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Pitfalls in the Modelling of Forward-Looking Price Setting and
Investment Behavior

by

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Pitfalls in the Modeling of Forward-Looking Price Setting and Investment Decisions*

Tommy Sveen[†] Lutz Weinke[‡]

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Abstract

We discuss some difficulties in a dynamic New-Keynesian model with staggered price setting à la Calvo and a convex capital adjustment cost at the firm level, as considered by Woodford (2003, Ch. 5). It is shown that the implied simultaneous price setting and investment decision has not been analyzed properly. Our work fills that gap by proposing a tractable solution to the key problem of describing the inflation dynamics associated with that structure. We use our framework to assess to what extent capital accumulation matters for inflation and output dynamics.

Keywords: Sticky Prices, Investments.

JEL Classification: E22, E31

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1 Introduction

By now there exists a large literature studying macroeconomic dynamics in general equilibrium models with sticky prices. However, it is generally assumed that labor is the only productive input¹ or alternatively that the capital stock in the economy is held constant.² Woodford (2003, p. 352) comments on these modeling choices: ‘[...] while this has kept the analysis of the effects of interest rates on aggregate demand quite simple, one may doubt the accuracy of the conclusions obtained, given the obvious importance of variations in investment spending both in business fluctuations generally and in the transmission mechanism for monetary policy in particular.’ Woodford (2003, Ch. 5) makes important progress in analyzing capital accumulation in general equilibrium models with staggered price setting. First, he observes that the widely used assumption of a rental market for capital could imply that a substantial part of the aggregate capital stock shifts each period from low demand to high demand producers. This is unrealistic, and more importantly, it has non-trivial implications for the determination of marginal costs at the firm level, hence for price setting decisions and for inflation dynamics. Second, he observes that the marginal return to capital is given by the marginal savings in a firm’s labor cost as opposed to its marginal revenue product of capital: with price staggering, firms are demand constrained. Hence, the return from having an additional unit of capital in place derives from the fact that this allows to produce the quantity that happens to be demanded at a lower marginal cost.

Assuming that firms make investment decisions implies that price setters face an intricate simultaneous choice problem. Woodford (2003, p. 357) notes: ‘The capital stock affects a firm’s marginal cost, of course; but more subtly, a firm considering how its future profits will be affected by the price it sets must also consider how its capital stock will evolve over the time that its price remains fixed.’ However, as

¹See, e.g., Clarida et al. (1999).

²Erceg et al. (2000) assume a constant aggregate capital stock combined with a rental market for capital, while Sbordone (2001) assumes a constant capital stock at the firm level.

we argue, Woodford (2003, Ch. 5) does not solve in a correct way the price setting problem in the presence of an investment decision at the firm level. In a nutshell: he appears not to have assessed correctly over what set of future states of the world an optimizing Calvo price setter forms expectations.³

We reconsider the structure in Woodford (2003, Ch. 5), i.e. our model features staggered price setting à la Calvo and convex adjustment costs in the process of capital accumulation at the firm level. We propose a tractable solution to the key problem of characterizing the inflation dynamics associated with that structure. In particular, we suggest a simple approximate inflation equation, and show that it can be used without any sizeable loss of accuracy.

Our ultimate goal is to assess the extent to which capital accumulation matters for inflation and output dynamics. To this end we compare impulse responses to a shock in the exogenous growth rate of money balances for two cases: our baseline model with endogenous capital (henceforth baseline) and a specification with decreasing returns to scale resulting from a constant capital stock at the firm level (henceforth DRS). We find the following: first, the response of output is higher in the former – both on impact and during the transition period. Second, the inflation dynamics are similar in the two models. The intuition is as follows: there are two opposite effects from endogenous capital accumulation on the determination of marginal costs. On the one hand, the additional production triggered by investment demand increases marginal costs in the baseline model with respect to the DRS specification. On the other hand, the resulting additional capital increases the economy’s productive capacity thereby decreasing marginal costs.

The remainder of the paper is organized as follows: section 2 outlines the baseline model. In particular, it is shown why the price setting problem associated with that structure has not been solved in a correct way in Woodford (2003, Ch. 5). In section 3 we conduct the above mentioned simulation exercise. At this step, we also check the accuracy of our approximation to the inflation equation. Section 4 concludes.

³The same critique applies to Casares (2002).

2 The Model

We follow the general equilibrium structure outlined in Woodford (2003, Ch. 5).⁴ Our focus is on the firms's behavior, while a short exposition of the household's problem is left to the Appendix.

2.1 Outline of the Model Structure

There are two sectors, households and firms. The latter produce differentiated goods and act under monopolistic competition. The only aggregate uncertainty comes from the growth rate of money balances, which we assume to follow an $AR(1)$ process:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t, \quad (1)$$

where m_t denotes the log of nominal money balances M_t at time t . The autoregressive parameter ρ_m is assumed to be strictly positive and less than one. Finally, ε_t is *iid* with zero mean and variance σ_ε^2 .

Households are modeled in a standard way. They choose labor supply and consumption demand with the objective of maximizing lifetime utility. Consumption is given by a Dixit-Stiglitz aggregate of all the goods produced in the economy. The elasticity of substitution between goods is constant and given by ε . Households have access to complete financial markets and supply labor in a perfectly competitive labor market.⁵

Firms are indexed on the unit interval. Each firm i produces a differentiated good with the objective of maximizing the present value of its dividend stream. Technology is given by a Cobb-Douglas production function:

$$Y_t(i) = K_t(i)^\alpha N_t(i)^{1-\alpha}, \quad (2)$$

⁴He considers a more general structure than ours. However, this is irrelevant for our discussion of the conceptual problem in his treatment of the simultaneous price setting and investment problem.

⁵For a formal statement of the household's problem and the associated optimality conditions, see Appendix 1.

where $K_t(i)$ and $N_t(i)$ denote, respectively, capital holdings and labor input used by firm i in its period t production denoted $Y_t(i)$.

Each firm i makes an investment decision at any point in time with the resulting additional capital becoming productive one period after the investment decision is made. It is assumed that the investment good is a Dixit-Stiglitz aggregate of all of the goods in the economy with the same constant elasticity of substitution as in the consumption aggregate. Firms are assumed to face convex adjustment costs of changing their capital holdings. Given firm i 's time t capital stock $K_t(i)$ the amount of the composite good $I_t(i)$ that has to be purchased by that firm at this point in time in order to have a capital stock $K_{t+1}(i)$ in place in the next period is given by:

$$I_t(i) = I\left(\frac{K_{t+1}(i)}{K_t(i)}\right) K_t(i). \quad (3)$$

The function $I(\cdot)$ has the following characteristics: $I(1) = \delta$, $I'(1) = 1$ and $I''(1) = \epsilon_\psi$. The parameter δ denotes the depreciation rate and $\epsilon_\psi > 0$ measures the convex capital adjustment cost in a log-linear approximation to the equilibrium dynamics.

Firms post sticky prices à la Calvo (1983), i.e. each period a measure $(1 - \theta)$ is randomly selected. Those firms change their prices and the remaining firms post their last period's nominal prices. The price index P_t in period t is given by:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}, \quad (4)$$

where $P_t(i)$ denotes the nominal price posted at time t by firm i . P_t has the property that the minimum expenditure required to purchase a bundle of goods resulting in $I_t(i)$ units of the composite good is given by $P_t I_t(i)$.

Cost minimization by firms and households implies that demand for each individual good i in period t can be written as follows:

$$Y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t^d, \quad (5)$$

where Y_t^d denotes aggregate demand at time t , which is given by:

$$Y_t^d \equiv C_t + I_t,$$

where $I_t \equiv \int_0^1 I_t(i) di$ and C_t denote, respectively, aggregate investment demand and the representative household's consumption demand at time t .

2.2 Price Setting and Investment

The probability that a firm cannot adjust its price in any given period is given by θ . Hence, with probability θ^k a price that was chosen at time t will still be posted at time $t + k$. When setting a new price $P_t^*(i)$ in period t firm i maximizes the current value of its dividend stream over the expected lifetime of the chosen price. This implies the following first order condition:⁶

$$\sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} Y_{t+k}^d(i) [P_t^*(i) - \mu MC_{t+k}(i)] \} = 0, \quad (6)$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ is the frictionless mark-up over marginal costs, $Q_{t,t+k}$ is the stochastic discount factor for random nominal payments, and $MC_t(i)$ denotes the nominal marginal cost of firm i in period t . The latter is given by:

$$\begin{aligned} MC_t(i) &= \frac{W_t}{MPL_t(i)} \\ &= \frac{W_t}{(1-\alpha)} \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon\alpha}{1-\alpha}} K_t(i)^{\frac{-\alpha}{1-\alpha}} (Y_t^d)^{\frac{\alpha}{1-\alpha}}, \end{aligned} \quad (7)$$

where W_t is the nominal wage and $MPL_t(i)$ denotes the marginal product of labor of firm i in period t . The last equality follows from imposing $Y_t(i) = Y_t^d(i)$ and combining it with equations (2) and (5).

Equation (6) is the familiar first order condition implied by the Calvo model: optimizing price setters behave in a forward-looking manner, i.e. they take into account not only current but also future expected marginal costs in those states of the world where the chosen price is still posted. A price setter's capital holdings in those same states of the world result from its investment decisions. We turn to this next.

⁶See Appendix 2 for a formal statement of the firms' price setting and investment problems.

The first-order condition for investment spending is given by:

$$\frac{dI_t(i)}{dK_{t+1}(i)} P_t = E_t \left\{ Q_{t,t+1} \left[MS_{t+1}(i) - \frac{dI_{t+1}(i)}{dK_{t+1}(i)} P_{t+1} \right] \right\}, \quad (8)$$

where $MS_{t+1}(i)$ denotes the nominal marginal savings in firm i 's labor cost associated with having one additional unit of capital in place in period $t + 1$. The intuition behind the last equation is the following: the marginal cost of installing an additional unit of capital at time t (including the adjustment cost) is equalized to the expected discounted marginal contribution to the firm's value associated with having that additional unit of capital in place at point in time $t + 1$. The latter is given by the marginal return from using it for production, $MS_{t+1}(i)$, and selling the remaining capital after depreciation (net of the change in the time $t + 1$ adjustment cost that is associated with the time t investment decision). As has been emphasized by Woodford (2003, Ch. 5), the relevant measure of the marginal return to capital is the marginal savings in a firm's labor cost: firms are demand constrained and hence the return from having an additional unit of capital in place results from the fact that this allows to produce the quantity that happens to be demanded using less labor.

The following relationship holds true:

$$\begin{aligned} MS_{t+1}(i) &= W_{t+1} \frac{MPK_{t+1}(i)}{MPL_{t+1}(i)} \\ &= \frac{\alpha W_{t+1}}{1 - \alpha} \left(\frac{P_{t+1}(i)}{P_{t+1}} \right)^{\frac{-\varepsilon}{1-\alpha}} K_{t+1}(i)^{\frac{-1}{1-\alpha}} (Y_{t+1}^d)^{\frac{1}{1-\alpha}}, \end{aligned} \quad (9)$$

where $MPK_{t+1}(i)$ denotes the marginal product of capital of firm i in period $t + 1$. The last equality follows from imposing $Y_{t+1}(i) = Y_{t+1}^d(i)$ and invoking equations (2) and (5). With probability θ the firm's nominal price $P_{t+1}(i)$ is the one that was posted the period before, with probability $(1 - \theta)$ it is $P_{t+1}^*(i)$. This aspect of a firm's investment decision implies that price setters face an intricate problem. As we argue next, the latter has not been solved in a correct way in the existing literature.

2.3 A Short Note on the Existing Literature

Woodford (2003, pp. 688 - 690) computes future expected capital holdings as far as they are relevant for a log-linear approximation to the price setting problem without considering that these depend to some extent on future expected *optimally* chosen prices. However, equations (8) and (9) state that a time t price setter's choice over its next period's capital stock takes rationally into account that its time $t + 1$ price might be optimally chosen. In other words: the *possibility* of choosing a new price at point in time $t + 1$ affects a price setter's time t investment decision and hence its time $t + 1$ capital holdings, *in particular*, in those states of the world that are relevant for the price setting decision.

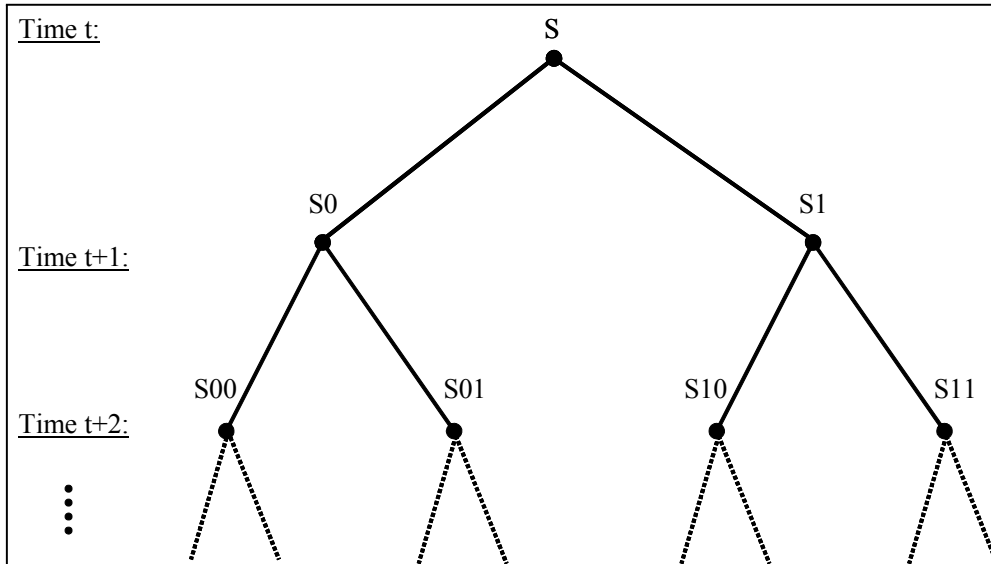


Figure 1: Decision tree for time t price setter.

To fix ideas we represent firm i 's price setting problem at time t by a simple tree, which consists of the states of the world that are consistent with the current state S . This is shown in Figure 1. Equations (6) and (7) prescribe that the relevant capital holdings are associated with those states of the world where the newly set price is still posted. These capital holdings are assumed to correspond to nodes S , S_0 , S_{00} ,... in the tree. Firm i 's capital stock at node S is predetermined. Now consider

firm i 's choice in period t over its next period's capital stock $K_{t+1}(i)$. Equations (8) and (9) state that this decision depends on the price setter's expectation of its time $t + 1$ relative price taking into account that this might be either the one associated with node $S0$ or the one that is chosen at node $S1$. Moreover, $K_{t+1}(i)$ depends on the price setter's expectation of its time $t + 2$ capital stock, which might be either the one that prevails at nodes $S00$ and $S01$ or the one that obtains at nodes $S10$ and $S11$.

Next we consider the equilibrium conditions associated with the baseline model, and in particular, we propose a tractable approximation to the inflation equation.

2.4 Equilibrium

We restrict attention to a log-linear approximation to the equilibrium dynamics around a symmetric equilibrium steady state with zero inflation. In what follows, the percent deviation of a variable with respect to its steady state value is denoted by a hat.

2.4.1 Market Clearing

Clearing of the labor market requires that hours worked, N_t , are given by the following equation, which holds for all t :

$$N_t = \int_0^1 N_t(i) di. \quad (10)$$

Moreover, it is useful to define aggregate capital for all t :

$$K_t \equiv \int_0^1 K_t(i) di. \quad (11)$$

For each good i in the economy supply $Y_t(i)$ must equal demand $Y_t^d(i)$:

$$Y_t(i) = C_t^d(i) + I_t^d(i), \quad (12)$$

where $C_t^d(i)$, $I_t^d(i)$ denote, respectively, consumption and investment demand for good i . Since an equation like (12) holds for each good in the economy we are

entitled to integrate on both sides over all firms in the economy. After invoking (2) and (5) this yields:

$$\int_0^1 N_t(i)^{1-\alpha} K_t(i)^\alpha di = (C_t + I_t) \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di. \quad (13)$$

Since we restrict attention to a log-linear approximation to the equilibrium dynamics around a zero inflation steady state, we can write the log-linearized goods market clearing condition in the following way:

$$\widehat{Y}_t = \frac{\rho + \delta(1 - \alpha)}{\rho + \delta} \widehat{C}_t + \frac{\alpha}{\rho + \delta} \left[\widehat{K}_{t+1} - (1 - \delta) \widehat{K}_t \right], \quad (14)$$

where Y_t is defined as follows:⁷

$$Y_t \equiv K_t^\alpha N_t^{1-\alpha}. \quad (15)$$

Based on the same argument the log-linearized aggregate production function is given by:

$$\widehat{Y}_t = \alpha \widehat{K}_t + (1 - \alpha) \widehat{N}_t. \quad (16)$$

2.4.2 Households

Log-linearizing and rearranging the first order conditions associated with the household's problem in Appendix 1 we obtain the following equilibrium conditions. The household's Euler equation is:

$$\widehat{C}_t = E_t \widehat{C}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho), \quad (17)$$

where σ is the household's relative risk aversion, or equivalently, the inverse of the intertemporal elasticity of substitution, and i_t denotes the nominal interest rate at time t . The time discount rate is $\rho \equiv -\log \beta$, with β denoting the discount factor.

The log-linearized labor supply equation is given by:

$$\widehat{\left(\frac{W_t}{P_t} \right)} = \phi \widehat{N}_t + \sigma \widehat{C}_t, \quad (18)$$

⁷The difference between Y_t and aggregate output in the economy is of the second order, so we can safely ignore it for the log-linear approximation to the equilibrium dynamics we are considering.

where ϕ can be interpreted as the inverse of the Frisch aggregate labor supply elasticity.

For convenience, we follow Galí (2000) and assume the following demand for real balances $\frac{M_t}{P_t}$:

$$\left(\frac{M_t}{P_t}\right) = \widehat{Y}_t - \eta(i_t - \rho), \quad (19)$$

where η is the semi-elasticity of demand for real balances with respect to the nominal interest rate.

2.4.3 Firms

First, we derive the law of motion of capital. A natural starting point is the log-linearized real marginal savings in the labor cost of a firm i :

$$\widehat{ms}_t(i) = \widehat{ms}_t - \frac{\varepsilon}{1 - \alpha} \widehat{p}_t(i) - \frac{1}{1 - \alpha} \widehat{k}_t(i), \quad (20)$$

where $p_t(i) \equiv \frac{P_t(i)}{P_t}$, $k_t(i) \equiv \frac{K_t(i)}{K_t}$, and ms_t denotes the average real marginal savings in labor costs at time t . The latter is given by:

$$ms_t = \frac{W_t MPK_t}{P_t MPL_t}, \quad (21)$$

where MPL_t and MPK_t denote, respectively, the average time t marginal products of labor and capital. They are obtained from (15).

Next we log-linearize the first order condition for investment (8) and average over all firms in the economy. Invoking the Euler equation (17) we obtain the following law of motion of the aggregate capital stock:

$$\begin{aligned} \widehat{K}_{t+1} &= \frac{1}{1 + \beta} \widehat{K}_t + \frac{\beta}{(1 + \beta)} E_t \widehat{K}_{t+2} \\ &\quad + \frac{1 - \beta(1 - \delta)}{\epsilon_\psi (1 + \beta)} E_t \widehat{ms}_{t+1} - \frac{1}{\epsilon_\psi (1 + \beta)} (i_t - E_t \pi_{t+1} - \rho). \end{aligned} \quad (22)$$

As the last equation shows, the existence of a capital adjustment cost implies that the aggregate capital stock in the economy is a forward looking variable.

Second, we characterize the inflation dynamics associated with the baseline model. To this end, we average and aggregate price setting decisions in the way described below. Our starting point is the log-linearized marginal cost at the firm level. Denoting $mc_t(i) \equiv \frac{MC_t(i)}{P_t}$ and log-linearizing yields:

$$\widehat{mc}_t(i) = \widehat{mc}_t - \frac{\varepsilon\alpha}{1-\alpha}\widehat{p}_t(i) - \frac{\alpha}{1-\alpha}\widehat{k}_t(i), \quad (23)$$

where mc_t is the average time t real marginal cost in the economy, which can be written as:

$$mc_t = \frac{W_t/P_t}{MPL_t}. \quad (24)$$

We refer to $\widehat{k}_t(i)$ as firm i 's capital gap at time t . The intuition behind equation (23) is the following: for a zero capital gap a firm that posts a higher than average price faces a lower than average marginal cost due to the decreasing marginal product of labor. This is reflected in the second term, and it is exactly as in Sbordone (2001) and Galí et al. (2001) for models with decreasing returns to scale and labor as the only variable input in production.⁸ With capital accumulation there is an extra effect coming from the firm's capital stock, which corresponds to the last term. Conditional on posting the average price in the economy a firm that has a higher than average capital stock in place faces a lower than average marginal cost. The reason is that the marginal product of labor increases with the capital stock used by the firm.

Invoking equations (6) and (23) the optimal relative price of firm i at time t , $p_t^*(i) \equiv \frac{P_t^*(i)}{P_t}$, can be log-linearized as:

$$\widehat{p}_t^*(i) = \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{mc}_{t+k} - \psi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{k}_{t+k}(i), \quad (25)$$

where $\xi \equiv \frac{(1-\beta\theta)(1-\alpha)}{1-\alpha+\varepsilon\alpha}$, and $\psi \equiv \frac{(1-\beta\theta)\alpha}{1-\alpha+\varepsilon\alpha}$.⁹ Hence, in addition to the usual inflation and average marginal cost terms a firm's optimal price setting decision does also

⁸For an early model, which features differences in marginal costs among producers, see Woodford (1996).

⁹The price setting problem is stated in terms of variables that are constant in the steady state.

depend on its current and future expected capital gaps over the (random) lifetime of the chosen price. We outline next how the key problem of describing the inflation dynamics associated with that structure can be solved.

We observe that in the zero inflation steady state a firm that is allowed to change its price will optimally choose not to do so. This implies: $\lim_{k \rightarrow \infty} E_t \widehat{p}_{t+k}^*(i) = 0$. Therefore, a time t price setter foresees that it will optimally choose a zero capital gap in the infinitely distant future. Formally: $\lim_{k \rightarrow \infty} E_t \widehat{k}_{t+k}(i) = 0$. We iterate on the following step:¹⁰ in the first round a price setter behaves in a myopic way, i.e. firm i posts a price $\widehat{p}_t^{*,1}(i)$ consistent with the expectation that it will choose to have a zero capital gap already from time $t + 1$ onward. The number in the superscript of the last variable is meant to indicate the round of the iteration or, more colorfully, the degree of sophistication in price setting that is assumed in its determination. This way we can solve for the newly set myopic price $\widehat{p}_t^{*,1}(i)$ in terms of aggregate variables only, except for the current predetermined capital gap of firm i . In the second round a price setter is a bit more rational and chooses $\widehat{p}_t^{*,2}(i)$ consistent with the expectation that it will close its capital gap from time $t + 2$ onward. The newly set price consistent with rational expectations is $\widehat{p}_t^*(i) = \lim_{k \rightarrow \infty} E_t \widehat{p}_t^{*,k}(i)$. At each step of the iteration we solve for the average newly set price. Since price setters are randomly selected the current average capital gap in the group of price setters is zero. Hence, the average newly set price in the economy is a function of aggregate variables only. Next we invoke the price index and solve for the implied inflation equation. The quantitative consequences of employing the different inflation equations associated with the steps of the iteration are analyzed in a simulation exercise. We turn to this next.

¹⁰The details of the first two steps of the iteration are given in Appendix 3. This also illustrates the way one can obtain arbitrarily many steps.

3 Simulation Results

Given the specification of monetary policy in (1), the equilibrium processes for the nominal interest rate, output, hours, consumption, real wage, real balances, capital, and inflation are given by equations (14), (16), (17), (18), (19), (22), and an inflation equation, which can be found by the iteration outlined above. The average marginal savings in labor costs and the average marginal cost in the economy are obtained from equations (21) and (24), respectively.¹¹

3.1 Calibration

The calibration of the model parameters in the baseline model is shown in Table 1. The period length is one quarter. The intertemporal elasticity of substitution is given by $\frac{1}{\sigma}$. Assuming $\sigma = 2$ is consistent with empirical estimates.¹² Consistent with a unit labor supply elasticity, we assume $\phi = 1$. The semi-elasticity of demand for real balances with respect to the nominal interest rate, η , is set to unity implying an empirically plausible value of about 0.05 for the interest rate elasticity. The capital share in the production function, α , is 0.36. We set $\beta = 0.99$ implying an average annual real return of about 4 percent. Setting $\theta = 0.75$ means that the average lifetime of a price is equal to one year. Consistent with the estimated autoregressive process for M1 in the United States we assume $\rho_m = 0.5$ and $\sigma_\varepsilon^2 = 0.1$.¹³ Setting $\varepsilon = 11$ implies a frictionless markup of 10 percent.¹⁴ We choose $\epsilon_\psi = 3$, as suggested by Woodford (2003, Ch. 5) and the references herein.

¹¹To solve the dynamic stochastic system of equations we use Dynare (<http://www.ceprenmap.cnrs.fr/dynare/>).

¹²See, e.g., Basu and Kimball (2003) and the references herein.

¹³Our calibration of ϕ , α , β , θ , ρ_m , and σ_ε^2 is justified in Galí (2000) and the references herein.

¹⁴This is consistent with the estimate in Galí et al. (2001).

Table 1: Calibration

The period length:	one quarter
Preference parameters:	$\sigma = 2, \phi = 1, \beta = 0.99$
Production function:	$\alpha = 0.36$
Elasticity of substitution between goods:	$\varepsilon = 11$
Capital accumulation:	$\delta = 0.025, \epsilon_\psi = 3$
Price stickiness:	$\theta = 0.75$
Money demand:	$\eta = 1$
Monetary policy:	$\rho_m = 0.5, \sigma_\varepsilon^2 = 0.1$

3.2 Results

We analyze impulse responses to a positive one standard deviation shock in the growth rate of money balances. Our first result regards the iteration outlined above. We find that the inflation equations at *each* step of the iteration are associated with almost identical equilibrium dynamics. This is illustrated in Figure 2, which shows that the difference in implied equilibrium dynamics between step 1 and step 20 of the iteration is negligible. Moreover, this result is remarkably robust with respect to the choice of the calibration. Therefore, we can use the inflation equation resulting from step 1 of the iteration in order to characterize the equilibrium dynamics implied by the baseline model.¹⁵ As we derive in Appendix 3, the latter equation takes the following simple form:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{m} c_t, \quad (26)$$

where $\kappa \equiv \frac{\xi(1-\theta)}{\theta}$. This is an interesting result: the Calvo assumption implies that the current average capital gap in the (randomly selected) group of price setters is

¹⁵The only parameter that has some influence on this result is the capital adjustment cost parameter ϵ_ψ . In fact, for ϵ_ψ smaller than one, it might be desirable to iterate more than once in order to characterize the resulting inflation dynamics. The special case without any capital adjustment cost is analyzed by Sveen and Weinke (2003).

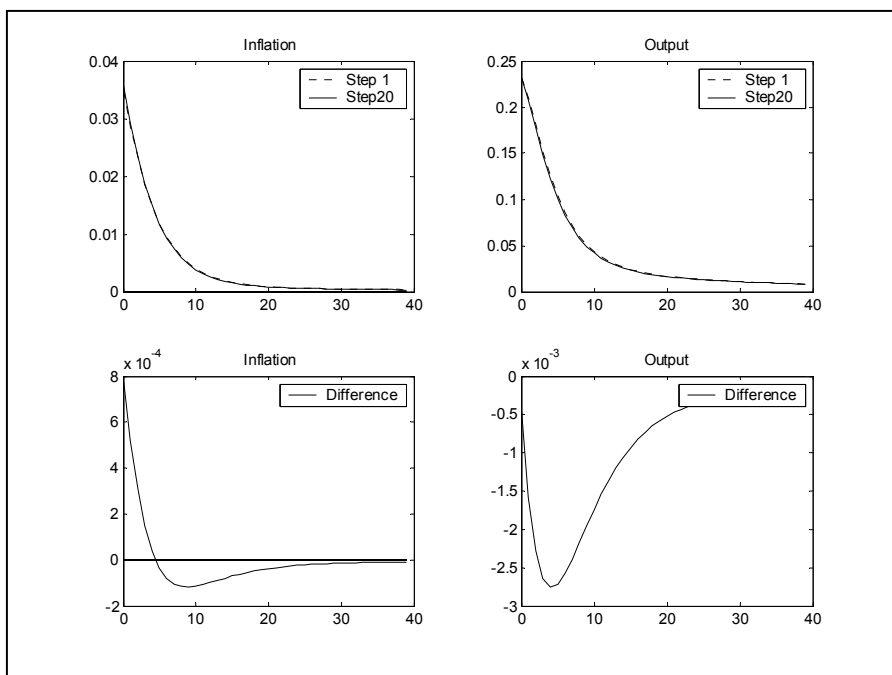


Figure 2: The difference between step 1 and step 20 in response to a monetary policy shock.

equal to zero. But, as has been emphasized by Woodford (2003, Ch. 5), this does not imply that future expected average capital gaps for this group of firms would be equal to zero as well. However, our first result shows that these future expected capital gaps can be treated as if they were zero without any sizable loss of accuracy in the determination of the equilibrium dynamics.

The intuition for this result is as follows: to the extent that there exists an adjustment cost for capital the firm's investment decision is forward-looking. If the planning horizon for the investment decision is long enough, a price setter and a non-price setter (both holding the same initial capital stocks) do not make very different investment decisions. The fact that they face the same probabilities of being allowed or restricted to change their prices over the relevant planning horizon leads to a small difference in their current investment decisions and, more generally, in their expected investment policies. Moreover, it should be noticed that our equation (26) takes the

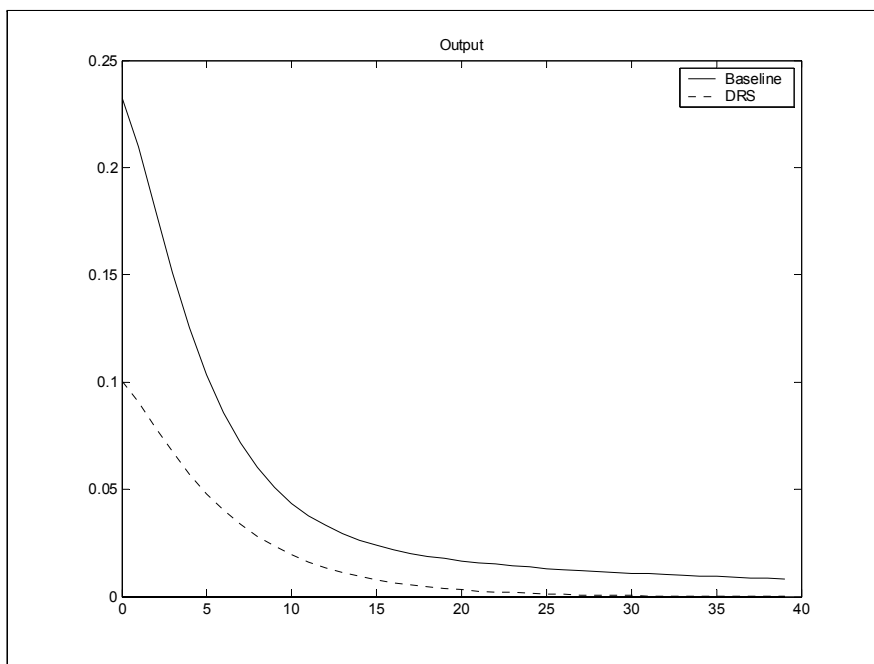


Figure 3: Output response to a monetary policy shock in the baseline model compared with the DRS specification.

same functional form as the inflation equation that has been derived by Sbordone (2001) and Galí et al. (2001) for models with decreasing returns to scale resulting from a constant capital stock at the firm level. Our first result therefore suggests that the main difference between the baseline model and the DRS specification lies in the determination of the average marginal cost in the economy. The functional form of the inflation equation itself is only affected to some negligible extent by the feature of endogenous capital accumulation at the firm level.

Second, we compare the responses to a monetary policy shock for the baseline model and the DRS specification. The result is shown in Figures 3 and 4: first, output is higher in the former – both on impact and during the transition period. Second, the inflation dynamics are similar in the two models. The intuition is as follows: there are two counteracting effects from endogenous capital accumulation on the determination of the marginal cost. First, investment spending adds to aggregate

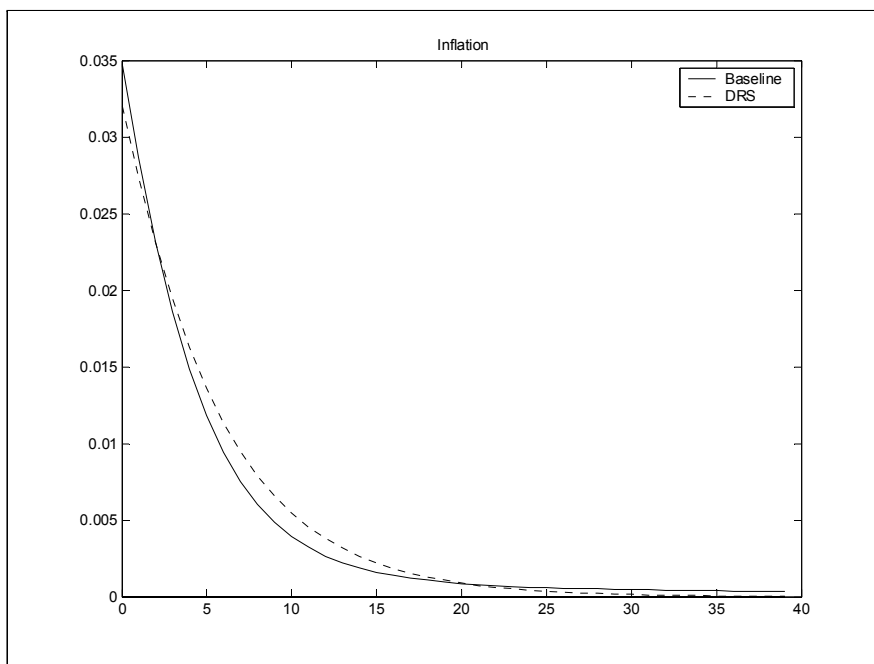


Figure 4: Inflation response to a monetary policy shock in the baseline model compared with the DRS specification.

demand, thereby implying higher production and an increase in the marginal cost in response to the shock. Second, the additional capital resulting from investment spending in one period increases the economy's productive capacity in subsequent periods. This implies a decrease in marginal costs. The latter is anticipated by forward-looking price setters.

4 Conclusion

We should emphasize the three contributions of our paper. First, we discuss some difficulties in a dynamic New-Keynesian model with staggered price setting à la Calvo and a convex capital adjustment cost at the firm level, as considered by Woodford (2003, Ch. 5). It is shown that the implied simultaneous price setting and investment decision has not been analyzed properly in the existing literature.

Second, our work fills that gap by proposing a tractable solution to the key problem of describing the inflation dynamics associated with that structure. Third, we use our framework to assess the extent to which the feature of endogenous capital accumulation at the firm level implies inflation and output dynamics that are different from the ones associated with a specification where the capital stock at the firm level is held constant. The difference lies primarily in the output dynamics, while the inflation dynamics are similar.

Appendix 1: Households

Throughout the Appendix we use the notation and the definitions that have already been introduced in the main text. A representative household maximizes expected discounted utility:

$$E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}), \quad (\text{A1})$$

where the consumption aggregator is defined as follows:

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (\text{A2})$$

We assume the following period utility function:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}. \quad (\text{A3})$$

The maximization is subject to the following sequence of budget constraints:

$$\int_0^1 P_t(i) C_t(i) di + E_t \{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t, \quad (\text{A4})$$

where D_{t+1} is the nominal payoff of the portfolio held at the end of period t , and T_t denotes profits resulting from ownership of firms. Cost minimization by households implies that the consumption demand functions for each type of goods are given by:

$$C_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t. \quad (\text{A5})$$

When combined with the assumptions stated in the text this structure implies the following first order conditions for the household's optimal choices:

$$C_t^\sigma N_t^\phi = \frac{W_t}{P_t}, \quad (\text{A6})$$

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}. \quad (\text{A7})$$

The first equation is the optimality condition for labor supply, and the second is a standard intertemporal optimality condition. The price of a risk-less one-period bond is given by $R_t^{-1} = E_t Q_{t,t+1}$, where R_t denotes the gross nominal interest rate.

Appendix 2: Price Setting and Investment

A time t price setter i chooses contingent plans for $\{P_{t+k}^*(i), K_{t+k+1}(i), N_{t+k}(i)\}_{k=0}^{\infty}$ in order to solve the following problem:

$$\max \sum_{k=0}^{\infty} E_t \{ Q_{t,t+k} [Y_{t+k}^d(i) P_{t+k}(i) - W_{t+k} N_{t+k}(i) - P_{t+k} I_{t+k}(i)] \}$$

s.t.

$$\begin{aligned} Y_{t+k}^d(i) &= \left(\frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}^d, \\ Y_{t+k}^d(i) &\leq N_{t+k}(i)^{1-\alpha} K_{t+k}(i)^\alpha, \\ I_{t+k}(i) &= I \left(\frac{K_{t+k+1}(i)}{K_{t+k}(i)} \right) K_{t+k}(i), \\ P_t(i) &= P_t^*(i), \\ P_{t+k+1}(i) &= \begin{cases} P_{t+k+1}^*(i) & \text{with prob. } 1 - \theta \\ P_{t+k}(i) & \text{with prob. } \theta, \end{cases} \\ &K_t(i) \text{ given.} \end{aligned}$$

Using the expressions for a firm's nominal marginal cost and nominal marginal savings in its labor cost given in equations (7) and (9), respectively, it follows that $P_t^*(i)$ and $K_{t+1}(i)$ must satisfy the first order conditions given in equations (6) and (8), respectively. A firm j that is restricted to change its price at time t solves the same problem, except for the fact that it takes $P_t(j)$ as given. Note that the first order condition associated with the investment decision takes the same functional form irrespective of whether a firm is allowed or restricted to change its price.

Appendix 3: Iteration for Inflation Dynamics

We stick to the notation introduced in the main text of indicating the step number in the superscript of each newly set relative price. For convenience, and since no ambiguity can arise, we do not indicate, however, the step number for all the other relevant variables.

We start by considering the log-linearized law of motion of the capital gap of an individual firm i . To this end, equation (8) is log-linearized and combined with the log-linearized law of motion of the aggregate capital stock in equation (22):

$$\widehat{k}_{t+1}(i) = \tau \widehat{k}_t(i) + \chi E_t \widehat{k}_{t+2}(i) - \varphi E_t \left\{ \theta (\widehat{p}_t^*(i) - \pi_{t+1}) + (1 - \theta) \widehat{p}_{t+1}^*(i) \right\}, \quad (\text{A8})$$

where: $\tau \equiv \frac{\epsilon_\psi}{\omega}$, $\chi \equiv \frac{\beta \epsilon_\psi}{\omega}$, $\varphi \equiv \frac{(1-\beta(1-\delta))\epsilon}{(1-\alpha)\omega}$, and $\omega \equiv \frac{\epsilon_\psi(1-\alpha) + (1-\beta(1-\delta)) + \beta \epsilon_\psi(1-\alpha)}{1-\alpha}$.

Step 1

A myopic price setter i sets a relative price $p_t^{*,1}(i)$ at time t consistent with the step one assumption that its capital gaps are expected to be closed already from the next period onward. This implies the following log-linearized price setting equation:

$$\widehat{p}_t^{*,1}(i) = \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+k} - \psi \widehat{k}_t(i). \quad (\text{A9})$$

Averaging the last equation over all price setting firms, solving forward, and invoking the price index we obtain equation (26) stated in the text:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{m}c_t. \quad (\text{A10})$$

Step 2

The step 2 assumption is that a price setter i chooses its relative price $p_t^{*,2}(i)$ consistent with the expectation that its capital gaps are closed from period $t + 2$ onward.

The log-linearized condition for price setting becomes:

$$\widehat{p}_t^{*,2}(i) = \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+k} - \psi \left[\widehat{k}_t(i) + \beta\theta \widehat{k}_{t+1}(i) \right]. \quad (\text{A11})$$

Combining the step 2 assumption with equation (A8) we obtain:

$$\widehat{k}_{t+1}(i) = \tau \widehat{k}_t(i) - \varphi E_t \{ \theta (\widehat{p}_t^{*,2}(i) - \pi_{t+1}) + (1 - \theta) \widehat{p}_{t+1}^{*,2}(i) \}. \quad (\text{A12})$$

An expression for $E_t \widehat{p}_{t+1}^{*,2}(i)$ is obtained by invoking the step 2 assumption again.

This yields:

$$E_t \widehat{p}_{t+1}^{*,2}(i) = \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+1+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+1+k} - \psi \widehat{k}_{t+1}(i). \quad (\text{A13})$$

Equations (A11), (A12), and (A13) show that the firm faces a simultaneous problem: price setting decisions and capital gaps depend on each other. We find it convenient to rewrite these equations using matrix notation:

$$\begin{bmatrix} \widehat{p}_t^{*,2}(i) \\ \widehat{k}_{t+1}(i) \\ E_t \widehat{p}_{t+1}^{*,2}(i) \end{bmatrix} = A \begin{bmatrix} \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+k} - \psi \widehat{k}_t(i) \\ \tau \widehat{k}_t(i) + \varphi \theta E_t \pi_{t+1} \\ \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+1+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+1+k} \end{bmatrix},$$

where

$$A^{-1} \equiv \begin{bmatrix} 1 & \psi\beta\theta & 0 \\ \varphi\theta & 1 & \varphi(1-\theta) \\ 0 & \psi & 1 \end{bmatrix}.$$

The elements in the first row of matrix A are given by the following expressions:

$$\begin{aligned} a_{11} &= \frac{1 - \varphi\psi(1-\theta)}{1 - \varphi\psi(1-\theta + \beta\theta^2)}, \\ a_{12} &= \frac{-\beta\psi\theta}{1 - \varphi\psi(1-\theta + \beta\theta^2)}, \\ a_{13} &= \frac{\beta\psi\theta\varphi(1-\theta)}{1 - \varphi\psi(1-\theta + \beta\theta^2)}. \end{aligned}$$

We, therefore obtain:

$$\begin{aligned} \widehat{p}_t^{*,2}(i) &= a_{11} \left[\sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+k} - \psi \widehat{k}_t(i) \right] \\ &\quad + a_{12} \left[\tau \widehat{k}_t(i) + \varphi \theta E_t \pi_{t+1} \right] \\ &\quad + a_{13} \left[\sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+1+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{m}c_{t+1+k} \right]. \end{aligned}$$

The resulting inflation equation is as follows:

$$\pi_t = \beta_{1,2} E_t \pi_{t+1} + \beta_{2,2} E_t \pi_{t+2} + \kappa_{0,2} \widehat{m}c_t + \kappa_{1,2} E_t \widehat{m}c_{t+1}, \quad (\text{A14})$$

where:

$$\beta_{1,2} \equiv \beta (\theta + a_{11} (1 - \theta)) + a_{12} (1 - \theta) \varphi,$$

$$\beta_{2,2} \equiv \beta (1 - \theta) (a_{13} - a_{12} \theta \varphi),$$

$$\kappa_{0,2} \equiv \frac{a_{11} \xi (1 - \theta)}{\theta},$$

$$\kappa_{1,2} \equiv \frac{a_{13} \xi (1 - \theta)}{\theta}.$$

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