Multiple unemployment equilibria: Do transitory shocks have permanent effects?

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Abstract

This paper tests for multiple equilibria in the Norwegian unemployment rate and investigates whether it displays asymmetric response to positive and negative shocks. Linear and nonlinear univariate models are employed to account for the unemployment behaviour over the period 1972-1997. Among these, only a logistic smooth transition autoregressive (LSTAR) model is data consistent. Accordingly, unemployment is a stationary variable that has switched between two stable equilibria during the sample period. It is shown that a large shock, or a sequence of small shocks, can cause a transition from one equilibrium level to another and thereby have a permanent effect on the unemployment rate. Moreover, unemployment recovers faster from a fall, relative to a given equilibrium level, than from a rise.

1. Introduction

This paper characterises the Norwegian unemployment rate in a framework that allows for multiple equilibria and asymmetric responses to positive and negative

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shocks. This approach may explain the unprecedented rise in the unemployment rate at the end of 1980s, and shed light on its dynamic behaviour, in particular its persistence. A study of unemployment from this perspective can also address the issue of monetary neutrality. If transitory shocks e.g. due to changes in monetary policy cause a movement from one unemployment equilibrium to another, then monetary policy can have permanent effects on the level of unemployment and activity. This is in contrast to unique equilibrium models where nominal shocks only cause temporary deviations from the unique equilibrium level (see e.g. Friedman, 1968; Layard *et al.*, 1991). The latter is assumed to be affected only by e.g. structural and institutional changes in the economy.

The paper proceeds as follows. Section 2 motivates our approach by providing the background facts and findings of previous studies. Section 3 summarises the main characteristics of multiple equilibria models and their implications for the actual (un)employment behaviour. Section 4 formalises the multiple equilibria approach in the Markov regime switching model of Hamilton (1989) and in a smooth transition autoregressive (STAR) model, see e.g. Teräsvirta (1994). These models are sufficiently general to encompass linear models that imply a unique unemployment equilibrium or hysteresis in the unit root sense. The empirical analysis is based on quarterly observations of seasonally non-adjusted data for the unemployment rate over the period 1972-1997. The series is described in section 5.1 that lays out its the main features. These appear to be easier to reconcile with the presence of multiple equilibria than with a unique equilibrium. Subsection 5.2 derives a linear autoregressive model of unemployment and explores its properties. This model serves as a reference model and is used to evaluate the nonlinear models developed in the following sections. Subsection 5.3 models the series in the Markov regime switching framework and show the sensitivity of the results to model specification. Section 6 is devoted to the specification, estimation and evaluation of a logistic STAR (LSTAR) model. It will be shown that the LSTAR model is quite successful in characterising the unemployment behaviour and hence provides a sound basis to address the issues of interest. The conclusions follow in the last section, section 7.

2. Background

The unemployment experience of (western) European countries in the last decades has revealed slow, if any, tendencies of actual unemployment to revert to a unique equilibrium rate, as prescribed by the natural rate or the NAIRU hypothesis. As noted by several authors, instead of providing an anchor for the actual unemployment rate, the estimated equilibrium level appears to track the actual rate (see for instance Layard *et al.*, 1991; Cromb, 1993; Elmeskov and MacFarland, 1993). However, this occurs without matching structural and institutional changes. In addition, approximately the same rate or even a constant rate of price and wage inflation is observed at significantly different levels of unemployment.

To account for these findings, and in particular the apparently ratcheting behaviour of unemployment, the hypothesis of unemployment hysteresis has been launched (see Blanchard and Summers, 1986; Lindbeck and Snower, 1988). The concept of hysteresis is associated with dynamic models. It denotes a situation in which the equilibrium state of a system depends on the past history of the system (see Amable *et al.*, 1995; Røed, 1997). Since the seminal work of Blanchard and Summers (1986), however, unemployment hysteresis has been commonly equated with unemployment following a random walk. In this case, any shock, irrespective of its size and whether it is transitory or permanent, has a permanent effect on the level of unemployment. Following this notion, formal tests of unemployment hysteresis have been conducted as unit root tests in linear dynamic models of unemployment (see e.g. Alogoskoufis and Manning, 1988; Cromb, 1993; Cross, 1995). Most of the empirical studies of European unemployment find estimates of a root close to one, which is consistent with a null hypothesis of a unit root but also with the null hypothesis of near unit root. The latter case, termed *unemployment persistence,* is consistent with a unique equilibrium but suggests weak equilibrium reversion, (cf. Bean, 1992).

The practice of equating hysteresis with the presence of a unit root in a linear model has been questioned (see e.g. Amable *et al.,* 1995; Cross, 1995; Røed, 1997). Firstly, it compels every shock, irrespective of its size and sign to have a permanent effect on the level of unemployment, disregarding the existence of (endogenous) stabilising mechanisms. Secondly, long series of unemployment rate data often show that it does not wander around randomly, but revert to its past levels, sooner or later, (cf. Layard et al., 1991 and Bianchi and Zoega, 1997). Indeed, the bounded nature of the unemployment rate series prevents it from taking values outside the 0-1 range. Thirdly, the empirical evidence in favour of a unit root, or high degree of persistence, in relatively smaller samples may be due to large shocks to the series. It is well known that standard unit root tests underreject the null hypothesis of unit root when there are breaks in the series, (see e.g. Banerjee *et al.,* 1993). Finally, the linear models imply symmetric dynamic behaviour when unemployment rises or falls. Observations of the unemployment behaviour, however, indicate that it rises faster than it declines. Such asymmetries are often explained by asymmetric adjustment costs or by the hiring and firing practices of firms (cf. Hamermesh and Pfann, 1996; Johansen, 1982).

Models of multiple equilibria appear to be capable of reconciling the empirical evidence from long and short time series and allow for more flexibility with regard to the effects of shocks, see section 3. In these models, a large shock may cause a movement from one equilibrium level to another while small shocks only cause a temporary deviation from a given equilibrium level. These properties seem to be consistent with the findings of a number of studies, which ascribe the appearance of high degree of persistence in European unemployment series to infrequent shifts in their mean levels, (e.g. Blanchard and Summers, 1987; and Bianchi and Zoega, 1996, 1997, 1998).

Nonlinear models are required to entertain the possibility of multiple equilibria, i.e. to allow for asymmetric responses to the size of a shock. Theories of multiple equilibria, as well as models of unique equilibrium, are often silent on whether the response towards shocks depends on their sign or not. This property, i.e. asymmetric response to the sign of a shock, can also be incorporated into nonlinear models and tested for. A number of studies associate multiple equilibria with hysteresis, interpreted as a nonlinear phenomenon, (see e.g. Røed, 1997 and the references therein). Accordingly, the multiple equilibria approach differs from the standard hysteresis approach only in the sense of regarding it as a nonlinear phenomenon.

The possibility of multiple equilibria in the Norwegian unemployment rate and in a number of other OECD unemployment series has previously been tested for by Bianchi and Zoega (1996, 1998), hereafter BZ (1996, 1998), and Skalin and Teräsvirta (1999). The findings in BZ (1996, 1998) are based on annual and quarterly data, respectively. These are consistent with the notion of unemployment swings between multiple equilibria and contradict the hypothesis of a unique equilibrium rate of unemployment for most of the OECD countries, including Norway. Indeed, the high degree of persistence in these unemployment series is accounted for by infrequent changes in the equilibrium (mean) rates of unemployment. The switching mean rates are estimated by using Markov regime switching models but with no autoregressive terms (see Hamilton, 1989). The degree of persistence in each series is measured once the series are adjusted for the estimated mean rates. However, one may argue for three shortcoming with this modelling approach and the results that follow.

Firstly, the models deliver results that seems to be too dependent on the frequency of the data series. For instance, on annual data for Norway, the model in BZ (1996) suggests two equilibria at 2.1% and 5.5% and a shift to the higher equilibrium level in 1988. On quarterly data in BZ (1998), the estimates are 1.83% and 4.72%, respectively. One may say that these are roughly the same as those on annual data. However, the model implies two periods in which unemployment is at the higher equilibrium level, in 1982:3-1984:4 and 1988:1-1995:4. The shift to the higher equilibrium in the beginning of 1980s appears to be at odds with the actual data that never exceed 4% before 1988, cf. figure 5.1. BZ(1998) recognise this as a problem with a number of other series too and deal with it outside the models to obtain interpretable results. Secondly, the results may not be invariant to the inclusion of possibly neglected dynamics in the models. The models appears to be too restrictive since they only contain regime dependent intercepts as regressors. As will be shown in subsection 5.3.1, the estimates of mean rates and the estimated number of switches between equilibria change with the inclusion of autoregressive terms and with their numbers. Similar results have been reported by Clements and Krolzig (1998) in their study of US GNP. Finally, one may argue that the employed Markov regime switching framework is inflexible since it imposes abrupt transitions between possible equilibria at the outset. Although firms may face dichotomous decisions with regard to employment adjustment, it is unlikely that they opt for the same decision simultaneously, unless they are exposed to huge shocks. Hence at the aggregate level and in absence of huge shocks, one is more likely to observe smooth rather than abrupt transitions between possible equilibria.

Skalin and Teräsvirta (1999) employ smooth transition autoregressive (STAR) models that allow for both abrupt and smooth transitions between equilibria, (see Granger and Teräsvirta, 1993). They use logistic STAR (LSTAR) models for the first differences of the quarterly OECD unemployment series, but with lagged level terms and seasonal dummies. First they test the significance of the apparent asymmetric dynamics in these series, which is supported for most of the OECD unemployment series, except for Norway and a few other countries. For the Norwegian unemployment rate, and the other series that are found to not exhibit asymmetry, they go on and test for multiple equilibria. For Norway they find an abrupt switch to the high unemployment equilibrium in 1988, as in BZ (1996). However, their model does not seem to be suitable for deriving reasonable estimates of unemployment equilibria. Taken at face value, the model implies two equilibria at $0.0012/0.22 \approx 0.006\%$ and $(0.0012 + 0.64)/0.22 \approx 2.91\%$, respectively. Moreover, Skalin and Teräsvirta (1999) use the *time* trend as a transition variable to account for the multiple equilibria. Use of a *time* trend implies permanent transition to the one or the other equilibrium level. A characteristic feature of theory models of multiple equilibria is that they allow for back and forth movements between the equilibria depending on the size and sign of shocks. This feature is lost when a time trend is used as a transition variable, though it may capture slows changes in the equilibrium rates due to gradual changes in the structural and institutional features of the labour markets, as in the case of e.g. Austria and Canada, (see Skalin and Teräsvirta, 1999). In their analysis of Norway, however, the transition occurs abruptly in 1988.

This paper aims to derive a data consistent univariate model to test for the possibility of multiple equilibria in Norwegian unemployment and asymmetric adjustment. As Skalin and Teräsvirta (1999) it will allow for both abrupt and smooth transitions between equilibria by adapting the STAR framework, but to model the level of unemployment. To preserve the possibility of back and forth movements between possible equilibria, the transition variable will be a lagged value of the unemployment rate.

The univariate framework adapted here and in general, does not shed light on the sources of shocks, of persistence and on mechanisms which may provide scope for multiple equilibria. However, it provides a convenient way to test for multiple equilibria and to draw out the characteristic features of the unemployment series. By this, it can provide stylised facts to be explained by theoretical and multivariate empirical models.

3. Multiple unemployment equilibria

Models that display multiple equilibria are often based on the existence of reciprocal externalities in various guises. These can arise from trading and exchange opportunities as in Diamond (1982) and Cooper and John (1985), due to spillovers of demand across markets as in e.g. Weitzman (1982) and Murphy *et al.* (1989) or due to costs associated with layoffs and hirings as in Saint-Paul (1995) and Moene *et al.* (1997), respectively. In the presence of reciprocal externalities, the level of activity of one agent depends positively on the level of activity of another agent and vice versa. The positive feedback may potentially lead to multiple equilibria, and "coordination failure" among the agents may cause the economy to get stuck in an equilibrium with an inefficiently low level of activity and employment.

Manning (1990) shows that multiple equilibria can arise from the presence of increasing returns to labour, or in labour and capital together, in a quite standard model of an imperfect economy, for instance as in Layard *et al.* (1991).

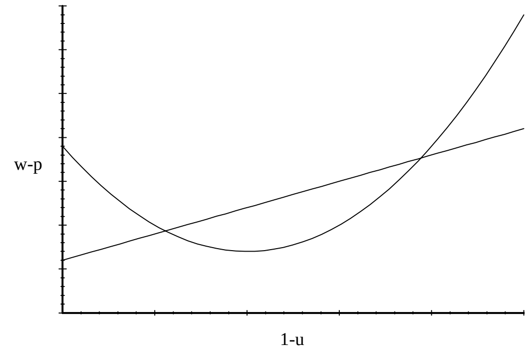

Figure (a)

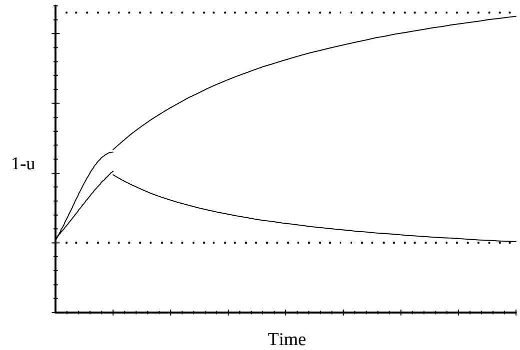

Figure (c)

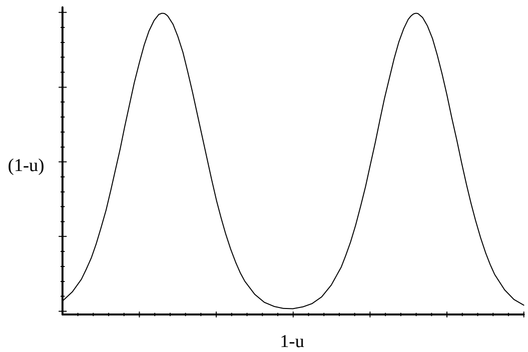

Figure (b)

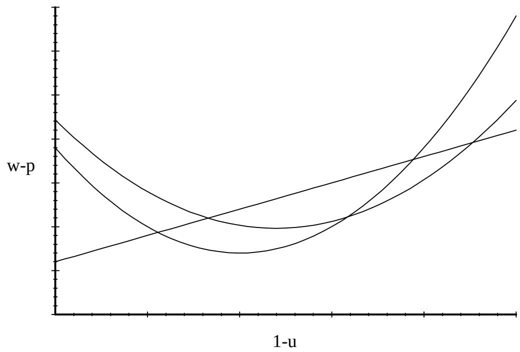

Figure (d)

Figure (a) can help us to illustrate the essence in models of multiple equilibria. Here a nonlinear price curve and a linear wage curve are sketched against the (un)employment rate, *1-u*. The price curve and the wage curve can also be interpreted as a labour demand curve and a labour supply curve, respectively. The curves intersect at two points that correspond to two stable (un)employment equilibria. Upon deviation from a given equilibrium, due to e.g. a small nominal shock, (un)employment reverts to the initial equilibrium. When exposed to a sufficiently large shock, however, the (un)employment converges towards the other equilibrium. The solid and dashed curves in figure (c) sketch the employment response to a small and a large shock, respectively, when it is at the lower equilibrium level. The shocks may stem from changes in fiscal and monetary policy or from foreign economies through the exposed sectors.

The solid line in figure (c) also indicate that employment adjusts at a lower pace towards its equilibrium than it rises. Such asymmetries are commonly ascribed to e.g. asymmetric adjustment costs. For instance, a number of studies suggest that costs of hiring usually exceed costs of separations, (see Hamermesh and Pfann, 1996 and the references therein). Hence employment will fall faster when exposed to a negative shock than it will rise upon a positive shock. The speeds of adjustment towards the different equilibria may also differ from each other and are dependent on the relative stability of the equilibria. The relative stability can be related to the sources of the sluggishness in the economy, e.g. to adjustment costs of different forms, whether there are nominal and real rigidities in the price-setting and or in the wage-setting and the governments policy towards unemployment, (see Manning, 1990). The latter study implies that the lower equilibrium is likely to be more stable if the rigidities are mainly nominal in nature and the government pursues a counter-cyclical policy.

The underlying data generating process in the labour market is unobservable but (un)employment data can provide some indications of the underlying process. If the underlying process in the labour market resembles the one sketched in figure (a), the (un)employment rates are likely to be concentrated around two distinct (un)employment rates. In this case one may observe a bimodal density for (un)employment rates as in figure (b). However, if the price and wage curves only intersect once, one will observe an unimodal density.

Models of multiple equilibria do not restrict the equilibria to be constant over time. They may vary following structural and institutional changes over time. Figure (d) illustrates the case where such changes shift the position of the price curve. The dotted curve indicates a movement in both of the equilibria. Tests for slow or abrupt changes in the parameters of a multiple equilibria model can provide indications of slow or abrupt movements in the equilibria.

4. Formalizing multiple equilibria

This section formalises the multiple equilibria approach in the Markov regime switching model of Hamilton (1989) and in the STAR model of Teräsvirta (1994). The linear models which imply unique equilibrium or hysteresis in the unit root sense, can be derived by imposing appropriate restrictions on these general models. The sections also touches upon ways to test these nonlinear models against the linear model.

4.1. Markov regime switching model

Equation (4.1) formalises the multiple equilibria approach in the Markov regime switching autoregressive model, hereafter MS-AR(q) model

$$U_t = \mu_{s_t} + \sum_{i=1}^{q} \phi_i (U_{t-i} - \mu_{s_{t-i}}) + \sigma_{s_t} \varepsilon_t \qquad \begin{array}{l} ; \varepsilon_t \sim N(0,1) \\ ; |\varrho = \sum_{i=1}^{q} \phi_i| < 1 \end{array} \tag{4.1}$$

The level of unemployment is assumed to evolve around a variable equilibrium level, μ_{s_t}, which takes on a finite number of values depending on the state, s, of the economy.[1] The regime or state variable s_t is assumed to be an unobserved discrete variable taking on a value in {1, 2, 3,...S}. This is governed by a first-order Markov chain with transition probabilities $\{p_{ij}\}_{i,j=1,2,...S}$. In a first-order Markov chain, the probability of state j in period t depends only on the past through the most recent value of s_t, i.e.

$$P\{s_t = j \mid s_{t-1} = i,\ s_{t-2} = k, ...\} = P\{s_t = j \mid s_{t-1} = i\} \equiv p_{ij}, \tag{4.2}$$

with $p_{i1} + p_{i2} ... + p_{iS} = 1$ for all i. The transition probability p_{ij} denotes the probability of state j conditional on the economy being in state i in the previous period. It is assumed that U_t depends only on the current and q most recent realization of s_t, cf. (4.1).

To fix ideas, consider the case of $q = 0$. When $s_t = 1$, U_t is presumed to have been drawn from a normal distribution with mean μ_1 and standard deviation σ_1, $N(\mu_1, \sigma_1)$, and from $N(\mu_2, \sigma_2)$ when $s_t = 2$, and so on. The transitions between these S regimes or distributions are assumed to be governed by a probability law specified as the Markov chain (4.2).

Shocks which do not change the state of the economy may be interpreted as affecting the level of unemployment through the epsilon, ε_t, in the expression for the residual, $\sigma_{s_t}\varepsilon_t$. In this general formulation, which allows for a state dependent standard error, the effect of such shocks on the level of unemployment may be state dependent. The restriction on the sum of autoregressive coefficients, ϱ, ensures that

[1] In a multivariate framework, one could interpret the model (4.1) as a reduced form equation for unemployment, derived from wage and price formation, and the term μ_{s_t} as a vector of conditional macroeconomic variables representing the state of the economy (cf. Layard *et al.*, 1991). In general, and in contrast to our purpose, the vector of conditional variables may take on an infinite number of values and imply an infinite number of unemployment equilibria. Therefor the multivariate framework would not be suitable to test for a finite number of equilibria.

these shocks only cause a temporary deviation from the state dependent unemployment equilibrium.

Since we do not know in which regime the process was at every date in the sample, neither do we know the distribution responsible for delivering the observed value of U_t at every date. Therefore, probabilistic inference is made about the value of s_t conditional on the history of unemployment and the estimated value of the parameter vector Θ, where: $\Theta = (\mu_1, \mu_2, ...\mu_S, \sigma_1, \sigma_2, ...\sigma_S, \phi_{1,}\phi_2, ...\phi_q, p_{11}, p_{12}, ...p_{1S}, p_{21}, p_{22}, ...p_{2S}, ..., p_{S1}, ...p_{SS})'$, i.e.

$$P(s_t = j \mid U_t, U_{t-1}, U_{t-2}, ...U_1; \widehat{\Theta}) \qquad ; j = 1, 2, ...S \tag{4.3}$$

The probability of being in regime j at time t given the observed data and the estimated value of Θ, $\widehat{\Theta}$, is called *filtered probability.* In contrast to filtered probabilities, *smoothed probabilities* are calculated by using the whole sample. Both filtered and smoothed probabilities are calculated for every date in the sample and are useful in dating the transition(s) between the regimes in the series. However, the number of possible regimes S characterising the series has to be determined beforehand. This issue is addressed in section 5.1.

Note that a change in any moment of the distribution can be interpreted as a change in the regime or state. For instance, two regimes may only be different from each other with respect to the variances. However, in the present context, a change in the regime or distribution will only correspond to a change in the equilibrium level of unemployment if there is a change in the mean value.

The linear autoregressive (AR) model (4.4) is a special case of (4.1) for $S = 1$, i.e. when there is only one possible regime or equilibrium.

$$U_t = \mu + \sum_{i=1}^{q} \phi_i (U_{t-i} - \mu) + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2) \tag{4.4}$$

In the more familiar transformation, where $\alpha = \mu(1 - \varrho)$ and $\varrho = \sum_{i=1}^{q} \phi_i$,

$$U_t = \alpha + \sum_{i=1}^{q} \phi_i U_{t-i} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2). \tag{4.5}$$

Note that for values of $|\varrho| < 1,$ the model is dynamically stable and unemployment is a stationary variable.[2] Thus a shock to the level of unemployment, represented by the residual, ε_t, will only cause a transitory deviation from a given unemployment equilibrium.

The special case of unemployment hysteresis in the unit root sense can be defined by $\varrho = 1$. In which case, the unemployment rate follows a random walk process, augmented by some stationary terms.[3] Thus every shock permanently changes the level of unemployment. The unemployment process does not revert to its initial equilibrium but remains at the post-shock level until disturbed by a new shock. Moreover, if an equilibrium is a state where there is no inherent tendency to change, unemployment is at equilibrium at every point in time, implying an infinite number of equilibria.

Thus one can test the hypothesis of a unique equilibrium against hysteresis or multiple equilibria by testing the null hypothesis of $\mu_s = \mu$ for all s, combined with restrictions on the sum of autoregressive coefficients.[4] The standard asymptotic distribution theory is, however, not valid under the null hypothesis of a constant parameter (linear) AR model. This is because the parameters $p_{11}, ...p_{SS}$ are unidentified, i.e. they may take on any value without affecting the likelihood value, under the null hypothesis of $\mu_1 = \mu_s$ and $\sigma_1 = \sigma_s$ (see Hansen, 1992 and 1996). However, Hansen (1992, 1996) suggests a likelihood ratio test that can be used to evaluate the MS-AR(q) against the AR(q) model.

4.2. Smooth transition autoregressive model

The Markov regime switching framework imposes abrupt transitions between equilibria. This assumption can be relaxed by basing the analysis on the smooth transition autoregressive (STAR) model (see e.g. Granger and Teräsvirta, 1993).

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[2]

$$U_t = \alpha + \sum_{i=1}^{q} \phi_i U_{t-i} + \varepsilon_t = \alpha + \sum_{i=1}^{q} \phi_i U_{t-1} - \sum_{j=1}^{q-1} \sum_{i=j+1}^{q} \phi_i \Delta U_{t-j} + \varepsilon_t$$

which may be transformed to

$$\Delta U_t = \alpha - (1 - \varrho) U_{t-1} - \sum_{j=1}^{q-1} \sum_{i=j+1}^{q} \phi_i \Delta U_{t-j} + \varepsilon_t$$

[3] Remember that $\alpha = 0$ when $\varrho = 1$.

[4] Note that we may have a unique equilibrium even when $\sigma_s \neq \sigma$.

A STAR model of order q can be formulated as follows:

$$U_t = \alpha + \sum_{i=1}^{q} \phi_i U_{t-i} + (\widetilde{\alpha} + \sum_{i=1}^{q} \widetilde{\phi}_i U_{t-i}) F(U_{t-d}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0,\ \sigma^2) \tag{4.6}$$

where $F(U_{t-d})$ is a transition function that monotonically increases with the level of unemployment, lagged d periods, and takes on values in the range [0, 1]. The transition function, interpreted as representing the phase or state of the economy, can be specified as a logistic function

$$F(U_{t-d}) = (1 + \exp[-\gamma\{U_{t-d} - c\}])^{-1}. \qquad \gamma > 0 \tag{4.7}$$

Here the transition parameter γ determines the speed of transition from 0 to 1, for a given deviation of U_{t-d} from a constant threshold value c. This logistic STAR (LSTAR) model allows both the constant term and the autoregressive coefficients to change with the value of $F(U_{t-d})$. Thus unemployment is allowed to evolve around distinct equilibria with different dynamics in expansions $U_{t-d} < c$ and contractions $U_{t-d} > c$. The change in parameters occurs with some delay which depends on the value of parameter d. The value of d can be determined together with the tests of a linear AR(q) model against a STAR model (see e.g. Teräsvirta, 1994).

The two extreme unemployment equilibria correspond to values of $F(U_{t-d}) = 0$ and $F(U_{t-d}) = 1$ and can be defined as $\mu_1 = \alpha/(1 - \sum_{i=1}^{q} \phi_i)$ and $\mu_2 = (\alpha + \widetilde{\alpha})/[1 - \sum_{i=1}^{q}(\phi_i + \widetilde{\phi}_i)]$, respectively. The sums $\varrho_1 \equiv \sum_{i=1}^{q} \phi_i$ and $\varrho_2 \equiv \sum_{i=1}^{q}(\phi_i + \widetilde{\phi}_i)$ measure the degree of persistence in each of the two states.

Note that both a two regime autoregressive model with abrupt transitions and the AR(q) model are nested in this LSTAR model. When $\gamma \longrightarrow \infty$, the transition function $F(U_{t-d}) = 0$ for $U_{t-d} \leq c$ and $F(U_{t-d}) = 1$ for $U_{t-d} > c$. In this case the LSTAR model reduces to a self-exciting threshold autoregressive (ESTAR) model with threshold value c. The LSTAR model is reduced to an AR(q) model for $\gamma \longrightarrow 0$. In this case $F(U_{t-d}) \longrightarrow 1/2$ for all values of U_{t-d}.

The generality of the LSTAR model makes it suitable to test hypotheses concerning the number of unemployment equilibria, the degree of persistence within each equilibrium and the speed of transition between the equilibria. However, as in the case of the Markov regime switching model, the parameters defining the LSTAR model are not identified under the null hypothesis. It follows from (4.6) and (4.7) that under the null hypothesis of an AR(q) model, i.e. under $\mathrm{H}_0 : \gamma = 0$, the $\widetilde{\alpha}$, $\widetilde{\phi}_i$s' and c may take on any value without changing the likelihood value. These parameters are only identified under the alternative hypothesis of $\gamma \neq 0$. In this case, Teräsvirta (1994) suggests a sequence of tests to evaluate the null hypothesis of an AR(q) model against the alternative of an LSTAR model. The tests are based on estimating the following auxiliary regression for a chosen value of the delay parameter d

$$U_t = \beta_0 + \sum_{i=1}^{q} \beta_{1i} U_{t-i} + \sum_{i=1}^{q} \beta_{2i} U_{t-i} U_{t-d} + \sum_{i=1}^{q} \beta_{3i} U_{t-i} U_{t-d}^2 + \sum_{i=1}^{q} \beta_{4i} U_{t-i} U_{t-d}^3 + \nu_t. \quad (4.8)$$

The test of an AR(q) model against a STAR model (both LSTAR and ESTAR) is equivalent to conducting a joint test of[5]

$$H_0 : \beta_{2i} = \beta_{3i} = \beta_{4i} = 0, \qquad i = 1, 2, ...q$$

The value of d can be determined by conducting the test for different values of d in the range $1 \leq d \leq q$. If linearity is rejected for more than one value of d, then the value which causes the strongest rejection of the null is chosen, i.e. the value corresponding to the lowest p-value of the joint test. If the AR(q) model is rejected, one needs to test the appropriateness of an LSTAR formulation against an ESTAR formulation. For this purpose, the following sequence of tests within the auxiliary regression is suggested

$$\begin{aligned} H_{04} &: \beta_{4i} = 0, \qquad i = 1, 2, ...q \\ H_{03} &: \beta_{3i} = 0 \mid \beta_{4i} = 0, \qquad i = 1, 2, ...q \\ H_{02} &: \beta_{2i} = 0 \mid \beta_{3i} = \beta_{4i} = 0, \qquad i = 1, 2, ...q \end{aligned}$$

An LSTAR model is chosen if H_{04} or H_{02} is rejected for at least one value of i and an ESTAR model is chosen if H_{03} is rejected for at least one i (see Teräsvirta,1994).

5. Data, AR(q) and MS-AR(q) models

This section offers some preliminary evidence against the unique equilibrium approach and mixed evidence in favour of two unemployment equilibria. It is organised as follows. Subsection 5.1 describes the unemployment data and plots its density to check whether its shape is consistent with multiple equilibria or not, see section 3.

[5] An exponential STAR (ESTAR) model is defined by the following exponential transition function $F(U_{t-d}) = 1 - \exp(-\gamma(U_{t-d} - c))^2$, see e.g. Teräsvirta (1994).

Moreover, whether unemployment tends to rise faster than it declines. Subsection 5.2 derives an AR(5) model and explores its properties. In particular, whether it can provide a reasonable estimate of a unique equilibrium or not. A number of parameter constancy tests are also conducted to test for parameter changes over time. These tests can be helpful in dating possible changes in the parameters and indicate the timing of possible transitions between equilibria. The AR(5) model will also serve as a reference model for the nonlinear models to be estimated later. Section 5.3 estimates a Markov regime switching model of order 5, MS-AR(5), model and examines its merits. Moreover, it replicates the results in BZ(1996, 1998) for Norway and demonstrate the sensitivity of results to model specification.

5.1. Data

The empirical analysis employs seasonally non-adjusted quarterly data for the open unemployment rate in Norway, see figure 5.1.[6] The data set covers the period 1972:1-1997:1. During this period, there are three noticeable upswings in unemployment relative to its low level in the early 1970s, in 1974/75, 1982/83 and 1988. It moves down relatively slowly to its initial level after the upswings except for the third upswing, which is also the most striking one. Unemployment continues to rise after the strong upswing, but at a slower pace until 1992. The movement is downward after that. The figure indicates asymmetry in the pace of unemployment between the low and high level. The movements from lower to higher levels seem to be more abrupt than vice versa.

The observed values lend themselves to interpretation in the light of the multiple equilibria or regime switching model. Figure 5.1 indicates two or possibly three different regimes in the sample. Most values of U_t in the period up to 1982 appear to have come from a "low-mean-distribution", while the values for the period after 1988 are more likely to have been drawn from a "high-mean-distribution". We are, however, more uncertain with regard to the values of U_t in the period from 1982 to 1985. Both regimes are likely to have delivered these observations. One may alternatively assume that these values are from a third distribution with a medium level of mean. Allowing for changes in the mean over the sample, there do not seem to be substantial differences in the magnitude of fluctuations between the regimes.

As noted in section 3, the unemployment rates should be concentrated around

[6] The unemployment figures are based on the Norwegian labour force survey (AKU). The source is the database OECD_MEI and the precise variable name is MEI_SQ0220_424000A0.

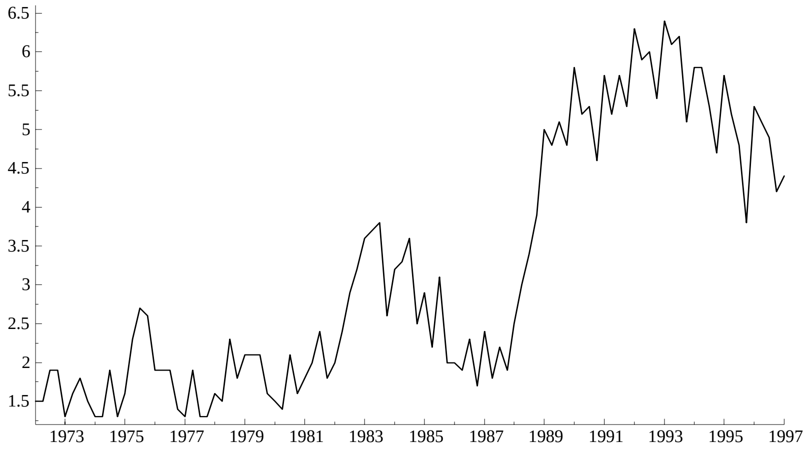

Figure 5.1: *Seasonally non-adjusted quarterly series of open unemployment in Norway, 1972:1-1997:1.*

two distinct rates, if they are generated by a labour market that has switched between two equilibria. Figure 5.2 plots the frequency distribution of unemployment, which appears to be bimodal with the modes centered at around 2% and 5%, respectively. The spread around the higher mode seems to be slightly larger than the spread around the lower mode. Although figure 5.2 indicates the presence of two unemployment equilibria, one cannot preclude the existence of a third mode with a value close to either 2 or 5 but with a relatively larger or smaller spread.[7] However, in order to estimate a parsimonious model section 5.3 assumes two possible unemployment equilibria, i.e. $S = 2$. This is in accordance with BZ (1996, 1998) who use a bootstrap method to identify the number of states.

5.2. An AR(5) model

The estimated AR model with five lags is presented in table 5.1.[8] Starting with eight lags, the autoregressive order of five was determined by excluding the statistically insignificant lags of a higher order. A lag order of five was also found to be optimal according to Akaike's information criterion (AIC). Except for signs of autocorrelation in the residuals, there do not seem to be significant violations of the other standard assumptions about residual properties. There are signs of autocorrelation

[7] See section 22.3 in Hamilton (1994).

[8] The model was also estimated with the method of maximum likelihood in order to derive the log likelihood value.

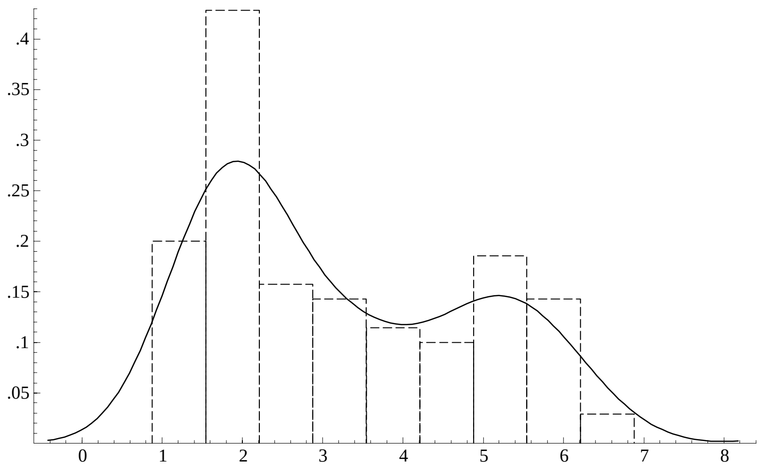

Figure 5.2: *Non-parametric density estimation of open unemployment in Norway (with histogram), 1972:1-1997:1.*

despite the inclusion of the second and the third lag that are insignificant at a 5% level, see the test summary in table 5.1. The autocorrelation remained a feature of the residuals even when a lag length of 8 was used. Note that a significant value for the autocorrelation test may also result when a linear functional form is imposed on a data-generating mechanism that can be more properly characterised by a nonlinear functional form or two or more linear segments. However, the regression specification test (RESET) does not indicate significant functional form mis-specification, at least not at standard levels of significance (see Ramsey, 1969). However, this test is constructed to have power against general forms of functional mis-specification and may thus have low power against specific nonlinear forms.

One-step ahead Chow tests in figure 5.3.a indicate noticeable changes in the model parameters in 1982/83 and in 1988. The recursive OLS estimates of the constant and the first autoregressive coefficient indicate non-constancies at these dates, see figure 5.3.b and 5.3.c. The recursively estimated standard deviations of the 1-step residuals in figure 5.3.d, indicate a slight increase in the variance around 1982 and then a further increase in 1988. The graphical presentations above do not seem to strongly contradict the assumption of two regimes characterizing the unemployment data.

Note that the estimated value of ϱ is not significantly different from 1. As noted earlier, a value of ϱ close to 1 is a common finding in studies of European unemployment series that use the linear AR framework, thus not contradicting the null

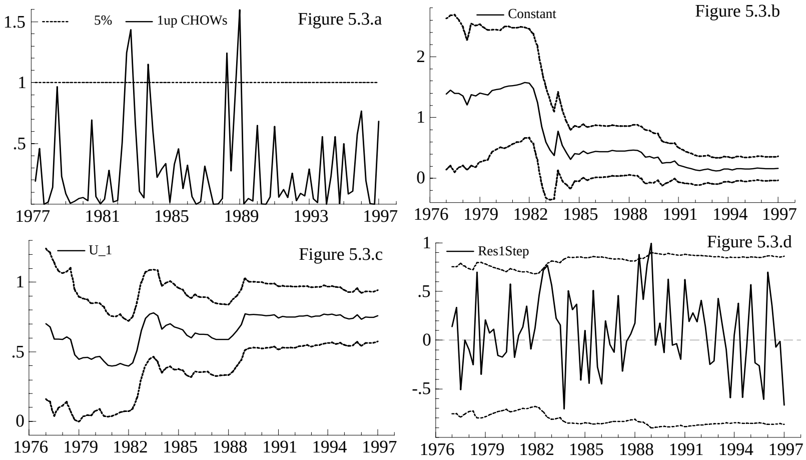

Figure 5.3: *One-step ahead Chow tests with critical values at 5% (5.3.a), recursive OLS estimates of the constant term, the first autoregressive coefficient and the one-step ahead residuals, respectively, each with ±2 SE, see (5.3.b), (5.3.c) and (5.3.d). Initial period 1973:2-1977:1.*

hypothesis of hysteresis in unemployment. The AR(5) model implies an equilibrium level of unemployment, μ, equal to 3.74 per cent, which can be questioned in the light of figure 5.1. Accordingly, the actual unemployment rate was systematically below the equilibrium level before 1988 and systematically above the equilibrium level afterwards.

5.3. A MS-AR(5) model

This section estimates the MS-AR(5) model, see section 4.1, and evaluates it against the AR(5) model. It is shown that the results from the relatively "well specified" MS-AR(5) model are largely inconclusive and are apparently at odds with the data. Subsection 5.3.1, however, shows that apparently interpretable results can be obtained if the autoregressive terms are omitted from the Markov regime switching model.

The maximum likelihood estimates (MLE) of equation (4.1) are presented in the second panel of table 5.1. The estimates are derived under the assumption of two regimes or equilibria. As in the AR(5) model, increasing the number of lags to five, $q = 5$, lead to a large increase in the log likelihood value, see table 5.2. The parameter estimates of the AR(5) model in the first panel were used as starting values. However, the results were found to be quite robust to changes in the starting values.

The MS-AR(5) model appears to provide a better fit to the historical unemployment record compared with the AR(5) model. The variances of residuals in the regime switching model, which are almost equal across the two regimes, are about half the value of those in the AR(5) model. The log likelihood value of the MS-AR(5) model is also higher compared with the log likelihood value of the AR(5) model. They are 35.8 and 40.5, respectively. The estimated equilibria are, however, not significantly different from each other at conventional levels of significance, implying weak evidence of two unemployment equilibria. Indeed, we are not able to reject the AR(5) model against the MS-AR(5) model when using the appropriate likelihood ratio (LR) test suggested by Hansen (1992, 1996). The standardised LR value is 2.19 with a p-value of 0.223.[9] Note that the estimated level of $\widehat{\mu}_1$ at 3.24% is higher and the estimate of $\widehat{\mu}_2$ at 3.86% is lower than what one would expect from a mere glance at figures 5.1 and 5.2 that presents the actual data.

The transition probabilities indicate that the probability of a switch from the high to the low unemployment equilibrium is smaller than vice versa, $\widehat{p}_{21} = 0.14$ versus $\widehat{p}_{12} = 0.21$. The unconditional probabilities of unemployment evolving around the low and high equilibrium levels are $\widehat{\pi}_1 = 0.4$ and $\widehat{\pi}_2 = 0.6$, respectively. These are long run functions of the transition probabilities (see Hamilton, 1994, pp. 681-84). This asymmetry in the transition from one level to another seems to comply with the observed movements of unemployment from one level to another, cf. figure 5.1. However, the transition probabilities also indicate that the equilibria are quite short dated, which appears to be in conflict with the impression from figure 5.1. The probability of remaining in the state of high or low unemployment equilibrium in the next period, given that one is in the high or low unemployment equilibrium is 0.86 and 0.79, respectively. Thus the expected duration of the high unemployment equilibrium is only $1/(1 - \widehat{p}_{22}) \approx 7$ quarters, while it is less than five quarters $(1/(1 - \widehat{p}_{11}) \approx 4.8)$ for the lower equilibrium.

Figure 5.4 displays the filtered and smoothed probabilities of being in a state with high unemployment equilibrium over the sample. The graph indicates large discrepancies between the filtered and smoothed probabilities. The smoothed probabilities, based on the full sample, seem to be more decisive regarding the state of unemployment than the filtered probabilities that are derived from observations up to period t, in which the observation is made. The smoothed probabilities are

[9] The Gauss software was downloaded from Bruce C. Hansen's home page.

Table 5.1: Estimated models of Norwegian unemployment

1. AR(5) model: OLS/ML estimates

Log likelihood = 35.8		$\widehat{\alpha} = \underset{(0.099)}{0.17}$	$\widehat{\mu} = 3.74$	$\widehat{\sigma}^2 = 0.19$	
$\widehat{\varrho} = 0.96$	$\widehat{\phi}_1 = \underset{(0.092)}{0.76}$	$\widehat{\phi}_2 = \underset{(0.104)}{0.19}$	$\widehat{\phi}_3 = \underset{(0.105)}{-0.12}$	$\widehat{\phi}_4 = \underset{(0.103)}{0.61}$	$\widehat{\phi}_5 = \underset{(0.093)}{-0.48}$

Test summary

AR $_{1-5}$ *F(5, 82)* = 2.85[0.02*], *ARCH 4 F(4, 79)* = 0.66[0.62],
Heterosced.: Xi ^{2} : *F(10, 79)* = 0.90[0.54], *XiXj*: *F(20, 69)* = 0.86[0.64]
Normality $\chi^2(2) = 0.57[0.75]$, *RESET: F(1, 89)* = 1.73[0.19]

2. MS-AR(5) model: MLE[1]

Log likelihood = 40.5		$\widehat{\mu}_1 = \underset{(0.844)}{3.24}$	$\widehat{\mu}_2 = \underset{(0.832)}{3.86}$	$\widehat{\sigma}_1^2 = \underset{(0.026)}{0.07}$	$\widehat{\sigma}_2^2 = \underset{(0.025)}{0.09}$
	$\widehat{\phi}_1 = \underset{(0.084)}{0.86}$	$\widehat{\phi}_2 = \underset{(0.087)}{0.07}$	$\widehat{\phi}_3 = \underset{(0.087)}{-0.09}$	$\widehat{\phi}_4 = \underset{(0.088)}{0.82}$	$\widehat{\phi}_5 = \underset{(0.086)}{-0.70}$
$\widehat{p}_{11} = \underset{(0.104)}{0.79}$	$\widehat{p}_{22} = \underset{(0.075)}{0.86}$	$\widehat{p}_{12} = 0.21$	$\widehat{p}_{21} = 0.14$	$\widehat{\pi}_1 = 0.40$	$\widehat{\pi}_2 = 0.60$

3. MS-AR(0) model with const. variance: MLE[2]

Log likelihood = −14.0		$\widehat{\mu}_1 = \underset{(0.082)}{2.13}$	$\widehat{\mu}_2 = \underset{(0.116)}{5.26}$	$\widehat{\sigma}^2 = \underset{(0.062)}{0.44}$	
$\widehat{p}_{11} = \underset{(0.010)}{0.99}$	$\widehat{p}_{22} = \underset{(0.016)}{0.99}$	$\widehat{p}_{12} = 0.01$	$\widehat{p}_{21} = 0.01$	$\widehat{\pi}_1 = 0.59$	$\widehat{\pi}_2 = 0.41$

4. MS-AR(0): MLE

Log likelihood = −16.8		$\widehat{\mu}_1 = \underset{(0.063)}{1.92}$	$\widehat{\mu}_2 = \underset{(0.163)}{4.81}$	$\widehat{\sigma}_1^2 = \underset{(0.042)}{0.20}$	$\widehat{\sigma}_2^2 = \underset{(0.234)}{1.02}$
$\widehat{p}_{11} = \underset{(0.021)}{0.97}$	$\widehat{p}_{22} = \underset{(0.026)}{0.97}$	$\widehat{p}_{12} = 0.03$	$\widehat{p}_{21} = 0.03$	$\widehat{\pi}_1 = 0.55$	$\widehat{\pi}_2 = 0.45$

The effective sample when 5 lags covers the period 1973:2-1997:1. The std. errors are in parentheses below the estimates. *Denotes significance at the 5% level. [1]Assuming one regime, the OLS estimates have been used as the starting values in the maximum likelihood (ML) estimation. [2]For the sake of convergence the initial values for the means were set equal to 2 and 4 here. The estimates have been obtained using PcGive 9.10, see Doornik and Hendry (1996) and Gauss 3.2. The Gauss software has been kindly provided by J. D. Hamilton.

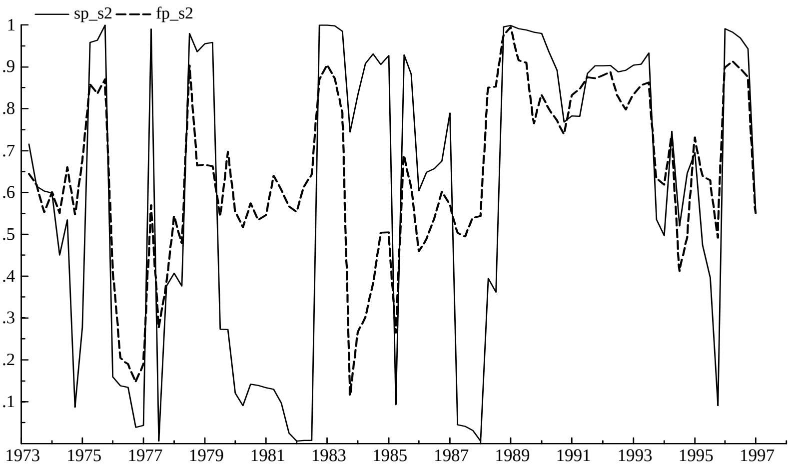

Figure 5.4: *Smoothed and filtered probabilities of being in regime 2, sp_ s2 and fp_ s2, respectively, 1973:2-1997:1.*

mostly close to 0 or 1, and in contrast to filtered probabilities, less often close to 0.5.

The smoothed probabilities indicate four brief periods of the high equilibrium state during the 1970s, one major period in the 1980s and another major period extending from the end of the 1980s to the end of the sample in 1997:1. The four brief periods are 1973:2-74:2, 1975:2-75:4, 1977:2 and 1978:3-79:2. The two major periods are 1982:4-87:1 and 1988:4-1997:1.[10] The major periods are, however, interrupted for one quarter and two quarters, in 1985:2 and 1995:3-95:4, respectively. The timing of switches in the mean, μ, during the 1980s is roughly consistent with our earlier observations, see figures 5.1 and 5.3.a-d. However, the implied changes in parameters during the 1970s, 1985:2, and 1995:3-95:4 are not apparent in these figures, see e.g. one-step ahead Chow tests in figure 5.3.a.

The MS-AR(5) model seems to attribute quite brief periods, e.g. even single isolated observations, to structural changes in the sense of a change in the parameters. These brief periods are perhaps better characterised as outliers within a given regime rather than representing structural changes. This problem of "spurious switches", low transition probabilities and small deviations between the estimated equilibria may be assigned to the non-rejection of the linearity hypothesis. The problem of "spurious switches" between regimes has also been remarked on by Clements and

[10] Observations at time t which are equally likely to have been generated by both regimes are assumed to belong to the regime at time $t-1$.

Table 5.2: Sensitivity of the results from the Markov switching model to number of autoregressive terms, q.

	q 0	1	2	3	4	5
$\widehat{\mu}_1$	1.92	3.66	4.83	4.04	2.82	3.24
$\widehat{\mu}_2$	4.81	4.29	5.40	4.66	3.74	3.86
$\widehat{\phi}_1$		0.97	0.43	0.62	0.50	0.86
$\widehat{\phi}_2$			0.56	0.71	0.26	0.07
$\widehat{\phi}_3$				-0.35	-0.20	-0.09
$\widehat{\phi}_4$					0.41	0.82
$\widehat{\phi}_5$						-0.70
$\widehat{p}_{11}$	0.99	0.34	0.38	0.79	0.60	0.79
$\widehat{p}_{22}$	0.99	0.59	0.84	0.38	0.99	0.86
$\widehat{\sigma}_1^2$	0.20	0.07	0.03	0.16	2.67	0.07
$\widehat{\sigma}_2^2$	1.02	0.15	0.21	0.28	0.19	0.09
likl.	-16.8	19.13	21.68	24.78	23.25	40.50

likl. = Log likelihood value, see table 5.1 for details.

Krolzig (1998) in their study of US GNP and dealt with by BZ(1998).

5.3.1. Comparison with existing studies

As noted in section 2, the Markov regime switching model has previously been employed by BZ (1996, 1998) to study unemployment rates in OECD countries. They have reported evidence of shifts between multiple equilibria for a number of countries, including Norway. However, BZ (1996, 1998) estimate the equilibrium rates without autoregressive terms, i.e. under the (a priori) restrictions of $\phi_i = 0 \;\forall\; i$, for all countries. In BZ (1996), the analysis of Norwegian unemployment rate is based on annual data for the period 1960-1993. Assuming constant residual variance across two possible regimes, they estimate the unemployment equilibria to be 2.1% and 5.5%, and find a one-time transition to the high unemployment equilibrium in 1988. On quarterly data for the period 1970:1-1995:4, BZ (1998) allow the residual variance to change across the two regimes. In this case they estimate the equilibrium levels to be 1.83% and 4.72% while the residual variance is estimated to be 0.12% and 1.16% in the low and high unemployment regime, respectively. In this case, their model predicts two periods of high unemployment equilibrium, 1982:3-1984:2 and 1988:1-1995:4.

We are almost able to replicate the results in BZ(1996, 1998) on our quarterly data set by following their course. Panel 3 of table 5.1 reports estimates quite close to BZ(1996) when the autoregressive terms are excluded from the model (4.1) and a constant variance across regimes is assumed. These restrictions also lead to a one-time transition to the state of high unemployment equilibrium in 1988:4, i.e. in 1988, as in BZ(1996). Panel 4 in table 5.1 reports the results when the assumption of constant residual variance is relaxed. These are close to the results in BZ (1998) for both the equilibria and the variances. Now the model also predicts two periods of high unemployment equilibrium, 1982:4-1984:3 and 1988:2-1997:1, with one period of low unemployment equilibrium in-between, i.e. from 1984:4 to 1988:1. These dates only differ from those of BZ (1998) by one quarter. The relatively fewer switches, compared with the more general model in the second panel of table 5.1, are also reflected in the transition probabilities. They suggest that movements between the equilibria are quite unlikely.

The models in panel 3 and 4 of table 5.1, as BZ(1996, 1998), provide estimates of the equilibria that are close to the values of modes observed in figure 5.2 and in accordance with the visual impression from figure 5.1. In addition, the number of switches and their datings appear to be consistent with the unemployment behaviour over time, cf. figure 5.1 and 5.3.*a*-*d*. However, these models have lower explanatory powers, i.e. lower log likelihood values, than the AR(5) model and the regime switching models for any number of autoregressive terms, from 1 to 5, cf. panel 1 and 2 in table 5.1 and table 5.2. Selecting a model that accounts for the dynamics by autoregressive terms, however, provide results that appear to be at odds with some of the observed features of the unemployment series, see table 5.2. This table prefers the MS-AR(5) model whose properties are discussed above. In addition, it demonstrates the sensitive of the estimates for the equilibria to the inclusion of autoregressive terms and their number. Similar results are reported in Clements and Krolzig (1998, table 6).

One might conclude that the Markov regime switching models provide mixed evidence in favour of multiple equilibria. Models that are better at accounting for the dynamics and have higher explanatory power provide results that are difficult to interpret, both within the unique equilibrium and the multiple equilibria approach. The contrary appears to be true for models that neglect the dynamics. In the latter case, the estimated levels of the equilibria and shifts between them seems to be reconcilable with the data and interpretable within the multiple equilibria approach.

The next section adapts an alternative framework to see if the evidence can be tilted in one or the other direction.

6. Smooth transition autoregressive model

This section analyses Norwegian unemployment in the STAR framework described in section 4.2. Section 6.1 tests the AR(5) model against a STAR model and makes inference on the form of nonlinearity, i.e. whether to use an ESTAR or an LSTAR model. It turns out that the tests favour an LSTAR model, which is derived in the rest of this section. Section 6.2 conducts a number of tests to assess the data consistency of the obtained model and section 6.3 explores its dynamic properties. In particular, it demonstrates that the long run effects of a shock depends on its size, as suggested by (theory) models of multiple equilibria, see section 3.

6.1. An LSTAR model

While the AR(5) model may not be rejected in favour of the MS-AR(5) model, the following tests reject the null hypothesis of an AR(5) model against a STAR model, and favour an LSTAR model against an ESTAR representation of the data. The results in table 6.1 rejects the AR(5) model at about 5% level of significance when $d = 5$. And for $d = 5$, the lower panel of the table shows rejection of H_{02} at 1% level of significance. This indicates that an LSTAR model can be a more appropriate characterisation of the unemployment process than an ESTAR model. Stronger rejection of the linearity hypothesis and of H_{02} can be achieved, if numerically small and statistically insignificant terms are excluded from the auxiliary regressions for values of $d \in [1, 5]$. In this case, the AR(5) model can be rejected in favour of a STAR model for all values of d at 10% level of significance, but still most strongly for $d = 5$ with a p-value of 0.3%. Moreover, H_{02} can also be rejected at the same p-value of 0.3% for $d = 5$. The F-tests may have more power in these parsimonious auxiliary regressions than in their general versions.

Table 6.2 presents an estimated LSTAR model with five autoregressive terms when $d = 5$. The model was estimated by nonlinear least squares (NLS). The results indicate that most of the lagged terms in the nonlinear part as wells of the linear part of the model are insignificant, see the upper panel of table 6.2. One reason for this might be the high degree of correlation between the regressors, the lagged terms of U_t. Indeed, when the general model is sequentially reduced to a parsimonious model, several of the lagged terms in the nonlinear part of the model become significant, see the parsimonious model in panel 2 of table 6.2.

Table 6.1: Testing the linear AR(5) against a nonlinear STAR model, and LSTAR against an ESTAR model

d	Testing linearity	p-values
1	F(15, 75) = 1.00	[0.46]
2	F(15, 74) = 0.98	[0.47]
3	F(15, 73) = 0.77	[0.70]
4	F(15, 72) = 1.16	[0.32]
5	F(15, 71) = 1.79	[0.05]*
	Testing the form of nonlinearity	p-values
H_{04}	F(5, 71) = 1.64	[0.16]
H_{03}	F(5, 76) = 0.57	[0.72]
H_{02}	F(5, 81) = 3.08	[0.01]**
Method: OLS, Initial sample: 1973:2-1997:1. * and ** denotes significance at 5% and 1%, respectively.		

The estimate of the transition parameter γ is associated with a high degree of uncertainty, see panel 1 or 2. This finding may be attributed to the problem of accurate estimation of γ when the transition variable U_{t-d} is close to the threshold value c and the transition function is steep. In this case the transition function $F(U_{t-d})$ rises rapidly for small deviations between U_{t-d} and c, and its shape becomes consistent with a broad range of values of γ. The high standard deviation of $\widehat{\gamma}$ is assumed to reflect this feature. In such cases, many observations in the neighbourhood of c are required to obtain precise estimates of γ (cf. Teräsvirta, 1994). In the present data set, however, most values of U_{t-d} are clustered around levels of 2% and 5% while c is (quite precisely) estimated at a value of about 3.6%, cf. figure 5.2.

The large value of $\widehat{\gamma}$ implies that even a 1 percentage point deviation of unemployment from $\widehat{c} \approx 3.6$ is sufficient to bring the estimated transition function $\widehat{F}(U)$ close to 0 or 1, see figure 6.1.[11] Consequently, the LSTAR model resembles a two regime threshold model because the unemployment rate has mainly been close to either 2% or 5% over the sample period.

Figure 6.2 shows that $\widehat{F}(U)$ moves quite fast from 0 to 1 during 1988 and stays close to 1 in the subsequent periods of the sample. In the period 1982/83 , however, $\widehat{F}(U)$ displays a transitory increase from 0 but falls short of reaching 1. The unemployment process in this period may be interpreted as being in between the

[11] To date the transition of unemployment process from regime to another, the values of the estimated transition function $\widehat{F}(U)$ are obtained by using U_t rather than U_{t-d}.

Table 6.2: LSTAR model and its properties

General LSTAR model

$$\widehat{U}_t = \underset{(0.23)}{0.45} + \underset{(0.12)}{0.79}\, U_{t-1} + \underset{(0.15)}{0.32}\, U_{t-2} - \underset{(0.16)}{0.14}\, U_{t-3} + \underset{(0.19)}{0.20}\, U_{t-4} - \underset{(0.16)}{0.36}\, U_{t-5}$$
$$+(\underset{(1.24)}{0.03} - \underset{(0.25)}{0.11}\, U_{t-1} - \underset{(0.25)}{0.32}\, U_{t-2} + \underset{(0.24)}{0.08}\, U_{t-3} + \underset{(0.24)}{0.75}\, U_{t-4} - \underset{(0.32)}{0.31}\, U_{t-5})\text{x}$$
$$[1+\exp\{-\underset{(2.54)}{3.70}\,(U_{t-5} - \underset{(0.59)}{3.63})\}]^{-1}$$

Sample 1973:2-1997:1, Log likelihood value = 43.81 and $\widehat{\sigma} = 0.42$

Parsimonious LSTAR model

$$\widehat{U}_t = \underset{(0.23)}{0.50} + \underset{(0.09)}{0.73}\, U_{t-1} + \underset{(0.12)}{0.37}\, U_{t-2} - \underset{(0.13)}{0.31}\, U_{t-5} +$$
$$(-\underset{(0.16)}{0.41}\, U_{t-2} + \underset{(0.15)}{0.95}\, U_{t-4} - \underset{(0.19)}{0.43}\, U_{t-5})\text{x}[1+\exp\{-\underset{(2.28)}{3.48}\,(U_{t-5} - \underset{(0.26)}{3.57})\}]^{-1}$$

Sample 1973:2-1997:1, Log likelihood value = 42.37 and $\widehat{\sigma} = 0.41$

Dynamic properties

$$\widehat{F}(U_{t-5}) = 0 : \widehat{\varrho}_1 = \textstyle\sum_{i=1}^{5} \widehat{\phi}_i = 0.78, \qquad \widehat{\mu}_1 \equiv \frac{0.5}{1-\widehat{\varrho}_1} \approx 2.3$$
$$\widehat{F}(U_{t-5}) = 1 : \widehat{\varrho}_2 = \textstyle\sum_{i=1}^{5} (\widehat{\phi}_i + \widehat{\widehat{\phi}}_i) = 0.90, \qquad \widehat{\mu}_2 \equiv \frac{0.5}{1-\widehat{\varrho}_2} \approx 5.1$$

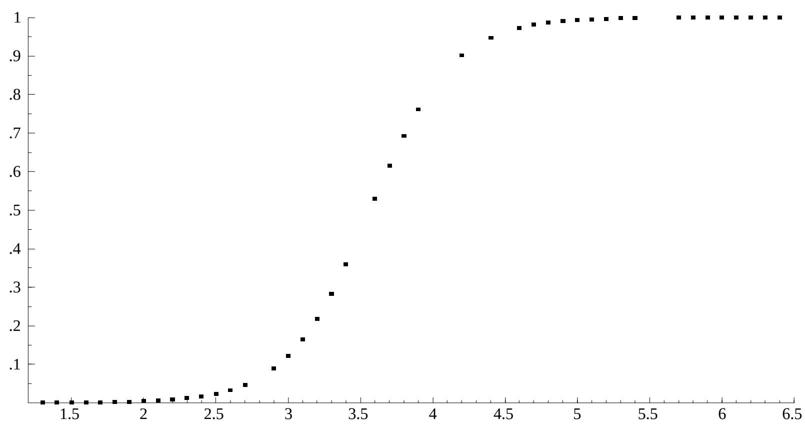

Figure 6.1: *A cross plot of the transition function $\widehat{F}$ (vertical axis) against the transition variable (horizontal axis). One dot represents at least one observation.*

Figure 6.2: *The transition function ($\widehat{F}$) from the parsimonious LSTAR model over the period 1972:1-1997:1.*

two regimes. This is in contrast to the results from the MS-AR(5) and MS-AR(0) models that implies full transition to the high unemployment regime in this period. However, the transition to the high unemployment regime during 1988 is consistent with these models and with Skalin and Teräsvirta (1998) who use a time trend as transition variable. The indicated parameter changes in the early 1980s and in 1988 are consistent with the results from the parameter constancy tests in figures 5.3.a-d.

Before presenting the dynamic properties of the LSTAR model, it would be appropriate to examine its explanatory power and investigate whether it offers an adequate description of the data or not.

6.2. Model evaluation

The (parsimonious) LSTAR model has slightly higher explanatory power than the AR(5) model and the MS-AR(5). The standard deviation of its residuals is smaller than that of the AR(5) model. The ratio between the standard deviations of residuals is about $0.93 = 0.41/0.44$. The log likelihood value of this model is 42.37 compared with 35.8 and 40.5 for the AR(5) model and the MS-AR(5), respectively. We are, however, not confident about the significance of the differences between these log likelihood values since the appropriate distributions are unknown.

The results in table 6.3 indicate that the LSTAR model is data consistent. None of the tests regarding the assumptions about residuals is rejected at 5% or 10% levels

Table 6.3: Testing the adequacy of the parsimonious LSTAR model

	Maximum lag q or d_2				
	1	2	3	4	5
AR(q), F(q, T-q-9):	[0.92]	[0.86]	[0.93]	[0.98]	[0.99]
ARCH (q), F(q,T-2q-9):	[0.68]	[0.29]	[0.46]	[0.48]	[0.57]
Nonlinearity(d_2), *F(15, 70)*:	[0.20]	[0.47]	[0.34]	[0.27]	[0.27]

Parameter constancy: $F(21, 66) = 1.54$ [0.10]
Heteroscedasticity (X_i^2): $F(16, 70) = 0.87$ [0.60]
Heteroscedasticity (X_iX_j): $F(41, 45) = 0.77$ [0.80]
Normality, $\chi^2(2)$ =1.15 [0.56]

The tests for no autocorrelation, no remaining nonlinearity of STAR type and parameter constancy are those suggested by Eitrheim and Teräsvirta (1996). p-values of test statistics in large brackets [.], and std. deviations in small brackets (.). In testing for no-autocorrelation, the (initial) missing values of residuals were set at zero, as recommended by Teräsvirta (1998). The tests for no remaining nonlinearity of STAR type were done for U_{t-d_2} as the transition variable.
The other tests for no heteroscedastisity and normality of the errors are the standard tests, as used in linear models, (see Doornik and Hendry, 1996).

of significance. The null hypotheses that are tested are: absence of autocorrelation up to 5 lags, no heteroscedasticity, including ARCH type up to order 5, and that the residuals have a normal distribution. Note that the residuals of the linear AR model were autocorrelated even if 8 lags were included, see table 5.1. The tests for no remaining nonlinearity of STAR type do not indicate any remaining nonlinearity in the model. The test of parameter constancy suggests that the initial non-constancies in the parameters have been modelled satisfactorily, though not completely since the p-value is 10%. The non-rejection of this test implies that there have not been significant changes in the two equilibria over time, see section 3. Although the alternative hypothesis is that of smooth changes in the parameters, the parameter constancy test has power even if the alternative hypothesis is that of abrupt changes in the parameters, (see Eitrheim and Teräsvirta, 1996).

6.3. Dynamic properties of the model

The LSTAR model suggests that the unemployment is stationary in both regimes, though quite persistent. The equilibria associated with the regimes $\widehat{F}(U) = 0$ and $\widehat{F}(U) = 1$ are at about 2.3% and 5.1%, respectively. The two regimes have different dynamics. The sums of the autoregressive coefficients during the expansion, $\widehat{F}(U) =$

0, and contraction, $\widehat{F}(U) = 1$, are 0.78 and 0.9, respectively. Thus, unemployment becomes more persistent when it rises to higher levels and its response towards a shock become more sluggish. It tends to revert more quickly towards the lower equilibrium upon a (small) deviation from it compared with when it deviates from the higher equilibrium. In other words, the lower equilibrium is relatively more stable than the upper one. This might be a reflection of the actual policy of the Norwegian government that has been aimed at low unemployment during the sample period, (cf. Manning, 1990). In addition, the wage bargaining institutions are relatively centralised making it difficult to leave the low unemployment regime, (cf. Calmfors and Driffill, 1988). The relative stability of the lower equilibrium implies that the asymmetries in the adjustment process are likely to increase with the level of unemployment and to be more apparent at the higher equilibrium than at the lower equilibrium.

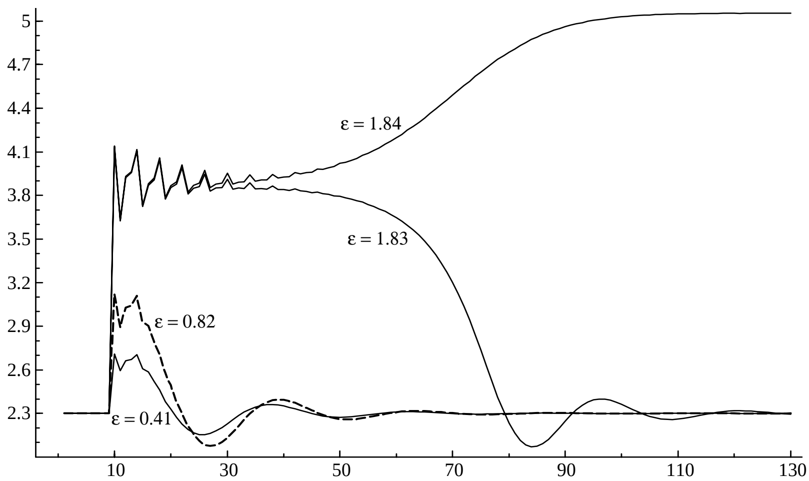

Figure 6.3: *Shocks (ε) of different sizes when in the low unemployment equilibrium of 2.3 %.*

Both equilibria are stable, implying that if unemployment is at one of the equilibria, it will remain there unless disturbed by a shock that cause a transition to the other equilibrium. Figures 6.3 and 6.4 display the response of unemployment when it is exposed to shocks of different sizes when in the lower and the higher equilibrium, respectively. In both figures the shocks hit in period 10. Figure 6.3 shows that unemployment returns to the lower equilibrium of 2.3% when exposed to positive single shocks ($\varepsilon > 0$) of sizes up to 1.84 i.e. about 4.5 times the standard error of residuals ($\widehat{\sigma}$). A larger shock, however, makes it converge towards the high

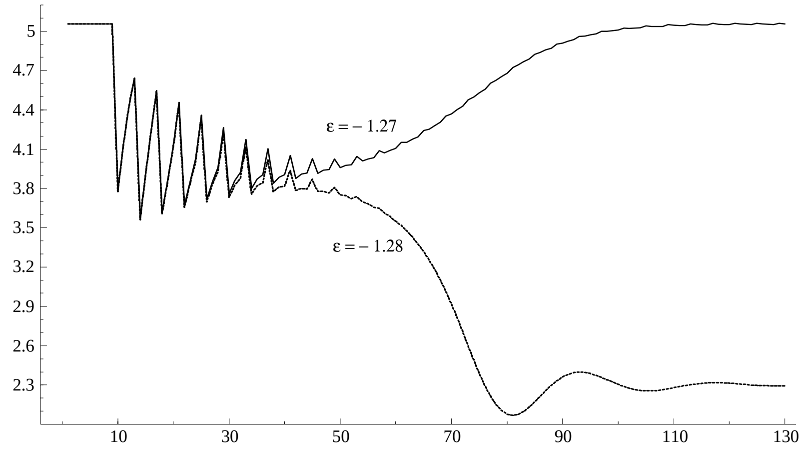

Figure 6.4: *Shocks (ε) of different sizes when in the high unemployment equilibrium of 5.1%.*

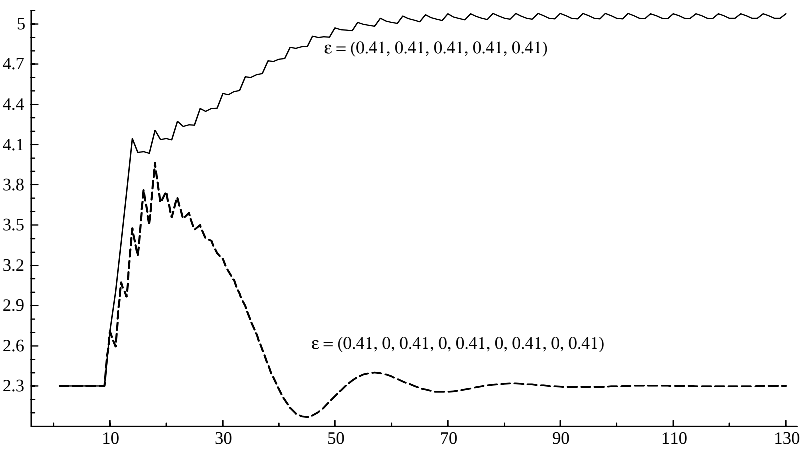

Figure 6.5: *A continuous sequence of five shocks, each equal to $\widehat{\sigma} = 0.41$, and a discontinuous sequence of the same shocks when in the low unemployment equilibrium.*

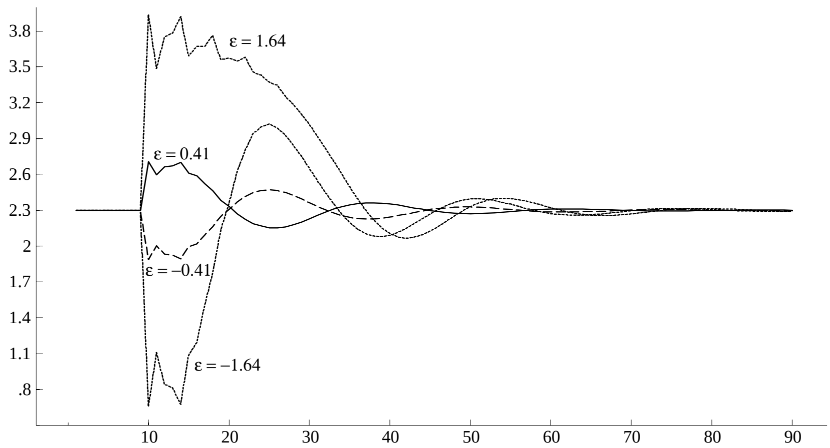

Figure 6.6: *Two positive shocks of sizes 0.41 and 1.64 when in the lower equilibrium and two negative shocks of the same magnitudes from the same initial level..*

unemployment equilibrium at 5.1%. From there, it requires a shock of size -1.28, i.e. of about -3$\widehat{\sigma}$ to revert to the lower equilibrium rate, see figure 6.4.[12] Otherwise, it will get stuck at the high unemployment equilibrium. Because of the weaker gravitation of the higher equilibrium compared with the lower equilibrium, unemployment requires a weaker impetus to leave the higher equilibrium.

Note that the model does not require a large single shock to cause a switch from one regime to another. A sequence of small shocks of the same sign may ultimately lead to a sufficient deviation of unemployment from its threshold value ($U_{t-5} - 3.57$) to cause a transition from one regime to another. However, they have to be larger in sum than a large single shock, because of slight reversions towards the initial equilibrium during the intervals between the shocks. Figure 6.5 displays the effects of a continuous sequence of five small shocks of size $\widehat{\sigma} = 0.41$ when unemployment is initially at the lower equilibrium. Unemployment converges to the higher equilibrium in this case too. The same figure lays out the response to a discontinuous sequence of five small shocks of the same size. In this case unemployment reverts to the initial equilibrium. This failure to leave the initial equilibrium in face of discontinuous small shocks is also related to the local mean reversion property of the process. When the second small shock arrives after the pause of one period, unemployment has

[12] For small shocks unemployment seems to display a limit cycle around the high unemployment equilibrium, see figure 6.5. This is to say that a set of values repeat themselves when approaching the high unemployment equilibrium.

already moved slightly back towards the initial equilibrium. Therefore, it requires larger shocks than 0.41 in the periods afterwards to converge towards the higher unemployment regime.

The model implies asymmetric unemployment response to sufficiently large positive and negative shocks. In the following, assume that the shocks fall short of causing a switch between the equilibria. Now, suppose that unemployment is at the lower equilibrium level. Then, for sufficiently large positive shocks the measure of persistence, $\widehat{\varrho}_1$, rises from 0.78 towards 0.91, but remains equal to 0.78 if the shock is negative, irrespective of its size in absolute terms. The opposite happens when unemployment is at the higher equilibrium. In that case the measure of persistence is $\widehat{\varrho}_2$, which remains at 0.91 if unemployment is exposed to a positive shock of any size, but falls from 0.91 towards 0.78 if the shock is large and negative. In both of these cases the recovery of unemployment is faster from the below of a given equilibrium than from the above. In other words, it rises faster than it declines. Figure 6.6 illustrates the case when unemployment is initially at the lower equilibrium and is disturbed by a positive and a negative shock of size 1.64 and -1.64, respectively. Its overshooting from the below stands in contrast with its sluggish recovery from the positive shock.

The importance of the sign of shocks is even more pronounced if they are sufficiently large to cause a switch between the equilibria. In that case, large positive shocks can cause a switch from the lower to the higher equilibrium, while negative shocks only cause a transitory deviation from it. The opposite happens when the unemployment is in the vicinity of the higher equilibrium. However, the effects of small positive or negative shocks are symmetric around a given equilibrium. Figure 6.6 shows this for the case of a positive and a negative shock of size 0.41 and -0.41, respectively.

The figures above demonstrate that the size and the sign of a shock matters for the unemployment dynamics and determines whether it leaves its initial equilibrium or not. Moreover, they elucidate the importance of the initial equilibrium for the response towards a given shock.

7. Conclusions

This paper aimed to derive a data consistent univariate model to test for the possibility of multiple equilibria in Norwegian unemployment and shed light on the issue of whether or not it adjusts asymmetrically when exposed to positive and negative shocks. To this end an LSTAR model has been derived. It has been shown that this model outperforms a linear AR(5) model in explanatory power and appears to be data congruent. The latter property has been established by testing the residual properties and the constancy of the model parameters.

The LSTAR model provides evidence for two stable unemployment equilibria at 2.3% and 5.1%, respectively. The degree of persistence in the vicinity of these equilibria is 0.78 and 0.90, respectively, which implies that the lower equilibrium is relatively more stable than the higher one. The unemployment process tend to exhibit equilibrium reversion around 2.3% before 1988 and around 5.1% afterwards, i.e. until the end of the sample in 1997:1. Although the LSTAR model allows smooth transitions between equilibria, the results indicate a rather abrupt transition from the low to the high unemployment regime in 1988. These results are largely consistent with the conclusions of Bianchi and Zoega (1996, 1998).

The model implies that large transitory shocks or a sequence of small shocks may cause a transition between equilibria and thus exert permanent effects on the level of unemployment. However, the small shocks must be larger in sum than the single large shock due to the equilibrium reversion property. In addition, due to the relatively lower degree of equilibrium reversion at higher unemployment levels, a smaller shock is required to make a transition from the higher equilibrium to the lower one than vice versa.

The model also implies that unemployment displays asymmetric response to large positive and negative shocks, while the response is symmetric to small positive and negative shocks. In other words, unemployment recovers faster from a fall than a rise, only when the disturbances are large. This result is intuitively appealing since it seems reasonable that small increases or decreases in the labour stock can be made with about the same costs. The findings regarding the asymmetries in the adjustment process are in contrast to Skalin and Teräsvirta (1999) for Norway, but consistent with their results for a number of other OECD countries.

This paper has also examined a linear AR(5), as noted above, and different versions of Markov regime switching models. The AR(5) model was found to be inconsistent with the data and offered an unrealistic estimate of the unique equilibrium at 3.74%. However, the high degree of implied persistence could also be interpreted as evidence of hysteresis in the unit root sense. The results from the Markov regime switching models were found to be highly sensitive to whether one attempted to capture the unemployment dynamics or not. Neglecting the dynamics lead to results that seemed to be consistent with the raw unemployment observations. These favoured two equilibria at around 2% and 5% and two periods of high unemployment equilibrium, as in Bianchi and Zoega (1998). However, attempts to control for dynamics lead to estimates of the equilibria that were close to each other and a quite a large number of switches between equilibria. Although such models offered a better fit, the implied results were difficult to reconcile with the main characteristics of the unemployment data.

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