A model for predicting aggregated corporate credit risk

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We present a model linking macroeconomic variables directly to an aggregate measure of credit risk in selected industries. The model is an alternative to an approach where firm-specific credit risk is predicted and then aggregated. The model performs well in backtests and it may be used to analyse the development in credit risk in macro stress tests of banks and financial systems.

Introduction

In financial institutions good estimates of credit risk are important both for pricing individual loans and for managing risk at the aggregate level. The authorities, central banks, and supervisors are concerned with the stability of banks and the financial system make assessments of credit risk at the aggregate level. Since credit risk is not directly observable, it is usually estimated by using statistical models. In this paper we present such a model linking macroeconomic variables to an aggregate measure of credit risk on loans made to corporate borrowers, where the risk is measured at the industry level. The alternative to this direct approach is to estimate credit risk for each firm and then aggregate the risk for all firms in the industry. The advantage with a direct modeling is that it is easier to link the development in macroeconomic variables to the development in credit risk. Firm-specific risk typically relies on financial ratios from the firms' financial statements and it is challenging to link macroeconomic variables to individual firms.

Credit risk is the risk of incurring a loss on a loan. The expected loss on a loan is determined by the probability of default and the loss provided that default has occurred. Default is here defined as the event when payments are not made according to the loan agreement. For many financial institutions credit risk, and in particular credit risk on loans to corporate borrowers, is the major source of risk. Other sources of risk are risks related to investments in securities (market risk), securing future financing (funding risk), and possible losses due to operational failures. Banks' exposures to credit risk in different industries vary. It is therefore important to both estimate the risk at the industry level and to take into account the exposure to different firms. We therefore use the debt-weighted probability of default (DWPD) per industry as our aggregated risk measure. When computing this measure we use the firm-specific estimates of probabilities of default (PDs) from Norges Bank's SEBRA model. The SEBRA model, see Eklund et al. (2001) and Bernhardsen and Larsen (2007), estimates the default risk for each firm based primarily on financial ratios computed from the firms' financial statements. The PDs are then aggregated by weighting each firm's PD with its debt and then taking the sum over all firms.[2]

Modeling of aggregate credit risk has become increasingly important when analysing the economic development in countries. Norges Bank monitors the stability of the financial system in Norway and follows closely the development in credit risk on corporate loans. Norges Bank's assessment of credit risk is also included in the Financial Stability report, which is published twice a year. One method for uncovering and identifying potential risks to the financial system or to individual financial institutions is to perform stress tests.[3] One type of stress test is a prediction of financial results and balance sheet items of banks or a group of banks based on a set of assumptions about future economic developments. In a macro stress test future economic developments are typically represented by key macroeconomic variables such as GDP growth, interest

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2 Other aggregated measures of credit risk may also be used. For example, Åsby Sommer and Shahnazarian (2009) use the median expected default frequency (EDF) from Moody's KMV for Swedish firms as a measure of economy-wide credit quality. Castren et al. (2008) use EDF for EURO-area firms and model credit risk for seven sectors and for all firms aggregated. As a proxy measure for credit risk within each portfolio they use the median EDF and the EDF for the median leveraged firm. Of course, other measures not based on default probabilities may be used. For example, Berge and Boye (2007) model problem loans and Jacobsen and Kloster (2005) model bankruptcy rates.

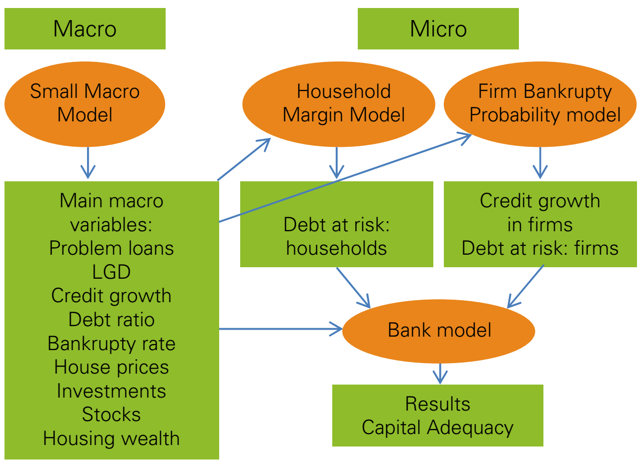
3 Foglia (2009) surveys authorities' approaches to stress testing.

rates, and exchange rates. The term *stress* test means that the set of assumptions (which may be termed a *scenario*) are chosen to represent a very negative development in the economy. Norges Bank uses several models when performing stress tests. The main models are a macro model and micro models covering the risks in the household and corporate sectors as well as a model for banks, see Figure 1. Andersen et al. (2008) presents this model framework in detail. Based on an assumed negative macroeconomic development, a model called the Small Macro Model is used to predict developments in future macroeconomic variables. These variables are then combined with micro information and separate micro models for households and firms. The output from the micro models, debt at risk, is then combined with the macroeconomic variables to predict banks' future income and capital adequacy.

The model presented in this paper may be used as a part of a stress test for estimating the development in corporate credit risk within the framework shown in Figure 1. Direct modeling of aggregate credit risk may be a supplement to the current micro approach. In the current approach estimated future macro variables are used to predict firms' future financial statements. In a second step, the default risk is then estimated for each firm based on these financial statements. This approach is described in detail in Bernhardsen and Syversten (2009).

We proceed as follows: We first present the data underlying the model in section two. Section three presents the model. In order to evaluate the model performance, we perform backtests at the industry level. These backtests are presented in section four. It is also important to make assessment of risk for portfolios that do not contain all firms in an industry. Section five therefore presents an analysis of the errors made when using aggregate estimates of risk on smaller loan portfolios.

Figure 1 The stress testing system in Norges Bank



Source: Figure 1 in Andersen et al. (2008)

Data

The debt-weighted default probability for an industry at time t is

$$DWPD_t = \sum PD_t^i w_t^i \tag{1}$$

where PD^i_t is the probability of default for firm i as estimated at time t. PD^i_t is estimated by using Norges Bank's default prediction model SEBRA Basic, see Bernhardsen and Larsen (2007). Each firm's weight is equal to the ratio of the firm's debt (D^i_t) to aggregated portfolio debt,

$$w_t^i = \frac{D_t^i}{\sum_i D_t^i} \tag{2}$$

Debt-weighted probabilities of default may be interpreted as the average expected fraction of 1 krone of loan in the portfolio that defaults next year. By using each firm's debt in the weight, we explicitly take into account the loan exposure of different firms. The risk-weighted debt (RWD) for the portfolio is the expected amount of debt that is expected to default the next year,

$$RWD_t = DWPD_t \cdot D_t \tag{3}$$

where D_t is the total amount of debt in the portfolio. In other words, risk-weighted debt is simply the debt-weighted probability of default scaled by the level of debt.

Table 1 reports selected descriptive statistics for the sample of firms for the years 1988–2008. The sample consists of Norwegian joint stock firms. Statistics are reported for 14 industries and for all industries aggregated. We also report statistics for all firms when we exclude oil-related firms (Oil services and Oil and gas). The industry classification is based on NACE Rev. 1.1.[4] The most important industries as measured by their share of bank debt are Commercial property, Shipping, and Manufacturing and mining with, respectively, 39.8, 13.8, and 12.7 of total bank debt at the end of 2008. Bank debt is here measured according to the information about the firms' debt in their balance sheets. The number of firms has been increasing during the sample period and the number of firms varies between the industries. Trade and

[4] EU's standard industry classification system.

retail has the highest average number of firms with above 27 000. Manufacturing and mining, Commercial property, and Business services have all average number of firms of above 10 000.

Table 1 also reports the descriptive statistics per industry for the yearly computed DWPDs and the yearly mean of the probabilities of default (MPDs). The average MPD was about 4.5 percent for all firms excluding oil-related firms while the average DWPD was about 2.5 percent. This highlights the importance of taking into account exposure when measuring relevant credit risk. Large firms have usually low probabilities of default and high levels of debt. We would therefore expect that DWPDs are lower than MPDs. We see from Table 1 that this is indeed the case for most industries. The only industries with a higher average DWPD than MPD are Construction and Commercial property. The period includes the banking crisis in the beginning of the nineties. During this period the credit quality was at its lowest. We see from Table 1 that the ranges between the highest and lowest levels of the DWPDs and MPDs are quite high. For example, the range in DWPD for the industry Hotels and restaurants is between 3.6 and 18.5 percent.

The model

A probability takes values only between zero and one. Since the model is estimated by OLS we therefore make a log-transform of the odds-ratio and use the variable

$$trp_t = \ln\left(\frac{DWPD_t}{1-DWPD_t}\right)$$

instead of the probability $DWPD_t$. We use an autoregressive distributed lag model (ADL) when modeling the development in DWPDs. The regression equation is

$$\Delta trpt = const + b \cdot trp_{t-1} + \sum_{k=2}^{K} c_k \Delta x_{k,t} + \sum_{m=K+1}^{M} c_m x_{m,t-1} + u_t \quad (4)$$

where Δtrp_t is the change from time t–1 to t in the transformed probability, Δx_t and x_{t-1} are, respectively, the change in the explanatory variable at time t and the level of the variable at time t–1. The error term is u_t. Better (worse) credit quality means that the transformed DWPD decreases (increases).

Table 1 Descriptive statistics – number of firms and probabilities of default (1988–2008[1])

Level of aggregation / Industry	Percent of bankdebt	Number of firms, 1000				Yearly debt weighted PD					Yearly mean of PD				
		Mean	Median	Max	Min	Mean	Median	Max	Min	SD	Mean	Median	Max	Min	SD
Agriculture	0.2	0.65	0.66	1.11	0.20	5.90	4.77	12.78	2.61	3.08	6.52	5.81	14.37	2.45	3.25
Fishing and fish farming	4.8	1.32	1.39	1.78	0.85	9.63	3.83	32.24	1.00	11.20	9.68	4.44	31.82	2.53	9.83
Manufacturing and mining	12.7	10.48	11.11	11.84	6.56	2.03	1.80	3.70	1.15	0.77	4.32	3.78	8.68	2.23	1.80
Power and water supply	3.9	0.36	0.39	0.75	0.07	0.43	0.31	1.85	0.11	0.38	1.19	0.97	3.11	0.52	0.63
Construction	2.3	8.90	9.29	13.69	3.42	4.32	2.82	13.82	1.36	3.40	3.99	2.94	11.14	1.45	2.69
Trade and retail	8.0	27.07	29.31	31.18	13.33	3.70	3.00	8.95	1.95	1.87	5.27	4.61	10.50	3.44	1.76
Hotels, restaurants	1.5	4.16	4.62	5.58	1.27	8.87	7.69	18.46	3.59	3.73	14.82	14.01	24.45	9.49	3.93
Shipping	13.8	1.47	1.33	2.12	0.88	1.65	1.37	4.82	0.45	1.10	3.28	2.81	8.08	1.00	1.93
Other transport	3.2	3.50	3.82	4.48	1.48	2.23	1.92	5.02	0.91	1.03	3.37	2.40	9.08	1.28	2.24
Telecom	0.4	0.24	0.21	0.50	0.03	7.69	6.46	16.25	0.94	4.97	8.18	7.24	19.90	2.22	3.40
Commercial property	39.8	19.56	19.60	32.28	8.28	1.66	1.12	4.31	0.70	1.13	1.41	1.13	3.04	0.75	0.68
Business services	5.6	18.68	19.71	27.94	6.00	4.11	3.39	9.08	1.57	2.33	4.35	3.78	8.13	2.05	1.82
Oil services	2.6	0.18	0.18	0.22	0.13	3.30	1.63	17.91	0.56	3.88	4.90	3.26	15.91	1.50	4.14
Oil and gas	1.3	0.08	0.07	0.12	0.05	1.75	1.43	3.37	0.71	0.92	4.79	3.33	11.44	1.62	3.14
All	100.0	96.64	102.22	128.94	42.56	2.34	1.98	4.82	1.04	1.08	4.49	3.99	9.24	2.47	1.81
All excluding oil	96.1	96.38	101.91	128.66	42.36	2.47	2.06	5.35	1.04	1.28	4.49	3.98	9.24	2.47	1.81

[1] Percent of bank debt is calculated on the basis of the firms' financial statements at year-end 2008.

Variable selection

Many central banks use only a few macroeconomic variables to link the development in the macro economy to credit risk.[5] When selecting variables there is a trade-off between a parsimonious and intuitive model on the one hand and the goodness of fit of the model on the other hand. The model's forecasting performance is evaluated by making backtests, which are presented in the next section. As a starting point we explored the explanatory variables described in Bernhardsen and Syversten (2009). These are GDP growth, inflation, the exchange rate, growth in wage income, and the interest rate. During the sample period the average GDP growth was 2.7 percent. The average unemployment rate and growth in wage income were, respectively, 3.5 and 5.9 percent. The variation in interest rates and rise in house prices was large. The highest interest rate was 16.6 percent and the lowest interest rate was 3.9 percent. The rise in house prices ranged from –12.1 to 15.1 percent.

Credit risk varies between industries and the link to the macroeconomic variables is therefore not so strong for all industries. In addition, the number and composition of firms within industries change over time. This adds noise to the observed credit risk not captured by macroeconomic variables. This is particularly pronounced in industries with relatively few firms. In the model different explanatory variables are therefore used depending on the industry. The following variables are included in the model:

- *gdp* is the percentage growth in GDP for mainland Norway measured at constant prices. This is a measure of the activity level in the economy and increased GDP growth is related to increased revenue in firms.
- *inc* is the percentage growth in household wage income. This variable is expected to be positively correlated with firms' payroll expenses. Increased wage growth will therefore increase firms' costs and is associated with a worsening of credit quality.
- *RX* is the real exchange rate. An increase in the variable *RX* means that NOK depreciates. For firms with sales abroad, a depreciation of NOK implies increased sales and thereby increased revenue. The opposite is true for firms that mainly import goods and sell them on the Norwegian market.
- *phinf* is the percentage rise in house prices.[6] House prices are important for household spending and consumption. Here, however, house prices are a proxy for prices of commercial real estate. Increased prices of real estate are related to revaluation and increased activity in the Commercial property industry.
- *BOR* is banks' lending rate. Increased interest expenses will reduce firms' bottom line and lead to a worsening of firms' credit quality.
- *loan ent* is an index for loans to enterprises. A change in this measure is a proxy for the change in the activity level in the corporate sector. Increased lending is related to increased investment activity, establishment of new firms and thereby also increased activity for firms in the Business industry.
- Since we model industry-specific risk, it is possible to use the risk level in one industry as an explanatory variable for the risk level in another industry. This approach is used for modeling the Shipping industry. High levels of sale and revenue in the Trade and retail industry implies a high level of imported goods. A part of these goods are imported by sea, explaining the positive correlation between the risk in the Trade and retail and the Shipping industries.

Estimation results

Based on the importance of the industries as measured by their share of bank debt (see Table 1), we decided to focus on the industries Commercial property, Manufacturing and mining, and Shipping. Business services and Trade and retail are also modeled separately since these industries are related to, respectively, Commercial property and Shipping. For the other industries we use a general model that is estimated on the sample of all firms, but where firms in Oil services and Oil and gas are excluded. Table 2 presents the estimated coefficients with accompanying t-values. Credit risk is mean-reverting and the development in risk therefore depends on the risk level. In order to make a comparison to a model without macroeconomic variables, Table 2 therefore reports the estimates for a model where the only variable is the previous year's risk level.

Commercial property is influenced by income growth, the interest rate, and the development in real estate prices as proxied by house prices. We see from Figure 2a that high interest rates and negative growth in real estate values contributed to high risk levels in the years 1989–1993 and that low interest rates and high growth in asset values contributed to low risk levels in the years after 1995. We also see that the persistence in risk (the contri-

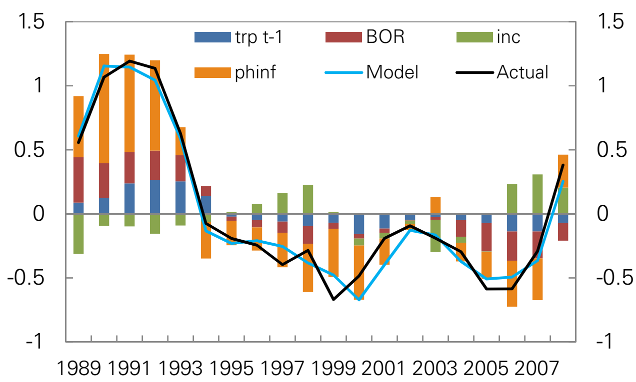
[5] See Foglio (2009).

[6] The sources for house prices are the Norwegian association of estate agents (Norges Eiendomsmeglerforbund), the Norwegian estate agent companies' association (Eiendomsmeglerforetakenes Forening), ECON Pöyry and Finn.no.

Table 2 Estimated coefficients

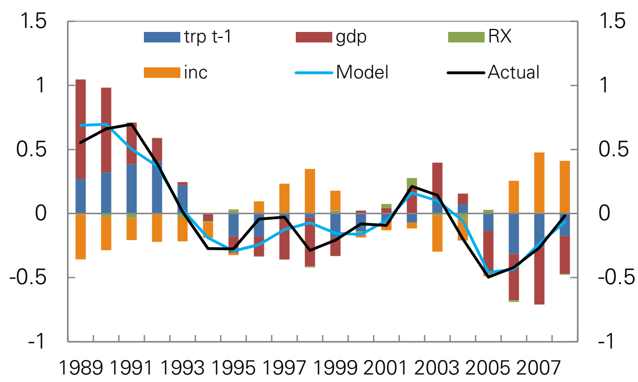
Variable	Manufacturing and mining		Commercial property		Business services		Trade and retail		Shipping		All excl. oil-related firms	
trp_{t-1}	−0.216	−0.402	−0.153	−0.773	−0.195	−0.475	−0.169	−0.383	−0.261	−0.422	−0.169	−0.265
	(−1.67)	(−2.79)	(−1.23)	(−5.84)	(−1.66)	(−3.22)	(−1.45)	(−3.18)	(−1.63)	(−2.42)	(−1.50)	(−2.36)
Δgdp_t		−0.069				−0.131		−0.105				−0.132
		(−1.81)				(−2.41)		(−2.31)				(−2.88)
gdp_{t-1}		−0.192				−0.295		−0.218				−0.211
		(−5.42)				(−5.28)		(−4.96)				(−4.87)
ΔRX_t		−0.009										
		(−0.85)										
Δinc_t		0.079		0.072		0.161		0.124		0.090		0.119
		(2.98)		(3.04)		(3.75)		(3.77)		(1.93)		(3.54)
inc_{t-1}		0.107		0.063		0.186		0.099		0.139		0.113
		(3.83)		(3.15)		(3.91)		(2.94)		(3.17)		(3.26)
BOR_{t-1}				0.046								
				(2.35)								
$\Delta phinf_t$				−0.034								
				(−6.48)								
$phinf_{t-1}$				−0.050								
				(−5.98)								
Δtrp_t^{Trade}										0.747		
										(2.20)		
trp_{t-1}^{Trade}										1.087		
										(4.16)		
$\Delta loan\ ent_t$						−0.269						
						(−1.63)						
$const$	−0.876	−1.690	−0.655	−3.786	−0.693	−1.776	−0.606	−1.293	−1.111	1.047	−0.672	−1.098
	(−1.72)	(−3.39)	(−1.22)	(−5.52)	(−1.78)	(−4.03)	(0.45)	(−3.64)	(−1.61)	(1.59)	(−1.57)	(−2.94)
R^2	0.135	0.766	0.077	0.897	0.133	0.741	0.105	0.680	0.129	0.685	0.111	0.703
DW	1.1	1.9	1.0	2.0	1.5	2.2	1.6	2.3	1.4	2.7	1.1	2.0
N	20	20	20	20	20	20	20	20	20	20	20	20

Figure 2a In-sample decomposition of credit quality (DWPD) in Commercial property. Deviation from means. Percent. 1989–2008



Source: Norges Bank

Figure 2b In-sample decomposition of credit quality (DWPD) in Manufacturing and mining. Deviation from means. Percent. 1989–2008



Source: Norges Bank

bution from previous year's risk level) kept the risk level high for the years 1991–1994.

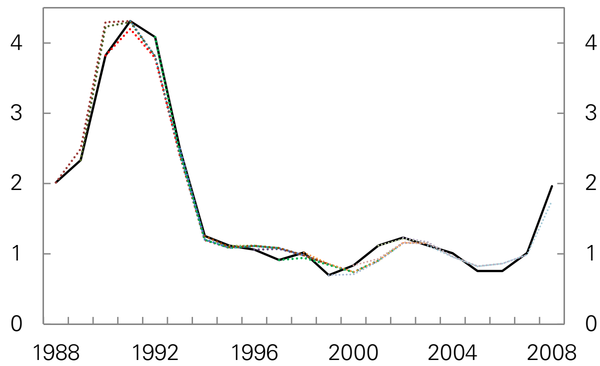
The equation for Manufacturing and mining contains variables for GDP growth, income growth, and the exchange rate. Increased GDP growth and a weakening of the Norwegian krone contribute to lower risk while income growth increases risk. Figure 2b shows the contribution of the different variables to risk as measured by the deviation from the mean of the transformed debt-weighted probability of default. For the years 1989–1993 the persistence of the high risk level of the previous year was an important explanation for continued high risk. The opposite was true during the years 2005–2008. The other main variable explaining a high risk level for the years 1989–1992 and low risk for the years 2005–2008 is GDP growth. The exchange rate contributed to an increase in risk in 2002.

The risk in Business services is influenced by GDP growth, income growth, and the activity in the corporate sector as measured by changes in loans made to the enterprise sector. The signs of the coefficients are as expected. The equation for Trade and retail includes only GDP growth and income growth. Increased GDP growth contributes to decreased risk while increased wage growth contributes to increased risk. The equation for Shipping includes income growth and the level and development in risk for the Trade and retail industry. An increase in risk for the Trade and retail industry coincides with an increase in risk for the Shipping industry. The shipping industry, as represented by Norwegian firms in the SEBRA database, reflects smaller and more home-based shipping than international shipping. For the other industries we use a common equation which only includes GDP growth and income growth. The constant in the equation is changed in order to match the mean of changes in the transformed debt-weighted probability of default during the sample period.[7]

Backtesting

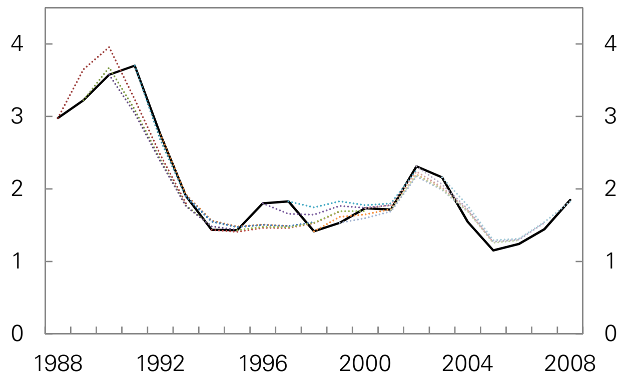
A backtest is a prediction of future credit risk using historical data for the explanatory variables and the actual DWPD in the base year for the prediction. We make five year predictions for the base years 1988–2003.[8] Figure 3a and b show the actual and predicted DWPDs for Commercial property and Manufacturing and mining, respectively. The forecasting performance is good, in particular

Figure 3a Backtest for Commercial property. The actual (solid line) and predicted (dotted lines) DWPDs. Percent. 1988–2008



Source: Norges Bank

Figure 3b Backtest for Manufacturing and mining. The actual (solid line) and predicted (dotted lines) DWPDs. Percent. 1988–2008.



Source: Norges Bank

for Commercial property. There is a tendency to overpredict the risk in Manufacturing and mining for the years 1998–1999.

Table 3 shows the average deviations for each of the five prediction years for the industry-specific DWPDs. As an example, the average prediction error for the Shipping industry two years ahead is 0.1 percentage point. This means that the model slightly overpredicts the risk on a two-year horizon. The sum of the yearly average of prediction errors over five years is below 1 percentage point for the industries with individual model equations. For the industries without individual equations, three of the industries have prediction errors of less than 1 percentage point.

7 The constant used for the industries are –1.581 (Power and water supply), –0.976 (Construction), –0.732 (Hotels and restaurants), –1.118 (Other transport), –1.222 (Oil services), and –1.142 (Oil and gas). We excluded the industries Agriculture, Fishing and fish farming, and Telecom from further analysis. The development in DWPDs for these industries was not well captured by the common equation. Since these industries are not large in terms of debt, we decided not to estimate tailored models for them.

8 Note that this is not an out-of-sample test since the model parameters are estimated on data for the years 1989–2008.

These are Power and water supply (0.0), Oil and gas (0.4), and Other transport (0.7). The industries where the sum of average prediction errors is above 1.5 percentage point are Construction (1.6) and Oil services (1.7). A high average prediction error indicates that the model including only GDP growth and growth in wage income probably misses important industry-specific variables that could have improved the predictions. Table 3 also reports an absolute measure of the prediction error ($\sqrt{MSE}$).[9] While over- and underpredictions may cancel each other out and give a low average deviation, this measure is always positive. The numbers reported in Table 3 show that the performance of the model is quite good for the industries Manufacturing and mining (1.0) and Commercial property (0.7). For Shipping, however, the error is quite large (8.4), even though the average deviation is only 0.3. This shows that there is a tendency for the model to either over- or underpredict the actual credit risk.

Even though the model may overshoot or undershoot the actual risk level in the prediction period, the model is still useful if it can correctly predict the direction of the change in credit risk. Table 4 reports how often the model wrongly predicts the sign of the change in DWPDs for different industries. For instance, if the model predicts wrongly the direction 20 percent of the time, you would on average expect to make 1 wrong prediction during a 5-year period. For first year prediction errors, industries with individual models have a prediction error of less than 20 percent. The lowest error, 12.5 percent, is achieved by the model for Commercial property. Four industries have below 20 percent prediction error the first year when the general model is applied. These are Power and water supply, Construction, Oil services, and Oil and gas.

Table 3 Average deviation between predicted and actual DWPDs in percentage for 5 year predictions (base years 1988–2003)

Industry	Measure	Prediction period ($t + k$) where k is 1	2	3	4	5	Sum
		Industries with individual models					
Manufacturing and mining	Average deviation	0.0	0.0	0.1	0.0	0.0	0.2
	$\sqrt{MSE}$	0.2	0.3	0.2	0.2	0.1	1.0
Commercial property	Average deviation	0.0	0.0	0.0	0.0	0.0	0.1
	$\sqrt{MSE}$	0.2	0.2	0.1	0.1	0.1	0.7
Business services	Average deviation	0.0	0.0	0.1	0.1	0.0	0.2
	$\sqrt{MSE}$	0.5	0.5	0.5	0.5	0.4	2.4
Trade and retail	Average deviation	0.1	0.1	0.1	0.1	0.0	0.4
	$\sqrt{MSE}$	0.4	0.4	0.4	0.4	0.4	2.1
Shipping	Average deviation	0.0	0.1	0.1	0.1	0.1	0.3
	$\sqrt{MSE}$	2.4	1.6	2.0	1.4	1.0	8.4
All excluding oil	Average deviation	0.0	0.1	0.1	0.0	0.0	0.2
	$\sqrt{MSE}$	0.3	0.3	0.3	0.3	0.3	1.5
		Selected industries with general model					
Power and water supply	Average deviation	0.0	–0.1	0.0	0.0	0.1	0.0
	$\sqrt{MSE}$	0.4	0.3	0.3	0.2	0.2	1.5
Construction	Average deviation	0.5	0.6	0.5	0.1	–0.1	1.6
	$\sqrt{MSE}$	1.5	1.8	1.9	0.9	0.8	7.1
Hotels, restaurants	Average deviation	0.1	0.3	0.3	0.3	0.2	1.2
	$\sqrt{MSE}$	1.2	1.1	1.1	1.2	1.2	5.7
Other transport	Average deviation	0.1	0.1	0.2	0.1	0.1	0.7
	$\sqrt{MSE}$	0.9	0.8	0.7	0.6	0.6	3.7
Oil services	Average deviation	0.0	0.3	0.5	0.5	0.4	1.7
	$\sqrt{MSE}$	1.9	1.7	1.2	1.0	1.0	6.8
Oil and gas	Average deviation	0.1	0.1	0.1	0.1	0.1	0.4
	$\sqrt{MSE}$	0.7	0.9	1.0	0.9	0.8	4.3

[9] For prediction year k the mean squared error, $\sqrt{MSE}$, is computed as $\sqrt{1/N\sum_{t=1}^{N}(\widehat{DWPD}_{t+k} - DWPD_{t+k})^2}$ where the base years for the prediction are 1,2,..N and $\widehat{DWPD}_{t+k}$ is the predicted level of DWPD

Table 4 Percent of years with wrongly predicted sign of the change in DWDP (base years 1988–2003)

Industry/aggregate	Prediction between time ($t+k-1$) and ($t+k$), where k is 1	2	3	4	5
	Industries with individual models				
Manufacturing and mining	18.8	18.8	31.3	25.0	18.8
Commercial property	12.5	18.8	18.8	18.8	18.8
Business services	18.8	12.5	31.3	31.3	25.0
Trade and retail	18.8	12.5	25.0	25.0	18.8
Shipping	18.8	12.5	25.0	25.0	18.8
All excluding oil	18.8	18.8	25.0	31.3	31.3
	Selected industries with general model				
Power and water supply	18.8	50.0	37.5	37.5	25.0
Construction	18.8	18.8	25.0	12.5	12.5
Hotels, restaurants	31.3	31.3	37.5	50.0	50.0
Other transport	31.3	31.3	37.5	31.3	25.0
Oil services	18.8	43.8	37.5	43.8	43.8
Oil and gas	18.8	18.8	31.3	37.5	37.5

Prediction of portfolio risk

The backtest is made for the portfolio consisting of all firms in an industry. The predictions are based on the whole sample of firms in the base year and it is assumed that the sample does not change during the prediction period. The predicted DWPD for this base-year portfolio is then compared to the DWPD for the actual sample at future dates. The backtest therefore measures the model's ability to predict the risk in the actual future sample. Financial institutions do not make loans to all firms in an industry. It is therefore of interest to know how predictions for the whole sample will perform when it is applied to a smaller portfolio. What will the error be if we assume that the risk in the smaller sample develops according to the industry-wide risk? We use the following procedure to build a small sample.

1) Select a random sample of N firms in industry S in year t.

2) Replace the firms that are not in the sample the following year (t+1). This is done by making a random draw of the available firms.

3) Compute the prediction error for year t+1 as

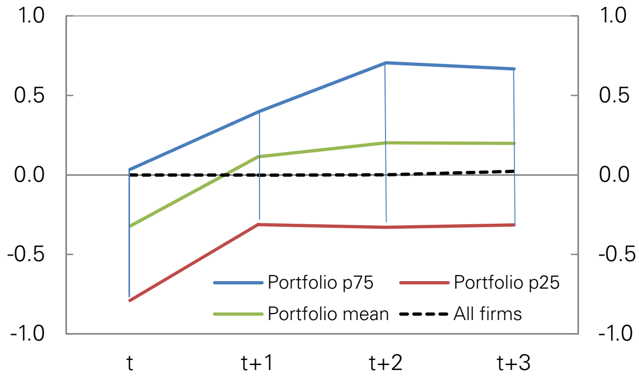
$$Error^{N}_{t+1} = [DWPD^{N}_{t} + \Delta\widehat{DWPD}^{S}_{t+1}] - DWPD^{N}_{t+1} \qquad (5)$$

where the term in brackets is the predicted future level of risk in the portfolio if the change in portfolio risk is equal to the change in the risk for the whole industry.

We make random draws of 2000 firms from the industries Commercial property and Manufacturing and mining. For every base year during the years 1988–2003 (16 years) we draw 1000 such random portfolios. We then compute the portfolio errors for the 16 000 random portfolios for the three first prediction years for each industry. Figure 4a and b summarise the results. The dotted line shows the prediction error from the backtest, i.e., for the portfolio consisting of all firms in the industry. The backtest shows that the model slightly overpredicts the actual risk. Figure 4a and b also show the mean (green line), the 75 percentile (blue line), and the 25 percentile (red line) for the randomly drawn portfolios' prediction errors. For both industries the average prediction error is on average positive, but not by much. A negative prediction error means that the predicted future level of portfolio risk is lower than the actual risk. From a prudential perspective it is worse to underpredict than overpredict portfolio risk. Figure 4a and b show that there is a 25 percent probability of underpredicting the portfolio risk by more than about 0.3 percentage point during the three prediction years in Commercial property. For Manufacturing and mining there is a 25 percent probability of underpredicting the portfolio risk by more than about 0.2, 0.2 and 0.15 percentage point, respectively.

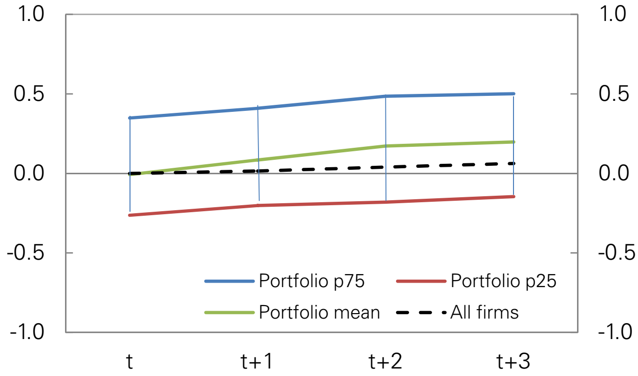
Figure 4 also shows the portfolio risk in the base year (year t) for the predictions. For Manufacturing and mining the average risk for the randomly drawn portfolios in the base year is similar to the risk level for the whole industry (the error is zero). For Commercial property, however, the average portfolio risk is lower than the industry-wide risk. The intuition behind this is that there are some firms with high levels of debt and high PDs that contribute to high DWPDs for the industry. These firms are, however, relatively seldom included in the randomly drawn portfolios. These portfolios will therefore on average have a lower DWPD than the industry-wide DWPD.

Figure 4a Prediction errors for DWPD of a portfolio of 2000 firms in Commercial property. The prediction errors in percentage points for all firms in the industry (the dotted line) and the sample of 16 000 randomly drawn portfolios (the solid lines). Year t is the base year for the predictions. Base years 1988–2003



Source: Norges Bank

Figure 4b Prediction errors for DWPD of a portfolio of 2000 firms in Manufacturing and mining. The prediction errors in percentage points for all firms in the industry (the dotted line) and the sample of 16 000 randomly drawn portfolios (the solid lines). Year t is the base year for the predictions. Base years 1988–2003



Source: Norges Bank

Concluding remarks

Direct modeling of measures of aggregate credit risk at the industry level by using macroeconomic explanatory variables makes it easier to assess the impact of macroeconomic scenarios on credit risk. The model presented in this paper performs well in backtests both with respect to the predicted future level and the direction of the change in credit risk. The model may be improved in at least two directions. First, industry-specific variables may improve the prediction performance. Examples of such variables are freight rates in the Shipping industry or prices of commercial real estate in the Commercial property industry. Second, a general model equation was used to predict risk in a selection of industries. This simplification was made in order to concentrate on industries with a large share of total debt. The prediction performance for an industry will increase if industry-specific model equations are used instead of the general model equation.

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Appendix

This appendix compares the prediction performance for the full model and the model including only previous year's risk level (the simple model). Table 5 shows the difference between the prediction errors $(\sqrt{MSE})$ for the full and simple model. As expected, the prediction error is lower (a negative difference) for the full model with individual equations. The simple model is slightly better for the industries Power and water supply and Oil services. Table 6 shows the difference between the percentage of wrongly predicted direction of changes in DWPD with the full model and the simple model. The general model with individual equations makes fewer mistakes than the simple model. An exception is, however, the prediction of change in risk between year 2 and 3 in the prediction period for Business services. With a few exceptions, the general model with a common equation predicts more correctly the change in risk than the simple model.

Table 5 Difference between prediction errors ($\sqrt{MSE}$) for the full model and the model with lagged dependent variable only (base years 1988–2003)

	Prediction period ($t + k$) where k is					
Industry	1	2	3	4	5	Sum
	Industries with individual models					
Manufacturing and mining	−0.2	−0.4	−0.4	−0.3	−0.3	−1.5
Commercial property	−0.5	−0.8	−0.9	−0.8	−0.6	−3.6
Business services	−0.5	−0.9	−0.9	−0.7	−0.6	−3.6
Trade and retail	−0.7	−1.1	−0.8	−0.6	−0.4	−3.4
Shipping	−1.0	−2.9	−2.8	−3.0	−1.8	−11.4
All excluding oil	−0.3	−0.6	−0.6	−0.4	−0.2	−2.1
	Selected industries with general model					
Power and water supply	0.0	0.1	0.0	0.0	0.0	0.1
Construction	−0.7	−1.1	−0.9	−0.5	−0.3	−3.5
Hotels, restaurants	−0.7	−1.5	−1.1	−0.8	−0.4	−4.5
Other transport	0.0	0.0	−0.3	0.0	−0.1	−0.5
Oil services	0.3	0.4	0.0	0.1	0.1	1.0
Oil and gas	−0.1	−0.2	−0.2	−0.2	−0.1	−0.7

Table 6 Difference between the percentage of wrongly predicted sign of the change in DWDP with the full model and the model with lagged dependent variable only (base years 1988–2003)

	Prediction between time ($t + k - 1$) and ($t + k$), where k is				
Industry/aggregate	1	2	3	4	5
	Industries with individual models				
Manufacturing and mining	−18.8	−12.5	−12.5	−18.8	−31.3
Commercial property	−43.8	−31.3	−18.8	−6.3	−12.5
Business services	−25.0	−12.5	12.5	−18.8	−31.3
Trade and retail	−18.8	−31.3	−12.5	−18.8	−31.3
Shipping	−12.5	−43.8	−25.0	−37.5	−31.3
All excluding oil	−12.5	−6.3	−18.8	−18.8	−37.5
	Selected industries with general model				
Power and water supply	−6.3	0.0	−6.3	0.0	0.0
Construction	−12.5	−6.3	−18.8	−31.3	−25.0
Hotels, restaurants	0.0	−6.3	−6.3	18.8	−12.5
Other transport	12.5	−12.5	−6.3	0.0	−12.5
Oil services	−6.3	6.3	0.0	0.0	0.0
Oil and gas	−31.3	−12.5	−18.8	−12.5	−31.3