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by

Ragna Alstadheim and Dale Henderson

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Price-Level Determinacy, Lower Bounds on the Nominal Interest Rate, and Liquidity Traps

Ragna Alstadheim
Norges Bank

Dale Henderson*
Federal Reserve Board

March 30, 2006

Abstract

We consider standard monetary-policy rules with inflation-rate targets and interest-rate or money-growth instruments using a flexible-price, perfect-foresight model. There is always a locally-unique target equilibrium. There are also below-target equilibria (BTE) with inflation always below target and constant or asymptotically approaching or eventually reaching a below-target value. Liquidity traps are neither necessary nor sufficient for BTE which can arise if monetary policy keeps the interest rate above a lower bound. We construct monetary-policy rules that preclude BTE, some of which are monotonic in inflation but all of which are non-differentiable at a point. For standard monetary-policy rules, there are plausible fiscal policies that insure uniqueness by precluding BTE; those policies exclude perpetual surpluses and, possibly, perpetual balanced budgets.

Keywords: Zero bound, liquidity trap, inflation targeting, determinacy
JEL-codes: E31, E41, E52, E62

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1 Introduction

In this paper we discuss a price-level indeterminacy problem caused by a lower bound on the (nominal) interest rate. The lower bound may arise either because of the behavior of (private) agents or because of monetary policy. For generality and relevance, our analysis is conducted in terms of the inflation rate instead of the price level.[1] In our terminology, a model exhibits (inflation-rate) indeterminacy if it has multiple equilibria.

Determinacy in flexible-price models is of both theoretical and practical interest. As regards theory, price-level determinacy is a standard topic. Furthermore, in models with synchronized price contracts, agents must be able to determine what expected inflation would be under price flexibility in order to set their contract prices. As regards practice, the Japanese experience with deflation and zero short-term interest rates, makes it more urgent to ascertain whether the existence of multiple equilibria is more than a theoretical curiosum.

We illustrate, modify, and extend recent analysis of the indeterminacy problem using a perfect-foresight, superneutral model with flexible prices which may have a liquidity trap. As conventionally defined, a liquidity trap is a region of the money-demand function in which bonds and money are perfect substitutes so that open-market operations in bonds cannot lower the nominal interest rate any further.[2] In our model, a liquidity trap may arise at a zero or at a strictly positive interest rate.

The indeterminacy problem considered in this paper is distinct from the well-known stabilization problem highlighted by Keynes.[3] A lower bound, such as a conventional liquidity trap, can keep the nominal interest rate from being reduced as much as necessary to meet stabilization objectives, at least for a while.[4] Many have argued that Japan faced just such a stabilization problem for several years beginning in the mid-1990s.

[1] Of course, inflation-rate determinacy and price-level determinacy are linked. If the inflation rate (defined as the percentage change in the price level between today and yesterday) is determined and yesterday's price level is known, then today's price level is determined.

[2] McCallum (2001) refers to a 'liquidity trap situation' as a situation in which 'the (usual) interest rate instrument is immobilized'. Svensson (2000) refers to a liquidity trap as a situation with a binding zero lower bound on the nominal interest rate. Krugman (1998) refers to a liquidity trap as a situation where 'monetary policy loses its grip because the nominal interest rate is essentially zero [and] the quantity of money becomes irrelevant because money and bonds are essentially perfect substitutes'. Our definition is the same as Krugman's except that, like Sargent (1987) among others, we explicitly allow for a liquidity trap at a positive interest rate.

[3] Eggertsson and Woodford (2003a) make particularly clear the distinction between the two kinds of problems.

[4] For example, suppose there is a temporary negative demand shock in an economy with one-period nominal wage contracts, flexible prices, a liquidity trap at a zero nominal interest rate, and strict targeting of a zero inflation rate. Suppose the shock is large enough that a negative expected real interest rate is required to keep output from falling below potential. Expected future inflation is zero. Because of the liquidity trap, the nominal interest rate cannot fall below zero, so output and inflation objectives cannot be achieved.

Recently, the indeterminacy problem has received much attention. Models with standard interest-rate rules or money-growth rules and a locally unique steady-state target equilibrium (TE) for the inflation rate may have additional equilibria. To be more precise, there may be multiple equilibrium paths along which the inflation rate is always below target and is constant or either asymptotically approaches or eventually reaches a below-target value.[5]Others have referred to these equilibria as "deflationary-trap equilibria" or "liquidity-trap equilibria".[6] We prefer to call them below-target equilibria (BTE) because they need not involve deflation and can arise without a liquidity trap.

It is useful to summarize what we do. We present integrated derivations of the central results regarding indeterminacy given the existence of a lower bound on the interest rate.[7] We distinguish clearly between a lower bound that arises because of monetary policy and one that arises because agents are in a liquidity trap. It may be surprising to some that a liquidity trap is neither necessary nor sufficient for BTE.

Benhabib, Schmitt-Grohe, and Uribe (2001c) present an interest-rate rule that precludes BTE. Under this rule the interest rate responds only to current inflation. It is unusual in that it is non-monotonic in inflation. We present an interest-rate rule that we find considerably less unusual that also precludes BTE. The interest rate is monotonically increasing in both current and expected future inflation. It is asymmetric: the interest rate responds more strongly to expected future inflation if the current inflation rate is below the target rate. When either of the two rules is combined with another relationship, the result is a difference equation in inflation. Benhabib, Schmitt-Grohe, and Uribe (2001c) noticed that for uniqueness the implied difference equations must be non-monotonic. What no one has noticed, as far as we know, is that they must also be non-differentiable at a point.

In addition, we provide a parsimonious account of how determinacy depends on fiscal policy.[8] For simplicity, we characterize fiscal policy by the growth rate of total nominal government debt.[9] There is always a growth rate of debt high enough to

[5] A 1999 version of Woodford (2003) cited by Benhabib, Schmitt-Grohe, and Uribe (2001a) contains an explanation of the possible existence of BTE with both interest-rate rules and money-supply rules. In Woodford (2003) and Benhabib, Schmitt-Grohe, and Uribe (2001c) the inflation rate may asymptotically approach a below-target steady state; in Schmitt-Grohe and Uribe (2000), Benhabib, Schmitt-Grohe, and Uribe (2001a) and Eggertsson and Woodford (2003b) it may eventually reach a below-target steady state.

[6] Eggertsson and Woodford (2003a, 2003b) use deflationary-trap, but Benhabib, Schmitt-Grohe, and Uribe (2001a, 2000c) use liquidity-trap.

[7] Since our model has flexible prices and exhibits superneutrality, a lower bound can give rise to only nominal indeterminacy. However, using a model with sticky prices, Benhabib, Schmitt-Grohe, and Uribe (2001c) show that a lower bound can give rise not only to nominal indeterminacy but also to real indeterminacy.

[8] Woodford (2001) and Benhabib, Schmitt-Grohe, and Uribe (2001a) consider fiscal policy. Schmitt-Grohe and Uribe (2000) analyze balanced-budget fiscal policy.

[9] The characterization of fiscal policy in Eggertsson and Woodford (2003b) is more general and includes ours as a special case.

preclude BTE no matter what the monetary policy because BTE paths would violate the transversality condition. As an illustration, a balanced-budget fiscal policy (a zero growth rate of debt) precludes BTE in which the interest rate is always at a zero lower bound. Within a range, the combination of a small deficit with a standard interest-rate or money-supply rule guarantees that the TE is the unique equilibrium because nominal debt growth precludes BTE.

In the next section we lay out our model and discuss two specific money-demand functions. Section 3 is a presentation of some results regarding the existence of BTE with interest-rate rules. We discuss indeterminacy under money-growth rules in section 4. In section 5, we present monetary-policy rules that assure determinacy. Section 6 is a discussion of some implications of fiscal policy for determinacy. Concluding remarks are provided in section 7.

2 The model

2.1 Agents

Our model economy is populated by a continuum of agents each of which acts simultaneously as a consumer and a producer. For simplicity, we assume that the product market is perfectly competitive, that prices are flexible, and that agents have perfect foresight. The problem of each agent is to find the

$$\underset{C_t,B_t,M_t,Y_t}{Max}\sum_{t=0}^{\infty}\beta^t\left\{U(C_t)+V(m_t)-(1/2)\,{Y_t}^2\right\}$$
$$U'(C_t)\;>\;0,\quad U''(C_t)<0,\quad m_t=M_t/P_t,\quad V'(m_t)\geq 0,\quad V''(m_t)\leq 0 \tag{1}$$

subject to the following period budget and positivity constraints:

$$P_tY_t+M_{t-1}+I_{t-1}B_{t-1}=T_t+P_tC_t+M_t+B_t$$
$$C_t>0, M_t>0, P_t>0,\qquad \forall\; t \tag{2}$$

Period utility is increasing in consumption, weakly increasing in real (money) balances m_t, and decreasing in output.[10]The agent takes as given the money price of goods (P_t) and the gross nominal interest rate (I_t) earned on a bond held from period t to period $t+1$, and chooses holdings of two nominal financial assets, money (M_t) and bonds (B_t); consumption (C_t); and output (Y_t). According to the period budget constraint, nominal income from production in this period plus nominal money balances and bond holdings inclusive of interest from last period must equal tax payments T_t plus

[10]We consider specific functional forms for $V(m_t)$ in section 2.3.

the sum of consumption and money and bond holdings for this period. In addition, each agent and, therefore, agents as a group are subject to a no-Ponzi-game condition:

$$\lim_{t\to\infty}(M_t + B_t)\Pi_{k=0}^{k=t-1} I_k^{-1} \geq 0, \quad \Pi_{k=0}^{k=-1} I_k^{-1} \equiv 1 \tag{3}$$

where $M_t + B_t$ is their net (nominal) financial assets in period t.

To simplify exposition, in sections 2 - 6 we express the nominal interest rate, the real interest rate and inflation rate in gross terms and refer to them as 'the interest rate', 'the real interest rate', and 'the inflation rate' respectively. In the introductory and concluding sections, we refer to the net nominal interest rate as 'the interest rate' in order to facilitate comparison of our results to those of others.

Three necessary conditions for an optimum are

$$U'(C_t) = \beta U'(C_{t+1})\frac{I_t}{\Pi_{t+1}} \qquad \text{(bonds)} \tag{4}$$

$$U'(C_t) = V'(m_t) + \beta U'(C_{t+1})\frac{1}{\Pi_{t+1}} \qquad \text{(money)} \tag{5}$$

$$C_t = U'(C_t) \qquad \text{(output)} \tag{6}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the (backward looking) gross inflation rate. A fourth necessary condition (the transversality condition) is that (3) hold with equality

$$\lim_{t\to\infty}(M_t + B_t)\Pi_{k=0}^{k=t-1} I_k^{-1} = 0, \tag{7}$$

Informally, since the marginal utility of consumption is always positive, it cannot be optimal for the present value of agents' 'end of horizon' net financial assets to be strictly positive. The first order conditions have been written as equilibrium conditions: Y_t has been set equal to C_t as it must be in equilibrium since there is no government spending, and desired asset stocks have been set equal to actual asset stocks.

The four equilibrium conditions (4), (5), (6), and (7) reduce to the Fisher equation, the money market equilibrium condition, the output determination equation, and the transversality condition:

$$I_t = R\Pi_{t+1} \tag{8}$$

$$I_t = \frac{U'(\bar{C})}{U'(\bar{C}) - V'(m_t)} \tag{9}$$

$$\bar{C} = U'(\bar{C}) \tag{10}$$

$$\lim_{t\to\infty}(M_t + B_t)\Pi_{k=0}^{k=t-1} I_k^{-1} = 0, \tag{11}$$

where $R \equiv \frac{1}{\beta} > 1$ is the constant gross real interest rate and $\bar{C}$ is the constant flexible-price value of consumption. $I_t \leq 0$ is ruled out by (8), given $R, \Pi_{t+1} > 0$. In turn, (9) implies that $U'(\bar{C}) > V'(m_t)$, so depending on the functional form of $V'(m_t)$ there may be a lower bound on m_t that is greater than zero. Since $U'(\bar{C}) > 0$ and $V'(m_t) \geq 0$, (9) implies $I_t \geq \Omega \geq 1$, where Ω is the lower bound on I_t implied by $V'(m_t)$. Since $R > 1$ and $I_t \geq \Omega$, (8) also implies $\Pi_t \geq \frac{\Omega}{R}$.

2.2 Policy

We assume that fiscal policy determines the total amount of nominal government bonds outstanding, D_t, through control of the budget deficit inclusive of interest payments. Monetary policy determines whether these bonds are held by the monetary authority as a match for the money supply, M_t, or directly by the public, B_t, through control of open-market operations. The consolidated government balance sheet implies that $D_t = M_t + B_t$. Except for section 6, this paper is devoted to the analysis of determinacy under alternative *monetary-policy* rules. For this analysis we assume that *fiscal policy* does not determine the price level because it is conducted so that (11) holds for any path of the nominal interest rate.[11] That is, we assume that fiscal policy is Ricardian according to the definition of Benhabib, Schmitt-Grohe, and Uribe (1998). For this reason, fiscal policy and equation (11) may be disregarded until section 6.

So that we can discuss indeterminacy in our stripped-down model, we assume that there is a target equilibrium (TE) with a target value for inflation (Π^*) given by $\Pi^* > \frac{\Omega}{R}$ because of considerations not included in the model. Since the target nominal interest rate (I^*) must satisfy $I^* = R\Pi^*$, it follows that $I^* > \Omega$. Absent such considerations, the optimal values for Π_t and I_t would be $\frac{\Omega}{R}$ and Ω, respectively. Our main focus is on the possible existence of below-target equilibria (BTE). We define BTE as weakly increasing or decreasing paths for inflation and the interest rate along which they are always below Π^* and I^* respectively, and are either constant at, asymptotically approach, or eventually reach values represented by Π^{BTE} and I^{BTE}, respectively, where $I^* > I^{BTE} = R\Pi^{BTE} \geq \Omega$.

2.3 Money demand and lower bounds on the interest rate

In order to have versions of the model with and without a liquidity trap, we consider two particular specifications of money demand. Under both specifications the gross nominal interest rate has a lower bound (possibly one). Equation (9) implies

$$I_t = \frac{U'(\bar{C})}{U'(\bar{C}) - V'(m_t)} \geq \frac{U'(\bar{C})}{U'(\bar{C}) - \lim\limits_{m_t \to \infty} V'(m_t)} = \Omega \geq 1 \tag{12}$$

Of course, a lower bound of one for I_t implies a lower bound of zero for the net nominal interest rate.

The lower bound Ω may be attainable or unattainable. An attainable lower bound (ALB) for I_t represents a liquidity trap as conventionally defined. To model an ALB

[11] A sufficient condition for (11) to hold is that the gross growth rate of total government debt is low enough. Since the nominal interest rate enters equation (11), the maximum permissible growth rate depends on monetary policy and the rest of the model.

we assume that the utility of real balances is given by

$$V(m_t) = \begin{cases} \underline{V}'m_t - \frac{1}{2}(\overline{m} - m_t)^2 & \text{for } m_t \leq \overline{m} \\ \\ \underline{V}'m_t & \text{for } m_t > \overline{m} \end{cases}, \qquad \overline{m} > 0 \tag{13}$$

$\underline{V}' \geq 0$ represents the minimum marginal utility of real balances. Equation (12) implies

$$I_t = \left\{ \begin{array}{ll} \frac{U'(\bar{C})}{U'(\bar{C})-(\overline{m}-m_t)-\underline{V}'} & \text{for } m_t \leq \overline{m} \\ \\ \frac{U'(\bar{C})}{U'(\bar{C})-\underline{V}'} & \text{for } m_t > \overline{m} \end{array} \right\} \geq \frac{U'(\bar{C})}{U'(\bar{C}) - \underline{V}'} = \Omega^{ALB}, \tag{14}$$

With an unattainable lower bound (ULB), there is no liquidity trap because the interest rate can always fall a little farther. To model an ULB we assume that the utility of real balances is given by

$$V(m_t) = \underline{V}'m_t + \gamma \ln m_t \qquad \gamma > 0, \quad \underline{V}' \geq 0, \quad m_t > \frac{\gamma}{U'(\bar{C}) - \underline{V}'} \tag{15}$$

where $\underline{V}'$ represents the lower limit of the marginal utility of real balances. With this functional form, equation (9) implies

$$I_t = \frac{U'(\bar{C})}{U'(\bar{C}) - \frac{\gamma}{m_t} - \underline{V}'} > \frac{U'(\bar{C})}{U'(\bar{C}) - \lim\limits_{m_t \to \infty} \frac{\gamma}{m_t} - \underline{V}'} = \frac{U'(\bar{C})}{U'(\bar{C}) - \underline{V}'} = \Omega^{ULB} \tag{16}$$

Money-demand functions with an ALB and an ULB are represented in figure 1.[12] In both cases, $\Omega = 1$ if and only if $\underline{V}' = 0$. At an ALB purchases of bonds with money can not lower the interest rate. With an ULB such purchases can always lower the interest rate, if only by an infinitessimal amount.

From (16) or(14), and $I > 0$ (from (4)) we know that there exists a minimum level for real money balances denoted by $\underline{m}$: $m \geq \underline{m} = \max\{\overline{m} + \underline{V}' - U'(\bar{C}), 0\}$ with an ALB model, and $m > \underline{m} = \frac{\gamma}{U'(\bar{C}) - \underline{V}'}$ with an ULB. In order for $\lim_{m \to \underline{m}} I = \infty$ in the ALB model, we need $\overline{m} + \underline{V} - U'(\bar{C}) \geq 0$.

3 Interest-rate rules and BTE

In this section we distill the essence of the central indeterminacy results for interest rate rules when there is a lower bound on the nominal interest rate. Using our two simple money demand functions, it is easy to show why these results hold with or without liquidity traps. One finding that may be surprising at first is that the BTE steady state can be above the lower bound if the lower bound is preference-determined.

[12]For figure 1, $\underline{V}' = 0.001$, $\gamma = 1$, $\overline{m} = 2$, and $U'(\bar{C}) = 1$.

We begin by assuming that monetary policy takes the form of interest-rate rules and consider two examples. The general form of the interest-rate rules is

$$I_t = g(I^*, \frac{Y_t}{\bar{Y}}, \frac{\Pi_t}{\Pi^*}) \tag{17}$$

where Π^* is the target-equilibrium (TE) inflation rate and $\bar{Y} = \bar{C}$ is the flexible-price output level. With flexible prices, output is always at its flexible-price level , $Y_t = \bar{Y}$, so from now on we omit $\frac{Y_t}{\bar{Y}}$. We assume that the elasticity of the interest rate with respect to the inflation rate evaluated at the target inflation rate is greater than one. If $\Pi_t = \Pi^*$, then $I_t = I^*$. Until stated otherwise, we assume that the interest-rate rule is differentiable everywhere above the lower bound, that it is at least weakly increasing in the inflation rate and that the interest rate has a lower bound, either a preference-determined lower bound or a policy-determined lower bound. A preference-determined lower bound might exist because of a liquidity trap. A policy-determined lower bound might exist, for example, because the monetary authority wants to keep money market funds economic.[13] Under these assumptions, interest-rate rules are associated with two steady-state equilibria: a TE with $\Pi_t = \Pi^*$ and a BTE, where $\Omega/R \leq \Pi^{BTE} < \Pi^*$ and $\Omega \leq I^{BTE} < I^*$.

3.1 A preference-determined lower bound

First, consider the case of an interest-rate rule under which the interest rate may go all the way to the *preference-determined* lower bound, Ω^{ALB}, associated with an ALB money-demand function:[14]

$$I_t = \max\left[\Omega^{ALB}, I^*\left(\frac{\Pi_t}{\Pi^*}\right)^{\lambda}\right], \qquad \lambda > 1 \tag{18}$$

The piecewise log-linear rule (18) is globally (weakly) increasing in the inflation rate, and the elasticity of the interest rate with respect to the inflation rate in the strictly increasing part is $\lambda > 1$.[15] We refer to this case as the liquidity-trap case.

The Fisher equation (8), the interest-rate rule (18), and $I^* = R\Pi^*$ imply a log-linear difference equation in Π_t when $I_t > \Omega^{ALB}$:

$$\begin{aligned} \Pi_{t+1} &= \tfrac{I^*}{R}\left(\tfrac{\Pi_t}{\Pi^*}\right)^{\lambda} = (\Pi^*)^{1-\lambda}(\Pi_t)^{\lambda} \\ \hat{\Pi}_{t+1} &= (1-\lambda)\hat{\Pi}^* + \lambda\hat{\Pi}_t \end{aligned} \tag{19}$$

[13]For these funds to survive, there must be some spread between the market rates they earn and the deposit rates they pay as noted by Bernanke and Reinhart (2004).

[14]In this sense, the rule (18) is similar to the one used in Schmitt-Grohe and Uribe (2000) and in the appendix of Benhabib, Schmitt-Grohe, and Uribe (2001a).

[15]We use an interest-rate rule of the form (18) so that the inflation-rate term in the strictly increasing range is directly comparable to the inflation-rate term in the money-supply rule (29).

where variables with hats over them represent logarithms. The solution is

$$\hat{\Pi}_{t+k} = \hat{\Pi}^* + (\lambda)^k[\hat{\Pi}_t - \hat{\Pi}^*] \tag{20}$$

One possible steady-state equilibrium is inflation equal to the target rate. If one could disregard the lower bound on the interest rate, this would be the only equilibrium. Deviations from the inflation target would result in explosive or implosive paths of inflation since $\lambda > 1$.

However, equation (20) applies only when it calls for an interest rate at or above Ω^{ALB}. The inflation rate cannot decline forever. If the inflation rate given by (20) calls for an interest rate below Ω^{ALB}, the inflation rate is determined by the Fisher equation (8) together with $I_t = \Omega^{ALB}$ instead of by (20). That is, the inflation rate will stop declining when it is equal to its lower bound:

$$\Pi_{t+1} = \frac{\Omega^{ALB}}{R} \tag{21}$$

The difference equation reflecting the lower bound has the form shown in figure 2:[16]

$$\Pi_{t+1} = \max\left[\frac{\Omega^{ALB}}{R}, (\Pi^*)^{1-\lambda}(\Pi_t)^{\lambda}\right] \tag{22}$$

The list of equilibria as indexed by Π_0 is

1. $\Pi_0 = \Pi^*$. Steady-state TE.
2. $\Pi_0 \in \langle\frac{\Omega^{ALB}}{R}, \Pi^*\rangle$. Π_t decreases to $\frac{\Omega^{ALB}}{R}$ in finite time.
3. $\Pi_0 = \frac{\Omega^{ALB}}{R}$. Steady-state BTE equilibrium.
4. $\Pi_0 \in \langle 0, \frac{\Omega^{ALB}}{R}\rangle$. Equilibria with $\Pi_0 < \Pi_t = \frac{\Omega^{ALB}}{R}$, $t > 0$.[17]

Along any BTE path, the inflation rate eventually reaches $\frac{\Omega^{ALB}}{R}$ so that I_t reaches its liquidity-trap value Ω^{ALB} at which the levels of the nominal and real money supplies are indeterminate.[18] Hence, the number of equilibria is even larger than indicated above. Let the liquidity trap be reached in period n at price level P_n, given a particular initial P_0. There is an infinity of equilibria associated with each initial $P_0 \in \langle 0, \Pi^* P_{-1}\rangle$. Once the liquidity trap is reached, the set of possible paths for M_k,

[16] For figure 2, $\Pi^* = 1.025$, $\Omega^{ALB} = 1.001$, $1/R = 0.975$, and $\lambda = 2$.

[17] The initial inflation rate is not constrained by the lower bound Ω^{ALB}/R. The reason is that the constraint on the inflation rate is implied by the Fisher equation in combination with the interest rate given by the interest-rate rule. The Fisher equation has no implications for the initial price level or the inflation rate $\frac{P_0}{P_{-1}} = \Pi_0$, but it has implications for the inflation rate $\frac{P_1}{P_0}$. Hence, $\Pi_1 \geq \Omega^{ALB}/R$.

[18] Otherwise, the nominal money supply is determined by the nominal interest rate and the money-demand function, given the unique inflation rate associated with the TE.

$k \geq n$ includes all paths for which $\frac{M_k}{P_k} \in [\overline{m}, \infty\rangle$, $k \geq n$ since agents are indifferent between money and bonds.

There may be more equilibria than those listed above. Beginning on any initial inflation rate above Π^* the inflation rate follows a divergent path. Such divergent paths have been referred to as speculative hyperinflations, for example, by Obstfeld and Rogoff, who have discussed ways of precluding them.[19] Throughout this paper we assume that paths with ever increasing inflation are precluded.

For the sake of comparison, we briefly consider the case of an interest-rate peg in which $\lambda = 0$ and the difference equation in figure 2 is a horizontal line. Suppose it is announced that I_t will equal I^* in period t and all future periods. The Fisher equation (8), implies that I^* would be associated with Π^* from period $t+1$ on. However, this interest-rate rule would not pin down the initial inflation rate. There would be a continuum of equilibria, indexed by the initial inflation rate $\Pi_t \in \langle 0, \Pi^* \rangle$. However, if the monetary authorities specify the initial level of the money supply in addition to the interest-rate peg, the initial inflation rate is determinate, since there is a unique level of real balances associated with $I_t = I^*$.

3.2 A policy-determined lower bound

Now consider interest-rate rules designed to keep the gross nominal interest rate from falling below a *policy-determined* lower bound, Λ, that may be above Ω. The policy-determined lower bound may be attainable, Λ^{ALB}, or unattainable, Λ^{ULB}. For example, with the interest-rate rule

$$I_t = \max \left[\Lambda^{ALB}, I^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\lambda} \right], \qquad \Lambda^{ALB} \geq \Omega, \qquad \lambda > 1 \tag{23}$$

the policy-determined lower bound is attainable. This rule and the difference equation in inflation that it implies are identical to the ones considered in the last subsection except that the lower bounds on the interest rate and inflation are determined by policy, not by preferences. The rule can be implemented with both ALB money demand ($\Lambda^{ALB} \geq \Omega^{ALB}$) and with ULB money demand ($\Lambda^{ALB} > \Omega^{ULB}$). The entire list of possible equilibria is given by items 1 through 4 in the last subsection except that Λ^{ALB} replaces Ω^{ALB} everywhere. In contrast to the liquidity trap case, when Π_t reaches $\frac{\Lambda^{ALB}}{R}$ along a BTE path, real balances and nominal balances are uniquely determined.[20]

[19] See Obstfeld and Rogoff (1983) and Obstfeld and Rogoff (1986).

[20] Consider the family of interest-rate rules given by

$$I_t = \max \left[\Upsilon, (I^* - \Upsilon) \left(\frac{\Pi_t - \Upsilon/R}{\Pi^* - \Upsilon/R} \right)^{\lambda R \left[\frac{\Pi^* - \Upsilon/R}{I^* - \Upsilon} \right]} + \Upsilon \right], \quad I^* > \Upsilon \geq \max\left(\Omega^{ALB}, \Lambda^{ALB}, 1\right), \quad \lambda > 1$$

With this family, the strictly increasing part of the difference equation for Π_t begins at the point $\left(\frac{\Upsilon}{R}, \frac{\Upsilon}{R}\right)$ on the 45° line in (Π_{t+1}, Π_t) space. If $\Upsilon = 1$, the implied difference equation is qualitatively identical to the one plotted in Figure 2.4 of Woodford (2003).

It is useful to consider a rule that is very similar to the continuously differentiable rules used in the seminal papers on the existence of BTE:

$$I_t = \left(I^* - \Lambda^{ULB}\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\lambda I^*}{I^*-\Lambda^{ULB}}} + \Lambda^{ULB}, \qquad \Pi_t > 0 \tag{24}$$

$$I^* = R\Pi^* > \Lambda^{ULB} \geq \Omega \geq 1, \qquad \lambda > 1$$

This rule has a policy-determined lower bound that is unattainable.[21] It can be implemented with both ALB and ULB money-demand functions. The interest rate rises with inflation, and the response is greater the higher is inflation.[22] As before, the elasticity of the interest rate with respect to the inflation rate at Π^* is $\lambda > 1$.

With the rule (24), there must be two steady-state equilibrium inflation rates: one is Π^* and the other is Π^{BTE} which is *below* Π^* and *above* Λ^{ULB}/R but which *may or may not involve deflation.* Combining the rule (24) with the Fisher equation (8) yields a difference equation in inflation of the form plotted in figure 3:[23]

$$\Pi_{t+1} = \frac{1}{R}\left[\left(I^* - \Lambda^{ULB}\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\lambda R}{I^*-\Lambda^{ULB}}} + \Lambda^{ULB}\right] \tag{25}$$

This equation is a convex function that has a lower bound of $\frac{\Lambda^{ULB}}{R} \geq \frac{\Omega}{R}$ and that crosses the 45° degree line from below at $\Pi_{t+1} = \Pi_t = \Pi^*$ where its slope is greater than one:

$$\left.\frac{d\Pi_{t+1}}{d\Pi_t}\right|_{\Pi_t=\Pi^*} = \lambda > 1, \tag{26}$$

that is, Π^* is an unstable steady state equilibrium. In addition its slope approaches zero as Π_t approaches 0 from above and rises continuously with Π_t:

$$\frac{d\Pi_{t+1}}{d\Pi_t} = \lambda\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\lambda R}{I^*-\Lambda}-1} > 0 \tag{27}$$

[21] The continuous-time policy rule used in Benhabib, Schmitt-Grohe, and Uribe (2001c) and the main text of Benhabib, Schmitt-Grohe, and Uribe (2001a) has a policy-determined unattainable lower bound. The discrete-time analogue of this rule is

$$I_t = \left(I^* - \Lambda^{ULB}\right)\exp\left[\frac{RA}{I^* - \Lambda^{ULB}}(\Pi_t - \Pi^*)\right] + \Lambda^{ULB}, \qquad A > 1$$

We employ the rule (24) because it is directly comparable to the other interest-rate rules and the money-supply rules (29) used in this paper. Rule (24) is also used by Evans and Honkapohja (2003), who assume that $\Lambda^{ULB} = 1$.

[22] That is

$$\frac{dI_t}{d\Pi_t} = \lambda R\left(\frac{\Pi_t}{\Pi^*}\right)^{\frac{\lambda I^*}{I^*-\Lambda}-1} > 0, \qquad \frac{d^2 I_t}{d{\Pi_t}^2} = \left(\frac{\lambda I^*}{I^* - \Lambda} - 1\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{-1}\frac{dI_t}{d\Pi_t} > 0$$

[23] For figure 3, $\Pi^* = 1.025$, $1/R = 0.975$, $\Lambda^{ULB} = 1.02$, and $\lambda = 2$.

Therefore, it must intersect the 45° line a second time at a point below Π^* represented by Π^{BTE} where its slope is less than one.[24] That is, Π^{BTE} is a stable equilibrium with deflation ($\Lambda^{ULB}/R < \Pi^{BTE} < 1$), stable prices ($\Lambda^{ULB}/R < \Pi^{BTE} = 1$), or inflation ($1 < \Pi^{BTE}$).

There is a continuum of equilibria, indexed by Π_0. Each Π_0 is associated with one of the two possible steady-state inflation rates.

1. $\Pi_0 = \Pi^*$: Steady-state TE.
2. $\Pi_0 \in \langle \Pi^{BTE}, \Pi^* \rangle$: Non-steady-state BTE; $\Pi_t \to \Pi^{BTE}$ from above
3. $\Pi_0 = \Pi^{BTE}$: Steady-state BTE.
4. $\Pi_0 \in \left\langle 0, \Pi^{BTE} \right\rangle$: Non-steady-state BTE; $\Pi_t \to \Pi^{BTE}$ from below.[25]

4 Indeterminacy under money-growth rules

In our view there has been to little emphasis on the point that price-level indeterminacy is every bit as much of a problem with money growth rules as it is with interest-rate rules. To illustrate this point, we use a money growth rule of the general form

$$\frac{M_t}{M_{t-1}} = h(\Pi^*, \frac{Y_t}{\bar{Y}}, \frac{\Pi_t}{\Pi^*}) \tag{28}$$

where as before $\frac{Y_t}{\bar{Y}}$ is always unity because of flexible prices. We assume that the money-growth rule is differentiable everywhere and that it is at least weakly increasing in the inflation rate. For completeness we consider both ALB and ULB money demand functions.

4.1 Indeterminacy with ULB money demand

When the money-demand function has an ULB, money-growth rules are consistent with the existence of both a TE and BTE in which real money balances are forever increasing and the interest rate is approaching its lower bound.[26]

Consider the money-growth rule:

$$\frac{M_t}{M_{t-1}} = \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{-\tau}, \qquad \tau > -1 \tag{29}$$

[24] If $\lambda = 1$ there would be a unique steady state at $\Pi_t = \Pi^*$, but there would be multiple initial equilibrium inflation rates because all paths that start below the target rate would approach it.

[25] Π_0 can be lower than $\frac{\Omega^{ULB}}{R}$, see footnote 17.

[26] This possibility is pointed out, for example, in Woodford (1994), Woodford (2003), Christiano and Rostagno (2001), and Benhabib, Schmitt-Grohe, and Uribe (2001a)). We begin with the case of ULB money demand because it is somewhat simpler.

If $\tau = 0$, money grows at the constant target gross growth rate of money which is equal to Π^*. In that case, since M_{-1} and P_{-1} are given, M_0 is determined by Π^*. If $\tau \neq 0$, there is one M_0 associated with each P_0.[27] In either case, there will be a range of equilibria, each with its own level of initial real balances.

The Fisher equation (8) and the money market equilibrium condition (9) with the functional form for $V(m_t)$ in equation (15) imply an expression for the inflation rate in terms of real balances:

$$\Pi_t = \left(\frac{1}{R}\right) \frac{U'(\bar{C})}{U'(\bar{C}) - (\frac{\gamma}{m_{t-1}} + \underline{V}')} \tag{30}$$

Furthermore, the money-growth rule can be rewritten as

$$m_t = m_{t-1} \left(\Pi_t\right)^{-(1+\tau)} \Pi^{*\,1+\tau} \tag{31}$$

Combining (31) and (30), yields a difference equation in real balances,

$$m_t = m_{t-1}(\Pi^*)^{1+\tau} \left[\left(\frac{1}{R}\right) \frac{U'(\bar{C})}{U'(\bar{C}) - \frac{\gamma}{m_{t-1}} - \underline{V}'}\right]^{-(1+\tau)} \tag{32}$$

with the form plotted in figure 4.[28] Given the definition of Ω^{ULB} in equation (16), it follows that the term in square brackets (Π_t) approaches Ω^{ULB}/R as $m_{t-1} \to \infty$. Therefore, the growth rate of real balances (m_t/m_{t-1}) approaches $\left(\frac{R\Pi^*}{\Omega^{ULB}}\right)^{1+\tau} > 1$ for $\tau > -1$.[29]

The unique steady-state solution for equation (32) is

$$m^* = \frac{\gamma}{U'(\bar{C}) - \frac{U'(\bar{C})}{I^*} - \underline{V}'} = \frac{\gamma}{\frac{I^*-1}{I^*}U'(\bar{C}) - \underline{V}'} \tag{33}$$

If the steady-state level of real balances is to be positive, it must be that

$$I^* > \frac{U'(\bar{C})}{U'(\bar{C}) - \underline{V}'} \equiv \Omega^{ULB} \tag{34}$$

The money-growth rule may be associated not only with the TE, but also with a range of BTE in which inflation declines forever and approaches the limit Ω^{ULB}/R. The equilibria may be indexed by m_0:

[27] With $0 > \tau > -1$, nominal balances increase when the inflation rate is above target. For further discussion of this case, see footnote 29.

[28] For figure 4, $\tau = 2$, $\Pi^* = 1.025$, $\gamma = 1$, $U'(\bar{C}) = 1$, $\underline{V}' = 0.001$, and $1/R = 0.975$, so $\underline{m} = \gamma/(U'(\bar{C}) - \underline{V}') = 1.001$.

[29] m^* is an unstable steady state since equation (32) and the expression for Π^* given by (30) imply that for all $\tau > -1$

$$\left.\frac{dm_t}{dm_{t-1}}\right|_{m_t=m^*} = (\Pi^*)^{1+\tau}\left[\left(\frac{1}{R}\right)\frac{U'(\bar{C})}{U'(\bar{C})-\frac{\gamma}{m^*}-\underline{V}')}\right]^{-(1+\tau)}\left[1+\frac{(1+\tau)m^*\frac{\gamma}{(m^*)^2}}{U'(\bar{C})-\frac{\gamma}{m^*}-\underline{V}'}\right] = 1+\frac{R(1+\tau)\gamma\Pi^*}{m^*U'(\bar{C})} > 1$$

1. $m_0 = m^*$: Steady state TE with $\Pi_0 = \Pi_t = \Pi^*$.
2. $m_t > m^*$: BTE with positive growth in m_t, $\Pi_0 \in \langle 0, \Pi^* \rangle$ and $\Pi_t \to \Omega^{ULB}/R$ from above, $t > 0$.

The money-growth rule has a representation as an interest-rate rule that is related to but somewhat different from the one discussed in section 3.2.[30]

4.2 Indeterminacy with ALB money demand

With ALB money demand money-growth rules are also consistent with the existence of both a TE and BTE. In the BTE, real money balances are forever increasing as with ULB money demand, but the interest rate reaches its lower bound. The Fisher equation (8) and the $m_t \leq \overline{m}$-part of the money-demand function (14) imply an expression for inflation in terms of real balances:

$$\Pi_t = \left(\frac{1}{R}\right)\left[\frac{U'(\bar{C})}{U'(\bar{C}) - \overline{m} + m_{t-1} - \underline{V}'}\right] \tag{35}$$

Combining the money-growth rule (31) with (35) we obtain a difference equation in real balances:

$$m_t = m_{t-1}\left[\left(\frac{1}{R}\right)\frac{U'(\bar{C})}{U'(\bar{C}) - \overline{m} + m_{t-1} - \underline{V}'}\right]^{-(1+\tau)} \Pi^{*(1+\tau)}, \qquad m_{t-1} \leq \overline{m} \tag{36}$$

Given the definition of Ω^{ALB} in equation (14) it follows that as m_{t-1} increases and reaches $\overline{m}$, the term in square brackets (Π_t) increases and reaches Ω^{ALB}/R. Therefore, the growth rate of real balances (m_t/m_{t-1}) increases and reaches $\left(\frac{R\Pi^*}{\Omega^{ALB}}\right)^{1+\tau} > 1$ for $\tau > -1$.

There exists a steady-state equilibrium with positive real balances equal to

$$m^* = \frac{U'(\bar{C})}{R\Pi^*} + \overline{m} + \underline{V}' - U'(\bar{C}) = \overline{m} + \underline{V}' - \left(\frac{I^* - 1}{I^*}\right)U'(\bar{C}) \tag{37}$$

under weak conditions that we assume are met.[31] This equilibrium is a TE in which $\Pi = \Pi^*$ and $I = I^*$.[32] Equation (36) applies only when $m_t \leq \overline{m}$. In cases in which

[30] Equation (16) can be used to obtain an expression for m_t in terms of I_t. Using this expression and assuming $\underline{V}' = 0$, it follows from equations (31) and (8) that

$$\frac{I_t - 1}{I_t} = \frac{I_{t-1} - 1}{I_{t-1}}\left(\frac{\Pi_t}{\Pi^*}\right)^{1+\tau}$$

[31] The positive steady state exists only if $\overline{m} + \underline{V}' - \frac{I^* - 1}{I^*}U'(\bar{C}) > 0$ or equivalently $\frac{U'(\bar{C})}{U'(\bar{C}) - \overline{m} - \underline{V}'} > I^*$. That is, the target interest rate must be smaller than the maximum possible interest rate. For parameter values for which $\lim_{m_t \to \underline{m}} I_t = \infty$, $\overline{m} + \underline{V}' - U'(\bar{C}) = \underline{m} > 0$, so the condition is definitely met. In order to exist, the strictly positive steady state equilibrium must also satisfy $\underline{m} \leq m^* \leq \overline{m}$, which means that $\overline{m} + \underline{V}' - U'(\bar{C}) \leq m^* = \overline{m} + \underline{V}' - \left(\frac{I^* - 1}{I^*}\right)U'(\bar{C}) \leq \overline{m}$. This condition is always satisfied as long as $\underline{V}'$ is not too large.

[32] m^* is an unstable steady state since (37) and the definition of I^* imply

$m_t > \overline{m}$, the difference equation in real balances is given by (31) with the inflation rate fixed at Ω^{ALB}/R.

Figure 5 is a diagram for the case where $\underline{m} > 0$ because $\overline{m} + \underline{V}' - U'(\bar{C}) > 0$.[33]

The list of equilibria indexed by m_0 is now

1. $m_0 = m_t = m^*$: TE steady state with $\Pi_0 = \Pi^*$
2. $\overline{m} > m_0 > m^*$: BTE with $\Pi_0 \in \left\langle \frac{\Omega^{ALB}}{R}, \Pi^* \right\rangle$, Π_t reaches $\frac{\Omega^{ALB}}{R}$ from above, $t > 0$. m_t reaches $\overline{m}$ from below,
3. $m_0 \geq \overline{m}$: BTE with $\Pi_0 \in \left\langle 0, \frac{\Omega^{ALB}}{R} \right]$ and $\Pi_t = \frac{\Omega^{ALB}}{R}$, $t > 0$. The m_t growth rate is constant at $\left(\frac{I^*}{\Omega^{ALB}}\right)^{1+\tau}$.

There is a second steady-state equilibrium with zero real balances when $\overline{m} + \underline{V}' - U'(\bar{C}) \leq 0$ so that $\underline{m} = 0$.[34] This case is not of particular interest to us because we want to focus on equilibria in which inflation is below target (real balances are above target). However, it is quite important for those considering the existence of hyperinflation equilibria.

The money-growth rule in the ALB model also has a representation as an interest-rate rule as long as $m_t \leq \overline{m}$.[35]However, $I = \Omega^{ALB}$ for all $m_t \geq \overline{m}$. Hence, the interest-rate rule representation requires an additional specification of policy when $I = \Omega^{ALB}$ in order to uniquely pin down the path of real money balances for each initial m_0. For example, Eggertsson and Woodford (2003b) add a rule for money growth that applies whenever the interest rate reaches its lower bound.

5 Interest-rate rules that imply uniqueness

As might be expected there are interest-rate rules that imply uniqueness.[36] The question is this: Are they too unusual to be taken seriously? In this section, we

$$\left.\frac{dm_t}{dm_{t-1}}\right|_{\bar{m}=m^*} = 1 + (1+\tau)m^*\Pi^* \frac{I^*}{U'(\overline{C})} > 1$$

[33]For figure 5, $\underline{V}' = 0.001$, $\tau = 2$, $\overline{m} = 1.5, U'(\bar{C}) = 1$, $1/R = 0.975$, $\Pi^* = 1.025$, so $\underline{m} = 0.5$.

[34]See section 2.3. In this case (not shown), the line representing the difference equation would start at the origin and have a slope that is below one at the origin, increases until it exceeds one, and is constant for $m_t > \overline{m}$.

[35]Using (31),(8) and (14), we get

$$\frac{I_t - 1}{I_t} U'(\bar{C}) = \left[\frac{I_{t-1} - 1}{I_{t-1}} U'(\bar{C}) - (\underline{V}' + \overline{m})\right] \left[\frac{I^*}{I_{t-1}}\right]^{1+\tau} + (\underline{V}' + \overline{m})$$

[36]There are also money-growth rules that imply uniqueness. In Alstadheim and Hendersonn (2004) we present such a money-growth rule that has unusual properties analogous to the those of the Benhabib, Schmitt-Grohe, and Uribe (2001c) interest-rate rule considered here.

first consider a rule of the type suggested by Benhabib, Schmitt-Grohe, and Uribe (2001c), hereafter BSGU. They argue that such rules are too unusual to be taken seriously. We then present a rule that we regard as being considerably less unusual. Our analysis yields a pair of conditions that are necessary for uniqueness.

5.1 A BSGU-type unusual rule

Consider the following BSGU-type unusual rule that is piecewise log-linear

$$I_t = \begin{cases} I_t^* \left(\frac{\Pi_t}{\Pi^*}\right)^\lambda & \text{if } I^* \left(\frac{\Pi_t}{\Pi^*}\right)^\lambda > \Omega \\ \widetilde{I} > I^* & \text{if } I^* \left(\frac{\Pi_t}{\Pi^*}\right)^\lambda \leq \Omega \end{cases} \tag{38}$$

This rule has two unusual properties: it is non-monotonic in the current inflation rate, and it is discontinuous. Although BSGU point out that discontinuity is not necessary for uniqueness, they assume it in constructing their example of a non-monotonic rule, and we assume it for comparability. This rule is similar to the one described in section 3.2 except that the interest rate is pegged at $\widetilde{I} > I^*$ instead of at the policy-determined lower bound value Λ^{ALB} when $I^* \left(\frac{\Pi_t}{\Pi^*}\right)^\lambda \leq \Omega$. It is feasible with both ALB and ULB money demands, since it never calls for $I_t = \Omega$.

The difference equation in the inflation rate that follows from (38) and the Fisher equation (8) is illustrated in figure 6.[37] It inherits discontinuity and non-monotonicity from the interest rate rule. Consider a situation where the inflation rate is so low that if I were given by $I^* \left(\frac{\Pi}{\Pi^*}\right)^\lambda$ it would be less than or equal to the lower bound, Ω. In such a situation, I_t jumps up to $\widetilde{I}$. In the next period, the inflation rate must be higher than the target inflation rate given the Fisher equation and the fact that $\widetilde{I} > I_t^*$. But such a path is not a possible solution, because it implies that the inflation rate increases without limit. Hence, the economy cannot start out on a path of declining inflation.

It is worth remarking that if the interest-rate rule under consideration were credible the economy would always be at the unique target equilibrium so that a high interest rate at low inflation rates would never be observed. Of course, the rule may not be credible precisely because it has quite unusual properties.

5.2 A considerably less unusual rule

As an example of a rule that also implies uniqueness but is considerably less unusual, consider the following:[38],

$$I_t = I_t^* \left(\frac{\Pi_t}{\Pi^*}\right)^\lambda \left(\frac{\Pi_{t+1}}{\Pi^*}\right)^\gamma, \qquad \lambda, \gamma \geq 0 \tag{39}$$

[37]For figure 6, $\Pi^* = 1.025, 1/R = 0.975$, $\Omega = 1.001$, $\lambda = 2$, and $\tilde{I} = 1.05/0.975 \approx 1.077$.

[38]Benhabib, Schmitt-Grohe, and Uribe (2001b) also consider interest-rate rules where the interest rate responds to both current and future inflation, but in a different context.

Note that the rule is monotonic in both current and expected future inflation. We assume that the rule is asymmetric. In particular, we make the plausible assumption that the response to expected future inflation is stronger when current inflation is below target.

Combining equation (39) with the Fisher equation (8) and using $I_t^* = R\Pi^*$, give a first-order difference equation in the inflation rate,

$$\Pi_{t+1} = \Pi^{*\frac{1-\lambda-\gamma}{1-\gamma}} \Pi_t^{\frac{\lambda}{1-\gamma}} \tag{40}$$

Taking logs, we have

$$\widehat{\Pi}_{t+1} = \alpha_0 \widehat{\Pi}^* + \alpha_1 \widehat{\Pi}_t \tag{41}$$

where $\alpha_0 = \frac{1-\lambda-\gamma}{1-\gamma}$ and $\alpha_1 = \frac{\lambda}{1-\gamma}$. With λ and γ chosen so that $|\alpha_1| > 1$, the equation is unstable and the unique solution for Π_t is Π^*.

As an example, suppose that

- If $\Pi_t < \Pi^*$, then $2 > \gamma > 1$ and $\lambda > 1$, so that $\alpha_0 > 1$ and $\alpha_1 < -1$.

- If $\Pi_t \geq \Pi^*$, then $\gamma = 0$ and $\lambda > 1$, so that $\alpha_0 < 0$ and $\alpha_1 > 1$.

For these parameter values, the difference equation for inflation (40) has the form shown in figure 7.[39] It is continuous, non-differentiable at $\Pi_t = \Pi^*$, and non-monotonic in current inflation. Π^* is a unique steady-state equilibrium, and $\Pi_0 = \Pi^*$ is the only equilibrium. If the initial inflation rate is in the interval $\Pi_0 \in \langle 0, \Pi^* \rangle$, the economy embarks on a path with ever-increasing inflation. The part of the difference equation that applies when $\Pi_t \in \langle 0, \Pi^* \rangle$ implies $\Pi_1 > \Pi^*$, and the inflation rate continues to increase because the part that applies when $\Pi_t \in [\Pi^*, \infty\rangle$ is now relevant. Thus, as pointed out by BSGU, it is clear that necessary conditions for uniqueness do not include discontinuity in the interest-rate rule or, by implication, discontinuity in the implied difference equation.

As stated by BSGU, non-monotonicity of the implied difference equation is a necessary condition for uniqueness. What has gone unnoticed, as far as we know, is that non-differentiability of the implied difference equation is also a necessary condition.[40] In establishing our claims, we refer frequently to a constraint on the inflation difference equation implied by a lower bound on the nominal interest rate. The nominal interest rate must always be at or above its lower bound ($I_t = R\Pi_{t+1} \geq \Omega$), and hence the path of inflation is also bounded from below. That is, $\Pi_t \geq \frac{\Omega}{R}$ for $t > 0$. Only

[39] For figure 7, $\lambda = 2$, $\Pi^* = 1.025$, and $1/R = 0.975$. $\gamma = 1.5$ when $\Pi_t < \Pi^*$, and $\gamma = 0$ otherwise.

[40] We thank an anonymous referee for drawing our attention to the importance of non-differentiability.

Π_0 is not constrained by the lower bound.[41] Strict inequalities apply in the case of an unattainable lower bound.

First, consider the necessity of the non-monotonicity of the implied difference equation in inflation. This equation cannot be monotonically increasing, start out above the 45° line (as it must because of the constraint), and cross the 45° line only once and from below (as it must for uniqueness). Whether non-monotonicity of the interest-rate rule is also required depends on the rules of the game. If current inflation is the only variable allowed on the right side of the interest rate rule, then it must be non-monotonic for uniqueness.[42] However, as we have shown, if both current and expected inflation are allowed, an interest-rate rule that is monotonically increasing in these two variables can generate the required non-montonic difference equation.

Now, consider the necessity of non-differentiability of the implied difference equation. Suppose that the difference equation is differentiable. Suppose that it cuts the 45° line from below at Π^* so that there is a locally unique steady state. Because of differentiability, the equation is continuous. Because of the constraint and continuity, the difference equation must cross the 45° line a second time at a value below Π^* but above $\frac{\Omega}{R}$, so there is a multiplicity of equilibria. Alternatively, if the only contact between the difference equation and the 45° line is at a point of tangency, that point is the only steady state. In this case the difference equation must lie above the 45° line everywhere except at the tangency point because of the constraint. If the difference equation is flat or increasing to the left of the tangency point, it follows that there is a multiplicity of equilibria because any inflation rate to the left of the tangency point can be the first point on a path to that point. If there is a point of contact between the difference equation and the 45° line at the minimum of the difference equation, the steady state will be locally unique, but the difference equation cannot be differentiable at the point of contact (the target inflation rate). We are left with two possibilities for uniqueness: either, the difference equation may be nondifferentiable at the target inflation rate but continuous as for example in figure 7, or the difference equation may be discontinous and nondifferentiable at some point as, for example, in figure 6. The difference equation can be non-differentiable and discontinous, respectively, only if the interest-rate rule is, since the other equation involved (the consumption Euler equation $I_t = R\Pi_{t+1}$) is continuous and differentiable.

Although non-montonicity and non-differentiability of the difference equation are necessary for determinacy, they are not sufficient. Non-monotonicity cannot be sufficient. For example, the non-monotonic quadratic equation $\Pi_{t+1} = \varepsilon_0 + \varepsilon_1 (\Pi_t - \Pi^*)^2$ would cross the 45° line twice and would not be associated with uniqueness. Non-differentiability cannot be sufficient either. For example, the non-differentiable difference equations implied by the interest rate rules (18) and (23) in section 3 do not rule out indeterminacy.

[41] See footnote 17.

[42] All it takes for equation (40) to be associated with a unique steady-state TE is that $\alpha_1 < -1$ when $\Pi_t < \Pi^*$ and $\alpha_1 > 1$ when $\Pi_t > \Pi^*$. One could, for example, let $\gamma = 0$ so that there was zero response to future inflation, and vary λ to meet these requirements.

6 Fiscal policy and BTE

6.1 Fiscal policy can always preclude BTE

If a monetary-policy rule generates both a TE and BTE, there are always fiscal policies that preclude BTE thereby guaranteeing uniqueness. Fiscal policy determines the path of total nominal government debt measured as total nominal government bonds, D_t. The consolidated government balance sheet implies that D_t must equal the money supply (which equals the government bonds held by the monetary authority) plus government bonds held by the public:

$$D_t \equiv M_t + B_t, \qquad D_0 \neq 0 \qquad t \geq 0 \tag{42}$$

For simplicity, we devote most of our attention to fiscal policies under which there is a constant gross growth rate (Γ) for total government bonds:

$$D_{t+1} = \Gamma D_t \tag{43}$$

where $\Gamma = 1$ is the case of a balanced-budget policy.

Fiscal policy and the transversality condition (11) taken together have implications for the possibility of BTE. First, consider a candidate steady-state BTE in which the interest rate is constant at I^{BTE}. The path $I_t = I^{BTE}\ \forall\ t$ can be a steady-state equilibrium only if

$$\lim_{t\to\infty} \left(\frac{\Gamma}{I^{BTE}}\right)^{t-1} \Gamma D_0 = 0 \tag{44}$$

Therefore if $\Gamma \geq I^{BTE}$, fiscal policy precludes such a path. With our parameterization, $\Gamma < I^{BTE}$ is a necessary condition for fiscal policy to be consistent with steady-state BTE.[43]

One implication of the previous paragraph is that a balanced-budget policy ($\Gamma =$ 1) rules out BTE in which the nominal interest rate is always zero ($I_t = 1\ \forall\ t$). Note that this policy is not expansionary enough if the interest rate is positive enough of the time either because of a positive lower bound or, for example, because of foreseen variation in productivity.[44] A closely related implication is that if $\Gamma \geq 1$, the Friedman rule ($I_t = 1\ \forall\ t$) can not be implemented exactly under any type of monetary policy. This result for the case of $\Gamma = 1$ is obtained by Schmitt-Grohe and Uribe (2000) in cases in which the monetary-policy rule is either an interest-rate rule or a money-growth rule. Yet a third implication is that there may be BTE with small deficits ($I^{BTE} > \Gamma > 1$).

The class of fiscal policies for which $I^* > \Gamma > I^{BTE}$ is especially interesting. It is clear from equation (44) that this class precludes BTE for any monetary policy. It

[43] Of course, there are many other parameterizations of fiscal policy which are consistent with the existence of BTE.

[44] I_t can be unity an infinite number of periods but it must also be positive for an infinite number of periods. For an example see Alstadheim and Hendersonn (2004).

follows that a combination of a member of this class with a standard interest-rate rule like equation (18) or money-supply rule like equation (29) is sufficient to insure that Π^* is the unique equilibrium value for the inflation rate.[45]These observations suggest that BTE may not be a matter for concern in OECD countries. Most, if not all, of these countries run positive but relatively small government deficits on average so their fiscal policies may preclude BTE.[46] This possibility is yet another reminder of the importance of analyzing monetary and fiscal policy jointly.[47]

For completeness, consider a candidate BTE path on which the interest-rate path is some weakly decreasing sequence $\{I_k\}$ with $\lim_{k\to\infty} I_k = I^{BTE}$. Let $\{I_k\} \equiv \{\Theta_k I^{BTE}\}$,where $\{\Theta_k\}$ is a weakly decreasing sequence with $\lim_{k\to\infty} \Theta_k = 1$. Let the value of Θ_k at some time $\hat{t}$ given by $\Theta_{\hat{t}} > 1$ be arbitrarily close to one. This sequence of interest rates can be an equilibrium only if

$$\lim_{t\to\infty} D_0 \Gamma^t \Pi_{k=\hat{t}}^{k=t-1} \left[\Theta_k^{-1} (I^{BTE})^{-1}\right] = 0 \tag{45}$$

The condition (45) cannot hold if $\Gamma > I^{BTE}$ and $\Theta_{\hat{t}}$ is arbitrarily close to one. Therefore, $\Gamma > I^{BTE}$ precludes BTE in which the interest rate asymptotically approaches or eventually reaches I^{BTE}. In addition, $\Gamma = I^{BTE}$ precludes equilibria in which I^{BTE} is approached asymptotically if $\lim_{t\to\infty} \Pi_{k=\hat{t}}^{k=t-1} \Theta_k^{-1} > 0$. This condition is satisfied if the nominal interest rate approaches I^{BTE} (and hence Θ_k approaches one) fast enough, for example if the nominal interest rate reaches I^{BTE} (Θ_k reaches one) in finite time.

6.2 Uniqueness with money growth through transfers

In subsection 4.2 we show that with an ALB money-demand function, the money-growth rule (29) is associated with multiple equilibria. There, as well as in all of the paper before section 6, we assume that B_t can take on any value implied by the open-market purchases used to increase the money supply. In particular, B_t can be as negative as necessary, or, in other words, government interest-bearing claims on the public can increase without limit.

In this subsection, we assume that there is a finite lower bound on B_t designated by $\underline{B}$ where $-\infty < \underline{B}$; that is, government claims on the public cannot exceed the finite amount $-\underline{B}$. Once this limit is reached, the money supply can be increased

[45]Using the terminology of Woodford (2001), fiscal policy is "locally Ricardian" in the neighborhood of Π^* but is "locally non-Ricardian" in the neighborhood of the BTE steady state.

[46]Using the growth of nominal public debt to rule out BTE may result in exploding real debt. However, the present value of real debt does not explode as long as the transversality condition holds. As a referee has pointed out, this case is possible if the inflation target $\Pi^* < R\Pi^{BTE} = I^{BTE} \leq \Gamma$. Whether it is of much interest is contentious. Clearly, it can be ruled out by choosing a higher inflation target.

[47]An early joint analysis of monetary and fiscal policy is Leeper (1991); more recent analyses include Canzoneri, Cumby, and Diba (2001) and Evans and Honkapohja (2002).

only by money-financed transfers ('money rain'). The required policy is best viewed as a combination of fiscal policy and monetary policy: the fiscal authority makes a bond-financed transfer to the private sector, and the monetary authority buys the bonds.

If BTE are to be precluded, the transversality condition (11) implies that Π^* (the target rate of inflation and money growth) must be chosen so that

$$\lim_{t\to\infty}\left(\frac{1}{\Omega^{ALB}}\right)^{t-1}\left[(\Pi^*)^{1+\tau}\left(\frac{\Omega^{ALB}}{R}\right)^{-\tau}\right]^t M_0 + \lim_{t\to\infty}\left(\frac{1}{\Omega^{ALB}}\right)^{t-1}\underline{B} \neq 0 \tag{46}$$

where $(\Pi^*)^{1+\tau}(\frac{\Omega^{ALB}}{R})^{-\tau}$ is the constant rate of growth of nominal balances implied by the money-growth rule when the interest rate is at Ω^{ALB} so that the inflation rate is $\frac{\Omega^{ALB}}{R}$. The condition (46) can always be met under the not very restrictive assumption that $(\Pi^*)^{1+\tau}\left(\frac{\Omega^{ALB}}{R}\right)^{-\tau} M_0 + \underline{B} \neq 0$. Since $M_0 > 0$, the first term is strictly positive and either unbounded or bounded at $(\Pi^*)^{1+\tau}\left(\frac{\Omega^{ALB}}{R}\right)^{-\tau} M_0$ if Π^* is chosen so that

$$(\Pi^*)^{1+\tau}\left(\frac{\Omega^{ALB}}{R}\right)^{-\tau} \geq \Omega^{ALB} \tag{47}$$

or, equivalently, so that

$$\Pi^* \geq \left(\frac{\Omega^{ALB}}{R}\right) R^{\frac{1}{1+\tau}} \tag{48}$$

The second term is either zero or bounded at $\underline{B}$ as $\Omega^{ALB} \geq 1$. Therefore, (46) is satisfied no matter whether the first term is bounded or not.[48] Note that equation (48) implies uniqueness can be consistent with $\Pi^* < \Omega^{ALB}$ if $\tau > 0$ and, in particular, with Π^* arbitrarily close to its Friedman-rule value of $\frac{1}{R}$ if $\Omega^{ALB} = 1$ and τ is large enough.[49]

7 Conclusions

Under many specifications of monetary policy, standard models exhibit price-level indeterminacy. That is, they have multiple equilibria that include both a locally-unique

[48] In a model in which the lower bound on the gross interest rate is one, Woodford (1994) shows that a money-growth rule in combination with the condition that $B_t = 0 \ \forall\ t$ is associated with uniqueness as long as gross money growth is equal to or larger than one ($\Pi^* \geq 1$ in our notation). When the public holds no government bonds, money must be increased by money-transfers. A money-growth rule with $\Pi^* > \Omega^{ALB}$ then corresponds to the class of fiscal policies discussed in is subsection with $\Gamma > \Omega^{ALB}$, since the nominal growth rate of money is equal to the nominal growth rate of government bonds. Benhabib, Schmitt-Grohe, and Uribe (2001a) show that multiple equilibria are precluded under money-growth rules when $B_t \geq 0$. As we have shown the lower bound on B_t can be negative. What is needed is that money be supplied with lump-sum transfers instead of open-market operations after some point

[49] Woodford (1994, 2003) restricts attention to the case of $\tau = 0$ (and $\Omega^{ALB} = 1$) and argues that the restriction on the range of the target inflation rate, $\Pi^* > 1$, necessary to preclude BTE is a limitation of money-supply rules relative to interest-rate rules.

steady-state target (inflation-rate) equilibrium (TE) and multiple below-target equilibria (BTE), equilibria in which the inflation rate is always below target and is constant or eventually reaches or asymptotically approaches a below-target value.

Rules with either the interest rate or money growth as instruments are consistent with price-level indeterminacy when there is a lower bound on the nominal interest rate. There may be such a lower bound because of a conventionally-defined liquidity trap in which bonds and money are perfect substitutes at either a positive or zero nominal interest rate. Also, there may be a lower bound even if there is no liquidity trap for one of two reasons. First, money demand may imply a lower bound on the interest rate which is approached asymptotically. Second, an interest-rate rule may keep the interest rate above a policy-determined lower bound.

We have found that implementing monetary policy by setting money growth instead of the interest rate makes a difference for determinacy only if money is injected without open-market operations, e.g. by lump sum transfers. Under these conditions, a money-growth rule, more accurately viewed as a combination of monetary and fiscal policy, may preclude BTE when an interest-rate rule does not.

The above results all apply when the interest rate and money growth respond monotonically to eliminate deviations in the current inflation rate from its target value. Benhabib, Schmitt-Grohe, and Uribe (2001c) present an interest-rate rule that precludes BTE. Under this rule the interest rate responds only to current inflation. The rule is unusual in that it is not monotonically increasing in inflation. We present an interest-rate rule that we find considerably less unusual that also precludes BTE. The interest rate is monotonically increasing in both current and expected future inflation. It is asymmetric: the interest rate responds more strongly to expected future inflation if the current inflation rate is below the target rate. When each rule is combined with same other relationship, the result is a difference equation in inflation. Benhabib, Schmitt-Grohe, and Uribe (2001c) noticed that for uniqueness this difference equation must non-monotonic. What no one has noticed, as far as we know, is that it must also be non-differentiable. If it is nondifferentiable at the target inflation rate, the difference equation may still be continuous. Otherwise, the difference equation and also the interest rate rule must be discontinous at a point in order to insure uniqueness.

Benhabib, Schmitt-Grohe, and Uribe (2001c) have observed that a unique steady-state inflation rate is implied by unusual interest-rate rules that are discontinuous at a point and non-monotonic in the current inflation rate. By constructing a less unusual rule, we show that neither of these properties are necessary. We find two properties that are necessary but not sufficient for determinacy: (1) the interest rate rule must be non-differentiable at a point and (2) although the interest-rate rule itself may be montonic in current and expected future inflation, the difference equation that it implies must be non-montonic in current inflation.

Conclusions about determinacy under alternative monetary rules depend on fiscal policy as measured by the growth rate of total nominal government debt. There is always a class of fiscal policies that can preclude BTE for standard monetary policies because with those fiscal policies BTE paths violate the transversality condition. Balanced budget fiscal policy can preclude BTE if the interest rate is always at a zero lower bound. However, it cannot do so if the lower bound on the interest rate is positive.

It is not yet clear whether the existence of BTE is an important problem or a curiosum. Some have argued that BTE are of little interest for theoretical reasons. McCallum (2001, 2003) uses his 'minimum state variable' criterion as one way of determining which equilibria are of interest. In terms of our model, he shows that the locally-unique TE meets this criterion while the multiplicity of BTE equilibria associated with the below-target steady state, often referred to as 'sun-spot' equilibria, do not. Both McCallum (2001, 2003) and Evans and Honkapohja (2002) use stability under a particular type of learning as an alternative criterion. In terms of our model, they show that the TE is stable under learning while the BTE are not. Although these theoretical arguments are attractive, not all analysts are completely convinced by them.

Our analysis suggests another more practical reason for focusing on the target equilibrium. The combination of any of a range of small deficits with a standard interest-rate or money-supply rule guarantees that the TE is the unique equilibrium because the deficit precludes BTE. This observation suggests that BTE may not be a matter for concern in most OECD countries. Most of these countries run positive but relatively small government deficits on average so their fiscal policies may preclude BTE. This possibility provides another illustration of the importance of analyzing monetary and fiscal policy jointly.

It might be argued that Japanese experience from the mid-90s until recently fits the description of a BTE, that is, a bad draw from a set of multiple equilibria.[50] Although this argument cannot be rejected out of hand, there are at least two reasons to question it and to argue instead that Japanese experience fits the description of the kind of stabilization problem described in Section 1. First, the collapses of the stock and land markets and the accompanying bank crises were big negative demand shocks. Second, Japan has come out of its slump arguably partly because the Bank of Japan announced and carried out its zero-interest-rate and quantitative easing policies. A good, but by no means airtight, case can be made that policies of this type are more likely to be helpful in solving a stabilization problem than an indeterminacy problem. For whatever reason, most analysts have concluded that what the Japanese really faced was not an indeterminacy problem but a stabilization problem.

For some policies, appropriateness depends crucially on whether one is concerned about stabilization or indeterminacy, but, for others, it does not. For example, it

[50] Advocates of the fiscal theory of the price level might argue that Japanese fiscal policy ruled out BTE. For some time, Japan had near-zero interest rates and deflation. During roughly the same period, the growth rate of nominal government debt exceeded nominal interest rates on government debt. If this kind of fiscal policy had been expected to continue, it would have been too expansionary to be consistent with a BTE. However, it was probably not expected to continue.

is often suggested that Japan should have announced a higher target inflation rate. Raising the target inflation rate (or increasing the money supply even though the interest rate is at the lower bound) can help an economy solve a stabilization problem by raising expected future inflation thereby lowering the real interest rate.[51] However, we have shown that this policy cannot help solve an indeterminacy problem because a wide range of target inflation rates are consistent with indeterminacy. In contrast, no matter which type of problem one is trying to solve, there is a strong case for a more aggressive response to inflation when it is below target. It has been stressed that more aggressive easing reduces the chances of having a stabilization problem in which policymakers must rely on less familiar instruments with more uncertain effects.[52] We have shown that responding more aggressively to expected inflation when current inflation is below target makes it possible to avoid multiple equilibria.

[51] See, for example, Krugman (1998).
[52] See, for example, Orphanides and Wieland (2000).

References

Alstadheim, R., and D. W. Hendersonn (2004) "Price Level Determinacy, Lower Bounds on the Nominal Interest Rate, and Liquidity Traps", International Finance Discussion Paper 795, Federal Reserve Board, (http://www.federalreserve.gov/pubs/ifdp/2004/795/ifdp795.pdf).

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (1998) "Monetary Policy and Multiple Equilibria", Finance and Economics Discussion Series 1998-29, Federal Reserve Board.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001a) "Avoiding Liquidity Traps", *Journal of Political Economy*, 110, 535–563.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001b) "Monetary Policy and Multiple Equilibria", *American Economic Review*, 91, 167–185.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001c) "The Perils of Taylor Rules", *Journal of Economic Theory*, 96, 40–69.

Bernanke, B. S., and V. Reinhart (2004) "Monetary Policy at Very Low Nominal Interest Rates", *American Economic Review*, 94, forthcoming.

Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2001) "Is the Price Level Determined by the Needs of Fiscal Solvency?", *American Economic Review*, 91, 1221–1238.

Christiano, L. J., and M. Rostagno (2001) "Money Growth Monitoring and the Taylor Rule", NBER Working Paper 8539, National Bureau of Economic Research.

Eggertsson, G., and M. Woodford (2003a) "Optimal Monetary Policy in a Liquidity Trap", NBER Working Paper 9968, National Bureau of Economic Research.

Eggertsson, G., and M. Woodford (2003b) "The Zero Bound on Interest Rates and Optimal Monetary Polic", *Brookings Papers on Economic Activity*, 1, 139–211.

Evans, G. W., and S. Honkapohja (2002) "Policy Interaction, Learning and the Fiscal Theory of Prices", Processed, University of Helsinki, (http://ideas.repec.org/e/pho12.html).

Evans, G. W., and S. Honkapohja (2003) "Policy Interaction, Expectations and the Liquidity Trap", Processed, University of Helsinki, (http://ideas.repec.org/e/pho12.html).

Krugman, P. (1998) "It's Baack: Japan's Slump and Return of the Liquidity Trap", *Brookings Papers on Economic Activity*, 2, 137–187.

Leeper, E. (1991) "Equilibria under 'Active' and 'Passive' Monetary Policies", *Journal of Monetary Economics*, 27, 129–147.

McCallum, B. T. (2001) "Inflation Targeting and the Liquidity Trap", in N. Loayza and R. Soto (eds.) *Inflation Targeting: Design, Performance, Challenges*, Central Bank of Chile, Santiago, 395–497 (NBER Working Paper 8225).

Obstfeld, M., and K. Rogoff (1983) "Speculative hyperinflations in maximizing models: Can we rule them out?", *Journal of Political Economy*, 91, 675–687.

Obstfeld, M., and K. Rogoff (1986) "Ruling Out Divergent Speculative Bubbles", *Journal of Monetary Economics*, 17, 349–362.

Orphanides, A., and V. Wieland (2000) "Efficient Monetary Policy Design Near Price Stability", *Journal of the Japanese and International Economies*, 14, 327–365.

Sargent, T. J. (1987) *Macroeconomic Theory*, Academic Press, New York, Second Edition.

Schmitt-Grohe, S., and M. Uribe (2000) "Price Level Determinacy and Monetary Policy under a Balanced-Budget Requirement", *Journal of Monetary Economics*, 45, 211–246.

Svensson, L. E. O. (2000) "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap", NBER Working Paper 7957, National Bureau of Econonmic Research.

Woodford, M. (1994) "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy", *Economic Theory*, 4, 345–380.

Woodford, M. (2001) "Fiscal Requirements for Price Stability", *Journal of Money, Credit and Banking*, 33, 669–728.

Woodford, M. (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton.

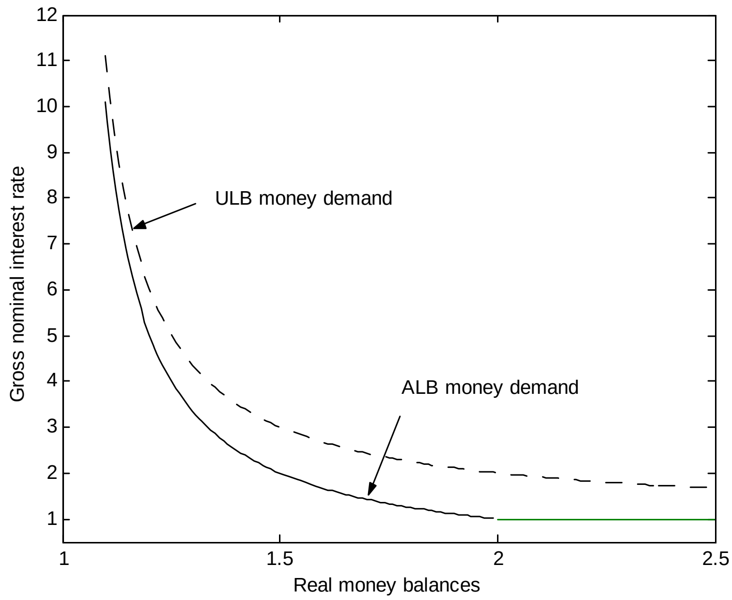

Figure 1: ULB and ALB money demands

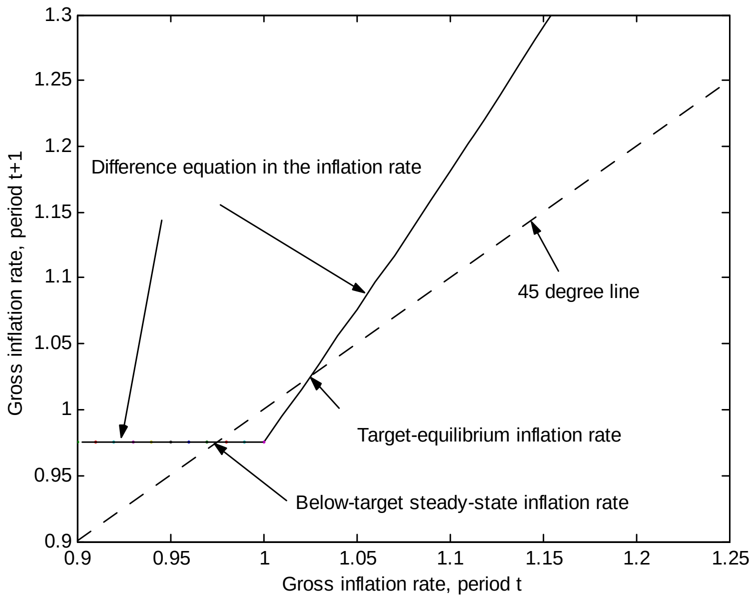

Figure 2: Interest-rate rule and preference-determined lower bound

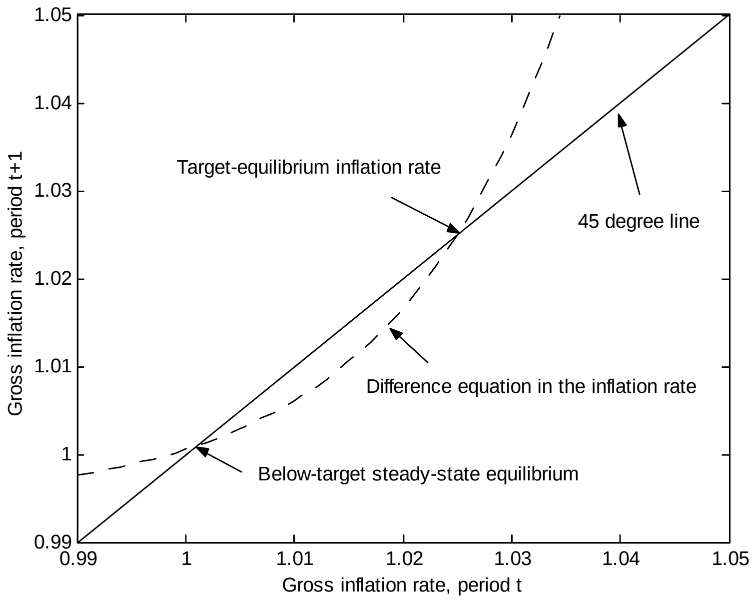

Figure 3: Interest-rate rule and policy-determined lower bound

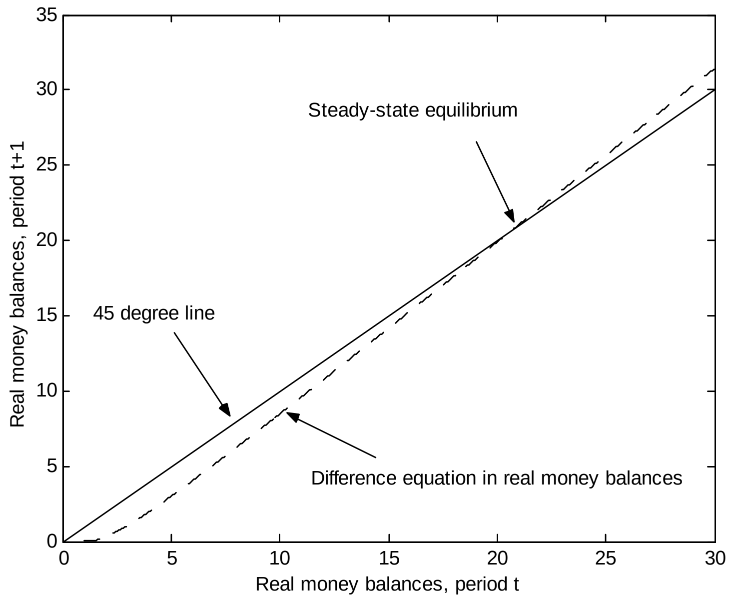

Figure 4: Money-growth rule with unattainable lower bound

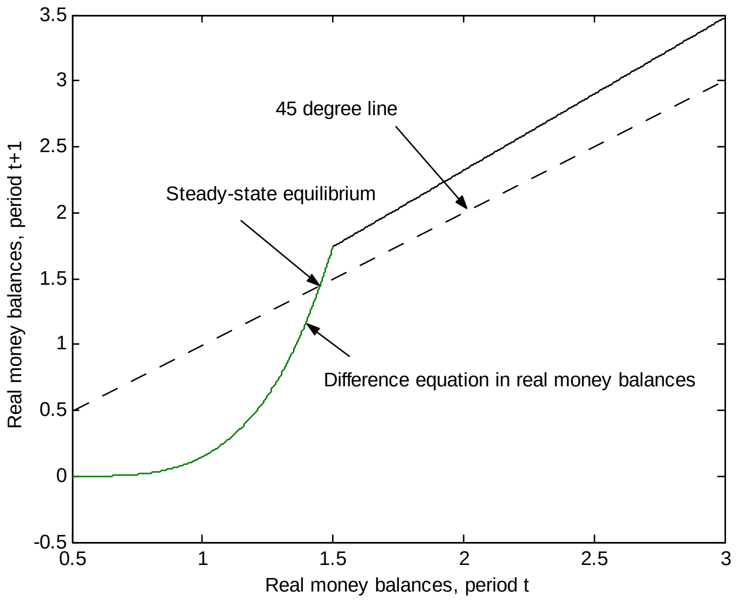

Figure 5: Money-growth rule with attainable lower bound

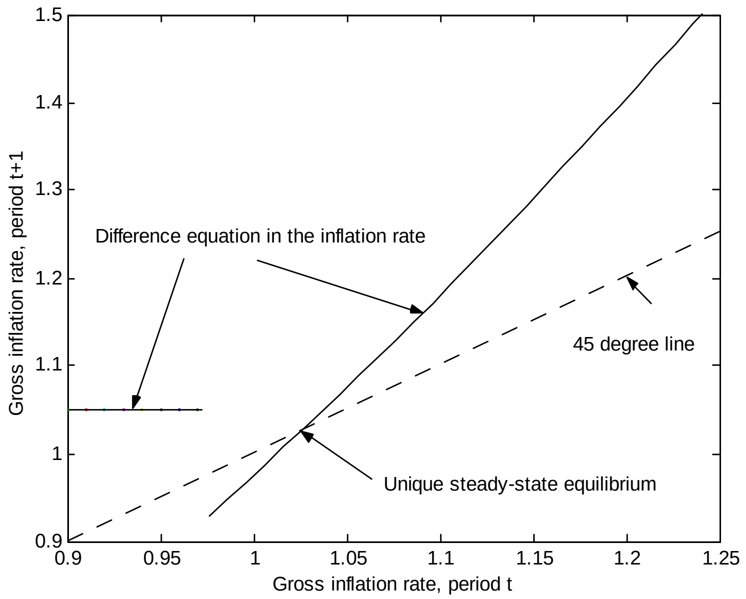

Figure 6: Non-monotonic interest-rate rule

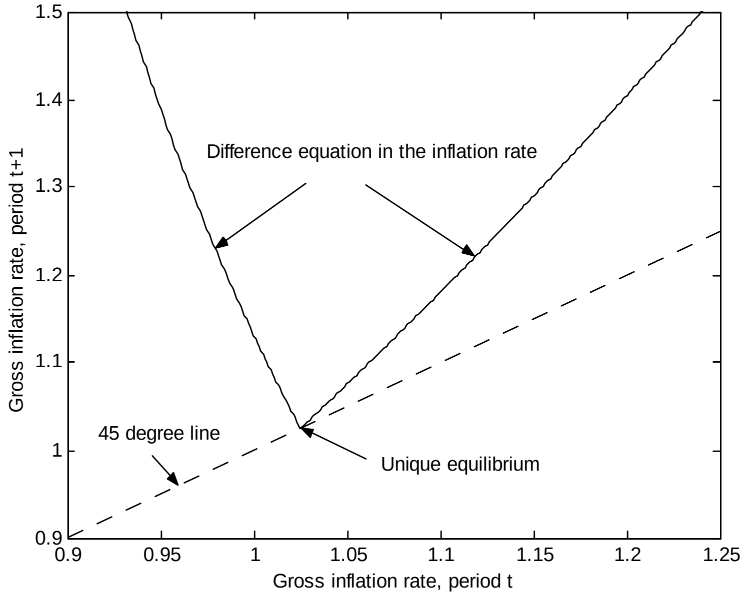

Figure 7: Asymmetric interest-rate rule

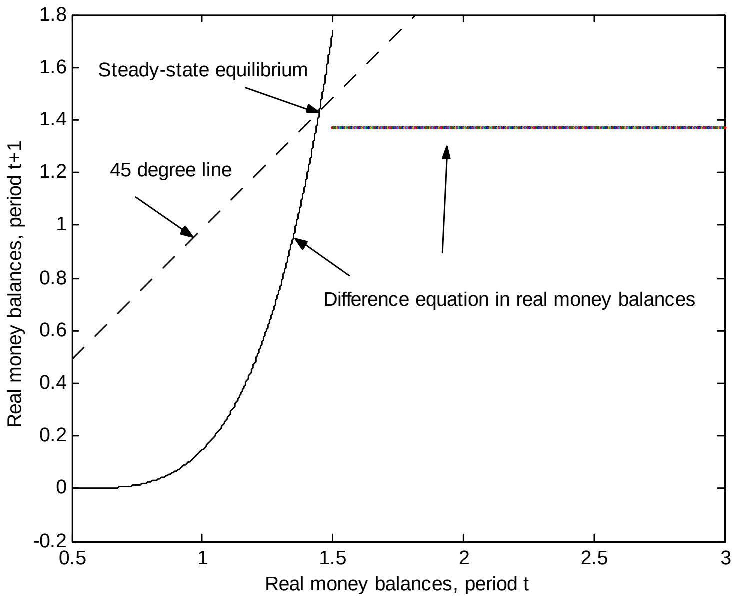

Figure 8: Non-monotonic money-growth rule

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