,t}$ and $u_{y,t}$ are unobservable variables representing supply and demand shocks, respectively. The model has been estimated by OLS on the US quarterly data for the period 1961q1–1996q2.

We assume that both of these shocks follow AR(1) processes:

$$u_{\pi,t} = \rho_{\pi} u_{\pi,t-1} + \varepsilon_{\pi,t}, \quad (15)$$

$$u_{y,t} = \rho_y u_{y,t-1} + \varepsilon_{y,t}, \quad (16)$$

where ρ_{π} and ρ_y are constant parameters representing persistence in the supply and demand shocks, respectively. We assume that $\rho_{\pi} \in [0, 1)$ and $\rho_y \in [0, 1)$. The ε 's are assumed to be IID-shocks.

Econometrically, a non-zero degree of persistence can also indicate omission of relevant variables as well as a misspecified functional form of the model. Uncertainty regarding shock persistence can therefore be interpreted broadly as uncertainty regarding the structure of the model and the shock process.

3.2 Monetary policy

We assume that monetary policy authorities have a standard quadratic loss function, which can be presented in terms of variance of the inflation, output gap and interest-rates, when the discount factor is close to one:

$$L = V(\pi - \pi^*) + \lambda V(y) + \phi V(\Delta r), \quad (17)$$

where $V(\cdot)$ denotes the unconditional variance of its argument; π^* is the constant inflation target; λ denotes the authority's preference for stabilization of the output-gap, y , relative to that for the inflation gap, $\pi - \pi^*$; and ϕ is the relative preference for interest-rate stability. In our empirical analysis we let λ be, say, 0.5 and $\phi = 0.1$. It is useful to express the loss function, eq.(17), as an explicit function of policy Ω and uncertain parameters of the models θ , as $L(\Omega, \theta)$.

We characterize monetary policy response by a simple Taylor-type interest rate rule:

$$r_t = \omega_r r_{t-1} + (1 - \omega_r)[rr^* + \pi^* + \omega_{\pi}(\pi_t - \pi^*) + \omega_y y_t], \quad (18)$$

where ω 's are constant coefficients representing the interest rate response to the lagged interest rate, the inflation gap and the output gap. rr^* denotes the steady state value of the real interest rate. The inflation target and the steady state real interest rate sum to the steady state nominal

interest rates.

Monetary policy maker chooses the parameters, $\Omega = (\omega_r, \omega_\pi \text{ and } \omega_y)$, in the interest rate rule eq.(18) while facing Knightian uncertainty regarding specific parameters, θ , of the economic model: (13)–(16). The robust-satisficing policy, $\Omega_s(L_s)$, maximizes the robustness and satisfies the relative loss at the value L_s , from a set R of available policies, as in eq.(5).

To limit the number of policies considered, we let robust satisficing policies $\Omega_s(L_s)$ be selected from a set $R = \{\Omega_{\theta_1}, \Omega_{\theta_2}, \dots, \Omega_{\theta_n}\}$. This set consists of policies that are optimal with respect to one from among n specific realizations of the model parameters, $\theta_1, \theta_2 \dots, \theta_n$.⁶ The optimal policy, $\Omega_{\tilde{\theta}}$, defined by minimizing the loss function (17), when θ is assumed to be $\tilde{\theta}$ ($= \theta_j; j = 1, 2, \dots, n$) is defined as in eq.(6). $\Omega_{\tilde{\theta}}$ as well as $\Omega_s(L_s)$ will depend on the degree of concern for stability in the real economy and the interest rate, expressed by λ and ϕ . Moreover, by letting $\Omega_s(L_s) \in R$, as defined above, the choice of $\Omega_{\tilde{\theta}}$ as $\Omega_s(L_s)$ becomes dependent on the acceptable loss L_s (and the set of feasible policies R).

We evaluate a policy in a given state relative to the optimal policy in that state. Therefore, we employ the relative loss function defined as:

$$dL(\Omega_{\tilde{\theta}}, \theta) \equiv \frac{L(\Omega_{\tilde{\theta}}, \theta)}{L(\Omega_{\theta}, \theta)} - 1. \quad (19)$$

$L(\Omega_{\theta}, \theta)$ expresses the loss under optimal policy conditioned on θ being the true value, while $L(\Omega_{\tilde{\theta}}, \theta)$ expresses the loss when policy is conditioned on $\tilde{\theta}$, the policy maker's choice, which can differ from θ . It follows that $dL(\Omega_{\tilde{\theta}}, \theta) > 0$ for $\Omega_{\tilde{\theta}} \neq \Omega_{\theta}$ while $dL(\Omega_{\tilde{\theta}}, \theta) = 0$ when $\Omega_{\tilde{\theta}} = \Omega_{\theta}$, assuming the loss function has a unique minimum. Examining the relative loss implied by a policy makes it possible to separate its contribution to the performance (loss level) from that of the realized value of the parameter. This makes it easy to compare the robustness of different policies.

We evaluate a policy $\Omega_{\tilde{\theta}}$ by inquiring for which set of realized values of θ the associated losses $dL(\Omega_{\tilde{\theta}}, \theta)$ s would not exceed a given satisfactory level dL_s . The robust-satisficing policy at a given dL_s is the policy for which the associated loss does not exceed dL_s for the largest set of θ values. Essentially, robust satisficing policy conditions on that possible value $\tilde{\theta}$ of θ , which would keep $dL(\Omega_{\tilde{\theta}}, \theta)$ equal or below some preferred level for the largest range of possible θ values. Thus, the choice of $\tilde{\theta}$ can be thought of as being based on strategic considerations in a game against an unpredictable nature.

⁶ In principle one can evaluate any policy, Ω , which is defined by values of ω_π, ω_y and ω_r in the Taylor-type rule (18).

4 Empirical analysis

We first employ the robust-satisficing approach to deal with uncertainty about key parameters, individually. These are the degree of persistence in the supply shock and the slope of the Phillips curve. In Section 4.3, we consider an extension of this approach and employ the approach to deal with uncertainty in two parameters jointly, persistence in the supply and demand shocks, ρ_π and ρ_y . The precise formulation of the info-gap model appears in appendix A.

For brevity, we do not present the results for uncertainty in the persistence of the demand shock as well as those for uncertainty in the effects of interest rates on the output gap. The results for uncertainty in persistence of the demand shock are qualitatively similar to those for persistence in the supply shocks while those for uncertainty in interest rate effects are qualitatively similar to those for uncertainty in the slope of the Phillips curve.⁷

4.1 Uncertain persistence of supply shocks, ρ_π

Here, we consider the case where the parameter space of the uncertain parameter, ρ_π , is bounded within the $[0, 1)$ -space. Specifically, in our simulations we assume that the degree of persistence in the supply shock ρ_π takes on one of the hundred possible values from the set $0, 0.01, 0.02, \dots, 0.99$ and investigate robustness of each of the corresponding (optimal) policies: $\Omega_0, \Omega_{0.01}, \Omega_{0.02}, \dots, \Omega_{0.99}$. For each $\rho_\pi \in [0, 0.99]$, the vector Ω_{ρ_π} contains optimal values of the response coefficients in the Taylor rule eq.(18), ω_r , ω_π and ω_y , obtained by minimizing the loss function eq.(17) conditional on the corresponding specific value of ρ_π . The interval $[0, 0.99]$ is also the maximum range of robustness values, or the maximum level of uncertainty conjectured ℓ_m .

To summarize our results, we find that the robust-satisficing policies turn out to be those which are based on ρ_π -values in the range 0.3–0.8. In general, an increase in the relative loss implies a policy based on a relatively higher degree of persistence. However, policies based on neither relatively low nor high ρ_π s are most robust. Hence, policies based on ρ_π -values below 0.3 and above 0.8 turn out not to be robust-satisficing policies. Moreover, we find that the min-max policy is the optimal policy based on $\rho_\pi = 0.8$ and hence coincides with the most robust robust-satisficing policy.⁸

In greater detail, Table 1 shows levels of robustness, $\widehat{\ell}(\Omega_{\rho_\pi}, dL_s)$, implied by selected policies at different levels of acceptable loss. Each column of the table evaluates a specific policy rule Ω_{ρ_π} in terms of robustness at different levels of relative loss dL_s . In the unidimensional case of uncertainty considered here, $\widehat{\ell}(\Omega_{\rho_\pi}, dL_s)$ is defined as the range of ρ_π -values for which the

⁷We have also employed the robust satisficing approach to deal with uncertainty in other parameters as well in the context of alternative models. While policy implications of uncertainty in the other parameters are different, presentation of these results would not add much to the illustration of the robust satisficing approach. The results are available upon request to the authors.

⁸Interestingly, Angeloni *et al* (2003) reach the same conclusion regarding the min-max policy using an estimated DSGE model of the euro area.

Table 1: Robustness of selected policies at different levels of acceptable loss when uncertain ρ_π

dL_s	$\widehat{\ell}(\Omega_{\rho_\pi}, dL_s)$											
50	0.93	0.94	0.94	0.94	0.95	0.95	0.96	0.97	0.99	0.52	0.10	
40	0.91	0.92	0.92	0.93	0.93	0.94	0.95	0.96	0.87	0.44	0.08	
35	0.9	0.9	0.91	0.92	0.92	0.93	0.94	0.95	0.8	0.39	0.07	
30	0.88	0.89	0.9	0.9	0.91	0.92	0.93	0.94	0.72	0.35	0.06	
25	0.86	0.87	0.88	0.88	0.89	0.9	0.92	0.93	0.63	0.31	0.05	
20	0.83	0.84	0.85	0.86	0.87	0.89	0.90	0.88	0.55	0.27	0.04	
15	0.8	0.81	0.82	0.83	0.85	0.86	0.88	0.74	0.46	0.22	0.03	
10	0.73	0.75	0.77	0.79	0.8	0.82	0.84	0.59	0.35	0.17	0.02	
5	0.61	0.64	0.67	0.7	0.73	0.76	0.57	0.4	0.24	0.11	0.01	
1	0.35	0.4	0.46	0.52	0.43	0.33	0.24	0.16	0.09	0.04	0.00	
0	0	0	0	0	0	0	0	0	0	0	0	
Ω_{ρ_π}	Ω_0	$\Omega_{0.1}$	$\Omega_{0.2}$	$\Omega_{0.3}$	$\Omega_{0.4}$	$\Omega_{0.5}$	$\Omega_{0.6}$	$\Omega_{0.7}$	$\Omega_{0.8}$	$\Omega_{0.9}$	$\Omega_{0.99}$	

Note: The policies, which are represented by Ω s containing the response coefficients in the Taylor rule, are optimal conditional on the subscripted values of persistence in the supply shock (ρ_π). Bold faced numbers correspond to the robust-satisficing policies at different levels of acceptable loss (in per cent).

requirement $dL(\Omega_{\rho_\pi}, \rho'_\pi) \leq dL_s$ is satisfied. Bold faced numbers correspond to the robust-satisficing policies at different loss levels. Such policies maximize the robustness at given levels of (relative) losses among the other policies evaluated in the table.

Robustness increases as one moves up each column of Table 1. This illustrates proposition 1: the robustness of a given policy increases as the desired performance deteriorates, i.e. as dL_s increases. However, robustness of a given policy increases with dL_s at a decreasing rate. This also applies to the robust-satisficing policies which vary with the level of the loss. Therefore, none of the robust-satisficing policies is robust-satisficing at all levels of relative loss. It results that robust-satisficing policies at relatively higher loss levels tend to be defined by policies conditional on relatively higher degrees of persistence. Table 1 also illustrates proposition 2, which asserts that the robustness for the aspiration, $dL_s = 0$, is zero.

Figure 1 presents robustness curves defined by $\widehat{\ell}(\Omega_{\rho_\pi}, dL_s)$ for the complete set of the policies evaluated versus ρ_π . Each robustness curve is defined by a given relative loss level and depicts the robustness of optimal policies conditional on the hundred possible values of ρ_π noted on the horizontal axis.

The costs of robustness increase with the level of robustness. At a given policy Ω_{ρ_π} , the robustness curves suggest a strongly concave relationship between robustness $\widehat{\ell}(\Omega_{\rho_\pi}, dL_s)$ and dL_s . We note that an increase in robustness is obtained by accepting an increasingly larger rise in dL_s . This is suggested by the decreasing vertical distance between the curves when we vary dL_s from 1% to 60%.

At any given loss level, the corresponding robustness curve also suggests a strongly concave relationship between robustness and the degrees of persistence on which the policies are conditioned upon; see Figure 1. Hence, a unique robust satisficing policy at a given loss level can be identified

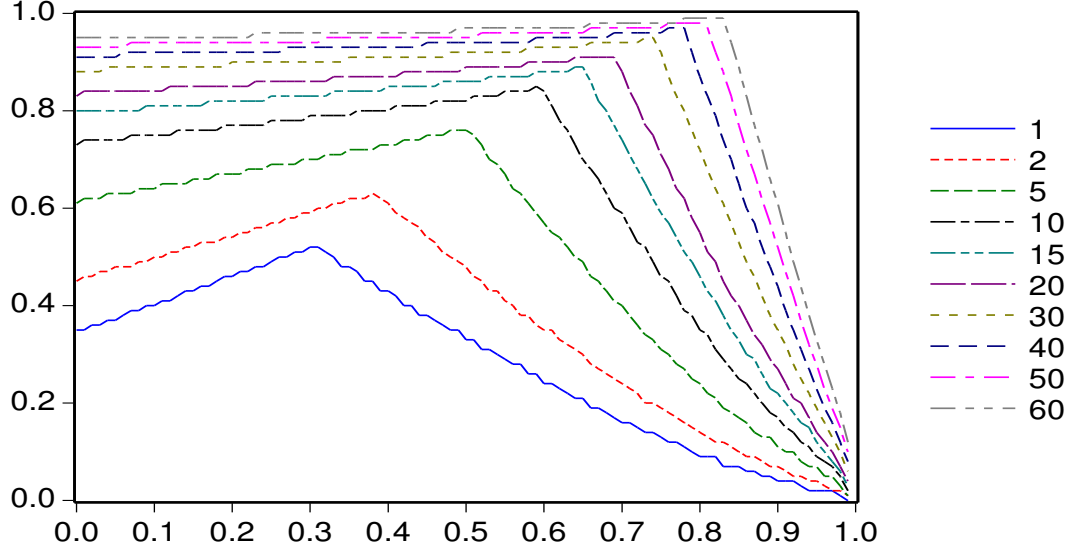


Figure 1: Robustness of all policies considered at different levels of acceptable loss when ρ_π is uncertain. Level of robustness $\hat{\ell}(\Omega_{\rho_\pi}, dL_s)$ is represented on the vertical axis, while ρ_π -values are denoted on the horizontal axis. Robustness is measured by the length of an interval containing ρ_π -values $\in [0, 0.99]$. The degrees of persistence on which the optimal policies are conditioned, are presented on the horizontal axis. Different levels of acceptable loss dL_s in per cent are indicated by line-style of the robustness curves.

among the set of policies considered. The robust-satisficing policies at different loss levels are defined by the peaks of the different robustness curves. We note that the peaks correspond to optimal policies conditional on ρ_π -values in the range 0.3–0.8.

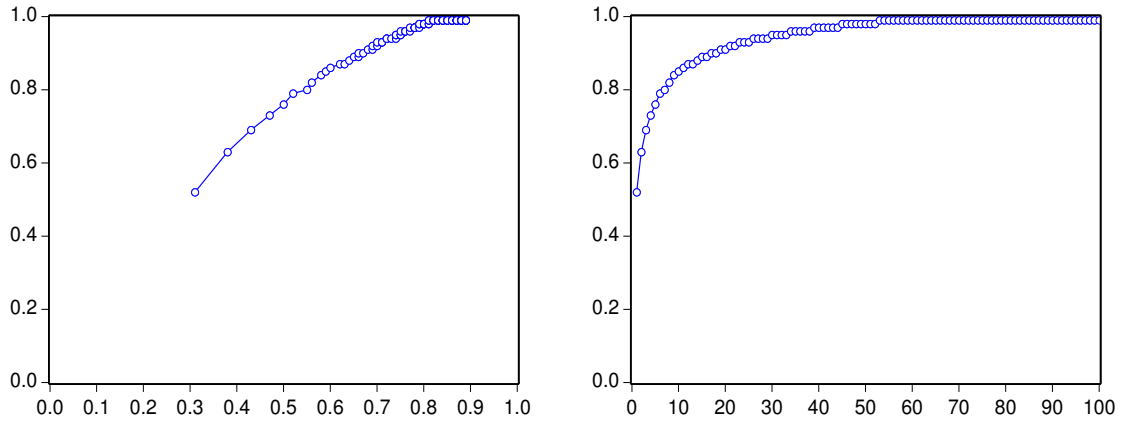


Figure 2: (a) Left frame: Robustness $\hat{\ell}(\Omega_{\rho_\pi}, dL_s)$ offered by robust-satisficing policies (and some other policies) is indicated on the vertical axis. Robustness is measured by the lengths of intervals containing ρ_π -values $\in [0, 0.99]$. The policies are identified by the degrees of persistence (ρ_π) indicated on the horizontal axis. The set of robust-satisficing policies are conditional on the following set of ρ_π -values: 0.30, 0.31, 0.32, ..., 0.80. (b) Right frame: Robustness (vertical axis) offered by the different policies considered in the left frame (a) at different levels of acceptable loss, dL_s , which is represented on the horizontal axis in per cent.

Figure 2.a shows robustness offered by the robust-satisficing policies. The circled line represents maximal robustness offered by the corresponding policy. The bold face numbers in Table 1 are points on this curve. It is seen that optimal policies conditional on values below 0.3 and above 0.8 are not robust-satisficing policies. The policy based on $\rho_\pi = 0.8$ offers maximum robustness (indicated by 1) at the lowest level of loss compared with policies based on ρ_π outside the range 0.3–0.8. Figures 2.a–b suggest that the policy conditional on $\rho_\pi = 0.8$ would not lead to a loss higher than 50% under any value of $\rho_\pi \in [0,0.99]$; see also Table 1. The other policies including those based on ρ_π -values strictly larger than 0.8 require willingness to accept higher loss than 50% for maximum robustness and will therefore not be adopted.

Figure 2.b also suggests that by accepting less than 10% per cent deviation from whatever would be the optimal loss level, one can perform satisfactorily under about 3/4 of the possible values of ρ_π . However, a higher level of robustness requires willingness to accept a substantially higher loss. To perform satisfactorily under any value of $\rho_\pi \in [0,0.99]$ one would have to accept a loss of at least 50%.

Table 2: Sets of ρ_π -values for which selected policies are robust at different levels of dL_s

dL_s	$\mathcal{U}(\Omega_{\rho_\pi}, dL_s)$											
50	0-.93	0-.94	0-.94	0-.94	0-.95	0-.95	0-.96	0-.97	0-.99	.47-.99	.89-.99	
40	0-.91	0-.92	0-.92	0-.93	0-.93	0-.94	0-.95	0-.96	.1-.97	.55-.99	.91-.99	
35	0-.9	0-.9	0-.91	0-.92	0-.92	0-.93	0-.94	0-.95	.17-.97	.6-.99	.92-.99	
30	0-.88	0-.89	0-.9	0-.9	0-.91	0-.92	0-.93	0-.94	.24-.96	.64-.99	.93-.99	
25	0-.86	0-.87	0-.88	0-.88	0-.89	0-.9	0-.92	0-.93	.32-.95	.67-.98	.94-.99	
20	0-.83	0-.84	0-.85	0-.86	0-.87	0-.89	0-.90	.04-.92	.39-.94	.71-.98	.95-.99	
15	0-.8	0-.81	0-.82	0-.83	0-.85	0-.86	0-.88	.16-.9	.47-.93	.75-.97	.96-.99	
10	0-.73	0-.75	0-.77	0-.79	0-.80	0-.82	01-.85	.29-.88	.56-.91	.79-.96	.97-.99	
5	0-.61	0-.64	0-.67	0-.7	0-.73	0-.76	.23-.8	.44-.84	.65-.89	.83-.94	.98-.99	
1	0-.35	0-.4	0-.46	0-.52	.15-.58	.31-.64	.46-.7	.61-.77	.75-.84	.88-.92	.99-.99	
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	
	Ω_0	$\Omega_{0.1}$	$\Omega_{0.2}$	$\Omega_{0.3}$	$\Omega_{0.4}$	$\Omega_{0.5}$	$\Omega_{0.6}$	$\Omega_{0.7}$	$\Omega_{0.8}$	$\Omega_{0.9}$	$\Omega_{0.99}$	

Note: The policies, which are represented by Ω s containing the response coefficients in the Taylor rule, are optimal conditional on the subscripted values of persistence in the supply shock (ρ_π). Bold faced sets of ρ_π -values correspond to the robust-satisficing policies at different levels of acceptable loss (in per cent).

Table 2 presents sets of ρ_π -values, $\mathcal{U}(\Omega_{\rho_\pi}, dL_s)$, for which selected policies would be robust at different levels of (relative) loss. Specifically, at level of robustness/uncertainty $\hat{\ell} = \hat{\ell}(\Omega_{\rho_\pi}, dL_s)$, the uncertainty set $\mathcal{U}(\hat{\ell}, dL_s)$ is an interval of ρ_π -values of length $\hat{\ell}$, where $\mathcal{U}(\hat{\ell}, dL_s)$ is alternatively represented as $\mathcal{U}(\Omega_{\rho_\pi}, dL_s)$. These sets correspond to the robustness measures in Table 1 and display similar properties.

Table 2 illustrates the contraction as well as the nesting axioms. It shows that sets corresponding to optimal policies for $dL_s = 0\%$ only contain the single conditioning ρ_π . Accordingly, highest aspirations have zero robustness: $\hat{\ell}(\Omega_{\rho_\pi}, dL_s) = 0$ while $\mathcal{U}(\Omega_{\rho_\pi}, dL_s) = \{\rho_\pi\}$ when $dL_s = 0$. The table also shows that robustness of policies considered increases as well as those of robust-satisficing policies with the (relative) loss levels, dL_s . The uncertainty sets are non-decreasing as one moves

up each column of Table 2 and across columns associated with the robust-satisficing policies (while raising dL_s). Figure 3 displays how the uncertainty sets vary with different robust-satisficing policies as well as with loss levels.

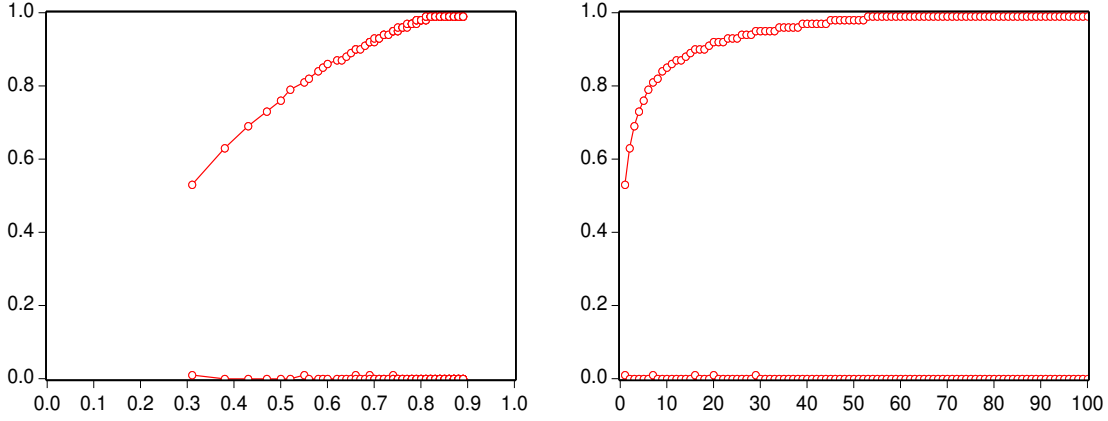


Figure 3: (a) Left frame: Robustness intervals offered by robust-satisficing policies (and some others) in terms of sets of ρ_π -values. The sets are defined by the minimum and the maximum value in their ranges; the values within the extreme values are also parts of the sets. The robust-satisficing policies are identified by values of ρ_π ($= 0, 0.30, 0.31, \dots, 0.80$) indicated on the horizontal axis. (b) Robustness (vertical axis) in terms of sets of ρ_π -values indicated by the minimum and maximum values offered by the different policies considered in (a) at different levels of acceptable loss, dL_s , which is represented on the horizontal axis in per cent.

Notably, the costs of robustness increase with the level of robustness and asymmetrically around the persistence values conditioned upon. Table 2 as well as Figure 3 show that the parameter sets, $\mathcal{U}(\Omega_{\rho_\pi}, dL_s)$ s, expand at a decreasing rate when we raise the relative loss levels, implying increasing costs of robustness. Hence, inclusion of relative extreme values of ρ_π in the parameter sets demands willingness to accept relatively high costs. It also appears that when the relative loss level increases, the parameter sets do not expand symmetrically around the parameter values conditioned on. Hence, the costs of expanding the parameter sets to include particularly high or low parameter values can be quite high.

We also note that the increase in robustness, i.e. expansions of the sets $\mathcal{U}(\Omega_{\rho_\pi}, dL_s)$, is highly policy dependent. For example, we see in Table 2 that an increase in the loss from 1% to 5% expands the parameter set associated with policy $\Omega_{0.1}$ from $[0, 0.35]$ to $[0, 0.61]$ while the parameter set associated with $\Omega_{0.9}$ changes from $[0.88, 0.92]$ to $[0.83, 0.94]$. Regarding policies that are not robust-satisficing, we note that policies conditional on degrees of persistence below 0.3 seem to be more robust, i.e. the associated sets $\mathcal{U}(\Omega_{\rho_\pi}, dL_s)$ are larger, than policies conditional on degrees of persistence higher than 0.8.⁹

⁹The robust-satisficing policies indicated in the Table 2 are relative to policies evaluated in this table. Some of

The min-max policy Ω_m is defined as the optimal policy conditioned on $\rho_\pi = 0.8$, $\Omega_{0.8}$, if we assume that the level of uncertainty ℓ_m coincides with the range 0–0.99. Then, this policy would offer robustness against any $\rho_\pi \in [0, 0.99]$ at the lowest loss level, which is 50%. The other policies do not offer robustness against the complete set of ρ_π -values at this level of relative loss. Specifically, optimal policies conditioned on $\rho_\pi \in [0, 0.8)$ imply higher loss than 50% if ρ_π turns out to be e.g. 0.99, while the policies conditioned on $\rho_\pi \in (0.8, 0.99]$ imply relatively higher loss than 50% if ρ_π turns out to be particularly low. Notably, robust-satisficing policies defined by $\rho_\pi \in [0.3, 0.8)$ would imply higher loss than the min-max policy for ρ_π -values slightly below and including 0.99. Except for these values, the robust-satisficing policies will imply lower relative loss than the min-max policy.

The min-max policy and the robust satisficing policy coincide, i.e. $\Omega_m = \Omega_{0.8}$, if $dL(\Omega_{\tilde{\rho}_\pi}, \rho_\pi) \leq dL_s = 50\%$. Moreover, at a given level of uncertainty, the robust-satisficing policy will coincide with the min-max policy even for $dL(\Omega_{\tilde{\rho}_\pi}, \rho_\pi) > dL_s = 50\%$. This is because any policy different from the min-max policy will imply higher loss than necessary for complete robustness and hence not be selected.

Thus, a min-max policy and a robust satisficing would be observationally equivalent if the robust satisficer may not incur more than the maximum loss level under the min-max policy. The robust-satisficing policy may, however, deviate from the min-max policy and offer higher robustness if relatively lower levels of loss are required, as shown above.

Theoretically, by assuming away uncertainty a min-max policy can be equated to any robust-satisficing policy. Moreover, by raising the acceptable level of loss to dL_m , any robust-satisficing policy can be equated to the min-max policy implying dL_m ; see proposition 3. Such an exercise may be unreasonable and hence seem artificial in practice, though.

Finally, robust-satisficing policies are found to be less aggressive than the min-max policy, in general. Figure 4 shows the response coefficients of inflation, ω_π , in the coefficient vectors Ω_0 – $\Omega_{0.99}$ defining the Taylor-type rule. The figure shows that ω_π increases with ρ_π . The range of ω_π -values corresponding to the robust satisficing policies, which are defined by $\rho_\pi \in [0.3, 0.8]$, is about 3–4.5. Hence, except for the robust-satisficing policy that coincides with the min-max policy, the robust-satisficing policies will be generally less aggressive than the min-max policy, for which $\omega_\pi = 4.5$. In particular, robust-satisficing policy for relatively low levels of loss which would be based on relatively low persistence values, will imply relatively weak response to the inflation gap. The response coefficient of the output gap (ω_y), associated with the response coefficients of the inflation gap (ω_π), displays similar properties. This varies in the range 1.45–2.30 for the robust-satisficing policies (not shown).

the policies are robust-satisficing at several levels of relative losses. This is an artefact of not evaluating policies based on a finer grid of ρ_π . When we use a finer grid, robust-satisficing policies would vary continuously over the range 0.30–0.80 with the different relative loss levels, as shown in Figures 1–3.

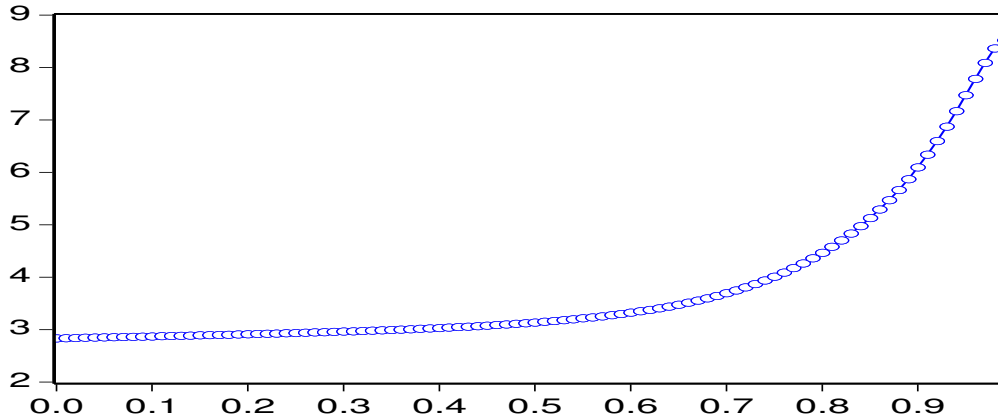


Figure 4: Optimal values of the response coefficient associated with the inflation gap in the Taylor rule, ω_π , conditional on different degrees of persistence in the supply shock $\rho_\pi = 0, 0.01, 0.02, \dots, 0.99$ (horizontal axis).

4.2 Uncertain slope of the Phillips curve

In the following we apply the robust-satisficing approach to the case when the coefficient corresponding to the output gap in the inflation equation (13), c_y , is uncertain. The results regarding uncertainty in the response of the output gap to interest rates in the demand equation (14) were found to be comparable to the case with uncertain slope coefficient c_y and are therefore not reported.

We let the parameter space of c_y be unbounded and investigate the robustness of different policies at different levels of relative loss. For illustration, we evaluate three policies $\Omega_{\tilde{c}_y} = \Omega_{.13}$, $\Omega_{.25}$ and $\Omega_{.30}$, where $\Omega_{\tilde{c}_y}$ is the optimal policy conditional on a specific c_y value.¹⁰ To calculate the robustness of a policy $\Omega_{\tilde{c}_y}$ we find the range and the set of c_y values for which $dL(\Omega_{\tilde{c}_y}, c_y) \leq dL_s$ holds, where $dL_s = 0, 5, 10, 20\%$. The range of c_y -values for which $dL(\Omega_{\tilde{c}_y}, c_y) \leq dL_s$ holds defines the degree of robustness of policy $\Omega_{\tilde{c}_y}$, $\ell(\Omega_{\tilde{c}_y}, dL_s)$, while the corresponding set of c_y -values defines the uncertainty set $\mathcal{U}(\Omega_{\tilde{c}_y}, dL_s)$. Table 3 presents the results where bold-faced numbers correspond to robust-satisficing policies.

Table 3 illustrates the characteristics of robust-satisficing policies consistent with propositions 1–2, as above; see Figures 2.a and 3.a and Table 2. The results are also consistent with the other properties of robust-satisficing policies observed above. In particular, the costs of robustness increase with the level of robustness and to some extent asymmetrically around the parameter values conditioned upon.

In greater detail, among the three policies evaluated, $\Omega_{.30}$ has relatively higher robustness for acceptable loss up to 10%. The policy defined by $\Omega_{.25}$, however, becomes slightly more robust than $\Omega_{.30}$ when the acceptable loss is raised to 20%. The right panel of the table shows the c_y

¹⁰Note that defining the set of feasible policies R is not the dual of defining the parameter space of c_y . A policy maker has more information about the set of feasible policies than the parameter space. Hence, it is not unreasonable to assume that the policy maker compares the robustness of feasible policies, without specifying the boundaries of the parameter space.

Table 3: Robustness and corresponding sets of c_y -values at different levels of dL_s

dL_s	$\widehat{\ell}(\Omega_{c_y}, dL_s)$			$\mathcal{U}(\Omega_{c_y}, dL_s)$		
20	0.310	0.430	0.420	.005–.305	.050–.480	.080–.500
10	0.240	0.315	0.345	.015–.255	.100–.415	.135–.480
5	0.170	0.220	0.245	.050–.220	.145–.365	.180–.425
0	0	0	0	.13	.25	.30
Ω_{c_y}	$\Omega_{.13}$	$\Omega_{.25}$	$\Omega_{.30}$	$\Omega_{.13}$	$\Omega_{.25}$	$\Omega_{.30}$

Note: The policies, which are represented by Ω s containing the response coefficients in the Taylor rule, are optimal conditional on the subscripted values of slope coefficients (c_y). Bold faced c_y -values correspond to the robust-satisficing policies at different levels of acceptable loss (in per cent).

values for which the losses will not exceed the acceptable levels. When the acceptable loss is 0%, i.e. one aspires for the optimal levels, the robustness of all policies is zero as any deviation from the value conditioned on, will lead to a higher loss than aspired. However, by being willing to accept up to 5% deviation from optimal levels, one can raise the robustness of all policies to a quite large range of possible c_y values. We also observe that robustness, i.e. expansion of $\mathcal{U}(\Omega_{c_y}, dL_s)$, does not increase symmetrically around the parameter values conditioned upon.

Moreover, the policy conditioned on $c_y = 0.13$, $\Omega_{0.13}$, which is the econometrically estimated value of 0.13, has relatively lower robustness than the other two policies. In general, a robust-satisficing policy would not be conditioned on the estimated value of a parameter. This is because when we assume Knightian parameter uncertainty, the estimated value of a parameter does not receive more weight than any other parameter value. One may also say that the choice of the parameter value for policy making is based on “strategic” rather than econometric considerations, in the robust-satisficing approach as well in the robust-control approach.

To ease comparison with the min-max policy, let us now assume that the slope coefficient c_y takes on a value in the range $[0.005, 0.5]$, which is fairly broad suggesting a relatively high level of uncertainty ℓ_m . Figure 5 presents robustness curves implied by optimal policies conditional on every value of the slope coefficient in the interval 0.005–0.5, differing from each other by just 0.005.

The results support the characteristics of robust-satisficing policies noted above; see Figure 5 and Table 4, which reports uncertainty sets for selected policies. At a given loss level, the robustness curves in Figure 5 also suggest a strongly concave relationship between robustness and the different values of the slope coefficient on which the policies have been conditioned on.

Specifically, it is seen that policies based on relatively low values of c_y are more robust than those based on relatively higher values of c_y . The robust-satisficing policies are defined by c_y -values in the range $[0.25, 0.44]$. The policy based on $c_y = 0.25$ offers complete robustness to any $c_y \in [0.005, 0.5]$ at a relative loss level of 40%, while the policy based on $c_y = 0.44$ is the robust-satisficing policy at the 1% level of loss and also the least robust policy, among the set of robust-satisficing policies.

The policy based on $c_y = 0.25$ is also the min-max policy for the full range of parameter

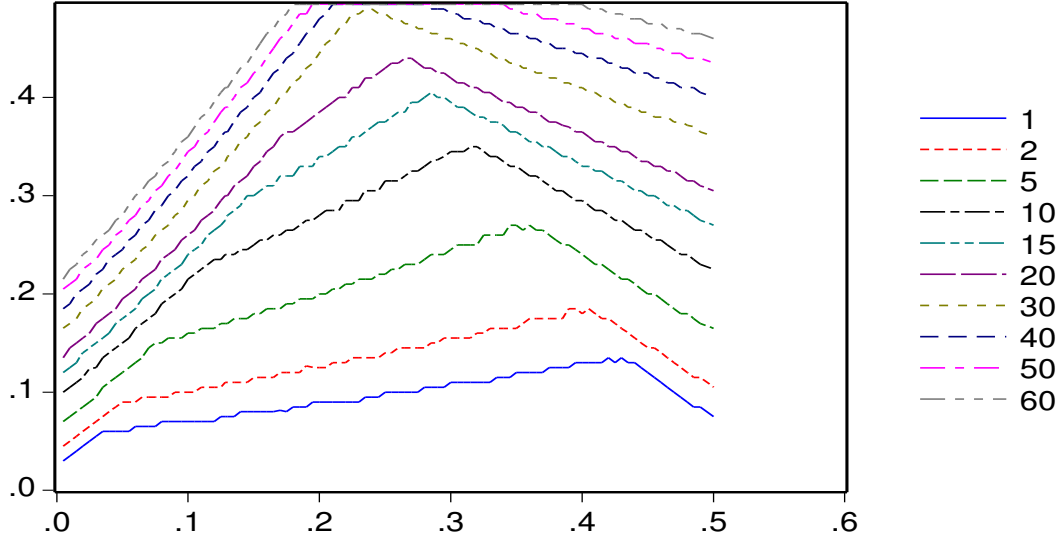


Figure 5: Robustness of all policies considered at different levels of acceptable loss when the slope of the Phillips curve, c_y , is uncertain. Level of robustness $\hat{\ell}(\Omega_{c_y}, dL_s)$ is represented on the vertical axis, while c_y -values are indicated on the horizontal axis. Robustness is measured by the length of an interval containing c_y -values $\in [0.005, 0.50]$. Policies that are optimal conditional on these slope coefficient values are represented by these values on the horizontal axis. Different levels of acceptable loss dL in per cent, are indicated by patterns of the robustness curves.

Table 4: Sets of c_y -values for which selected policies are robust at different levels of dL_s

dL^s	$\mathcal{U}(\Omega_{c_y}, dL_s)$							
40	.005-.190	.005-.370	.005-.5	.015-.5	.035-.5	.055-.5	.100-.5	.100-.5
30	.005-.170	.005-.345	.015-.5	.040-.5	.065-.5	.090-.5	.140-.5	.140-.5
20	.005-.140	.005-.305	.050-.48	.080-.5	.110-.5	.135-.5	.195-.5	.195-.5
15	.005-.125	.005-.285	.075-.450	.105-.5	.135-.5	.170-.5	.230-.5	.230-.5
10	.005-.105	.015-.255	.100-.415	.135-.480	.170-.5	.205-.5	.275-.5	.275-.5
5	.005-.075	.050-.220	.145-.365	.180-.425	.220-.490	.260-.5	.335-.5	.335-.5
2	.005-.050	.075-.185	.180-.320	.225-.380	.270-.435	.310-.490	.395-.5	.395-.5
1	.005-.035	.095-.170	.200-.300	.245-.355	.290-.410	.335-.465	.425-.5	.425-.5
0	0.005	0.13	0.25	0.3	0.35	0.4	0.5	0.5

Note: The policies, which are represented by Ω s containing the response coefficients in the Taylor rule, are optimal conditional on the subscripted values of slope coefficients (c_y). Bold faced sets of c_y -values correspond to the robust-satisficing policies at different levels of acceptable loss (in per cent).

uncertainty conjectured, as any other policy implies relatively higher loss for some of the possible values of c_y within its assumed range. For example, the optimal policies conditional on $c_y = 0.30$ and $c_y = 0.13$ would imply a higher loss than 40% for parameter values in the ranges $[0.005, 0.015]$ and $(0.370, 0.5]$, respectively; see Table 3.

It should be noted that when the level of uncertainty, here represented by the range $[0.005, 0.5]$, is given, the robust-satisficing policies (as well as the min-max) policy may be affected. This is because a rise or reduction in the level of uncertainty, here widening or narrowing of the range, increases or reduces the set of parameter values for which the robustness of a policy is evaluated. This is, however, not the case when the parameter space is unbounded.

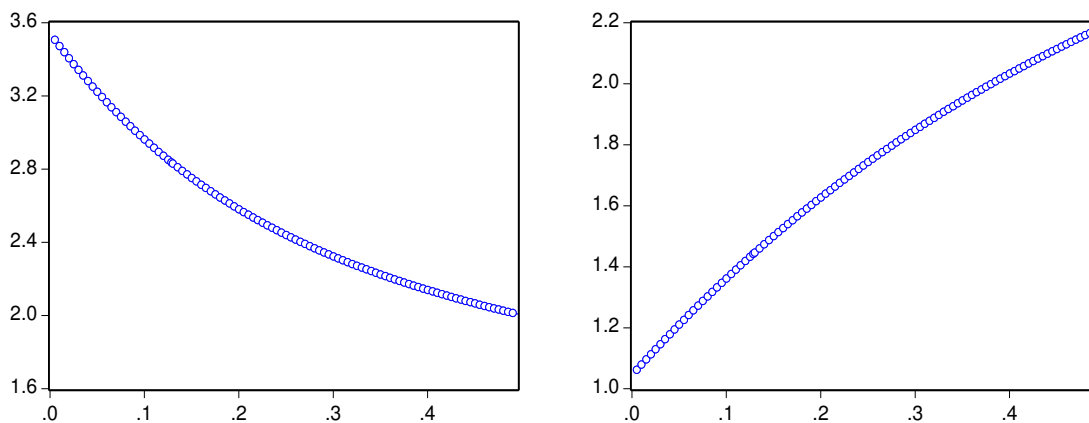


Figure 6: Left frame: Optimal values of the response coefficient associated with the inflation gap in the Taylor rule, ω_π , conditional upon different values of the slope coefficient $c_y = 0.005, 0.01, 0.015, \dots, 0.50$ (horizontal axis). Right frame: Optimal values of the response coefficient associated with the output gap in the Taylor rule, ω_y , conditional on the c_y values (horizontal axis). Subsets of these response coefficients define robust-satisficing policies.

Uncertainty in the slope coefficient implies that the policy is more influenced by the inflation gap than the output gap in the Taylor rule. Figure 6 presents (optimal) values of the response coefficients associated with the inflation and the output gaps. These values are optimal conditional on values of the slope coefficient denoted on the horizontal axes. We note that the response coefficient associated with the inflation gap increases with the quest for robustness while the response coefficient associated the output gap decreases. The response coefficients associated with $c_y \in [0.25, 0.44]$ increase from 2 to 2.5 for the inflation gap and decrease from 2 to 1.3 for the output gap.

The reason for these choices of the response coefficients is that the policy is conditioned on a lower value of the slope coefficient c_y , within the range 0.25–0.45, the higher robustness one seeks. Accordingly, policy becomes more effective if interest rates are less influenced by the output gap than the inflation gap. Thus, the weight on the output gap declines with the values of the slope coefficient conditioned on.

The relatively higher weight on the inflation gap at the expense of relatively lower weight on the output gap in the interest rate rule is consistent with earlier studies based on the min-max approach; see e.g. Smets (2002) and the references therein. However, within the robust-satisficing approach alteration of the weights depends on the how much robustness one seeks, which depends on the acceptable level of loss.

As noted above, results for the case when the interest rate effect on the output gap is uncertain are comparable to those for uncertainty in the slope of the Phillips curves. In the former case,

robustness was increased by basing policy on a relatively weak response of the output gap to interest rates. In both cases, there is uncertainty regarding the policy maker's ability to control inflation. And in both cases, the degree of robustness increases if policy is based on understating the policy maker's ability to control inflation. Moreover, the higher the robustness one seeks, the weaker control does one assume; up to some limit, though, as the extreme case would be to assume no control at all.

4.3 Uncertain persistence in the demand and the supply shocks

This section employs the robust satisficing approach to deal with uncertainty in two parameters, ϱ_π and ϱ_y . In this case, θ becomes a vector and $\mathcal{U}(\ell, \tilde{\theta})$ becomes a two-dimensional parameter space, defined in Appendix A. This space will be defined by the optimal policy based on value $\tilde{\theta}$ of θ , $\Omega_{\tilde{\theta}}$. We represent the robustness of a policy, $\hat{\ell} (= \ell(\Omega_{\tilde{\theta}}, dL_s))$, by the fraction of all possible vectors θ for which the given policy does not imply a higher loss than some specific level dL_s .

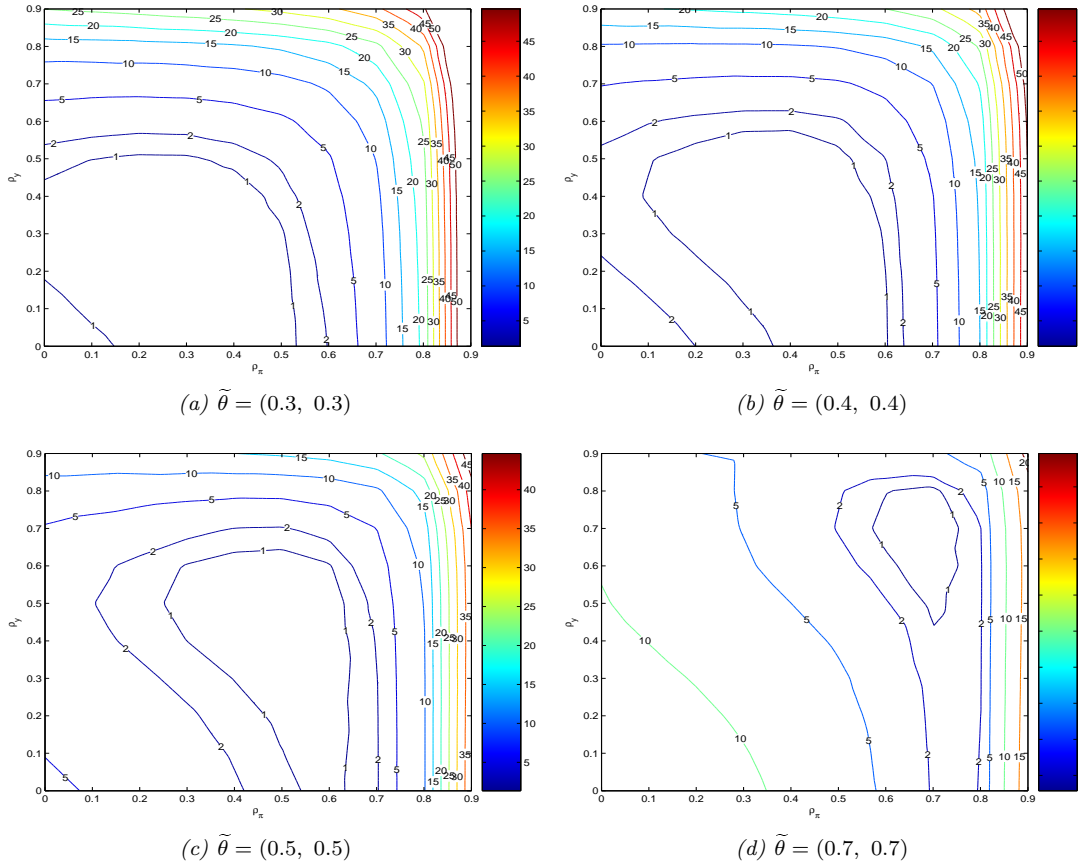


Figure 7: Sets of ρ_π and ρ_y (vertical axis) for which selected policies are robust at different levels of dL_s in per cent, noted on curves defining the boundaries of the sets. The policies are optimal conditional on the given values of ρ_π and ρ_y in parentheses below the figures.

In the two-dimensional case considered here, $\hat{\ell}$ is represented by the fraction of the area defined by all possible parameter values: $\varrho_\pi \times \varrho_\pi$. In our simulations, we let each of the degrees of persistence take on the following values: 0, 0.1, 0.2,...0.9 and evaluate robustness of policies conditional

on 100 different values of the θ vector.

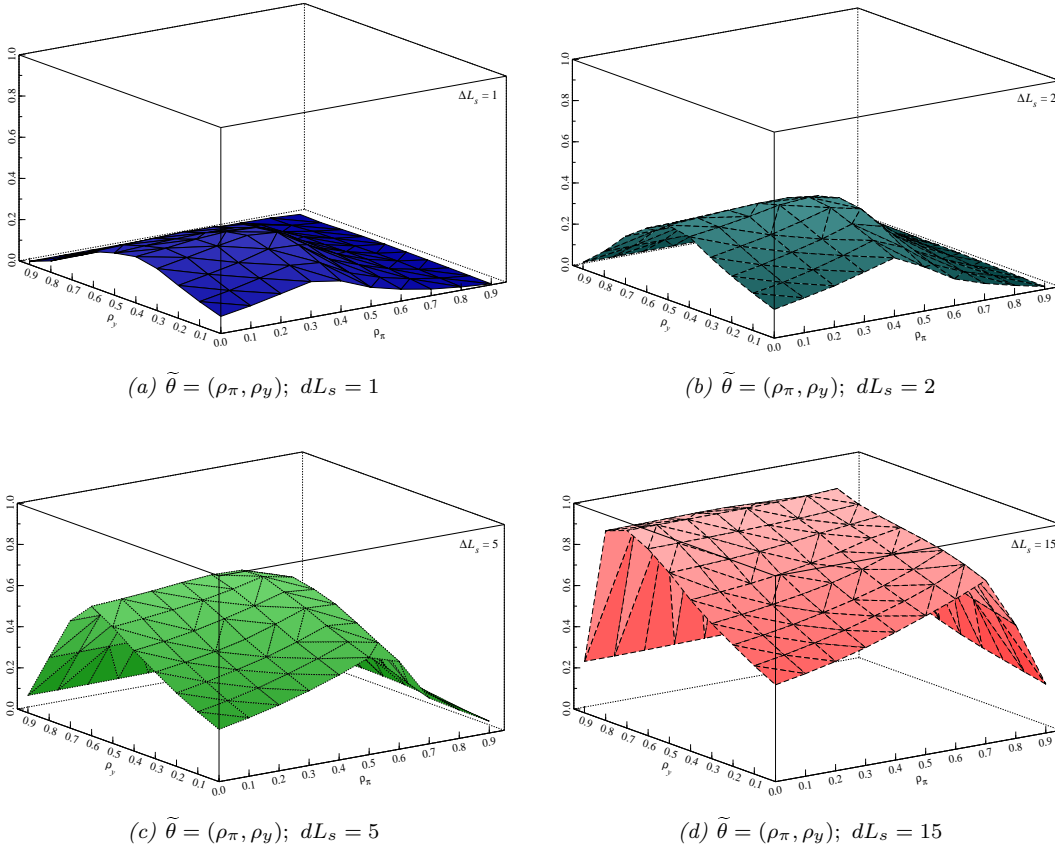


Figure 8: Robustness of all policies considered at indicated levels of acceptable losses. The policies considered are optimal conditional on 100 different combinations of ρ_π and ρ_y values, where both ρ_π and ρ_y take on the following values: 0, 0.1, 0.2, ..., 0.9. The policies are identified by the ρ_π - and ρ_y -values indicated on the axes. Robustness is measured as the share of the 100 different parameter combinations for which the loss remains below the acceptable losses.

Figures 7.a-d show the parameter spaces for which the losses do not exceed acceptable loss levels when policies are conditional on given parameter vectors. The parameter spaces vary highly with the policies as well as with the relative loss levels; see also Figure 8. The figures display the main properties of the robust-satisficing approach consistent with the results for the case of uncertainty in single parameters. For example, $\mathcal{U}(\hat{\ell}, \tilde{\theta})$ representing areas associated with a given robustness level increase with relative loss level dL_s and at a decreasing rate. This suggests increasing costs of robustness. In particular, one would have to accept a relatively large loss to perform satisfactorily under every possible combination of ρ_π - and ρ_y -values within their assumed ranges. In particular, the cost of the robust-satisficing policy coinciding with the min-max policy would be high.

Figures 8.a-d compare the robustness of different policies at selected levels of relative loss. Robustness is portrayed on the z-axis, while the x- and the y-axis indicate the parameter values on which policies $\Omega_{\tilde{\theta}}$ s are conditioned. The surface plots correspond to the strongly concave robustness curves in Figures 1 and 4.

The surface plots display concavity conditional on given loss levels as in the single parameter

cases. We observe that the robustness increases with the level of persistence up to some loss-specific level, as in the case of the uncertainty regarding the supply shock.

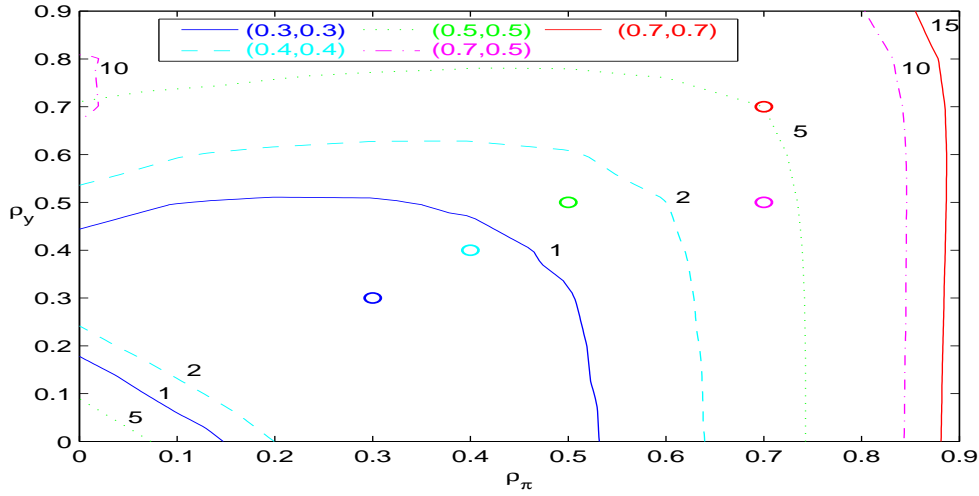


Figure 9: Robustness offered by robust-satisficing policies in terms of sets of ρ_π - and ρ_y -values. The largest set would include all values of $\rho_\pi \in [0, 0.9]$ and $\rho_y \in [0, 0.9]$. The robust-satisficing policies are identified by values of ρ_π and ρ_y presented in parentheses. The levels of acceptable losses are noted close to the boundaries defining the sets. Every set has been selected among 100 different sets implied by the (100) different policies.

The heights of the surface plots representing robustness rise up to some acceptable loss-specific level when policy is conditioned on a relatively higher ρ_π and/or ρ_y . Also, the robustness increases at a decreasing rate as suggested by the concavity of the surface plots.

The robust-satisficing policies at the four different loss levels, 1, 2, 5 and 15%, are defined by the maximum values on these surface plots and are conditioned on the parameter values on the x- and the y-axis. Figure 9 shows the parameter vectors and the corresponding parameter spaces defining robust-satisficing policies at different levels of relative losses. For example, the robust satisficing policies for $dL_s = 1\%$ and 5% would be optimal policies conditional on $(0.3, 0.3)$ and $(0.5, 0.5)$, respectively. We also note that if the acceptable loss level is 15%, the optimal policies defined by $(0.7, 0.7)$ would perform satisfactorily for almost all parameter values within the parameter spaces of ρ_π and ρ_y . This also defines the min-max policy.

Figure 10 shows the policies corresponding to different combinations of parameter values. Values of ω_π are given on the z-axis, while the corresponding persistence values are given on the x- and the y-axis.

Values of the persistence parameters defining robust-satisficing policies are in the range 0.3–0.7 for both ρ_π and ρ_y . In particular, the min-max policy is equals the robust-satisficing policy conditional on $\rho_\pi = 0.7$ and $\rho_y = 0.7$. We note that the min-max policy in the case of uncertainty about persistence in a single shock was based on relatively higher persistence: $\rho_\pi = 0.8$. The

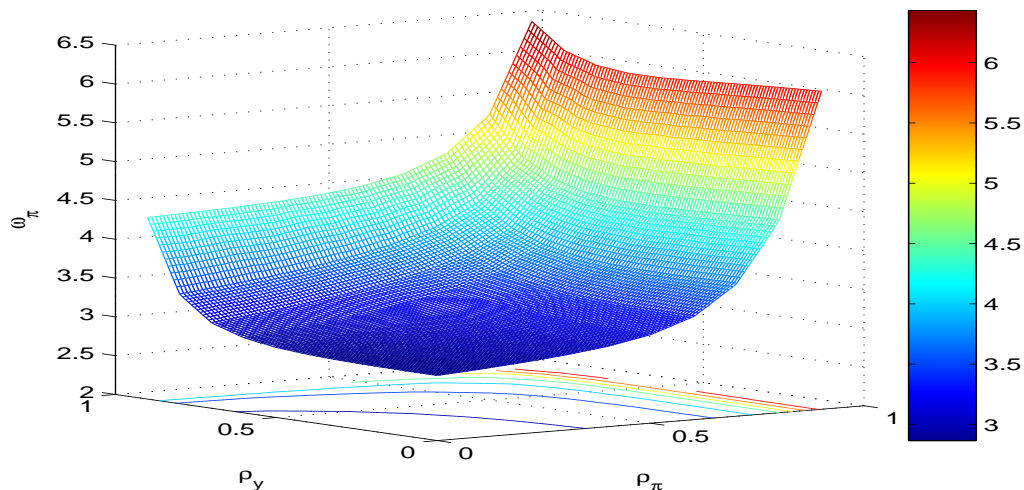


Figure 10: Optimal values of the response coefficient associated with the inflation gap in the Taylor rule, ω_π , conditional on different values ρ_π and ρ_y . Both ρ_π and ρ_y take on the following values: 0, 0.1, 0.2, ..., 0.9. Robust-satisficing policies are identified by ρ_π - and ρ_y -values within the range 0.2, 0.3, ..., 0.7.

reason for the difference between the single parameter case and the two parameter case is that we have not evaluated policies for values of ρ_π and ρ_y higher than 0.9.

5 Conclusions

We have employed the robust-satisficing approach to formulate robust monetary policy under Knightian parameter uncertainty within the framework of a small macroeconomic model. Robustness can be interpreted as the extent of deviations from a policy's underlying premises under which its performance will not deteriorate beyond some acceptable level. The robust-satisficing policy maximizes robustness at a given level of acceptable loss, measured relative to the ex-post optimal loss level. The robust-satisficing approach enables one to focus on attaining satisfactory performance in the preponderance of plausible scenarios, while also managing unbounded and non-probabilistic uncertainty.

The empirical analysis has illustrated key features of the robust-satisficing approach and properties of robustness measures. First, robustness of any given policy, as well as of explicitly robust-satisficing policies, increases with the level of acceptable loss. Second, robust-satisficing policies lead to the evaluation of the range of parameter values within which the performance is acceptable. Third, robustness of a policy increases with the acceptable loss at a decreasing rate. Thus, a relatively high degree of robustness is particularly costly as it requires willingness to accept relatively high levels of potential loss. Fourth, the increase in robustness may not extend equally in all directions in the space of uncertain parameters. For example, the range of parameter values, for which a given policy will not imply higher loss than some accepted loss, may not increase equally

in both directions. Possible asymmetry in the extension depends on the loss function, the model used and the uncertain parameter. And fifth, by raising the level of acceptable loss sufficiently, one can derive a robust-satisficing policy that would be robust to any parameter value from a specified set of parameter values.

By raising the level of acceptable loss sufficiently, one eventually reaches the point at which the robust-satisficing policy coincides with the min-max policy. Equivalently, reducing the min-max estimate of the worst case also causes concurrence of the robust-satisficing and min-max policies. We have referred to this concurrence of min-max with robust-satisficing as observational equivalence. However, when the worst-case is sufficiently large, or the required level of loss is sufficiently low, then min-max and robust-satisficing policies differ, which we have called the behavioral difference. In this situation, the robust-satisficing policy would be generally more robust than the min-max policy for a less-than-complete set of possible parameter values. However, the loss under such robust-satisficing policies would be higher than the loss under the min-max policy for extreme parameter values.

Our empirical results suggest that higher robustness can be achieved by overstating challenges to the economy and understating the abilities to meet them. How much to overstate or understate depends on the robustness sought and the performance aspirations. More precisely, we find that higher robustness can be achieved by basing policy on relatively high degrees of persistence in the shocks and relatively weak effects of the output gap on inflation and of interest rates on the output gap. How much to raise the degree of persistence and lower the effects of the output gap and the interest rate depends on how much robustness one seeks and how much relative loss one can tolerate.

The level of robustness, and thereby the extent of the over- and understatement of the challenges and the abilities, respectively, depends on the level of acceptable loss. In particular, robustness to a broad set of possible parameter values is particularly costly and requires willingness to accept a relatively high level of potential loss. Furthermore, when the sets of possible parameter values have been specified, policies based on the assumption of overly high persistence have relatively low robustness. Similarly, policies based on assuming overly weak response of the inflation to the output gap or of the output gap to interest rates have relatively low robustness.

The main policy implications of parameter uncertainty in terms of the response coefficients in the Taylor-type rule are qualitatively the same as for assuming a relatively high degree of shock persistence and relatively weak responses of the inflation to the output gap and of the output gap to interest rates. These assumptions imply that the response coefficient associated with the inflation gap increases with desired robustness. In the case of uncertainty regarding persistence in the shocks, the response coefficient associated with the output gap also increases with robustness sought. In the case of the response coefficients, however, the output gap coefficient decreases with

the robustness. This is because it is less effective to respond to the output gap as its importance in the model is down played, by assuming weaker and weaker response coefficients. We also observe that the robust-satisficing policies are less aggressive than the min-max, except when they coincide.

Finally, our analysis points to the importance of using available information efficiently to reduce the level of uncertainty. The potential costs of robust policies informed by a high level of uncertainty can be substantial. More information can be helpful in determining how much robustness one actually needs.

A Appendix: Info-Gap Model of Uncertainty

We now formulate the info-gap model, $\mathcal{U}(\ell, \Omega)$, for uncertainty in a vector θ . The analyst has complete Knightian uncertainty: the probability distribution of θ is unknown. One has no estimate, in any statistical sense, of the value of θ .¹¹

Even though the true value of θ is not known, it is still useful to talk about values of θ that would motivate any particular choice of the parameters $\Omega = (\omega_r, \omega_\pi, \omega_y)$ of the policy rule. For instance, if we are considering a specific choice of Ω , one may ask: given our economic models, what should θ be in order to make this a good choice of Ω ? The value of θ which, were it the true value, would justify a particular Ω , will be denoted $\tilde{\theta}(\Omega)$ and is defined by the solution for θ in the relation:

$$\Omega = \tilde{\Omega}(\theta), \tag{20}$$

where $\tilde{\Omega}(\theta)$ is the loss-minimizing policy if the uncertain parameter equals θ , defined in eqs.(6).

We now posit a “weak convexity” of the loss function. We assume that, for any fixed Ω , the loss function $dL(\Omega, \theta)$ rises increasingly above the full-knowledge value, $dL[\Omega, \tilde{\theta}(\Omega)] = 0$, as θ deviates from $\tilde{\theta}(\Omega)$. This does not assert that $dL(\Omega, \theta)$ is convex vs. θ , but only that it has a unique minimum vs. θ , for any given Ω . We posit that this weak convexity holds for $dL(\Omega, \theta)$.

The weak convexity property implies that, for any non-negative bound on the loss function, the corresponding set of θ -values is a connected region, which we define as:

$$\mathcal{D}(x, \Omega) = \{\theta \in \Theta : dL(\Omega, \theta) \leq x\}, \tag{21}$$

where Θ is the set of meaningful values for the vector θ . For any level of loss x , the set of θ values for which the loss (with policy Ω) does not exceed x is the set $\mathcal{D}(x, \Omega)$. Because the loss function has the property of weak convexity, this set is a connected region whose length (or area, or volume, etc., depending on the dimension of θ) we denote by $|\mathcal{D}(x, \Omega)|$. Since $dL(\Omega, \theta) = 0$ at its unique minimum, $\tilde{\theta}(\Omega)$, we note that $\mathcal{D}(0, \Omega) = \{\tilde{\theta}(\Omega)\}$.

¹¹This example could be extended to consider info-gap uncertainty in the probability distribution of θ . Examples of info-gap analysis of uncertainty probability distributions are found in Ben-Haim (2005, 2006).

The weak convexity property implies that the sets $\mathcal{D}(x, \Omega)$ are nested:

$$x < x' \quad \text{implies} \quad \mathcal{D}(x, \Omega) \subseteq \mathcal{D}(x', \Omega). \quad (22)$$

This states that any value, θ , whose loss is no less than x , also has loss no less than x' , when the same policy is used.

We use this concept to define an info-gap model for uncertainty in θ . For any contemplated policy parameters Ω , the info-gap model is the following family of nested sets of uncertain θ -values:

$$\mathcal{U}(\ell, \Omega) = \{\theta : \theta \in \mathcal{D}(x, \Omega), x \geq 0, |\mathcal{D}(x, \Omega)| \leq \ell\}, \quad \ell \geq 0. \quad (23)$$

At any level of uncertainty ℓ , the set $\mathcal{U}(\ell, \Omega)$ contains all values θ which belong to sets $\mathcal{D}(x, \Omega)$ no larger than ℓ , regardless of the loss value x . The level of uncertainty is unknown so ℓ can take any non-negative value. The info-gap model is a family of nested sets and obeys the axioms of contraction and nesting (Ben-Haim 2006).

In light of eq.(22) we see that, for any ℓ , there exists an $x(\ell)$ such that:

$$\mathcal{U}(\ell, \Omega) = \mathcal{D}(x(\ell), \Omega). \quad (24)$$

Combining eqs.(21) and (24) we see that the uncertainty set, evaluated at a level of uncertainty equal to the robustness, is:

$$\mathcal{U}[\widehat{\ell}(\Omega, dL_s), \Omega] = \mathcal{D}(dL_s, \Omega). \quad (25)$$

We emphasize that this info-gap model depends on a contemplated policy parameters Ω . The sets $\mathcal{U}(\ell, \Omega)$ are an expression of epistemic (rather than objective or ontological or aleatoric) uncertainty: if we use policy Ω , then the info-gap model contains all θ -values for which the loss will not exceed some specific value. We don't know which θ value will occur or the value of ℓ , so there is no known worst case (other than the limits on meaningful values of θ , such as unbounded persistence, expressed by the set Θ).

References

- Angeloni, I., G. Coenen and F. Smets, 2003, Persistence, the transmission mechanism and robust monetary policy, *Scottish Journal of Political Economy*, 50: 527-549.
- Ben-Haim, Y., 2000, Robust rationality and decisions under severe uncertainty, *Journal of the Franklin Institute*, 337: 171-199.
- Ben-Haim, Y., Akram, Q.F. and Eitrheim, Ø., 2007, Monetary policy under uncertainty: Min-

- max vs robust-satisficing strategies. Working paper ANO 2007/6. Oslo, Norges Bank.
- Ben-Haim, Y., 2005, Value at risk with info-gap uncertainty, *Journal of Risk Finance*, vol. 6, #5, pp.388–403.
- Ben-Haim, Y., 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty*, 2nd edition, Academic Press, London.
- Carmel, Y. and Ben-Haim, Y., 2005, Info-gap robust-satisficing model of foraging behavior: Do foragers optimize or satisfice?, *American Naturalist*, vol.166: 633–641.
- Coenen, G., 2007, Inflation persistence and robust monetary policy design, *Journal of Economic Dynamics & Control*, 31: 111-140.
- Cogley, T. and Sargent, T.J., 2005, The conquest of US inflation: Learning and robustness to model uncertainty, *Review of Economic Dynamics*, 8: 528–563.
- Epstein, L.G. and Miao, J., 2003, A two-person dynamic equilibrium under ambiguity, *Journal of Economic Dynamics and Control*, 27: 1253–1288.
- Giannoni, M.P., 2002, Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model, *Macroeconomic Dynamics*, 6: 111–144.
- Gilboa, I., and Schmeidler, D., 1989, Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics*, 18 141—153.
- Halpern, B.S., Regan, H.M., Possingham H.P. and McCarthy, M.A., 2006, Accounting for uncertainty in marine reserve design, *Ecology Letters*, 9: 2–11.
- Hansen, L.P., Maenhout, P., Rustichini, A., Sargent, T.J. and Siniscalchi, M.M., 2006, Introduction to model uncertainty and robustness, *Journal of Economic Theory*, 128:1–3.
- Hansen, L.P. and Sargent, T.J., 2001, Acknowledging misspecification in macroeconomic theory, *Review of Economic Dynamics*, 4: 519–535.
- Knight, F.H., 1921, *Risk, Uncertainty and Profit*, Hart, Schaffner and Marx. Re-issued by Harper Torchbooks, New York, 1965.
- Leitemo, K. and Söderström, U., 2004, Robust monetary policy in the New-Keynesian framework, Discussion Paper No. 4805, Centre for Economic Policy Research. Forthcoming, *Macroeconomic Dynamics*.
- Levin, A. and Williams, J., 2003, Robust monetary policy with competing reference models, *Journal of Monetary Economics*, 50: 945–975.

- Moffitt, L.J., Stranlund, J.K. and Field, B.C., 2005, Inspections to Avert Terrorism: Robustness Under Severe Uncertainty, *Journal of Homeland Security and Emergency Management*, Vol.2, #3. <http://www.bepress.com/jhsem/vol2/iss3/3>
- Moffitt, L.J., Stranlund, J.K. and Osteen, C.D., 2007, Robust detection protocols for uncertain introductions of invasive species, *Journal of Environmental Management*, to appear.
- Moilanen, A. and Wintle, B.A., 2006, Uncertainty analysis favours selection of spatially aggregated reserve structures, *Biological Conservation*, Volume 129, Issue 3, May 2006, Pages 427–434.
- Onatski, A. and Williams, N., 2003, Modeling model uncertainty, *Journal of the European Economic Association*, 1: 1087–1122.
- Pierce, S.G., Ben-Haim, Y., Worden, K. and Manson, G., 2006, Evaluation of neural network robust reliability using information-gap theory, *IEEE Transactions on Neural Networks*, vol.17, #6, pp.1349–1361.
- Regev, S., Shtub, A. and Ben-Haim, Y., 2006, Managing project risks as knowledge gaps, *Project Management Journal*, vol.37, #5, pp.17–25.
- Smets, F., 2002, Output gap uncertainty: Does it matter for the Taylor rule? *Empirical Economics*, vol. 27, #1, pp.113-129.
- Simon, A.H., 1959, Theories of decision-making in economics and behavioral science, *American Economic Review*, 49: 253–283.
- Simon, A.H., 1979, Rational decision making in business organizations, *American Economic Review*, 69: 493–513.
- Stranlund, J.K. and Ben-Haim, Y., 2007, Price-based vs. quantity-based environmental regulation under Knightian uncertainty: An info-gap robust satisficing perspective, *Journal of Environmental Management*, forthcoming.
- Taylor, J.B. (ed.), 1999, *Monetary Policy Rules, A National Bureau of Economic Research Conference Report*, University of Chicago Press, Chicago.
- Tetlow, R. and von zur Muehlen, P., 2001, Robust monetary policy with misspecified models: does model uncertainty always call for attenuated policy?, *Journal of Economic Dynamics and Control*, 25 (6-7): 911–949.
- Rudebusch, G.D. and Svensson, L.E.O., 1999, Policy rules for inflation targeting, in J. Taylor, ed., *Policy Rules for Inflation Targeting*, University of Chicago Press for NBER, pp.203–246.
- Vinot, P., Cogan, S. and Cipolla, V., 2005, A robust model-based test planning procedure, *Journal of Sound and Vibration*, 288: 571–585.

WORKING PAPERS (ANO) FROM NORGES BANK 2003-2007

Working Papers were previously issued as Arbeidsnotater from Norges Bank, see Norges Bank's website <http://www.norges-bank.no>

- 2003/1 Solveig Erlandsen
Age structure effects and consumption in Norway, 1968(3) – 1998(4) Research Department, 27 p
- 2003/2 Bjørn Bakke og Asbjørn Enge
Risiko i det norske betalingssystemet
Avdeling for finansiell infrastruktur og betalingssystemer, 15 s
- 2003/3 Egil Matsen and Ragnar Torvik
Optimal Dutch Disease Research Department, 26 p
- 2003/4 Ida Wolden Bache
Critical Realism and Econometrics Research Department, 18 p
- 2003/5 David B. Humphrey and Bent Vale
Scale economies, bank mergers, and electronic payments: A spline function approach
Research Department, 34 p
- 2003/6 Harald Moen
Nåverdien av statens investeringer i og støtte til norske banker
Avdeling for finansiell analyse og struktur, 24 s
- 2003/7 Geir H.Bjønnes, Dagfinn Rime and Haakon O.Aa. Solheim
Volume and volatility in the FX market: Does it matter who you are? Research Department, 24 p
- 2003/8 Olaf Gresvik and Grete Øwre
Costs and Income in the Norwegian Payment System 2001. An application of the Activity Based Costing framework Financial Infrastructure and Payment Systems Department, 51 p
- 2003/9 Randi Næs and Johannes A.Skjeltorp
Volume Strategic Investor Behaviour and the Volume-Volatility Relation in Equity Markets
Research Department, 43 p
- 2003/10 Geir Høidal Bjønnes and Dagfinn Rime
Dealer Behavior and Trading Systems in Foreign Exchange Markets Research Department, 32 p
- 2003/11 Kjersti-Gro Lindquist
Banks' buffer capital: How important is risk Research Department, 31 p
- 2004/1 Tommy Sveen and Lutz Weinke
Pitfalls in the Modelling of Forward-Looking Price Setting and Investment Decisions
Research Department, 27 p
- 2004/2 Olga Andreeva
Aggregate bankruptcy probabilities and their role in explaining banks' loan losses
Research Department, 44 p
- 2004/3 Tommy Sveen and Lutz Weinke
New Perspectives on Capital and Sticky Prices Research Department, 23 p
- 2004/4 Gunnar Bårdsen, Jurgen Doornik and Jan Tore Klovland
A European-type wage equation from an American-style labor market: Evidence from a panel of Norwegian manufacturing industries in the 1930s Research Department, 22 p
- 2004/5 Steinar Holden and Fredrik Wulfsberg
Downward Nominal Wage Rigidity in Europe Research Department, 33 p
- 2004/6 Randi Næs
Ownership Structure and Stock Market Liquidity Research Department, 50 p
- 2004/7 Johannes A. Skjeltorp and Bernt-Arne Ødegaard
The ownership structure of repurchasing firms Research Department, 54 p
- 2004/8 Johannes A. Skjeltorp
The market impact and timing of open market share repurchases in Norway
Research Department, 51 p

- 2004/9 Christopher Bowdler and Eilev S. Jansen
Testing for a time-varying price-cost markup in the Euro area inflation process Research Department, 19 p
- 2004/10 Eilev S. Jansen
Modelling inflation in the Euro Area Research Department, 49 p
- 2004/11 Claudia M. Buch, John C. Driscoll, and Charlotte Østergaard
Cross-Border Diversification in Bank Asset Portfolios Research Department, 39 p
- 2004/12 Tommy Sveen and Lutz Weinke
Firm-Specific Investment, Sticky Prices, and the Taylor Principle Research Department, 23 p
- 2004/13 Geir Høidal Bjønnes, Dagfinn Rime and Haakon O.Aa. Solheim
Liquidity provision in the overnight foreign exchange market Research Department, 33 p
- 2004/14 Steinar Holden
Wage formation under low inflation Research Department, 25 p
- 2004/15 Roger Hammersland
Large T and small N: A three-step approach to the identification of cointegrating relationships in time series models with a small cross-sectional dimension Research Department, 66 p
- 2004/16 Q. Farooq Akram
Oil wealth and real exchange rates: The FEER for Norway Research Department, 31 p
- 2004/17 Q. Farooq Akram
En effisient handlingsregel for bruk av petroleumsinntekter Forskningsavdelingen, 40 s
- 2004/18 Egil Matsen, Tommy Sveen and Ragnar Torvik
Savers, Spenders and Fiscal Policy in a Small Open Economy Research Department, 31 p
- 2004/19 Roger Hammersland
The degree of independence in European goods markets: An I(2) analysis of German and Norwegian trade data Research Department, 45 p
- 2004/20 Roger Hammersland
Who was in the driving seat in Europe during the nineties, International financial markets or the BUBA? Research Department, 35 p
- 2004/21 Øyvind Eitrheim and Solveig K. Erlandsen
House prices in Norway 1819–1989 Research Department, 35 p
- 2004/22 Solveig Erlandsen and Ragnar Nymo
Consumption and population age structure Research Department, 22 p
- 2005/1 Q. Farooq Akram
Efficient consumption of revenues from natural resources – An application to Norwegian petroleum revenues Research Department, 33 p
- 2005/2 Q. Farooq Akram, Øyvind Eitrheim and Lucio Saro
Non-linear dynamics in output, real exchange rates and real money balances: Norway, 1830-2003 Research Department, 53 p
- 2005/3 Carl Andreas Claussen and Øistein Røisland
Collective economic decisions and the discursive dilemma Monetary Policy Department, 21 p
- 2005/4 Øistein Røisland
Inflation inertia and the optimal hybrid inflation/price level target Monetary Policy Department, 8 p
- 2005/5 Ragna Alstadheim
Is the price level in Norway determined by fiscal policy? Research Department, 21 p
- 2005/6 Tommy Sveen and Lutz Weinke
Is lumpy investment really irrelevant for the business cycle? Research Department, 26 p
- 2005/7 Bjørn-Roger Wilhelmsen and Andrea Zaghini
Monetary policy predictability in the euro area: An international comparison Economics Department, 28 p
- 2005/8 Moshe Kim, Eirik Gaard Kristiansen and Bent Vale

	<i>What determines banks' market power? Akerlof versus Herfindahl</i>	Research Department, 38 p
2005/9	Q. Farooq Akram, Gunnar Bårdsen and Øyvind Eitrheim <i>Monetary policy and asset prices: To respond or not?</i>	Research Department, 28 p
2005/10	Eirik Gard Kristiansen <i>Strategic bank monitoring and firms' debt structure</i>	Research Department, 35 p
2005/11	Hilde C. Bjørnland <i>Monetary policy and the illusionary exchange rate puzzle</i>	Research Department, 30 p
2005/12	Q. Farooq Akram, Dagfinn Rime and Lucio Sarno <i>Arbitrage in the foreign exchange market: Turning on the microscope</i>	Research Department, 43 p
2005/13	Geir H. Bjønnes, Steinar Holden, Dagfinn Rime and Haakon O.Aa. Solheim <i>"Large" vs. "small" players: A closer look at the dynamics of speculative attacks</i>	Research Department, 31 p
2005/14	Julien Garnier and Bjørn-Roger Wilhelmsen <i>The natural real interest rate and the output gap in the euro area: A joint estimation</i>	Economics Department, 27 p
2005/15	Egil Matsen <i>Portfolio choice when managers control returns</i>	Research Department, 31 p
2005/16	Hilde C. Bjørnland <i>Monetary policy and exchange rate interactions in a small open economy</i>	Research Department, 28 p
2006/1	Gunnar Bårdsen, Kjersti-Gro Lindquist and Dimitrios P. Tsomocos <i>Evaluation of macroeconomic models for financial stability analysis</i>	Financial Markets Department, 45 p
2006/2	Hilde C. Bjørnland, Leif Brubakk and Anne Sofie Jore <i>Forecasting inflation with an uncertain output gap</i>	Economics Department, 37 p
2006/3	Ragna Alstadheim and Dale Henderson <i>Price-level determinacy, lower bounds on the nominal interest rate, and liquidity traps</i>	Research Department, 34 p
2006/4	Tommy Sveen and Lutz Weinke <i>Firm-specific capital and welfare</i>	Research Department, 34 p
2006/5	Jan F. Qvigstad <i>When does an interest rate path „look good“? Criteria for an appropriate future interest rate path</i>	Norges Bank Monetary Policy, 20 p
2006/6	Tommy Sveen and Lutz Weinke <i>Firm-specific capital, nominal rigidities, and the Taylor principle</i>	Research Department, 23 p
2006/7	Q. Farooq Akram and Øyvind Eitrheim <i>Flexible inflation targeting and financial stability: Is it enough to stabilise inflation and output?</i>	Research Department, 27 p
2006/8	Q. Farooq Akram, Gunnar Bårdsen and Kjersti-Gro Lindquist <i>Pursuing financial stability under an inflation-targeting regime</i>	Research Department, 29 p
2006/9	Yuliya Demyanyk, Charlotte Ostergaard and Bent E. Sørensen <i>U.S. banking deregulation, small businesses, and interstate insurance of personal income</i>	Research Department, 57 p

- 2006/10 Q. Farooq Akram, Yakov Ben-Haim and Øyvind Eitrheim
Managing uncertainty through robust-satisficing monetary policy Research Department, 33 p
- 2006/11 Gisle James Natvik:
Government spending and the Taylor pinciple Research Department, 41 p
- 2006/12 Kjell Bjørn Nordal:
Banks' optimal implementation strategies for a risk sensitive regulatory capital rule: a real options and signalling approach Research Department, 36 p
- 2006/13 Q. Farooq Akram and Ragnar Nymoen
Model selection for monetary policy analysis – importance of empirical validity Research Department, 37 p
- 2007/1 Steinar Holden and Fredrik Wulfsberg
Are real wages rigid downwards? Research Department, 44 p
- 2007/2 Dagfinn Rime, Lucio Sarno and Elvira Sojli
Exchange rate forecasting, order flow and macroeconomic information Research Department, 43 p
- 2007/3 Lorán Chollete, Randi Næs and Johannes A. Skjeltorp
What captures liquidity risk? A comparison of trade and order based liquidity factors Research Department, 45 p
- 2007/4 Moshe Kim, Eirik Gaard Kristiansen and Bent Vale
Life-cycle patterns of interest rate markups in small firm finance Research Department, 42 p
- 2007/5 Francesco Furlanetto and Martin Seneca
Rule-of-thumb consumers, productivity and hours Research Department, 41 p
- 2007/6 Yakov Ben-Haim, Q. Farooq Akram and Øyvind Eitrheim
Monetary policy under uncertainty: Min-max vs robust-satisficing strategies Research Department, 28 p
- 2007/7 Carl Andreas Claussen and Øistein Røisland
Aggregating judgments on dependent variables: an (im)possibility result Research Department, 17 p
- 2007/8 Randi Næs, Johannes Skjeltorp og Bernt Arne Ødegaard
Hvilke faktorer driver kursutviklingen på Oslo Børs? Forskningsavdelingen, 68 s
- 2007/9 Knut Are Astveit and Tørres G. Trovik
Nowcasting Norwegian GDP: The role of asset prices in a small open economy Research Department, 29 p
- 2007/10 Hilde C. Bjørnland, Kai Leitemo and Junior Maih
Estimating the natural rates in a simple new Keynesian framework Economics Department, 33 p
- 2007/11 Randi Næs and Bernt Arne Ødegaard
Liquidity and asset pricing: Evidence on the role of investor holding period Research Department, 31 p
- 2007/12 Ida Wolden Bache
Assessing estimates of the exchange rate pass-through Research Department, 60 p
- 2007/13 Q. Farooq Akram
What horizon for targeting inflation? Research Department, 45 p
- 2007/14 Q. Farooq Akram, Yakov Ben-Haim and Øyvind Eitrheim
Robust-satisficing monetary policy under parameter uncertainty Research Department, 33 p

KEY WORDS:

Robust monetary policy
Knightian uncertainty
Parameter uncertainty
Info-gap decision theory