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Monetary Policy Rules for an Open Economy

by

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Abstract

The most popular simple rules for the interest rate, due to Taylor (1993a) and Henderson and McKibbin (1993), are both meant to inform monetary policy in economies that are closed. On the other hand, their main open economy alternative, i.e. Ball's (1999) rule based on a Monetary Conditions Index (MCI), may perform poorly in the face of specific types of exchange rate shocks and thus cannot offer guidance for the day-to-day conduct of monetary policy. In this paper we specify and evaluate a comprehensive set of simple monetary policy rules that are suitable for small open economies in general, and for the UK in particular. We do so by examining the performance of a battery of simple rules, including the familiar Taylor and Henderson and McKibbin rules and MCI-based rules à la Ball. This entails comparing the asymptotic properties of a two-sector open-economy dynamic stochastic general equilibrium model calibrated on UK data under different rules. We find that an inflation forecast based rule ('IFB'), i.e. a rule that reacts to deviations of expected inflation from target is a good simple rule in this respect, when the horizon is adequately chosen. Adding a separate response to the level of the real exchange rate (contemporaneous and lagged) appears to reduce the difference in adjustment between output gaps in the two sectors of the economy, but this improvement is only marginal. Importantly, an IFB rule, with or without exchange rate adjustment, appears robust to different shocks, contrary to naïve or Ball's MCI-based rules.

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1. Introduction

The literature on simple rules for monetary policy is vast.[1] It contains theoretical research comparing rules that respond to alternative intermediate and final targets, backward- and forward-looking rules, and finally, rules which include or exclude interest rate smoothing terms. It also contains work on historical estimates of monetary policy rules for various countries.

However, the literature does not contain a thorough analysis of simple rules for open economies, i.e. for economies where the exchange rate channel of monetary policy plays an important role in the transmission mechanism. The most popular simple rules for the interest rate — due to Taylor (1993a) and Henderson and McKibbin (1993) — for example, were both designed for the United States and, thus, on the assumption that the economy is closed. And the main open economy alternatives, (for example, the rule by Ball (1999) based on a Monetary Conditions Index (MCI)), may perform poorly in the face of specific types of exchange rate shocks and thus cannot offer guidance for the day-to-day conduct of monetary policy. So at present we only have a choice of ignoring the exchange rate channel of monetary transmission completely (Taylor, Henderson and McKibbin) or including it in an ad hoc way that may not always prove right (MCI-based rules).

In this paper we specify and evaluate a family of simple monetary policy rules that may stabilize inflation and output in small open economies at a lower social cost than existing rules. These rules parsimoniously modify alternative closed- or open-economy rules to analyse different ways of explicitly accounting for the exchange rate channel of monetary transmission. We compare the performance of these rules to that of a battery of alternative rules when the model economy is buffeted by various shocks. The alternatives we consider include the Taylor and Henderson and McKibbin closed-economy rules, naïve MCI-based rules as well as Ball's MCI-based rule, and inflation forecast-based rules. Some of the rules in the family we consider appear to be robust across a set of different shocks, including shocks to the domestic economy emanating from the rest of the world. This is in contrast to closed-economy rival simple rules, which ignore the exchange rate channel of monetary transmission, and naïve or Ball's MCI-based rules, the performance of which can be highly shock-specific.

To test the rules, we stylise the economy — that we calibrate to UK data — as a two-sector open-economy dynamic stochastic general equilibrium model. The export/non-traded sector split is important because it allows us to discern different impacts of the same shock on output and inflation in the two sectors. Identification of sectoral inflation and output dynamics is a key element on which to base the design of efficient policy rules. More generally, it also makes it possible for the monetary authority to consider the costs of price stabilization on each sector of the economy.

Because it is theoretically derived on the assumption that consumers maximise utility and firms maximise profits, the model has a rich structural specification. This enables us to contemplate shocks that could not be analysed in less structural or reduced form small macro-models.

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[1] See Bryant *et al* (1993) and Taylor (ed.) (1999).

In particular, with our model, we can examine the implications of shocks to aggregate demand such as a shock to households' preferences, or a shock to the rest of the world's income. On the supply side, we can consider shocks overseas inflation. We can analyse the impact of a relative productivity shock on the two sectors and investigate how this affects the real exchange rate by altering the price of the non-tradables relative to export goods. We can also look at the effects of a change in the price of imported intermediate goods. We can examine the effects of shocks to the foreign exchange risk premium. Finally, we can look at the implications of a monetary policy shock, both home and abroad.

The ability to examine all these different shocks is important when comparing alternative policy rules for an open economy, because the efficient policy response to changes in the exchange rate will typically depend on what shock has hit the economy — with different shocks sometimes requiring opposite responses. For this purpose our small economy general equilibrium model is sufficient. A two-country model would enable us to look at these same shocks, but we believe the small-economy assumption is more realistic for the UK.

In short, this model is well suited to our analysis for three reasons. First it is a structural, theoretically based model. The structural nature of the model is important because it implies that our policy analysis (i.e. comparison of different rules/regimes) is less subject to the Lucas critique than a more reduced-form model. Second, it offers a more disaggregated picture of the economy than many existing models. This allows us to identify the different dynamics of output and inflation after a shock — a valuable input to the efficient design of rules. Third, because it is structural and built from micro-principles, it allows us to consider shocks (such as preference or relative productivity shocks) which are key for the design of a rule meant to be a 'horse for all courses' in an open economy setting.

The rest of the paper is organised as follows. In section 2 we lay out the model that we employ throughout and describe its steady state properties. The solution and calibration of the model are discussed in section 3. In section 4 we study some properties of the model. In section 5 we specify a family of open-economy simple rules and present results comparing the stabilisation properties of these rules against those of a battery of alternative simple rules, in the face of various disturbances. Finally, section 6 concludes. The Technical Appendix contains further details about the model's non-linear and log-linear specifications.

2.   A two-sector open-economy optimising model

The model we use is a calibrated stochastic dynamic general equilibrium model of the UK economy with a sectoral split between exported and non-traded goods. Its specification draws on the literature on open-economy optimising models by Svensson and van Wijnbergen (1989), Correia, Neves and Rebelo (1994), Obstfeld and Rogoff (1996), and more recent work by McCallum and Nelson (1999). In this sense, the model is close in spirit to a number of open-economy models developed at or after the time of writing by Monacelli (1999), Gali and Monacelli (1999), Ghironi (2000), Smets and Wouters (2000), Benigno and Benigno (2000) and Devereux and Engle (2000). However, it extends upon all of these, individually (and other closed-economy optimising models), by introducing several novel features that are described in detail below.

The model describes an economy that is 'small' with respect to the rest of the world. In practice, this means that the supply of domestically produced traded goods does not affect the price of these goods internationally. It also means that the price of imported foreign goods, foreign interest rates and foreign income are exogenous in this model, rather than being endogenously determined in the international capital and goods markets, as would happen in a multiple-country, global-economy model. This assumption considerably simplifies our analysis; and because we are not interested here in studying either the transmission of economic shocks across countries or issues of policy interdependence, it comes at a relatively small price.

As we are interested in evaluating alternative monetary policy rules, we specify monetary policy within the model as a rule for the nominal interest rate (the policy instrument). We look at alternative rules in order to see whether responding to some 'open-economy' variables such as the exchange rate or the balance of trade can improve the stabilisation properties of rules designed for a closed economy context.

2.1 Household preferences and government policy

The economy is populated by a continuum of households of unit mass. Each household is infinitely lived and has identical preferences defined over consumption of a basket of (final) imported and non-traded goods, leisure and real money balances at every date. Households differ in one respect: they supply differentiated labour services to firms. Preferences are additively log-separable and imply that household $j \in (0,1)$ maximises:

$$E_0 \sum_{t=0}^{\infty} \boldsymbol{b}^t \left(\exp(\boldsymbol{n}_t) \ln(c_t(j) - \boldsymbol{x}_c c_{t-1}(j)) + \boldsymbol{d} \ln(1 - h_t(j)) + \frac{\boldsymbol{c}}{1-\boldsymbol{e}} \left(\frac{\Omega_t(j)}{P_t} \right)^{1-\boldsymbol{e}} \right) \qquad \textbf{(1)}$$

where $0 < \boldsymbol{b} < 1$; $\boldsymbol{d}$, $\boldsymbol{c}$ and $\boldsymbol{e}$ are restricted to be positive and E_0 denotes the expectation based on the information set available at time zero. In equation **(1)**, $c_t(j)$ is total time t real consumption of household j, $\boldsymbol{n}_t$ is a white noise shock to preferences — essentially a demand shock, described in more detail in sections 3 and 4 — and $h_t(j)$ is labour supplied to market activities, expressed as a fraction of the total time available. So the term $(1 - h_t(j))$ captures the utility of time spent outside work. The last term $\Omega_t(j)/P_t$ represents the flow of transaction-facilitating services yielded by real money balances during time t (more on this later). Hence here, as in the standard Sidrauski-Brock model, money enters the model by featuring directly in the utility function.

In addition, since $\boldsymbol{x}_c \in [0,1)$, preferences over consumption exhibit habit formation, with the functional form used in **(1)** similar to that of Carrol *et al.* (1995) and Fuhrer (2000). This implies that preferences are not time-separable in consumption, so that households' utility depends not only on the level of consumption in each period, but also on their level in the previous period.

Total consumption is obtained by aggregating the consumption of imported and non-traded goods $c_{M,t}$ and $c_{N,t}$ via the geometric combination $c_t = c_{M,t}^{\boldsymbol{g}} c_{N,t}^{1-\boldsymbol{g}}$, where $\gamma \in (0,1)$. Here $c_{M,t}$ and $c_{N,t}$ represent imported and non-traded goods purchased by the consumer from

retailers at prices $P_{M,t}$ and $P_{N,t}$, respectively. It is easily shown that the consumption-based price deflator is given by $P_t = \dfrac{P_{M,t}^{g} P_{N,t}^{1-g}}{g^{g}(1-g)^{1-g}}$.[2]

Households have access to a state contingent bond market. Bond $b(s)$ in this market is priced in units of consumption, has price $r(s)$ in period t, and pays one unit of consumption in state s in period $t+1$. In practice, this means that households within the domestic economy can insure themselves perfectly against idiosyncratic shocks. In equilibrium, consumption and real money balances are equal across households. So households differ only because labour supply varies across the population.

In addition to this bond market, each household can also access a domestic and a foreign nominal government bond market at interest rates i and i_f, respectively. For the time being, we assume that both kinds of bond are riskless, but we investigate alternative assumptions later (see sub-section 2.4). Money is introduced into the economy by the government. Under Ricardian equivalence, we can assume without loss of generality a zero net supply of domestic bonds. Then the public sector budget constraint requires that all the revenue associated with money creation must be returned to the private sector in the form of net lump-sum transfers in each period:

$$M_t - M_{t-1} = T_t - t_t \tag{2}$$

where M_t is end-of-period t nominal money balances, T_t is a nominal lump-sum transfer received from the home government at the start of period t and t_t is a lump sum tax levied on consumers. For simplicity we assume the tax is constant at its steady state level.

The household's dynamic budget constraint in each period is given by equations **(3)** and **(4)** below. Equation **(3)** describes the evolution of nominal wealth. Equation **(4)** defines the nominal balances available to consumers to spend at time t. This reflects the assumption that consumers participate in the financial markets before spending money on goods and services. As suggested by Carlstrom and Fuerst (1999), entering money balances as defined in **(4)** in the utility function, gives a better measure of period utility; one in which we account exclusively for the services of balances that are actually available to households when spending decisions are taken.

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[2] Formally, P_t defines the minimum cost of financing a unit of consumption, c_t. See Obstfeld and Rogoff (1996, pp) for a simple example.

$$M_t(j) + B_t(j) + \frac{B_{f,t}(j)}{e_t(j)} + P_t \int r_t(s) b_t(s,j) ds = M_{t-1}(j) + (1 + i_{t-1}) B_{t-1}(j) + (1 + i_{f,t-1}) \frac{B_{f,t-1}(j)}{e_t} +$$

$$P_t \int b_{t-1}(s,j) ds + W_t(j) h_t(j) + D_t + T_t - P_t c_t(j)$$

$$\text{(3)}$$

$$\Omega_t(j) = M_{t-1}(j) + T_t + (1 + i_{t-1}) B_{t-1}(j) + (1 + i_{f,t-1}) \frac{B_{f,t-1}(j)}{e_t} - B_t(j) - \frac{B_{f,t}(j)}{e_t} \qquad \text{(4)}$$

where M_{t-1} is nominal money balances at time $t-1$, $B_{t-1}(j)$ and $B_{f,t-1}(j)$ are time $t-1$ holdings of domestic and foreign bonds, respectively and D_t are lump sum dividends from shares held in (domestic) firms. Household j's holdings of (state contingent) bond $b_t(s)$ are $b_t(s,j)$. With e_t we denote the nominal exchange rate, expressing domestic currency in terms of units of foreign currency.[3] Finally, $W_t(j)$ is the nominal wage rate received by household j. Because each household supplies differentiated labour services, it has some market power over the wage rate. So we assume that household j chooses $c_t(j)$, $B_{t-1}(j)$, $B_{f,t-1}(j)$, $\Omega_t(j)$, $M_t(j)$ and $b(s,j)$ to maximise **(1)** subject to **(3)** and **(4)**. The choice of wage $W(j)$ is discussed in section 2.3.2.

2.2   Technology and market structure

This sub-section describes the supply side of the economy by sector.

We assume that in our economy there are two kinds of producing firms: non-traded goods producers and export producers. By definition, non-traded goods are only consumed domestically, while we assume that exports produced at home are consumed only abroad. To produce, the exports and non-traded goods producers buy intermediate non-labour inputs for production (labour is purchased domestically from the households) from a group of 'imported intermediate input retailers'. Since consumers also purchase their final imports and non-traded goods via 'retailers', the economy has a total of three groups of retailing firms: imported intermediates retailers, non-traded good retailers and final imports retailers. Finally, both final imports retailers and imported intermediates retailers originally purchase their 'input' from a group of 'importers', who in turn, acquire goods from the world markets. There are two types of importers, one for each import. We refer to the first group as 'final goods importers' and to the second group as 'intermediate inputs importers'.

Chart 1 depicts the goods market structure of the model.

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[3] So that an increase in e_t represents an appreciation of the domestic currency.

Chart 1: Goods Markets Structure

This seemingly complicated representation of the supply side is desirable because, as we discuss later (sub-section 2.4) enables us to easily introduce nominal rigidities, which are essential for monetary policy to affect real variables in the economy. In what follows, we describe each sector in turn, starting from the non-traded goods sector. By 'sector' we mean a larger group of firms, which includes producers and retailers operating in the market of the same good. The behaviour of the two groups of 'importers' is described in the 'Final Imports Sector' and in the 'Intermediate Goods Sector' sub-sections, rather than in separate sub-sections. Next, we discuss the way in which the labour market is organised (sub-section 2.2.5), and then we focus more specifically on price and wage setting behaviour (sub-section 2.3).

2.2.1 *Non-traded goods sector*

We assume that non-traded goods retailers are perfectly competitive. These retailers purchase differentiated goods from a unit continuum of monopolistically competitive non-traded goods producers and combine them using a CES technology:

$$y_{N,t} = \left[\int_0^1 y_{N,t}(k)^{1/(1+q_N)} \, dk \right]^{1+q_N} \tag{5}$$

Profit maximisation implies that the demand for non-traded goods from producer $k \in (0,1)$ is given by

$$y_{N,t}(k) = \left(\frac{P_{N,t}(k)}{P_{N,t}} \right)^{-\frac{1+q_N}{q_N}} y_{N,t} \tag{6}$$

where $P_{N,t}(k)$ is the price of the non-traded good set by firm k. The assumption of perfect competition implies that retailers' profits are zero. This requires that:

$$P_{N,t} = \left[\int_0^1 P_{N,t}(k)^{-1/q_N} \, dk \right]^{-q_N} \tag{7}$$

Producers of non-traded goods use a Cobb-Douglas technology with inputs of an intermediate good (I) and labour (h):

$$y_{N,t}(k) = A_{N,t} h_{N,t}(k)^{a_N} I_{N,t}(k)^{1-a_N} \tag{8}$$

Non-traded goods producers are price takers in factor markets and purchase inputs from imported intermediates retailers (more on this later). So non-traded goods producers choose factor demands and a pricing rule (discussed in section 2.3) subject to technology **(5)** and demand function **(6)**.

2.2.2 *Export sector*

The export sector produces using a Cobb-Douglas technology:

$$y_{X,t} = A_{X,t} h_{X,t}^{a_X} I_{X,t}^{1-a_X} \tag{9}$$

where $A_{X,t}$ is a productivity shock. We assume that production is efficient in the export sector, i.e.,that marginal cost is equal to price in equilibrium.

We assume that the scale of exports is determined by a downward sloping demand curve:

$$X_t = \left(\frac{e_t P_{X,t}}{P_t^*}\right)^{-h} y_{f,t}^b, \tag{10}$$

where P_t^* is the exogenous foreign currency price of exports and $y_{f,t}$ is exogenous world income.[4] This is the same formulation of export demand as McCallum and Nelson (1999). The exogenous foreign price of exports is the same as the exogenous foreign currency price of imports used in equation **(14)** below. This simplification reduces the number of exogenous shock processes in the model.[5]

2.2.3 *Intermediate goods sector*

Intermediate goods are sold to export and non-traded producers by retail firms that operate in the same way as the firms which retail final imports and non-traded goods to consumers. These 'imported intermediate retailers' purchase inputs from 'intermediate goods importers' who buy a homogenous intermediate good in the international markets and then costlessly transform it into a differentiated good that they sell to retailers. This yields a nominal profit for firm *k* of:

$$D_{I,t}(k) = \left[P_{I,t}(k) - \frac{P_{I,t}^*}{e_t}\right] y_{I,t}(k) \tag{11}$$

where $y_{I,t}(k) = \left(\frac{P_{I,t}(k)}{P_{I,t}}\right)^{-\frac{1+q_I}{q_I}} y_{I,t}$, as in previous sections and $P_{I,t}^*$ is the exogenous foreign currency price of the intermediate good. The firm chooses a pricing rule (discussed in sub-section 2.3) to maximise the discounted future flow of real profits.

2.2.4 *Final imports sector*

We assume that retailers of final imports are perfectly competitive, purchase differentiated imports from 'final goods importers' and combine them using a technology analogous to that used by non-traded retailers. Following the analysis of section 2.2.1 we get:

$$y_{M,t}(k) = \left(\frac{P_{M,t}(k)}{P_{M,t}}\right)^{-\frac{1+q_M}{q_M}} y_{M,t}$$

$$\tag{12}$$

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[4] Note that firms in the export sector cannot exploit the downward sloping demand curve if the price elasticity of demand is less than unity, as we assume in the model.

[5] This is important because, as discussed in section 3, every exogenous foreign currency price must be deflated by a numeraire foreign price for the system of exogenous shocks to have stable properties (in terms of our model).

and

$$P_{M,t} = \left[\int_0^1 P_{M,t}(k)^{-1/q_M} \, dk \right]^{-q_M} \tag{13}$$

As for intermediate imported goods, final imported goods are purchased from world markets by importers who buy a homogenous final good from overseas and costlessly convert it into a differentiated good.[6] Nominal profits for these importers in period t are then given by

$$D_{M,t}(k) = \left[P_{M,t} - \frac{P_t^*}{e_t} \right] y_{M,t}(k) \tag{14}$$

where P_t^* is the exogenous foreign currency price of the imported good. Firms choose a pricing rule (discussed in section 2.3) to maximise the discounted flow of real profits subject to demand **(12)**.

2.2.5   *Labour market*

As discussed in section 2.1, households set the nominal wage that must be paid for their differentiated labour services. We assume that a perfectly competitive firm combines these labour services into a homogenous labour input that is sold to producers in the non-traded and export sector. This set-up follows Erceg, Henderson and Levin (2000) and relies on an aggregation technology analogous to those discussed in previous sections:

$$h_t = \left[\int_0^1 h_t(j)^{1/(1+q_W)} \, dj \right]^{1+q_W} \tag{15}$$

This implies a labour demand function for household j's labour of the form:

$$h_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+q_W}{q_W}} h_t. \tag{16}$$

Households take the labour demand curve (16) into account when setting their wages, as discussed in the next section.

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[6] Intuitively, this can be thought of as 'branding' a product.

2.3 Price and wage setting

As we have anticipated, the supply-side structure described in section 2.2 facilitates the introduction of nominal rigidities in the specification of our model economy. Our intent is in fact to assume that in both goods and labour markets prices are sticky in the sense of Calvo (1983).[7] Below we discuss what this implies for the pricing decisions facing different economic agents, starting with the pricing decisions of non-traded goods producers.

2.3.1 *Price setting*

We assume that the non-traded goods producers solve the following optimisation problem:

$$\max E_t \sum_{s=0}^{\infty} (\boldsymbol{b}\boldsymbol{f}_N)^s \Lambda_{1,t+s} \left(\frac{(1+\boldsymbol{p})^s P_{N,t}(k)}{P_{t+s}} - V_{t+s} \right) y_{N,t+s}(k)$$

$$\text{subject to} \quad y_{N,t+s}(k) = \left(\frac{(1+\boldsymbol{p})^s P_{N,t+s}(k)}{P_{N,t+s}} \right)^{-\frac{(1+\boldsymbol{q}_N)}{\boldsymbol{q}_N}} y_{N,t+s}$$

where $\boldsymbol{f}_N$ is the probability that the firm cannot change its price in a given period, and Λ_1 is the consumer's real marginal utility of consumption. The steady state gross inflation rate is $(1+\boldsymbol{p})$ and prices are indexed at the steady state rate of inflation. So when a firm sets a price at date t, the price automatically rises by $\boldsymbol{p}$% next period if the firm does not receive a signal allowing it to change price. The parameter $\boldsymbol{q}_N$ represents the net mark-up over unit costs that the firm would apply in a flexible-price equilibrium. Finally V (expressed below) is the minimised unit cost of production (in units of final consumption) that solves:

$$V_{t+s} = \min \left\{ \frac{W_{t+s}}{P_{t+s}} h_{N,t+s}(k) + \frac{P_{I,t+s}}{P_{t+s}} I_{N,t+s}(k) \right\} \text{ subject to } A_{N,t+s} h_{N,t+s}(k)^{\boldsymbol{a}_N} I_{N,t+s}(k)^{(1-\boldsymbol{a}_N)} = 1$$

The first order condition for the firm's pricing decision can be written as:

$$E_t \sum_{s=0}^{\infty} (\boldsymbol{b}\boldsymbol{f}_N)^s \Lambda_{1,t+s} \left(\frac{-\boldsymbol{q}_N (1+\boldsymbol{p})^s P_{N,t}(k)}{P_{t+s}} + (1+\boldsymbol{q}_N)V_{t+s} \right) y_{N,t+s}(k) = 0. \tag{17}$$

Importers of the final import good for consumption and importers of the intermediate good used in production face the same pricing problem confronting non-traded goods producers. But because we want to introduce sluggishness in the passthrough of exchange rate changes to import prices, here we assume that pricing decisions are based on the information set

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[7] For more details, see the Technical Appendix.

available in the previous period. This is the assumption made by Monacelli (1999). Given this additional assumption, the first order conditions become:

$$E_{t-1} \sum_{s=0}^{\infty} (bf_M)^s \Lambda_{1,t+s} \left(\frac{-q_M (1+p)^s P_{M,t}(k)}{P_{t+s}} + (1+q_M)V_{M,t+s} \right) y_{M,t+s}(k) = 0 \qquad (18)$$

$$E_{t-1} \sum_{s=0}^{\infty} (bf_I)^s \Lambda_{1,t+s} \left(\frac{-q_I (1+p)^s P_{I,t}(k)}{P_{t+s}} + (1+q_I)V_{I,t+s} \right) y_{I,t+s}(k) = 0 \qquad (19)$$

where the notation is analogous to that used above. The trivial production structures in these sectors imply that unit costs are simply given by $V_{M,t} = \dfrac{P_t^*}{e_t P_t}$ and $V_{I,t} = \dfrac{P_{I,t}^*}{e_t P_t}$.

2.3.2 *Wage setting*

The wage setting behaviour of households is based on Erceg *et al* (2000) and is closely related to the price setting behaviour of non-traded goods producing firms. Following Erceg *et al* (2000), we suppose that household *j* is able to reset its nominal wage contract with probability $(1-f_W)$. If the household is allowed to reset its contract at date *t*, then it chooses a nominal wage $W_t(h)$ that will be indexed by the steady state inflation rate until the contract is reset once more. The household chooses this wage rate to maximise discounted expected utility for the duration of the contract, subject to the budget constraint **(3)** and the labour demand function **(16)**. Hence, the first order condition is:

$$E_t \sum_{s=0}^{\infty} (bf_W)^s \left[\frac{(1+p)^s W_t(j)}{P_{t+s}} \Lambda_{1,t+s} - (1+q_W) \frac{d}{1-h_{t+s}(j)} \right] h_{t+s}(j) = 0. \qquad (20)$$

2.4 The Balance of Payments

Combining the first-order conditions for domestic and foreign bonds from the household's optimisation problem gives the familiar uncovered interest parity condition. A first-order approximation gives:

$$E_t \log e_{t+1} - \log e_t = i_{f,t} - i_t + z_t \qquad (21)$$

where we have added a stochastic risk premium term (z_t) to reflect temporary but persistent deviations from UIP, as in Taylor (1993b).

Despite the fact that domestic nominal bond issuance is assumed to be zero at all dates, domestic households can intertemporally borrow or save using foreign government bonds assets. As a result it is not necessary for the trade balance to be zero in each period as would be the case if we had imposed an equilibrium in which all government liabilities are held by residents of the issuing country. In practice, positive holdings of foreign bonds mean that the

domestic economy can run a trade deficit in every period financed via the interest payments that it receives on the foreign assets held.

In addition, since the economy is small, the foreign interest rate is exogenous in the model. So the supply of foreign government bonds is perfectly elastic at the exogenous world nominal interest rate. This means that steady state foreign bond holdings are indeterminate in our model. As a result, temporary nominal shocks can shift the real steady state of the model through the effects on nominal wealth (see Obstfeld and Rogoff (1996)). This means that the steady state around which log-linear approximations are taken is moving over time.

This is a common feature of small open economy monetary models and can be avoided in a number of ways. One approach is to make assumptions about the form of the utility function (see, for example, Correia et al, 1995) or the way in which consumption is aggregated. This is difficult to implement in our model if we wish to retain a rich structural specification. Another approach is to impose a global equilibrium condition on asset holdings (and restrict the trade balance to be zero in all periods). But this seems too restrictive. So instead, we substitute foreign bond holdings out of the model and concentrate on the movements of the other variables, as in McCallum and Nelson (1999).

2.5   The Transmission Mechanism

In an open economy, the exchange rate is an important channel of monetary transmission. This channel has a number of effects. First, and most obviously, the demand for exports is directly affected by exchange rate movements. Exporters also feel the effect of exchange rate changes through the price of imported intermediate goods. Importers of intermediate goods face an increase in their nominal unit costs as the nominal exchange rate depreciates. This is passed onto producers (including producers of non-traded goods) gradually, reflecting the fact that importers are required to set prices one period in advance and only a fraction of them are able to change price in any particular quarter.

Exchange rate changes also affect the consumer price index through the direct impact on the prices of imported consumption goods. Again this occurs with a lag because of the assumptions reflecting importers' pricing decisions. And the exchange rate affects consumer prices as non-traded goods producers pass on changes in production costs gradually (reflecting the Calvo pricing assumption).

It is clear from this discussion that the exchange rate affects different sectors unevenly. In summary, there are two channels of monetary transmission in this model. There is a standard interest rate channel, that influences the consumption-saving decision and hence the output gap and inflation. In addition, there is an exchange rate channel that directly affects export sector prices; and indirectly affects exports and non-traded goods' prices through changes in the cost of the intermediate imported inputs.

3. Model Solution and Calibration

3.1 Solving the Model

To solve the model we first derive the relevant first order conditions discussed in section 2. We then solve for the non-stochastic flexible price steady state and take the log-linear approximation of each non-linear first-order condition around this steady state. This procedure is presented in the Technical Appendix.

As shown in the Technical Appendix, the model can be cast in first order form:

$$\mathbf{A}E_t z_{t+1} = \mathbf{B}z_t + \mathbf{C}x_t \tag{22}$$

$$x_{t+1} = \mathbf{P}x_t + ?_t \tag{23}$$

where $\mathbf{A}$ and $\mathbf{B}$ are 31×31 matrices, while $\mathbf{C}$ is a 31×8 matrix. $\mathbf{P}$ is an 8×8 matrix containing the first order cross-correlation coefficients of the exogenous variables, whose white noise i.i.d. innovations are expressed by the vector $?_t$.

Let f_t and k_t denote the endogenous and pre-determined parts of the vector z_t respectively. Then the rational expectations solution to (11)-(12), expressing the vector of endogenous variables f_t as functions of predetermined (k_t) and exogenous (x_t) variables, can be written as:

$$f_t = \Xi_1 k_t + \Xi_2 x_t \tag{24}$$

$$\begin{bmatrix} k_{t+1} \\ x_{t+1} \end{bmatrix} = ? \begin{bmatrix} k_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ ?_t \end{bmatrix} \tag{25}$$

In this paper we computed this solution using Klein's (1997) algorithm.

3.2 Calibration

We calibrate the model to match key features of UK macroeconomic data. For this purpose, we set the discount factor, b, to imply a steady-state annual real interest rate of 3.5%. This is equal to the average ten-year real forward rate derived from the index-linked gilt market in the United Kingdom since these were first issued in March 1983. The steady state inflation rate was set at 2.5% per year: the current UK inflation target.

We assume that steady state foreign inflation was equal to steady state domestic inflation; that is, 2.5% per year. An implication is that the nominal exchange rate is stationary. We normalise the steady state prices of traded goods and intermediate goods (in foreign currency) to unity.

To set the parameter in the utility function reflecting preferences for imports vis-à-vis non-traded goods, g, we use data on consumption spending on traded versus non-traded goods. To do so, we equate consumption of non-traded goods with output of non-traded goods and set consumption of imports equal to output of traded goods less exports of traded goods. We

set g equal to 0.103, so that the implied constant share of consumption spending on traded versus non-traded goods matched the average value seen in the available data.[8] We set the habit formation parameter such that the persistence of the output response to shocks in the model is similar to that in the UK data. The value chosen is $x_c = 0.7$.

The weight on leisure vis-à-vis consumption in the utility function, d, is set to ensure that steady-state hours were equal to 0.3 in the absence of 'distortions'.[9] The required value is 1.815. Though essentially a normalisation, this choice corresponds to an 18 hour day available to be split between work and leisure time and workers, on average, working fifty 40-hour weeks in a year. We set $q_W = 0.165$ as this is consistent with steady state hours of 0.273 when habit formation and monopolistic supply of labour are accounted for. This level of hours represents a deviation from 'distortion-free' steady hours equal to 9% - the average level of UK unemployment using the LFS measure. We set $f_W = 0.75$ as this implies that wage contracts are expected to last for one year.

We set the weight on money in the utility function to $\chi = 0.005$. This implies that the ratio of real money balances to GDP is around 30% in steady state. Though this is somewhat higher than the ratio of M0 to nominal GDP, it is not clear that 'money' in our model is best proxied by M0 in the data. The ratio of M4 to quarterly nominal GDP is larger – the average for 1963 Q1-2000 Q1 is around 1.4. So our calibration fixes the ratio of steady state real money balances to GDP at an intermediate level. We set $\varepsilon = 1$ which implies a unit elasticity of money demand. This is consistent with findings for the UK (see QMA 1999).

To calibrate parameters on the production side of the model requires sectoral data. A description of the assumptions needed to do this is given in the Appendix. We first calibrate the mark-ups that firms in each sector apply to unit marginal costs, using the results of Small (1997). Weighting these mark-ups with the respective shares in value added output,[10] we obtain a value for the non-traded sector gross mark-up of 1.17. Gross mark-ups for the traded and intermediates goods sectors are found to be 1.183 and 1.270. These calibrations imply values for f_N, f_T and f_I of 0.17, 0.183 and 0.270, respectively.

Computing elasticities of non-traded and traded goods output with respect to employment gives estimates of a_N and a_X, of 0.763 and 0.636, respectively. To calibrate the probabilities that firms in a particular sector receive signals allowing them to change price, we use data on the average number of price changes each year for different industries. Hall, Walsh and Yates (1997) find that the median manufacturing firm changes price twice a year, the median construction firm 3 or 4 times a year, the median retail firm 3 or 4 times a year and the median 'Other Services' firm once a year. On this basis, we assume an average duration of prices of six months for firms in the import goods and intermediate goods sectors and an average duration of four months for firms in the non-traded goods sector. This implies values for f_M, f_I and f_N of 0.33, 0.33 and 0.43, respectively.

---

[8] The only reliable data we could obtain on output in current prices by industry is annual and covers only the period 1989 to 1998.

[9] This involved setting the habit formation parameter (x) to zero and assuming that the elasticity of substitution between labour types tended to infinity ($q_W=0$).

[10] Using weights from the 1985 ONS Blue Book.

The export demand function requires us to set the income and price elasticities. We set the income elasticity to unity and the price elasticity (h) to 0.2. The latter assumption approximates the one-quarter response of the UK export equation in the Bank of England's Medium Term Macroeconomic Model (see Bank of England (1999, pp50-51)).

To derive series for 'total factor productivity' in each sector, we use quarterly data on gross value added by industry at constant 1995 prices from 1983 onwards (ETAS Table 1.9) and 'workforce jobs' by industry for the same period.[11] We calculate our productivity series as:

$$\ln A_{Z,t} = \ln y_{Z,t} - a_Z \ln h_{Z,t} \tag{26}$$

where Z indexes the sector, y is value added and h is workforce jobs. An implicit assumption is that movements in intermediate inputs are 'small' relative to movements in output and employment. This is required to equate this measure of A with 'total factor productivity'.

After HP-filtering the two productivity series obtained from **(15)** we estimate the stochastic processes for the productivity terms using a vector autoregressive (VAR) system:

$$\begin{pmatrix} \hat{A}_t^T \\ \hat{A}_t^N \end{pmatrix} = R_A \begin{pmatrix} \hat{A}_{t-1}^T \\ \hat{A}_{t-1}^N \end{pmatrix} + \begin{pmatrix} e_{T,t} \\ e_{N,t} \end{pmatrix} \tag{27}$$

The disturbances $e_{T,t}$ and $e_{N,t}$ are normally distributed with variance-covariance matrix $\mathbf{V}_D$. Given that the model has zero productivity growth in steady state, $\hat{A}^Z$ refers to 'log-deviations of productivity in sector Z from a Hodrick-Prescott trend'. Our estimation results imply:

$$R_A = \begin{pmatrix} 0.705 & 0.227 \\ -0.066 & 0.784 \end{pmatrix} \text{ and } \mathbf{V}_D = 10^{-5} \times \begin{pmatrix} 3.19 & 1.43 \\ 1.43 & 7.044 \end{pmatrix} \tag{28}$$

To calibrate the forcing processes associated with overseas shocks we estimate another VAR. We derive processes for the shocks to the one-quarter change in the world price of traded goods and the world price of imported materials, as well as to foreign interest rates, the exchange rate risk premium and world demand. We construct a series for the foreign interest rate as a weighted average of three-month Euromarket rates for each of the other G6 countries, using the same weights used to construct the UK Effective Exchange Rate Index. For intermediate goods imports we follow Britton, Larsen and Small (1999) and construct an index based on the imported components of the Producer Price Index. For the world price of traded goods we use the G7 (excluding the United Kingdom) weighted average of exports of goods and services deflators where the weights match those in the UK Effective Exchange Rate index. For world output, we use the G7 (excluding the United Kingdom) average GDP weighted by the countries' share in total UK exports of goods and services in 1996.

We estimate the following VAR:

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[11]We adjusted the workforce jobs series prior to 1995Q3 to take account of a level shift of about 350,000 in total workforce jobs when the series was rebased. To do this, we added to the figure for each industry a share of the 350,000 workers equal to the industry's share in the published total. We combined the output data using the 1995 weights to get real value added for each of our two sectors (where, again, the traded goods sector consisted of 'manufacturing' and 'transport and communications').

$$\begin{pmatrix} i_{f,t} - i_f \\ \log(P^*_{I,t}/P^*_t) - \log(P^*_I/P^*) \\ \Delta \log P^*_t - \Delta \log P^* \\ \hat{y}_{F,t} \end{pmatrix} = R_F \begin{pmatrix} i_{f,t-1} - i_f \\ \log(P^*_{I,t-1}/P^*_{t-1}) - \log(P^*_I/P^*) \\ \Delta \log P^*_{t-1} - \Delta \log P^* \\ \hat{y}_{F,t-1} \end{pmatrix} + \begin{pmatrix} e_{i_f,t} \\ e_{P_I,t} \\ e_{P^*_t} \\ e_{y_F,t} \end{pmatrix} \tag{29}$$

where variables without time subscripts refer to their averages in the data and $\hat{y}_{F,t}$ is the log-deviation of world demand from its Hodrick-Prescott trend. The disturbances $e_{i,t}$, $e_{P_I,t}$, $e_{P^*,t}$ and $e_{y_F,t}$ are normally distributed with variance-covariance matrix $\mathbf{V}_F$. The VAR is specified in this way because the rest of the world is modeled in a reduced form way that does not place restrictions on the long run behaviour of variables. In particular if we included inflation of foreign intermediates prices as a separate variable then there would be no reason to expect the long-run responses of foreign intermediates prices and the general foreign price level to be equal. If this restriction did not hold, then temporary shocks could shift the steady state relationships between (exogenous) world variables. This would destabilise the relationships between the endogenous variables in our model. Rather than place restrictions on a VAR including foreign inflation rates, we estimate the system in (29).

Using data over the period 1977 Q3 − 1999 Q2 we obtained the following results:

$$R_F = \begin{pmatrix} 0.448 & -0.006 & 0.083 & 0.140 \\ 2.392 & 0.902 & 0.290 & -1.07 \\ -0.359 & -0.019 & 0.711 & -0.019 \\ -0.357 & 0.003 & 0.079 & 0.962 \end{pmatrix}$$

$$\mathbf{V}_F = 10^{-6} \times \begin{pmatrix} 3.82 & 4.47 & 3.08 & 0.54 \\ & 760 & 31.9 & -22.3 \\ & & 27.6 & 0.49 \\ & & & 7.79 \end{pmatrix}.$$

We derived a measure of the sterling exchange rate risk premium derived from the Consensus Survey[12] and estimated the following process:

$$\boldsymbol{z}_t = 0.261 \boldsymbol{z}_{t-1} + \boldsymbol{e}_{z,t}, \boldsymbol{s}_z = 0.009 \tag{30}$$

Finally, in line with McCallum and Nelson (*op. cit.*) we assumed that the preference shock $\boldsymbol{n}_t$ is white noise, and, for simplicity, we set its standard deviation equal to 0.011 as they do for the US.

4.   Properties of the model

To analyse the dynamic properties of the model, we have derived impulse response functions for the key endogenous variables when the model is hit by shocks.

Throughout, we closed the model with a policy rule for the nominal interest rate i_t. The rule used here was estimated using UK data over the period 1981Q2-1998Q2. We estimated a reduced-form model in which there were also equations determining (log) aggregate output $\hat{y}_t$, (the log of) the annualised log-change in the RPIX index inflation measured in terms of deviations from target ($4\Delta\hat{P}_t$) and changes in the (log of the) nominal trade-weighted effective exchange rate ($\Delta \ln e_t$). The model which is similar to that in Batini and Nelson (2000a), also contains two dummies ($DERM_t$ and $D92_t$) to capture the years of the UK membership of the ERM and the shift in policy regime which occurred in 1992 Q4.

To compute the impulse responses we need to identify the shocks. When the nominal interest rate is estimated as part of a VAR, a standard way of doing so is to orthogonalise the shocks using a Cholesky decomposition with a causal ordering that places the nominal interest rate last. Typically, however, estimating the equation for the nominal interest rate using a conventional VAR gives a reaction function where the nominal rate responds to lags of itself and lags of other variables in the VAR. This is unsatisfactory if we want to compare the estimated rule with Taylor-type rules that react to contemporaneous variables.

To overcome this problem, Rotemberg and Woodford (1997) obtain a similar dynamic specification of the estimated policy rule by leading the other variables in the vector auto-regression model (inflation and output in their case): in effect they estimate a VAR with a vector of endogenous variables equal to [$4i_t$, $4\Delta\hat{P}_{t+1}$, $\hat{y}_{t+1}$]. Even if it gives an estimated equation for the interest rate that responds to contemporaneous realisations of output and inflation, as we want, their approach may be unreliable. It, in fact, implies very restricted dynamic specifications for the other two variables in the model, where the leads of inflation and output depend only on *lags* of the interest rate and not also on the level of the interest rate at time t.

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[12] The measure is equal to the percentage point difference between the expected 24-month depreciation of the sterling ERI (derived from the responses of survey participants) and the two-year nominal interest rate differential.

For this reason, following the methodology in Ericsson, Hendry and Mizon (1998), we re-parameterised the system $Q_t = [\,i_t, 4\Delta\hat{P}_t, \hat{y}_t, \Delta\ln e_t\,]$ as the conditional and marginal models $i_t = f(4\Delta\hat{P}_t, \hat{y}_t, \Delta\ln e_t, Q_{t-1})$ and $(4\Delta\hat{P}_t, \Delta\ln e_t, \hat{y}_t) = (Q_{t-1}, \chi)$, where χ is the vector of estimated parameters. In effect, this orthogonalises the shocks, so that the nominal interest rate is not affected by time-t changes in the other variables. However, contrary to a VAR estimation approach, this method allows us to derive an estimated equation for the nominal interest rate in which i_t depends on contemporaneous values of inflation, output and changes in the exchange rate, rather than on lags of those variables. The model's estimates are available on request. For convenience, we reproduce here the estimate of the nominal interest rate equation, which we interpret as being the monetary policy reaction function over that period:

$$4i_t = c + k_1 4i_{t-1} + k_2 4\Delta\hat{P}_t + k_3 \hat{y}_t + k_4 \Delta\ln e_t + k_5 DERM_t + k_6 D924_t + e_{i,t} \qquad \textbf{(31)}$$

where $4i_t$ is the annualised interbank lending rate, and $e_{i,t}$ are the equation's estimated residuals. The estimated coefficients (standard errors in parenthesis) are:

$c = 0.0423, \quad \kappa_1 = 0.605, \quad \kappa_2 = 0.406, \quad \kappa_3 = 0.184, \quad \kappa_4 = -0.065 ,$

$\quad\;(0.008) \qquad\;\; (0.074) \qquad\quad (\text{----}) \qquad\quad (0.039) \qquad\qquad (0.027)$

$\kappa_5 = -0.014, \quad \kappa_6 = -0.015,$

$\quad\;\;(0.003) \qquad\quad (0.004)$

with SE $= 0.00821$.

To ensure that the log-run nominal interest rate response to inflation is larger than 1, we restrict $k_2 /(1 - k_1) = 1.01$. For this reason, no standard error is reported for that coefficient. The LR test of over-identifying restrictions cannot reject the null implied by this restriction $[\chi^2(1)=0.5032, p\text{-value} = 0.4781]$.

Since the endogenous variables in the model feature as deviations from their respective long-run values — or enter as first-differences — they are comparable to variables in the log-linearised first-order approximation version of the model.

Figure 1: Impulse responses following a 100 basis point monetary policy shock

Panel 1: Output Response
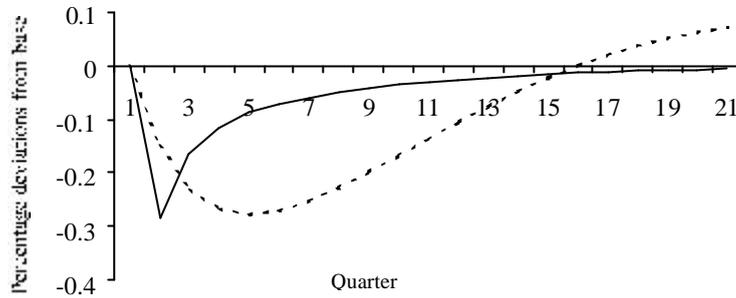
Quarter

Panel 2: Inflation Response

Quarter

Panel 3: Nominal Interest Rate Response
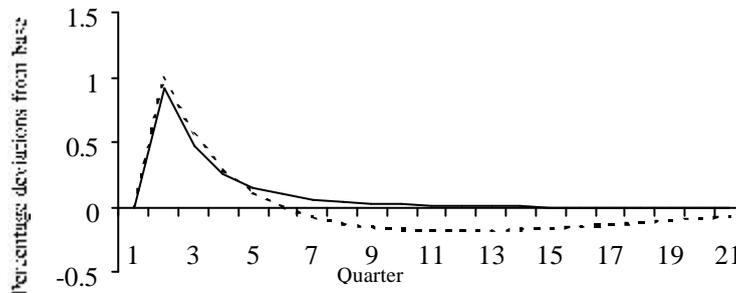
Quarter

Figure 1 shows output, (four-quarter) inflation and the nominal interest rate impulse response functions to a unit start shock to the monetary policy rule **(31)** over 20 periods (calendar quarters). The solid line depicts the analytical model's responses and the dashed line gives the estimated model's responses.
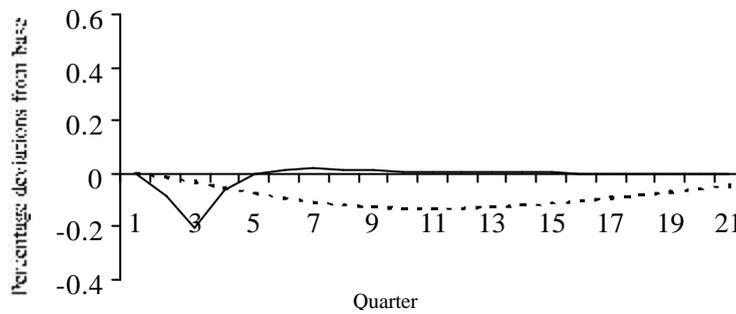
Both the estimated and our model's responses broadly agree with conventional wisdom: following a temporary rise in the interest rate, output declines, but ultimately reverts to base; and inflation also falls. Our estimated inflation equation exhibits no price puzzle (i.e. the finding in many empirically estimated models a rise in the nominal interest rate is associated with a *rise* — rather than a *fall* — in the rate of inflation in the periods immediately after the rise). However, we expect there to be rather wide error bands around the estimated model's impulse responses (not shown here) indicating that these effects cannot be estimated with

great precision, particularly those on inflation. So, the comparison of the two sets of responses should not be taken too literally.

Panel 1 indicates that, in our analytical model, output falls on impact by around 0.25%, following an unanticipated 100 basis point rise in the nominal interest rate — the same order of magnitude of that of the estimated model. The policy shock response in the data is slightly more sluggish than that in the model and in the data, the trough in output following the shock occurs later than in our model. The speedier response of output in our model reflects the volatility of the net trade component of aggregate output in our model. The consumption component of aggregate output is sluggish and 'hump shaped', which reflects the high value of the habit formation parameter (x). This result accords with the findings of Fuhrer (2000).

Panel 2 compares the RPIX inflation responses of the theoretical and estimated models. In our model inflation responds earlier and more intensely than the estimated model. There, inflation touches its *nadir* around ten quarters after the shock, and returns smoothly back on track over a period of about two to three years. The difference between the two responses probably reflects the fact that our model, even accounting for the built-in persistence, is still a forward-looking, 'jumpier' model, whereas the estimated model is entirely backward looking.
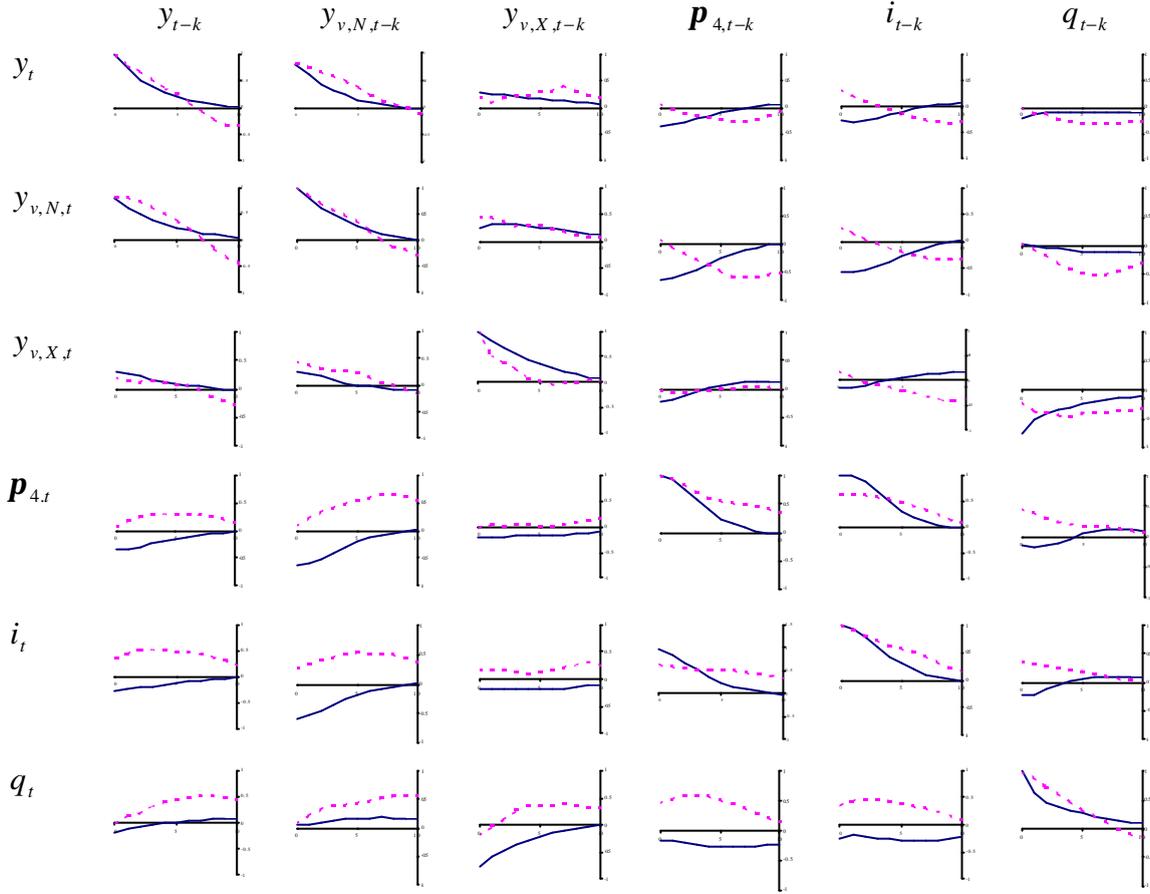
Panel 3 depicts how the (nominal) interest rate responds. While it rises by a full 1% in the estimated model, the nominal interest rate by slightly less in our model. There are two reasons why this happens. First, in our model, inflation expected at time $t + 1$ falls on impact one period after the shock; by contrast, in the estimated model, inflation is almost unchanged in the first quarters after the shock. This implies that, in practice, the real interest rate response is harsher on impact in our model than in the estimated model. Second, inflation and output (the feedback variables in the estimated policy rule) are forward-looking in our model; thus the interest rate response will be more muted than in the estimated model, inasmuch as those variables will themselves have already adjusted pre-emptively to the shock.

A second way of evaluating the correspondence between UK data and our model is to compare the dynamic cross-correlations of key variables from the data with those from the model.[13] Figure 2 shows this comparison for (log deviations of) aggregate output (y), value added sectoral outputs ($y_{v,N}$ and $y_{v,X}$), annual CPI inflation (p_4), the nominal interest rate (i) and the real exchange rate (q). In each of the thirty-six panels, the solid line illustrates the theoretical cross-correlation function and the dashed line the cross-correlation function from the data.

Figure 2 indicates that our model seems to account for the auto-correlations of the data to a reasonable extent (see charts on the diagonal). In particular, our model can in part replicate the degree of persistence of inflation seen in the data, although this is mainly driven by persistence in the exogenous shocks. The model is perhaps less successful at capturing cross-correlations: for example, the dynamic relationship between the real exchange rate and some of the other variables in the panel.

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[13] For the model, the cross-correlations were computed using a variant of the Hansen-Sargent doubling algorithm discussed in section 5.

Figure 2: Cross-correlations of selected endogenous variables

	y_{t-k}	$y_{v,N,t-k}$	$y_{v,X,t-k}$	$p_{4,t-k}$	i_{t-k}	q_{t-k}
y_t						
$y_{v,N,t}$						
$y_{v,X,t}$						
$p_{4,t}$						
i_t						
q_t						

5. Results: a comparison of alternative simple rules

In this section we present results from the model when it is closed with alternative monetary policy rules. In what follows we assume that deviations of the nominal interest rate from base are a linear function of deviations of endogenous variables (current, lagged or expected) from base. So we consider rules of the form,

$$i_t = Rg_t, \quad g_t \subseteq z_t. \tag{32}$$

where g is the set of feedback variables in the rule and R is a row vector of coefficients.[14] A simple rule therefore consists of two components, the vector of feedback variables, g, and the vector of coefficients, R. We define generic classes of rules by the g vector, that is, by the set of variables on which they feed back. To carry out the comparison, for each rule we consider two kinds of coefficients vectors, R.

First, we look at the rules in their original specification. In this case, the vector of coefficients, R, is that suggested for those rules. For example, the first group of rules includes a Taylor rule

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[14] Note that by using lag and lead identities within the model, the set of variables that could be included in the rule is large. For example, for the inflation forecast based rule considered below, we include conditional expectations of inflation up to five quarters ahead.

with the original coefficients advocated by Taylor (1993a). We call these rules 'non-optimised' because their coefficients are not set optimally for our model.

Second, we consider simple 'optimised' rules. In this case, the R vectors are those that minimise the policymaker's loss function, L_1, for each rule.

As a measure of loss, L_1, we choose a standard quadratic loss function in asymptotic variances ($AVar$) of inflation deviations from target and output deviations from potential. This is often used as a metric for capturing policymakers' preferences in studies that attempt to evaluate the performance of alternative policy rules [see Taylor (1999)]. Algebraically, L_1 can be written as:

$$L_1(\boldsymbol{p}, y, R) = w_{\boldsymbol{p}} \, AVar(4\boldsymbol{p}) + w_y \, AVar(y) + w_i \, AVar(\Delta 4i), \qquad \textbf{(33)}$$

which is a linear combination of the asymptotic variances ($AVar$) of annualised inflation and output, and the change in the (annualised) nominal interest rate. Following Batini and Nelson (2000a) we set $w_{\boldsymbol{p}} = w_y = 1$.

The inclusion of a term in the variability of the nominal interest rate is designed to address the fact that optimised coefficients for simple rules often imply very aggressive policy responses. In practice, this leads to large movements in the policy instrument. Casual empiricism suggests that policymakers prefer stability in the instrument, which implies that nominal interest rate variability should be included in the loss function.[15] Perhaps more importantly, when taken literally, aggressive policy rules often imply that policymakers should set a negative nominal interest rate, despite the general presumption that nominal rates cannot fall below zero. (See McCallum (2000) and Goodfriend (1999).) This issue is discussed in Williams (1999).

Including a term in the loss function is one way to ensure that rules with optimised coefficients do not imply that there is a high probability that the zero bound on the nominal interest rate is violated. The choice of the weight w_i depends on the model being used. Following Rudebusch and Svensson (1999), Batini and Nelson (2000a) set $w_i = 0.5$. We set $w_i = 0.25$ which ensures that there is a relatively low probability of violating the zero bound for the optimised rules we consider. We discuss this further below.

Turning to the vector of 'optimised' coefficients ($\tilde{R}$), this is chosen as follows:

$$\tilde{R} = \arg\min_R \ L_1(\boldsymbol{p}, y, R) \qquad \textbf{(34)}$$

To derive it, we employ a simplex search method based on the Nelder-Mead algorithm.[16]

In addition to loss L_1 upon which we optimise to get coefficients in the $\tilde{R}$ vector, we consider a second measure of loss, i.e. a utility-based loss function, which we denote L_2. However,

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[15] The fact that the interest rate smoothing term in the estimated rule – equation **(31)** – is high and significant, suggests that – historically - interest rates have not responded aggressively in the UK.
[16] The method is contained in the MATLAB Optimization Toolbox and detailed by Lagarias *et al* (1997).

we do not derive a second vector of optimal coefficients from this loss. Rather, we use it as a metric to measure the amount of utility loss associated with each rule when the authorities derive coefficients for the rules by optimising a set of preference described by the first, standard quadratic loss function L_1.

Following Woodford (1999), we derive L_2 by taking a second order log-linearisation of the utility function (**1**) around the steady state. We ignore the constant and first order terms (the latter are zero in expectation) and focus on the unconditional expectation of the second order terms, so that:

$$L_2 = \frac{1}{2}\begin{bmatrix} \dfrac{(1+\boldsymbol{x})^2}{1-\boldsymbol{x}} AVar(c) - \dfrac{2}{(1-\boldsymbol{x})^2} ACov(c,c_{-1}) - \log\big((1-\boldsymbol{x})c\big) AVar(\boldsymbol{n}) - \dfrac{2}{1-\boldsymbol{x}} ACov(c,\boldsymbol{n}) \\ + \dfrac{\boldsymbol{d}h^2}{(1-h)^2} AVar(h) + \boldsymbol{ec}\left(\dfrac{\Omega}{P}\right)^{1-e} AVar\left(\dfrac{\Omega}{P}\right) \end{bmatrix}$$

(**35**)

As equation (**35**) makes clear, this first measure of loss depends on six terms:

(i) the variance of (log) consumption; (ii) the first-order correlation of log consumption (this comes from the habit formation assumption); (iii) the variance of log hours; (iv) the variance of log real money balances; (v) the variance of preference shocks; and (vi) the covariance between preference shocks and consumption. Since this loss function is derived from the utility of the households, it seems to be a good way of judging the welfare effects of monetary policy rather than using an arbitrary loss function as has been common in this literature. However, it is not necessarily ideal in it requires us to make some judgements about how to measure welfare in a model with heterogeneous households.

Finally, to obtain the asymptotic variances in equations (**33**) and (**35**), we first write the solution of our model as:

$$z_t = \Gamma(R)z_{t-1} + \Phi(R)\boldsymbol{u}_t$$

(**36**)

where the coefficients in the Γ and Φ matrices potentially depend on the rule coefficients, R. The asymptotic variance of the state vector, z, is given by:

$$V = \sum_{j=0}^{\infty} \Gamma^j \Phi \Omega \Phi' \Gamma'^j$$

(**37**)

where Ω is the covariance matrix of the shocks, υ. We then compute V by the doubling algorithm of Hansen and Sargent (1998), given the covariance matrix, Ω, calibrated in section 3. The asymptotic variances of output and inflation are given by the relevant elements of V.[17]

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[17] This approach is also used by Williams (1999).

5.1 A battery of rules

We evaluate the relative performance of the following classes of rules:

(i) the estimated policy rule (see section 4);

(ii) a Taylor/Henderson-McKibbin rule;

(iii) an inflation-forecast based (IFB) rule;

(iv) a naïve MCI-based rule;

(v) Ball's (1999) rule;

(vi) a family of alternative 'open-economy' rules.

This battery of rules encompasses the mainstream of the literature on simple policy rules for both closed and open economies, but adds a series of new simple rules that may prove even more suited for open economies. The estimated rule enables us to assess the remaining rules vis-à-vis history, and to infer whether, using these other rules, it may have been possible to do better than historically. We discuss the remaining classes of rules in turn.

The Taylor/Henderson –McKibbin rule

This section considers rules of the following form:

$$i_t = \boldsymbol{l_p}\boldsymbol{p_t} + \boldsymbol{l}_y y_t \tag{38}$$

where i_t denotes the percentage point deviation of the short-term nominal interest rate from steady state, and $\boldsymbol{p}_t$ and y_t are log-deviations of inflation and GDP from base. Rules of this form are often associated with Taylor (1993a) and Henderson and McKibbin (1993). More precisely, Henderson and McKibbin place more weight on the inflation feedback term than Taylor does (i.e. $\boldsymbol{l}_p = 2$ rather than 1.5) and place a weight on the output gap that is four times that in Taylor (i.e. $\boldsymbol{l}_y = 0.5$ rather than 0.125). In all cases, rule **(38)** may be augmented with a lag of the nominal interest rate, to capture interest rate smoothing.

These rules were devised for a closed economy (the US), where the exchange rate channel of monetary transmission has a negligible role in the propagation of monetary impulses. So we would expect them to do relatively badly when compared with rules that account for that channel, or allow for the diverse way in which monetary impulses are transmitted across 'internationally exposed' and 'internationally sheltered' sectors.

Inflation forecast based rules

Inflation forecast-based (hereafter 'IFB') rules imply that the interest rate should respond to deviations of expected, rather than current, inflation from target.[18] In the presence of transmission lags, this has the benefit of aligning the policy instrument with the target variable (i.e., is said to be 'lag-encompassing'), which miminises the output costs of inflation stabilisation relative to more myopic rules. IFB rules typically do not respond to output deviations from potential: the inflation forecast used in the rules already encompasses the information contained in the current output gap (i.e. they are 'output-encompassing').

Batini and Haldane (1999) compare rules that respond to different horizons of inflation forecasts and assume that policymakers have a tendency to smooth rates. So algebraically, an IFB rule like those in Batini and Haldane (*op. cit.*) can be written as this:

$$i_t = \boldsymbol{l}_i i_{t-1} + \boldsymbol{l}_p E_t \boldsymbol{p}_{t+j} \qquad\qquad (39)$$

In their small scale macroeconomic model calibrated on UK data, an IFB rule responding to inflation expected 5 quarters ahead with a feedback parameter equal to 5, and an interest rate smoothing parameter equal to 0.5 appears optimal (so that $j = 5$, $\boldsymbol{l}_i = 0.5$ and $\boldsymbol{l}_p = 5$ in the above equation). When looking at the performance of rule with non-optimised coefficients (Table 1) we will adopt the same parametrisation of the rule used by Batini and Haldane (*op. ci*t.). However, since these rules tend to be highly model-specific – see Levin, Wieland and Williams (1998) – we would not expect them to do well in our model for the same choice of horizon and feedback parameters that was efficient in Batini and Haldane (*op. cit.*). Indeed, the low degree of inflation persistence in our model suggests that a shorter horizon is probably more adequate. Hence, in the experiments using optimised coefficients we also select the horizon optimally.

The naïve MCI-based rule

A Monetary Condition Index (MCI) is a weighted average of the domestic interest rate and the (log) exchange rate.[19] A MCI can be expressed in real or nominal terms. Because it has the potential to quantify the degree of tightness (ease) that *both* the interest rate and the exchange rate exert on the economy, MCIs are often used to measure the stance of monetary policy in an open economy.

A naïve simple rule based on a MCI could then be one that entails adjusting the nominal interest rate to ensure that real monetary conditions are unchanged over time:

$$i_t = \boldsymbol{p}_t - \boldsymbol{m} q_t \qquad\qquad (40)$$

where q_t is the real exchange rate and $\boldsymbol{m}$ is the MCI weight.[20] Setting $\boldsymbol{m}$=1/3 — a value consistent with the weights used by the Bank of Canada to construct an MCI — implies that

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[18] See Batini and Haldane (*op. cit.*).

[19] See Batini and Turnbull (2000) for a thorough discussion of MCIs and their possible uses for the UK.

[20] We do not consider nominal MCIs as they are likely to perform poorly in our model. The reason is that the level of the nominal exchange rate can shift permanently following a transitory nominal shock. This suggests that a simple nominal MCI rule could lead to instability.

a 3% appreciation in the real exchange rate is equivalent to a 100 basis points increase in the real interest rate.[21]

In practice, MCIs have been criticised on both empirical and theoretical grounds.[22]

One conceptual shortcoming of a MCI, when used as an operating target, is that different types of shocks have different implications for monetary policy. By construction, a MCI obscures the identification of exchange rate shocks because this requires focusing on movements in the exchange rate and interest rates in isolation, rather than aggregated together (see King (1997)). This shortcoming carries over to any MCI-based rule that recommends a level for the interest rate conditioning on the existing level of the exchange rate, when the latter can change for shocks that the central bank may not want to affect monetary conditions. For this reason, we would expect the performance of MCI-based rules to be shock-specific, doing poorly in the face of shocks that affect the exchange rate but do not ask for a compensating change in interest rates (e.g. shocks to the real exchange rate).

Ball's (1999) rule

Less naïve specifications of a rule which use a monetary conditions index as a policy instrument may potentially perform better. Ball (1999) proposes a rule of this kind where policymakers alter a combination of interest and exchange rates in response to deviations of (an exchange-rate-adjusted or 'long-run' measure of) inflation from target and output from potential. When the rule is re-arranged so that only the nominal interest rate features on the LHS of the equation, this rule indeed resembles a Taylor/Henderson-McKibbin rule with added real exchange rate (contemporaneous and lagged) terms:

$$i_t = \boldsymbol{l}_y y_t + \boldsymbol{l}_p \boldsymbol{p}_t + \boldsymbol{l}_{q1} q_t + \boldsymbol{l}_{q2} q_{t-1} \qquad\qquad (41)$$

In this sense, Ball's rule is an 'open economy' rule because responding also to the exchange rate, it expands parsimoniously upon 'closed-economy' rules to account for openness of the economy. So we would imagine that it outperforms closed-economy counterparts when utilised to control our open-economy model. This rule is in fact optimal in Ball's (1999) model, a model that contains only three states (inflation, output and the exchange rate). The coefficients that Ball suggests, conditioning on his dynamic constraints, are: $\boldsymbol{l}_y = 1.93$, $\boldsymbol{l}_p = 2.51$, $\boldsymbol{l}_{q1} = -0.43$ and $\boldsymbol{l}_{q2} = 0.3$. These are the coefficients we use in the experiments with non-optimised coefficients. As we will see below, three rules in our 'family' of open economy rules can be considered extensions of Ball's rule.

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[21] In practice, the actual MCI may be compared with a 'desired' MCI level, MCI*, say. MCI* is the level of monetary conditions compatible with the inflation target and non-inflationary economic growth. In this sense, the desired MCI can be viewed as an open economy extension of Blinder's (1998) concept of a 'neutral rate', an interest rate at which the monetary stance is neither dampening nor stimulating economic activity. In a closed economy, the monetary authority will want the actual nominal rate to depart from its neutral level, whenever the economy is out of equilibrium and vice-versa. In an open economy, the monetary authority may want the actual MCI to deviate from MCI* for the same reason. But it is not entirely clear from the existing literature how MCI-based rules expressed in terms of deviations of actual from desired should be constructed. Basically this is because to do so requires knowledge of how desired monetary conditions will evolve.

[22] See, among others, Ericsson *et al* (1997).

A family of alternative 'open economy' simple rules

Finally, we turn to our set of alternative open economy rules. As anticipated, these rules are meant to be rules for an economy that is open.

Ideally, following Ball (1999), we want these rules to do two things. First, alongside the standard output gap channel, the rules should also exploit the exchange rate channel of monetary transmission. This should make policy more effective by letting sectors in the economy that are affected unevenly by the two major channels of transmission adjust in the most efficient way following a shock. Second, they should do so by augmenting its closed-economy counterpart rules specifications (e.g., Taylor and Henderson and McKibbin) in a *parsimonious* way. This is because, both on credibility and monitorability grounds, there is a clear merit in having a rule that is simple to compute – that is a rule that does not introduce any extra uncertainty in the measurement of its arguments – and that can be easily understood by the public.

For this purpose we consider four different rules, which account for the openness of the economy in various ways. Three of these can be considered variants of Ball's (1999) rule. More specifically, the first variant ('OE2') adds to the standard feedback terms in Ball's rule a term responding to disequilibria in the balance of trade. The second variant ('OE3') replaces aggregate output with output gaps in the two sectors; this takes explicit account of the fact that components of GDP differ in their international exposure. And the third variant ('OE4') has the interest rate responding to the same variables as in Ball (1999), but imposes a restriction on the contemporaneous and lagged real exchange rate terms, so that their coefficients are equal and opposite. In practice, this implies that the policymakers respond to time-*t changes* in the real exchange rate rather than levels of it. The fourth and final rule in the family ('OE1') instead, is a modification of the inflation forecast based rule of Batini and Haldane (1999), which adds to that an explicit response to the real exchange rate (again contemporaneous and lagged, unrestricted). In principle, an IFB rule already accounts for the exchange rate channel of monetary transmission, inasmuch as this underlies the equations that inform the forecast for inflation. So evaluating this rules enables us to understand whether incorporating a separate exchange rate term in an IFB rule provides information over and above that already contained in the inflation forecast. [23]

As with the IFB and Ball's rules, we expect rules in this family (especially rules OE2, OE3 and OE4) to do better on average than their closed-economy counterparts. This is because they take explicit account of the fact that in an open economy there are multiple channels of monetary transmission that can be simultaneously exploited in an effective way. However, since the IFB rule already exploits the exchange rate channel of policy transmission, accounting for this channel explicitly may not add much to the stabilisation properties of this rule. In general, other things being equal, and like with the IFB and Ball's rules, we expect at least some of the OE rules to reduce the disparities in the costs of adjustment faced by the two sectors in the economy, relative to the closed economy simple rules case. Lastly, note that under optimisation these rules may do better than Taylor, Henderson and McKibbin and the IFB rules also because they typically react to more state variables than these. We will discuss this issue in more detail below.

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[23] See also Batini and Nelson (2000b) for a comprehensive study of this issue using a small-scale structural model of the UK economy.

5.2 Results

Table 1 below contains values of the two loss functions (L_1 and L_2) and asymptotic second order moments of inflation, output, the nominal interest rate, sectoral outputs and the real exchange rate. These are reported for the estimated rule, the Taylor and Henderson and McKibbin rules, the IFB rule, the naïve MCI-based rule and Ball's rule with the original weights under our model specification. Table 2, in turn, reports analogous statistics for these rules (excluding the estimated rule) and for the OE1, OE2, OE3 and OE4 rules when coefficients are optimally derived. This table also reports corresponding optimised rules' coefficients. Finally, Table 3 offers a test for relative robustness, by showing the same statistics for each rule when the model is hit by individual shocks rather than by a combination of shocks.

5.2.1 Results under an 'all shocks' scenario

Table 1 suggests that, when coefficients are not optimised and all the shocks in the model are operative, the best performing rule according to loss function L_1 is, surprisingly, the naïve MCI-based rule, which ensures the lowest volatility of the real exchange rate and of exports. This in turn gives better inflation control than Taylor and Henderson and McKibbin rules. Among these two, the latter comes second being more successful than the Taylor rule at stabilising both output and inflation — a consequence of its stronger feedback coefficients.

The estimated rule ranks fourth. Thanks to its term for interest rate smoothing, this rule responds gradually to inflationary pressures, and thus minimises interest rate and output volatility compared to Taylor/Henderson-McKibbin rules. On the other hand, this makes the estimated rule less successful at stabilising inflation than those rules. When coefficients are not optimally derived for this model, Ball's rule gives a high loss (abstracting from the results under the IFB rule). This is mainly a consequence of its huge coefficient on deviations of output from potential: something that turns out not to be optimal for this model (sees table 3a and 3b below). This, in turn, leads to large interest rate gyrations and a high variability in the first difference of the nominal interest rate — a variable that L_1 penalises. As expected, the IFB rule, with a horizon and feedback coefficients originally suggested for a model with significantly more inflation inertia than ours, triggers considerable volatility in the interest rate, the result of which is destabilised output and, thus, a very large loss. Finally, with non-optimal coefficients, the naïve MCI-based rule and the estimated rule give a lower probability of hitting a zero bound with the nominal interest rate, followed by the IFB rule.[24] All these rules imply a low variability of the level of the nominal interest rate.

Table 1: Comparison of simple monetary policy rules (non-optimised coefficients)

	Rules:	Taylor	H-McK	IFB	MCI	Ball	Est'd
Coeffs:	p_t	1.5	1.5	-	**1**	2.51	0.407
	y_t	0.125	0.5	-	-	1.93	0.046

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[24] The probability of hitting the lower bound on interest rates was derived as follows. We first note that, in the log-linearised model with Gaussian shocks, the nominal interest rate is normally distributed. It has a mean equal to the steady state value and the variance is given by the relevant element of V (see equation (**37**)). This estimate is likely to understate the true probability of hitting the lower bound because, once interest rates have fallen to zero and the economy is in a liquidity trap, it becomes less likely than implied by the asymptotic distribution that interest rates will be moved away from zero.

i_{t-1}		-	-	0.5	-	-	0.597
$E_t p_{t+5}$		-	-	5	-	-	-
q_t		-	-	-	-0.33	-0.43	-
q_{t-1}		-	-	-	-	0.33	-
Δe_t		-	-	-	-	-	-0.016
Welfare	Loss L_1	6.399	5.290	117.3	5.006	7.049	5.736
	Loss L_2	8.951	9.088	213.6	1.761	10.236	3.332
Avars	p	0.339	0.265	4.573	0.221	0.277	0.339
	y	0.301	0.297	27.436	0.592	0.168	0.198
	Δi	0.169	0.187	4.177	0.219	0.611	0.029
	$y_{v,N}$	0.436	0.529	42.585	1.139	0.386	0.301
	$y_{v,X}$	1.077	1.058	14.125	0.719	0.920	0.987
	c	0.374	0.486	34.022	1.295	0.409	0.327
	i	0.721	0.754	22.981	0.308	0.847	0.208
	q	5.157	4.776	338.863	1.423	3.171	4.545
Prob($i < 0$), %		3.9	4.2	37.7	0.3	5.2	0.1

Table 2 below lists analogous statistics and optimal coefficients for these rules and for the alternative open-economy rules (OE1 to OE4) when coefficients are optimally selected. The optimisation indicates that Taylor and Henderson-McKibbin rules for the UK economy, as modelled here, require stronger weights on inflation relative to output than those suggested for the US. This suggests that a mechanical application of the Taylor and/or Henderson and McKibbin rules in the UK context with coefficients designed for the US is not ideal. Moreover, our model seems to favour a stronger weight on inflation relative to output, even when the policymakers' preferences are symmetric between inflation and output stabilisation.

Similarly, for our model economy, the optimal coefficient for the MCI-based rule is smaller than one third – the value commonly used in the MCI literature – suggesting that a greater weight than that used in practice should be placed on interest rates vis-à-vis the exchange rate when altering monetary conditions.

Table 2: Comparison of simple monetary policy rules (optimised coefficients)

	Rules	Taylor/ H-McK	IFB	MCI	Ball	OE1	OE2	OE3	OE4
Coeffs:	p_t	5.94	-	**1**	5.117	**-**	4.111	5.193	5.869
	y_t	0.23	-	-	0.036	-	0.053	-	0.228
	i_{t-1}	-	0.913	-	-	0.947	-	-	-
	$E_t p_{t+1}$	-	0.808	-	-	1.093	-	-	-
	q_t	-	-	0.220	0.110	0.015	0.071	0.064	-
	q_{t-1}	-	-	-	0.047	0.024	0.054	0.039	-
	$y_{v,N,t}$	-	-	-	-	-	-	0.035	-
	$y_{v,X,t}$	-	-	-	-	-	-	0.148	-
	BT_t	-	-	-	-	-	-	-	0.029
	Δq_t	-	-	-	-	-	6.476	-	-
Welfare	Loss L_1	2.68	2.08	4.82	2.59	1.987	2.548	2.580	2.654
	Loss L_2	2.32	-3.04	2.31	2.26	-2.30	1.807	2.123	2.211
Avars	p	0.029	0.029	0.234	0.031	0.029	0.033	0.030	0.029
	y	1.608	1.540	0.449	1.489	1.425	1.452	1.522	1.614
	Δi	0.151	0.018	0.156	0.152	0.023	0.144	0.147	0.145
	$y_{v,N}$	2.958	2.832	0.859	2.742	2.646	2.708	2.799	2.972
	$y_{v,X}$	1.178	1.137	0.768	1.120	1.078	1.105	1.108	1.172
	c	2.679	2.668	1.001	2.525	2.525	2.517	2.583	2.692
	i	0.613	0.170	0.287	0.582	0.198	0.544	0.581	0.607
	q	6.431	5.332	1.923	5.503	4.592	5.257	5.512	6.327
Prob($i < 0$), %		2.8	0.0	0.3	2.5	0.0	2.1	2.5	2.7

Looking at the actual stabilisation properties of each rule, the first thing to notice is that Taylor/HendersonMcKibbin, IFB and Ball's rules perform much better than their counterparts with non-optimised coefficients. This is not true of the naïve MCI-based rule, since its performance is almost unaffected by the optimisation of its (unique) coefficient, making it the worse rule in the set. Indeed, when coefficients are optimised, rules in the Taylor/Henderson-McKibbin class now outperform the naïve MCI-based rule by ensuring lower inflation volatility.

Second, with optimally chosen horizon and feedback coefficients (both significantly reduced from the non-optimal case), the IFB rule performs extraordinarily well. When coefficients are optimal, the IFB appears to be extremely successful at minimising inflation volatility for a level of output volatility that is now comparable, if not lower, than that of other rules in the table. This is a consequence of the fact that, under this rule, the interest rate now moves optimally and solely to correct low-frequency changes in inflation. (The asymptotic standard deviation of changes in the nominal interest rate for this rule is, in fact, around a tenth of that under all

other rules if we exclude the OE1 rule, a modification of the IFB.) This result is not altogether surprising. IFB rules have now been found to perform well in a number of studies: essentially for the reasons noted in Batini and Haldane (1999).

Third, Ball's rule provides a lower than average variability of output when compared to Taylor/Henderson-McKibbin and to the IFB rule. Relative to them, it also reduces the disparity between output sector volatilities in the two sectors, other things being equal. However, it produces more interest rate, exchange rate and ultimately inflation variability than the IFB rule because it reacts to current, rather than expected inflation and hence necessitates greater aggressiveness. This rule is, unsurprisingly, more efficient than the simplistic MCI-based rule which does not react either to inflation nor output, and which thus is unsuited to cope with inflationary shocks that did not originate in a shock to the exchange rate.

Additional interesting results emerge when we look at our 'family' of open economy rules.

In general, it seems as if parsimonious modifications of either Ball's or the IFB rule do not gain much in terms of inflation or output control. The reduction in the value of L_1 is indeed negligible and most certainly ascribable to the fact that rules in the OE family typically react to more state variables. By construction, this gives them a performance 'bonus' relative to non-OE rules. By the same logic, the opposite is true of rule OE4 — a restricted version of Ball's rule — which hence does worse than Ball's rule itself.

However, on the whole, rules OE2 (Ball plus response to balance of trade), OE3 (Ball with separate response to sectoral outputs) and OE1 (IFB with additional exchange rate terms) do seem capable of reducing *further* the disparity between output sector volatilities in the two sectors, others things being equal.

When we look at each rule individually, the following emerges. In rules OE2 and OE3 which feed back on the balance trade and on sectoral output gaps respectively, introducing extra terms has the effect of lowering slightly the response to exchange rate terms as the optimised Ball's rule would imply in the absence of those terms. And in OE2, adding a feedback term on the trade balance inverts the sign on the output gap term (which was, somewhat counterintuitively, negative in Ball's optimised rule). This is possibly a consequence of the fact that when policymakers respond separately to the net trade component of the aggregate output gap, policy no longer needs to give a procyclical response to the output gap. The implications of these changes in existing coefficients combined with the effect of new coefficients are that: (i) OE2 gives a marginally better control than Ball's rule of output and interest rates, but a slightly worse control of inflation; (ii) OE3 gives a marginally better control of inflation and interest rates than Ball's rule, but worsens slightly the control of output; and (iii) only OE2 ensures more symmetry than Ball's rule in the adjustment of sectoral outputs after a shock.

Adding separate exchange rate terms to the IFB rule ('OE1') also moderately improves its stabilisation properties. The rule now delivers a lower volatility of output than without the exchange rate terms. Exchange rate volatility also falls. OE1 is in fact the best rule in the table. Compared to non-OE rules, OE1 gives considerably lower output, exchange rate and interest rate variability than Taylor/Henderson-McKibbin rules and also than Ball's rule. If we abstract from the naïve-MCI that smooths the costs of adjustment across sectors but does

badly in term of inflation variability, OE1 produces the minimum disparity between the volatility of the output gap in the two sectors. Note that, thanks to their ability of minimising the volatility of the policy instrument, IFB and OE1 rules also give the lowest probability of hitting a zero bound with the nominal interest rate. These results on the relative performance of the rules are confirmed by our second measure of loss, the utility-based loss function L_2. According to this metric, households would be better off if policymakers followed an OE1 rule or an IFB rule with coefficients optimised over the objective function L_1, rather than other rules that we consider. The worse possible rule according to L_2 is instead the Taylor/Henderson-McKibbin.

In summary:

- IFB rules appear to be efficient open economy rules because they capture all channels of transmission. They outperform closed economy rules like Taylor and Henderson-McKibbin in terms of both output and interest rate control. But they also prove superior to Ball's rule, which reacts to current, rather than expected inflation; this makes policy myopic rather than pre-emptive, and hence requires more aggressive changes in the interest rate, which in turn affect the exchange rate and thereby inflation. For these reasons, IFB rules also help stabilising the economy in a more 'symmetric way', demanding less adjustment from the the internationally exposed sector than that required by closed-economy rules which ignore differences in adjustment across sectors;

- Modifications to these rules to include explicit feedbacks on the level of the exchange rate (contemporaneous and lagged) improve the performance of these rules only marginally. But since they help reduce further the disparity in adjustment between traded and exports sectors, they may be desirable if the authorities have a specific preference for symmetry in adjustment, other things equal.

- Relative to other rules in the battery, IFB and exchange-rate-adjusted IFB (OE1) minimise the probability of hitting a zero bound with nominal interest rates, and thereby increase the chances of policy remaining operational under particularly severe deflationary shocks;

- Ball's rule and variants of it (notably OE2 which allows a response to disequilibria in the trade balance) are second-best options. Because they also account for the exchange rate channel of transmission, as expected they are significantly more efficient than Taylor/Henderson-McKibbin rules in stabilising inflation and output in our open economy model of the UK. In this respect they are indeed by far preferable to naïve MCI-based rule, which gives more stable output outcomes at the price of massive inflation variance (the loss associated with Ball's rule is a sixth of the loss associated with the naïve MCI-based rule).

5.2.2   Robustness analysis to individual shocks

In order to provide more intuition about why certain rules perform better than others, we have re-assessed the performance of the rules assuming that the economy was hit by one type of shock at a time. In particular, we are looking to see which rules seem to produce 'sensible' responses to each of the different shocks and analyse whether or not the rules that perform well do so because they are robust to many different shocks. In each case, the coefficients in the rules are again those optimally derived for the 'all-shocks' case (shown in table 2), so this is a test of robustness of the exact rule specification.[25]

Results from this experiment are summarised in Tables 3a and 3b below.

The tables suggest that the OE1 (i.e. the modified IFB) rule and the IFB rule itself are still the 'best' rules under most shocks. The OE1 (and to some extent also the original IFB) rule seems to perform particularly well in the face of shocks from overseas. However, both the OE1 and the IFB rules are outperformed by the OE4 (restricted Ball) rule and by their 'closed-economy' counterparts under productivity shocks to the exports sector. (OE1 and IFB are also inferior to OE4 and Taylor/Henderson-McKibbin under a shock to non-traded goods sector productivity but this is less severe than in the case of a shock to productivity to the export sector). The reason why this happens is that a shock to productivity in the export sector will affect both export prices and output. Since export prices do not enter the calculation of CPI inflation, rules like OE1 or IFB that respond only to consumer price inflation will not perform well because they fail to respond to the first round effects of this shock. This is in line with the general intuition that a simple rule can be a good guide for policy in the face of some — but not all — shocks. Crucially, however, our rule seems more robust to different shocks than a naïve and Ball's MCI-based rules. This is particularly evident for overseas shocks (e.g. foreign interest rate shocks and shocks to the risk premium), but also to shocks to the price of intermediate inputs and to shocks to world output and inflation. In the case of Ball's rule, this is also true for shocks to productivity to both the export sector and the non-traded goods sector.

A comparison of the losses associated with each shock in turn reveals that the most costly shock by far is that to intermediates prices. This is because this shock not only has a higher variance than other shocks but it is also highly cross-correlated with other overseas shocks. In fact, intermediate prices are a large proportion of unit costs in both sectors. For instance, since non-traded producers set prices as a mark up over unit costs, changes in these prices feed directly through non traded price inflation. On the other hand, shocks to the export sector seem to be relatively unimportant given the size of this sector and the openness of the economy. This is because shocks to this sector are largely absorbed by the price of exports which is not a component of CPI inflation.

Given that a shock to intermediate prices is the most costly of our shocks, we would ideally wish to use a monetary policy rule that generated the appropriate response to this shock. In particular, we know that optimal policy would want to absorb the first-round effects of this shock but would want to make sure that there was no long-run effect on inflation. A standard

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[25] To perform this test, we have re-derived losses and asymptotic second moments of the variables of interest by setting the variances of the remaining shocks to zero.

rule that feeds back off current inflation may lead to policy that was too tight and cause a fall in output. By contrast, the IFB and OE1 rules act to stabilise future inflation rather than current: exactly the policy response that seems appropriate for this sort of shock.

The results of this section suggest that OE1 and IFB rules manage to dominate all other rules in an 'all-shocks' scenario because they are efficient at stabilising the economy in the face of overseas shocks (among which are, notably, shocks to intermediate prices).

Table 3a: Comparison of simple monetary policy rules (individual shocks)

	Taylor/ H-McK	IFB	MCI	Ball	OE1	OE2	OE3	OE4
Non-traded productivity shock								
Loss L_1	0.0233	0.0240	0.0267	0.0277	0.0238	0.0268	0.0277	0.0233
Avars p	0.0000	0.0000	0.0005	0.0000	0.0000	0.0000	0.0001	0.0000
y	0.0226	0.0237	0.0188	0.0243	0.0233	0.0254	0.0235	0.0227
Δi	0.0000	0.0000	0.0001	0.0007	0.0000	0.0002	0.0008	0.0000
$y_{v,N}$	0.0046	0.0054	0.0007	0.0052	0.0048	0.0088	0.0042	0.0047
$y_{v,X}$	0.0117	0.0119	0.0100	0.0121	0.0118	0.0126	0.0118	0.0117
c	0.0017	0.0021	0.0000	0.0018	0.0017	0.0045	0.0012	0.0017
i	0.0001	0.0001	0.0004	0.0005	0.0001	0.0004	0.0006	0.0001
q	0.0036	0.0050	0.0001	0.0069	0.0045	0.0069	0.0060	0.0036
Export productivity shock								
Loss L_1	0.0492	0.0507	0.0487	0.0518	0.0505	0.0579	0.0562	0.0485
Avars p	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0000	0.0000
y	0.0445	0.0507	0.0483	0.0517	0.0505	0.0359	0.0547	0.0445
Δi	0.0010	0.0000	0.0000	0.0000	0.0000	0.0044	0.0002	0.0008
$y_{v,N}$	0.0001	0.0004	0.0000	0.0005	0.0003	0.0013	0.0009	0.0001
$y_{v,X}$	0.0550	0.0507	0.0510	0.0507	0.0508	0.0615	0.0487	0.0549
c	0.0001	0.0001	0.0000	0.0002	0.0001	0.0015	0.0005	0.0001
i	0.0005	0.0000	0.0000	0.0000	0.0000	0.0022	0.0001	0.0004
q	0.0023	0.0002	0.0000	0.0003	0.0002	0.0146	0.0012	0.0022
FX risk premium shock								
Loss L_1	0.1013	0.0187	0.2486	0.1287	0.0146	0.1150	0.1204	0.0891
Avars p	0.0006	0.0009	0.0007	0.0004	0.0005	0.0004	0.0004	0.0007
y	0.0013	0.0019	0.0010	0.0007	0.0033	0.0008	0.0008	0.0015
Δi	0.0226	0.0006	0.0592	0.0305	0.0009	0.0269	0.0282	0.0190
$y_{v,N}$	0.0006	0.0001	0.0001	0.0004	0.0045	0.0003	0.0004	0.0008
$y_{v,X}$	0.0372	0.0483	0.0309	0.0300	0.0389	0.0316	0.0310	0.0363
c	0.0004	0.0002	0.0003	0.0004	0.0052	0.0004	0.0004	0.0004
i	0.0219	0.0008	0.0440	0.0377	0.0023	0.0324	0.0347	0.0232
q	0.9686	1.2620	0.8071	0.7787	1.0156	0.8226	0.8067	0.9445
Preference shock								
Loss L_1	0.1310	0.1253	0.1336	0.1275	0.1258	0.1347	0.1277	0.1281
Avars p	0.0001	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0001
y	0.1121	0.1252	0.1324	0.1272	0.1257	0.1330	0.1274	0.1121
Δi	0.0044	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0037
$y_{v,N}$	0.1718	0.1893	0.2000	0.1923	0.1900	0.2004	0.1926	0.1718
$y_{v,X}$	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004
c	0.1867	0.2041	0.2148	0.2072	0.2048	0.2152	0.2074	0.1867
i	0.0022	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0019
q	0.0117	0.0007	0.0000	0.0004	0.0006	0.0002	0.0004	0.0112

Table 3b: Comparison of simple monetary policy rules (individual shocks)

	Taylor/ H-McK	IFB	MCI	Ball	OE1	OE2	OE3	OE4
Intermediate relative (world) price shock								
Loss L_1	1.3742	1.1519	2.5288	1.3274	1.1515	1.3131	1.3100	1.3777
Avars p	0.0227	0.0253	0.1337	0.0248	0.0226	0.0254	0.0223	0.0223
y	0.7648	0.6859	0.1690	0.6804	0.7089	0.6519	0.7122	0.7682
Δi	0.0616	0.0152	0.0552	0.0623	0.0205	0.0637	0.0601	0.0631
$y_{v,N}$	1.5420	1.4270	0.4322	1.3934	1.4660	1.3441	1.4511	1.5496
$y_{v,X}$	0.1502	0.0693	0.0340	0.1310	0.0756	0.1210	0.1340	0.1484
c	1.4231	1.4029	0.5089	1.3070	1.4372	1.2740	1.3594	1.4305
i	0.4815	0.1388	0.1548	0.4373	0.1637	0.4083	0.4337	0.4715
q	2.3738	0.5994	0.2366	1.9364	0.7385	1.7180	2.0106	2.3236
World inflation shock								
Loss L_1	0.4298	0.2179	0.2712	0.4605	0.1912	0.4914	0.4887	0.4340
Avars p	0.0031	0.0032	0.0093	0.0028	0.0026	0.0035	0.0030	0.0032
y	0.1894	0.1590	0.0550	0.2042	0.1402	0.2285	0.2183	0.1912
Δi	0.0477	0.0020	0.0170	0.0531	0.0024	0.0517	0.0558	0.0481
$y_{v,N}$	0.3721	0.3120	0.1230	0.3988	0.2797	0.4410	0.4238	0.3753
$y_{v,X}$	0.0119	0.0232	0.0169	0.0109	0.0198	0.0121	0.0108	0.0116
c	0.3690	0.3198	0.1419	0.3962	0.2914	0.4357	0.4195	0.3719
i	0.0906	0.0126	0.0212	0.0957	0.0113	0.0913	0.1010	0.0926
q	0.3366	0.6156	0.2875	0.3219	0.4713	0.3941	0.3332	0.3340
World interest rate shock								
Loss L_1	0.1251	0.1187	0.6265	0.1398	0.1119	0.1530	0.1484	0.1242
Avars p	0.0014	0.0012	0.0319	0.0025	0.0012	0.0036	0.0031	0.0015
y	0.0827	0.0974	0.0048	0.0674	0.0862	0.0548	0.0610	0.0816
Δi	0.0049	0.0007	0.0278	0.0081	0.0016	0.0101	0.0093	0.0047
$y_{v,N}$	0.1418	0.1686	0.0087	0.1159	0.1488	0.0952	0.1051	0.1402
$y_{v,X}$	0.1711	0.1699	0.1123	0.1663	0.1702	0.1604	0.1643	0.1704
c	0.1066	0.1310	0.0111	0.0859	0.1134	0.0701	0.0772	0.1054
i	0.0263	0.0053	0.0573	0.0383	0.0126	0.0424	0.0425	0.0261
q	0.3897	0.4337	0.2006	0.3429	0.4063	0.2959	0.3255	0.3841
World output shock								
Loss L_1	0.2636	0.1730	0.3944	0.1879	0.0962	0.1419	0.1528	0.2516
Avars p	0.0022	0.0051	0.0225	0.0012	0.0038	0.0010	0.0009	0.0022
y	0.0849	0.0771	0.0320	0.0563	0.0274	0.0479	0.0477	0.0859
Δi	0.0357	0.0034	0.0005	0.0280	0.0022	0.0196	0.0227	0.0325
$y_{v,N}$	0.1494	0.1180	0.0877	0.1055	0.0518	0.0943	0.0954	0.1513
$y_{v,X}$	0.6645	0.7375	0.5664	0.6544	0.6804	0.6437	0.6392	0.6633
c	0.1033	0.0766	0.1437	0.0779	0.0468	0.0743	0.0753	0.1048
i	0.0815	0.0155	0.0011	0.0578	0.0099	0.0513	0.0555	0.0818
q	1.3761	1.8257	0.5741	1.2551	1.3493	1.1713	1.1489	1.3707

6.   Conclusions

Existing closed economy rules like those advocated by Taylor (1993a) and Henderson and McKibbin (1993) may not account for the exchange rate channel of monetary transmission because they only respond to inflation deviations from target and output deviations from potential. In this paper we have explored alternative simple monetary policy rules for an economy that is open like the UK. This entailed considering existing rules for open economies like a naïve MCI-based rule and Ball's (1999) rule. It also entailed looking at parsimonious modifications of these and closed-economy rules that account for the exchange rate transmission channel in various ways.

We concluded that a good rule in this respect is an inflation forecast based rule ('IFB'), i.e. a rule that reacts to deviations of expected inflation from target is a good simple rule in this respect, when the horizon is adequately chosen. This rule is associated with a lower than average variability of inflation when compared to the alternative open and closed-economy rules. Relative to those, it also generally appears to reduce the disparity between output sector volatilities in the two sectors, other things being equal. Adding a separate response to the level of the real exchange rate (contemporaneous and lagged) appears to reduce further the difference in adjustment between output gaps in the two sectors of the economy, but this improvement is only marginal. These results on the relative performance of the rules are confirmed by results obtained comparing the utility losses faced by the households in our model economy under each rule.

Importantly, an IFB rule, with or without exchange rate adjustment, appears quite robust to different shocks, in contrast to naïve MCI-based rules or Ball's rule. Finally, relative to other open and closed-economy rules that we have analysed, an IFB rule (and OE1, an exchange-rate-adjusted IFB rule) seems to minimise the probability of hitting a zero bound with nominal interest rates, and thereby may increase the chances of policy remaining operational under particularly severe deflationary shocks.

References

Ball, L, (1999), 'Policy Rules for Open Economies', in J B Taylor (ed) (1999*) Monetary Policy Rules,* Chicago: University of Chicago Press.

Bank of England (1999), *Economic models at the Bank of England*, London: Bank of England.

Batini, N and Haldane, A G (1999), 'Forward-looking rules for monetary policy', *in* JB Taylor (ed) (1999*) Monetary Policy Rules,* Chicago: University of Chicago Press.

Batini and Nelson (2000a), 'Optimal horizons for Inflation Targeting', *Journal of Economic Dynamics and Control*, forthcoming.

Batini and Nelson (2000b), 'When the Bubble Bursts: Monetary Policy Rules and Foreign Exchange Rate Market Behaviour', *mimeo*.

Batini and Turnbull (2000), 'Monetary Condition Indices for the UK: A Survey', *External MPC Unit Discussion Paper*, No. 1, Bank of England.

Benigno, G and Benigno, P (2000), 'Monetary policy rules and the exchange rate,' *mimeo*.

Blinder, A S (**1998**), *Central banking in theory and practice*, Cambridge, Massachusets: MIT Press.

Britton, E, Larsen, J D J, and Small, I (**1999**), 'Imperfect competition and the dynamics of mark-ups', *Bank of England Working Paper*, No 110.

Bryant, RC, Hooper, P and Mann, C (**1993**), *Evaluating Policy Regimes*, Washington: The Brookings Institution.

Carlstrom, C T, and Fuerst, T S (1999) 'Timing and Real Indeterminacy in Monetary Models', *Journal of Monetary Economics*, forthcoming.

Carrol, C D, Overland, J, and Weil, D N (**1995**), 'Saving and Growth with Habit Formation', *Federal Reserve Board of Governors Finance and Economics Discussion Paper*, No 1995-42.

Calvo, G (**1983**), 'Staggered prices in a utility maximising framework', *Journal of Monetary Economics*, 12.

Correia, I, Neves, J C and Rebelo, S (**1995**), 'Business Cycles in a Small Open Economy,' *European Economic Review*, 39, pp1089-1113.

Devereux, M and C Engle (2000), 'Monetary policy in the open economy revisited: Price setting and exchange rate flexibility', *National Bureau of Economic Research Working Paper*, No 7665.

Erceg, C, Henderson, D and Levin, A (2000), 'Optimal monetary policy with staggered wage and price contracts,' *Journal of Monetary Economics*.

Ericsson, N, Hendry, D and Mizon, G (1998), 'Exogeneity, cointegration and economic policy analysis', *Journal of Business and Economic Statistics*, October.

Fuhrer, J C (2000), 'An Optimisation-Based Model for Monetary Policy Analysis: Can Habit Formation Help?' *American Economic Review*, forthcoming.

Gali, J and Monacelli, T (1999), 'Optimal monetary policy and exchange rate variability in a small open economy', *mimeo*.

Ghironi, F (2000), 'Alternative monetary rules for a small open economy: The case of Canada', *mimeo*.

Goodfriend M (1999), 'Overcoming the zero bound on interest rate policy' paper for the Federal Reserve Bank of Boston conference on 'Monetary policy in a low inflation environment'.

Hall, Walsh and Yates (1997), 'How do UK companies set prices?', *Bank of England Working Paper,* No 67.

Hansen, L P and Sargent, T J (1998), *Recursive Linear Models of Dynamic Economies*, Chicago: University of Chicago Press.

Henderson, D W and McKibbin, W J (1993), 'A comparison of some basic monetary policy regimes for open economies: implications of different degrees of instrument adjustment and wage persistence', *Carnegie-Rochester Conference Series on Public Policy*, 39, pages 221-317.

King, M A, (1997), 'Monetary Policy and the Exchange Rate', *BEQB*, May 1997, pages 225-7.

Klein, P (1997), 'Using the generalized Schur form to solve a system of linear expectational difference equations', *mimeo*.

Lagarias, J C, Reeds, J A, Wright, M H and Wright, P E (1997), 'Convergence properties of the Nelder-Mead simplex algorithm in low dimensions', to appear in the *SIAM Journal of Optimization*.

Levin, A, Wieland, V and Williams, J C (1998), 'Robustness of simple monetary policy rules under model uncertainty', *Federal Reserve Board of Governors Finance and Economics Discussion Paper*, No 1998-45.

McCallum (2000), 'Alternative monetary policy rules: A comparison with historical settings for the United States, the United Kingdom and Japan', *National Bureau of Economic Research Working Paper*, No 7725.

McCallum, B and Nelson, E (1999), 'Nominal income targeting in an open-economy optimizing model', *Journal of Monetary Economics*, 43(3), pages 553-78.

Monacelli, T (1999), 'Open economy policy rules under imperfect pass-through', *mimeo*.

Obstfeld, M and Rogoff, K (1996), *Foundations of international macroeconomics*, Cambridge, Massachusets: MIT press.

Rotemberg, J J and Woodford, M (1997), 'An optimisation-based econometric framework for the evaluation of monetary policy', *NBER Macroeconomics Annual 1997*, pages 297-346, Cambridge, Massachusets: MIT Press.

Small, I (1997), 'The cyclicality of mark-ups and profit margins: some evidence for manufacturing and services', *Bank of England Working Paper*, No 72.

Smets and Wouters (2000), 'Optimal monetary policy in an open economy: An application to the Euro area', *mimeo*.

Svensson, L E and van Wijnbergen, S (1989), 'Excess capacity, monopolistic competition, and international transmission of monetary disturbances', *Economic Journal*, 99(397), pages 785-805.

Taylor, J B (1993a), 'Discretion versus Policy Rules in Practice', *Carnegie Rochester Conference Series on Public Policy*, 39, pages 195-214.

Taylor, J B (1993b), *Macroeconomic Policy in a World Economy*, New York: W.W. Norton.

Taylor, J B (Ed) (1999), *Monetary Policy Rules*, Chicago: University of Chicago Press.

Williams, J C (1999), 'Simple rules for monetary policy', *Federal Reserve Board of Governors Finance and Economics Discussion Paper*, No 1999-12.

Woodford (1999), 'Inflation stabilisation and welfare', *mimeo*.

Technical Annexes for 'Monetary Policy Rules for an Open Economy' by Nicoletta Batini, Richard Harrison and Stephen Millard

Annex A: First order conditions

Following the discussion of the model in sections 2.1-2.3 of the main text, here we consider the problems facing agents in each of sector in turn.

Households

Household $j \in (0,1)$ solves the following problem:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left(\exp(v_t) \ln(c_t(j) - \xi_c c_{t-1}(j)) + \delta \ln(1 - h_t(j)) + \frac{\chi}{1-\varepsilon} \left(\frac{\Omega_t(j)}{P_t} \right)^{1-\varepsilon} \right)$$

Subject to

$$M_t(j) + B_t(j) + \frac{B_{f,t}(j)}{e_t(j)} + P_t \int r_t(s) b_t(s, j) ds =$$

$$M_{t-1}(j) + (1+i_{t-1}) B_{t-1}(j) + (1+i_{f,t-1}) \frac{B_{f,t-1}(j)}{e_t} + P_t \int b_{t-1}(s, j) ds + W_t(j) h_t(j) + D_t + T_t - P_t c_t(j)$$

$$\tag{A1}$$

$$\Omega_t(j) = M_{t-1}(j) + T_t + (1+i_{t-1}) B_{t-1}(j) + (1+i_{f,t-1}) \frac{B_{f,t-1}(j)}{e_t} - B_t(j) - \frac{B_{f,t}(j)}{e_t} \tag{A2}$$

$$c_t = c_{M,t}^{\gamma} c_{N,t}^{1-\gamma} \tag{A3}$$

$$P_t = \frac{P_{M,t}^{\gamma} P_{N,t}^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}} \tag{A4}$$

where the variables are defined as in the text. The household chooses c_M, c_N, Ω, M, B, $b(s)$ and B_f to solve the maximisation problem.

To solve this problem we substitute the definitions of aggregate consumption and the aggregate price level into the utility function, the budget constraint and the definition of

'money' (A2). We let the Lagrange multipliers on these two constraints be denoted λ_1 and λ_2, respectively. Suppressing the j index throughout, we differentiate to get:

$$\frac{\gamma \exp(v_t)}{c_t - \xi c_{t-1}}\left(\frac{c_{N,t}}{c_{M,t}}\right)^{1-\gamma} - \lambda_{1,t}P_{M,t} = \beta\gamma\xi\left(\frac{c_{N,t}}{c_{M,t}}\right)^{1-\gamma} E_t\left(\frac{\exp(v_{t+1})}{c_{t+1} - \xi c_t}\right) \tag{A5}$$

$$\frac{(1-\gamma)\exp(v_t)}{c_t - \xi c_{t-1}}\left(\frac{c_{N,t}}{c_{M,t}}\right)^{\gamma} - \lambda_{1,t}P_{N,t} = \beta(1-\gamma)\xi\left(\frac{c_{N,t}}{c_{M,t}}\right)^{\gamma} E_t\left(\frac{\exp(v_{t+1})}{c_{t+1} - \xi c_t}\right) \tag{A6}$$

$$\lambda_{1,t} + \lambda_{2,t} = \beta(1+i_t)E_t\left(\lambda_{1,t+1} + \lambda_{2,t+1}\right) \tag{A7}$$

$$\frac{\lambda_{1,t} + \lambda_{2,t}}{e_t} = \beta(1+i_{f,t})E_t\left(\frac{\lambda_{1,t+1} + \lambda_{2,t+1}}{e_{t+1}}\right) \tag{A8}$$

$$\lambda_{2,t} = \frac{\chi}{P_t}\left(\frac{\Omega_t}{P_t}\right)^{-\varepsilon} \tag{A9}$$

$$\lambda_{1,t} = \beta E_t\left(\lambda_{1,t+1} + \lambda_{2,t+1}\right) \tag{A10}$$

The choice of the nominal wage discussed in section 2.3.2. The first order condition is:

$$E_t\sum_{s=0}^{\infty}(\beta\phi_W)^s\left[\frac{(1+\pi)^s W_t(j)}{P_{t+s}}\Lambda_{1,t+s} - (1+\theta_W)\frac{\delta}{1-h_{t+s}(j)}\right]h_{t+s}(j) = 0. \tag{A11}$$

Equation (A12) features the real marginal utility of consumption, Λ_1, which is related to the marginal utility of nominal consumption in a simple manner: $\Lambda_1 = P\lambda_1$. This is discussed in more detail below.

Non-Traded Sector

As described in section 2.3.1 producer $k \in (0,1)$ in the non-traded sector choose prices to solve the following problem.

$$\max E_t\sum_{j=0}^{\infty}(\beta\phi_N)^j\Lambda_{1,t+j}\left(\frac{(1+\pi)^j P_{N,t}(k)}{P_{t+j}} - V_{t+j}\right)y_{N,t+j}(k)$$

subject to $y_{N,t+j}(k) = \left(\dfrac{(1+\pi)^j P_{N,t+j}(k)}{P_{N,t+j}} \right)^{-\frac{(1+\theta_N)}{\theta_N}} y_{N,t+j}$.

The first order condition is:

$$E_t \sum_{j=0}^{\infty} (\beta\phi_N)^j \Lambda_{1,t+j} \left(\frac{-\theta_N (1+\pi)^j P_{N,t}(k)}{P_{t+j}} + (1+\theta_N)V_{t+j} \right) y_{N,t+j}(k) = 0 . \tag{A12}$$

The real unit cost, V, in units of final consumption is given by:

$$V_{t+s} = \min\left\{ \frac{W_{t+s}}{P_{t+s}} h_{N,t+s}(k) + \frac{P_{I,t+s}}{P_{t+s}} I_{N,t+s}(k) \right\} \text{ subject to } A_{N,t+s} h_{N,t+s}(k)^{\alpha_N} I_{N,t+s}(k)^{(1-\alpha_N)} = 1$$

The first order conditions to this problem imply that:

$$\frac{W_t}{P_{I,t}} = \frac{\alpha_N}{1-\alpha_N} \frac{I_{N,t}(k)}{h_{N,t}(k)} ,$$

for all $k \in (0,1)$ at all dates t. Because non-traded producers are price takers in the factor market, the equilibrium ratio of intermediates to labour is constant across firm in this sector:

$$\frac{W_t}{P_{I,t}} = \frac{\alpha_N}{1-\alpha_N} \frac{I_{N,t}}{h_{N,t}} . \tag{A13}$$

The constancy of the intermediate:labour ratio implies that the aggregate output in the non-traded producers is given by:

$$y_{N,t} = \left[\int_0^1 y_{N,t}(k)^{1/(1+\theta_N)} dk \right]^{1+\theta_N}$$

$$y_{N,t} = \left[\int_0^1 y_{N,t}(k)^{1/(1+\theta_N)} dk \right]^{1+\theta_N}$$

$$= A_{N,t} \left[\int_0^1 [h_{N,t}(k)^{\alpha_N} I_{N,t}(k)^{(1-\alpha_N)}]^{1/(1+\theta_N)} dk \right]^{1+\theta_N} = A_{N,t} \left(\frac{h_{N,t}}{I_{N,t}} \right)^{\alpha_N} \left[\int_0^1 I_{N,t}(k)^{1/(1+\theta_N)} dk \right]^{1+\theta_N}$$

So,

$$y_{N,t} = A_{N,t} h_{N,t}^{\alpha_N} I_{N,t}^{1-\alpha_N} . \tag{A14}$$

The minimised unit cost for all firms in the non-traded sector is found to be:

$$V_t = \frac{1}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} \frac{W_t^{\alpha_N} (P_{I,t})^{1-\alpha_N}}{A_{N,t} P_t} . \tag{A15}$$

Export sector

As described in section 2.2.2 exports are produced using a Cobb-Douglas technology:

$$y_{X,t} = A_{X,t} h_{X,t}^{\alpha_X} I_{X,t}^{1-\alpha_X} \tag{A16}$$

Efficient production implies that factor demands are given by:

$$\frac{W_t}{P_{X,t}} = \alpha_X A_{X,t} \left(\frac{I_{X,t}}{h_{X,t}} \right)^{1-\alpha_X} \tag{A17}$$

$$\frac{P_{I,t}}{P_{X,t}} = (1-\alpha_X) A_{X,t} \left(\frac{h_{X,t}}{I_{X,t}} \right)^{\alpha_X} \tag{A18}$$

Export demand is:

$$X_t = \left(\frac{e_t P_{X,t}}{P_t^*} \right)^{-\eta} y_{f,t}^b . \tag{A19}$$

Intermediate goods sector

Producers in both the non-traded and export sectors purchase imported intermediates from retailers who solve a pricing problem described in section 2.3.1. The first order condition is:

$$E_{t-1} \sum_{s=0}^{\infty} (\beta \phi_I)^s \Lambda_{1,t+s} \left(\frac{-\theta_I (1+\pi)^s P_{I,t}(k)}{P_{t+s}} + (1+\theta_I) \frac{P_{I,t+s}^*}{e_{t+s} P_{t+s}} \right) y_{I,t+s}(k) = 0 \qquad \text{(A20)}$$

Final imports sector

The first order condition for the pricing problem of retailers of final imported goods is given by:

$$E_{t-1} \sum_{s=0}^{\infty} (\beta \phi_M)^s \Lambda_{1,t+s} \left(\frac{-\theta_M (1+\pi)^s P_{M,t}(k)}{P_{t+s}} + (1+\theta_M) \frac{P_{t+s}^*}{e_{t+s} P_{t+s}} \right) y_{M,t+s}(k) = 0 . \qquad \text{(A21)}$$

Government

The government operates monetary policy by setting nominal interest rates according to a rule (described below) and prints as much money as is demanded at this level of nominal interest rates. Any seignorage revenue is distributed as a lump-sum transfer to consumers. For simplicity, we assume a zero supply of domestic bonds. Hence:

$$M_t - M_{t-1} = T_t - \tau_t . \qquad \text{(A22)}$$

Market Clearing

We have the following market clearing conditions in factor markets, goods markets and asset markets:

$$h_t = h_{X,t} + h_{N,t} \qquad \text{(A23)}$$

$$c_{N,t} = y_{N,t} \qquad \text{(A24)}$$

$$X_t = y_{X,t} \qquad \text{(A25)}$$

$$\int \int b_t(s, j) ds dj = 0 \qquad \text{(A26)}$$

Net foreign assets

The evolution of net foreign assets can be found by evaluating the household's budget constraint (A2) at market equilibrium and then aggregating across households. As discussed in section 2.4, the net foreign asset position (under our assumptions this is equal to the domestic holdings of foreign bonds) is non-stationary. To deal with this problem we do not include this equation in our system. Instead we use the equation to substitute foreign bond holdings out of the definition of 'money' (A3).

Annex B: Flexible-price steady state

We use the following notation. Variables without time subscripts are the steady state values. Lower case letters represent nominal variables expressed relative to the CPI (we also define the real value of foreign bond holdings as $b_f = B_f/eP$). We express nominal variables relative to the general price level in order to solve for steady state variables that are not trended (in steady state all nominal variables will follow the same trend path). In addition, the Lagrange multipliers λ_1 and λ_2 are homogenous of degree -1 so we scale them by the CPI, to give stationary multipliers $\Lambda_1 = P\lambda_1$ and $\Lambda_2 = P\lambda_2$. Throughout we use the real exchange rate definition, $q_t = \dfrac{e_t P_t}{P_t^*}$.

To construct a steady state, we first assume that all domestic nominal variables are growing at an annual rate of 2.5%. This means that, in steady state, the government is meeting an inflation target of 2.5%. For simplicity, we also assume that the steady state growth of foreign nominal variables is 2.5%. The implied steady state value of nominal interest rates at home and abroad will be given by:

$$i = i_f = \frac{1+\pi}{\beta} - 1.$$

In what follows, we use equations (A23) and (A27) before evaluating the steady state. We assume that steady state taxes are set to exactly offset steady state dividends. Finally, we choose a flexible price equilibrium so that, although price setters retain some monopoly power, they simply set prices as a mark up over unit costs.

Then, the first order conditions imply the following equations defining steady state values of the variables:

$$\lambda_1 p_M = \frac{(1-\beta\xi)\gamma}{(1-\xi)c}\left(\frac{c_N}{c_M}\right)^{1-\gamma} \tag{A27}$$

$$\lambda_1 p_N = \frac{(1-\beta\xi)(1-\gamma)}{(1-\xi)c}\left(\frac{c_M}{c_N}\right)^{\gamma} \tag{A28}$$

$$\chi\omega^{-\varepsilon} = \lambda_2 \tag{A29}$$

$$\lambda_1 = \frac{\beta(\lambda_1+\lambda_2)}{1+\pi} \tag{A30}$$

$$\lambda_1 w = \frac{(1+\theta_W)\delta}{(1-h)} \tag{A31}$$

$$\frac{1-\beta}{\beta}b_f = p_X X - p_M c_M - p_I\left(I_X + I_N\right) \tag{A32}$$

$$\omega = m + \frac{1-\beta}{\beta}b_f \tag{A33}$$

$$c = c_M^{\gamma} c_N^{1-\gamma} \tag{A34}$$

$$p_N^{1-\gamma} p_M^{\gamma} = \gamma^{\gamma}(1-\gamma)^{1-\gamma} \tag{A35}$$

$$p_N = (1+\theta_N)v \tag{A36}$$

$$\frac{w}{p_I} = \frac{\alpha_N}{1-\alpha_N}\frac{I_N}{h_N} \tag{A37}$$

$$y_N = A_N h_N^{\alpha_N} I_N^{1-\alpha_N} \tag{A38}$$

$$v = \frac{w^{\alpha_N} p_I^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \tag{A39}$$

$$y_X = A_X h_X^{\alpha_X} I_X^{1-\alpha_X} \tag{A40}$$

$$\frac{w}{p_X} = \alpha_X A_X\left(\frac{I_X}{h_X}\right)^{1-\alpha_X} \tag{A41}$$

$$\frac{p_I}{p_X} = (1-\alpha_X)A_X\left(\frac{h_X}{I_X}\right)^{\alpha_X} \tag{A42}$$

$$X = \left(\frac{q}{p_X}\right)^{\eta} y_f^b \tag{A43}$$

$$p_I = (1+\theta_I)\frac{p_I^*}{q} \tag{A44}$$

$$p_M = (1+\theta_M)\frac{1}{q} \tag{A45}$$

$$c_N = y_N \tag{A46}$$

$$h = h_N + h_X \tag{A47}$$

$$X = y_X \tag{A48}$$

Annex C: A log-linear representation of the model

To solve the model we log-linearise the first order conditions of the model around the non-stochastic steady state defined by equations (A27) to (A48). As described in the main text we use (A2) evaluated at market equilibrium to substitute foreign bond holdings out of the model. As in Annex 2, we also substitute out for taxes, transfers and dividends. Log-linearising the consumers' first order conditions (equations (A5) to (A10)) gives us:

$$E_t\left(\frac{\beta\xi}{(1-\beta\xi)(1-\xi)}\hat{c}_{t+1} - \frac{\beta\xi}{1-\beta\xi}v_{t+1}\right) = \hat{\Lambda}_{1,t} + \hat{p}_{M,t} - \frac{1}{1-\beta\xi}v_t + \hat{c}_{M,t}$$
$$- \left(1 - \frac{1+\beta\xi^2}{(1-\beta\xi)(1-\xi)}\right)\hat{c}_t - \frac{\xi}{(1-\beta\xi)(1-\xi)}\hat{c}_{t-1} \tag{A49}$$

$$E_t\left(\frac{\beta\xi}{(1-\beta\xi)(1-\xi)}\hat{c}_{t+1} - \frac{\beta\xi}{1-\beta\xi}v_{t+1}\right) = \hat{\Lambda}_{1,t} + \hat{p}_{N,t} - \frac{1}{1-\beta\xi}v_t + \hat{c}_{N,t}$$
$$- \left(1 - \frac{1+\beta\xi^2}{(1-\beta\xi)(1-\xi)}\right)\hat{c}_t - \frac{\xi}{(1-\beta\xi)(1-\xi)}\hat{c}_{t-1} \tag{A50}$$

$$E_t\left(\hat{\pi}_{t+1} - \frac{\Lambda_1}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{1,t+1} - \frac{\Lambda_2}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{2,t+1}\right) = (i_t - i) - \frac{\Lambda_1}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{1,t}$$
$$- \frac{\Lambda_2}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{2,t} \tag{A51}$$

$$E_t\left(\hat{\pi}_{t+1}^* + \hat{q}_{t+1} - \frac{\Lambda_1}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{1,t+1} - \frac{\Lambda_2}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{2,t+1}\right) = (i_{f,t} - i_f) + \hat{q}_t - \frac{\Lambda_1}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{1,t}$$
$$- \frac{\Lambda_2}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{2,t} + \zeta_t \tag{A52}$$

$$\hat{\Lambda}_{2,t} + \varepsilon\hat{\omega}_t = 0 \tag{A53}$$

$$E_t\left(\hat{\pi}_{t+1} - \frac{\Lambda_1}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{1,t+1} - \frac{\Lambda_2}{\Lambda_1 + \Lambda_2}\hat{\Lambda}_{2,t+1}\right) = \hat{\Lambda}_{1,t} \tag{A54}$$

where for any variable x, $\hat{x} = \ln\left(\frac{x_t}{x}\right)$ where x is its steady state value and ζ is an exogenous 'foreign exchange risk premium' shock.

The definition of Ω (equation (A3)) becomes:

$$\hat{\omega}_t - \frac{m}{\omega}\hat{m}_t + \frac{wh}{\omega}\hat{h}_t + \frac{wh}{\omega}\hat{w}_t - + \frac{c}{\omega}\hat{c}_t = 0 . \tag{A55}$$

The definitions of consumption and the price indices are:

$$\hat{c}_t = \gamma\hat{c}_{M,t} + (1-\gamma)\hat{c}_{N,t} , \tag{A56}$$

$$0 = \gamma\hat{p}_{M,t} + (1-\gamma)\hat{p}_{N,t} . \tag{A57}$$

Wage setting is given by the following two equations (the derivation follows Erceg *et al* (1999, p25)).

$$\Delta\hat{W}_t = \beta E_t \Delta\hat{W}_{t+1} + \frac{(1-\phi_W)(1-\beta\phi_W)h}{\phi_W(1-h)[1+\frac{h(1+\theta_W)}{\theta_W(1-h)}]}\hat{h}_t - \frac{(1-\phi_W)(1-\beta\phi_W)}{\phi_W[1+\frac{h(1+\theta_W)}{\theta_W(1-h)}]}\hat{\Lambda}_{1,t}$$
$$- \frac{(1-\phi_W)(1-\beta\phi_W)}{\phi_W[1+\frac{h(1+\theta_W)}{\theta_W(1-h)}]}\hat{w}_t \tag{A58}$$

$$\hat{w}_t = \hat{w}_{t-1} + \Delta\hat{W}_t - \hat{\pi}_t . \tag{A59}$$

Pricing decisions by non-traded goods producers are described by:

$$\Delta\hat{P}_{N,t} = \beta E_t \Delta\hat{P}_{N,t+1} + \frac{(1-\phi_N)(1-\beta\phi_N)}{\phi_N}\hat{v}_t - \frac{(1-\phi_N)(1-\beta\phi_N)}{\phi_N}\hat{p}_{N,t}, \tag{A60}$$

$$\hat{p}_{N,t} = \hat{p}_{N,t-1} + \Delta\hat{P}_{N,t} - \hat{\pi}_t, \tag{A61}$$

$$\hat{v}_t = \alpha_N \hat{w}_t + (1-\alpha_N)\hat{p}_{I,t} - \hat{A}_{N,t}. \tag{A62}$$

Efficient production by non-traded producers implies that:

$$\hat{w}_t - \hat{p}_{I,t} = \hat{I}_{N,t} - \hat{h}_{N,t}, \tag{A63}$$

$$\hat{y}_{N,t} = \hat{A}_{N,t} + \alpha_N \hat{h}_{N,t} + (1-\alpha_N)\hat{I}_{N,t}. \tag{A64}$$

The first order conditions for export producers become:

$$\hat{w}_t - \hat{p}_{X,t} - \hat{A}_{X,t} - (1-\alpha_X)\hat{I}_{X,t} + (1-\alpha_X)\hat{h}_{X,t} = 0, \tag{A65}$$

$$\hat{p}_{I,t} - \hat{p}_{X,t} - \hat{A}_{X,t} + \alpha_X \hat{I}_{X,t} + \alpha_X \hat{h}_{X,t} = 0. \tag{A66}$$

Export production is given by:

$$\hat{y}_{X,t} - \hat{A}_{X,t} - \alpha_X \hat{h}_{X,t} - (1-\alpha_X)\hat{I}_{X,t} = 0. \tag{A67}$$

Export demand can be written as:

$$\hat{X}_t + \eta\hat{q}_t + \eta\hat{p}_{X,t} - b\hat{y}_{F,t} = 0. \tag{A68}$$

Pricing of intermediates is described by:

$$\begin{aligned}\Delta\hat{P}_{I,t} &= \beta E_{t-1}\Delta\hat{P}_{I,t+1} + \frac{(1-\phi_I)(1-\beta\phi_I)}{\phi_I}E_{t-1}\hat{p}_{I,t}^* \\ &\quad - \frac{(1-\phi_I)(1-\beta\phi_I)}{\phi_I}E_{t-1}\hat{q}_t - \frac{(1-\phi_I)(1-\beta\phi_I)}{\phi_I}E_{t-1}\hat{p}_{I,t}\end{aligned} \tag{A69}$$

$$\hat{p}_{I,t} = \hat{p}_{I,t-1} + \Delta\hat{P}_{I,t} - \hat{\pi}_t. \tag{A70}$$

Pricing of final imports is described by:

$$\Delta \hat{P}_{M,t} = \beta E_{t-1} \Delta \hat{P}_{M,t+1} - \frac{(1-\phi_I)(1-\beta\phi_I)}{\phi_I} E_{t-1} \hat{q}_t, \tag{A71}$$

$$\hat{p}_{M,t} = \hat{p}_{M,t-1} + \Delta \hat{P}_{M,t} - \hat{\pi}_t. \tag{A72}$$

The relevant market-clearing conditions can be written as:

$$\frac{h_X}{h} \hat{h}_{X,t} + \frac{h_N}{h} \hat{h}_{N,t} - \hat{h}_t = 0, \tag{A73}$$

$$\hat{c}_{N,t} - \hat{y}_{N,t} = 0, \tag{A74}$$

$$\hat{X}_t - \hat{y}_{X,t} = 0. \tag{A75}$$

Together with some obvious lag identities and log-linearised definitions (for example the GDP identity) the model can be cast in the form of equations (22) and (23) in the main text. The calibration of the forcing processes is described in section 3.2 of the main text.