The neutral real interest rate

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The concept "neutral real interest rate" is generally associated with the real interest rate level, which implies that monetary policy is neither expansionary nor contractionary. We define the neutral real interest rate as the real interest rate level which in the medium term is consistent with a closed output gap. We consider in more detail how the neutral real interest rate in a small, open economy is influenced by global conditions. The neutral real interest rate cannot be observed, and estimates are uncertain. Different methods for estimating the neutral real interest rate are presented in this article. An overall assessment implies that it will normally lie in the range of about $2\frac{1}{2}-3\frac{1}{2}$ per cent in Norway. In recent years, with low real interest rates globally, we cannot exclude the possibility that the neutral real interest rate in Norway may be even lower. The neutral real interest rate has probably been falling since the 1980s and early 1990s, partly as a result of lower inflation risk premia.

1 Introduction

The interest rate is the most important monetary policy instrument. It may be set so that monetary policy is expansionary, contractionary or neutral. The concept "neutral real interest rate" is generally associated with the real interest rate level, which implies that monetary policy is neither expansionary nor contractionary. If the central bank aims to stimulate economic activity, the interest rate must be set so that the real interest rate is lower than the neutral rate. If the central bank aims to dampen activity, the interest rate must be set so that the real interest rate is higher than the neutral rate.²

The concept "neutral real interest rate" stems from the Swedish economist Knut Wicksell³, who maintained about a hundred years ago that the general price level would rise or fall indefinitely as long as the real interest rate deviated from the neutral interest rate⁴. The neutral real interest rate cannot be observed, however, and estimates are uncertain. Blinder (1998) states that: "... the neutral real rate of interest is difficult to estimate and impossible to know with precision. It is therefore most usefully thought of as a concept rather than as a number, as a way of thinking about monetary policy rather than as the basis for a mechanical rule ..."

The neutral real interest rate is an important concept, nonetheless, for assessing the monetary policy stance. Central banks must have a perception of how expansionary or contractionary monetary policy is. This requires an assessment of the level of the neutral real interest rate.

There are a number of real interest rate concepts. It is

particularly important to distinguish between the *long*term equilibrium real interest rate, the neutral real interest rate and the actual real interest rate. The long-term equilibrium real interest rate is determined by economic fundamentals such as growth potential and private saving behaviour. The neutral real interest rate is in addition determined by various disturbances that affect the supply and demand side of the economy in the medium term. The neutral real interest rate may deviate from the long-term equilibrium real interest rate, but will move around and towards it over time. The actual real interest rate is largely determined by the level of the central bank's official policy rate, and therefore depends on the objectives of monetary policy and the disturbances to which the economy is exposed. The actual real interest rate may therefore differ from the neutral real interest rate for shorter or longer periods of time.⁵

The long-term equilibrium real interest rate is discussed in the next section. The neutral real interest rate and the relationship between the different real interest rate concepts are then considered in more detail. First, the concepts for a closed economy are discussed and in Section 4 the neutral real interest rate in a small, open economy is considered in more detail. Free movement of capital across countries implies that interest rates – including the neutral real interest rate – are influenced by global conditions. Section 5 investigates how the neutral real interest rate can be estimated empirically, and what may be regarded as reasonable estimates of the neutral real interest rate, globally and in Norway. Section 6 provides a summary.

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 $^{^{2}\,}$ A more precise definition of the neutral real interest rate is provided in Section 3.

³ Wicksell (1907) wrote the following: "If, other things remaining the same, the leading banks of the world were to lower their rate of interest; say 1 per cent below its ordinary level, and keep it so for some years, then the prices of all commodities would rise and rise and rise without any limit whatever; on the contrary, if the leading banks were to raise their rate of interest, say 1 per cent above its normal level, and keep it so for some years, then all prices would fall and fall without any limit except Zero".

⁴ The concepts "neutral real interest rate", "natural real interest rate" and "normal real interest rate" are used interchangeably in the literature. The expression "neutral real interest rate" is used in this article.

The expected real interest rate is defined as $r^e = i - \pi^e$, where r^e is the expected real interest rate, i is the nominal interest rate and π^e is inflation expectations (any risk premia are disregarded). Changes in the short-term real interest rate are largely determined by changes in the short-term nominal interest rate, which in turn is determined by the central bank's official policy rate. The nominal interest rate is deflated by expected inflation over the term of the nominal interest rate. In some contexts, the nominal interest rate is deflated by actual inflation during the period. These two deflation methods give rise to the concepts "ex ante" and "ex post" real interest rate. Inflation can also be measured in several ways, for example in terms of consumer prices, or in terms of an expression for underlying inflation. These factors may be of considerable significance for an empirical estimation of the real interest rate, but are of less importance for understanding the theoretical aspects of the various real interest rate concepts.

2 The long-term equilibrium real interest rate

Economic growth theory may shed light on what determines the real interest rate in the long term. In the Ramsey model, the long-term real interest rate is determined by economic fundamentals such as productivity and population growth and household saving preferences. Prices are assumed to be flexible, and input factors to be mobile. All markets are therefore in equilibrium. Under a number of simplified assumptions it can be shown that:

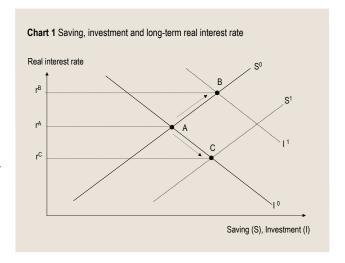
(1)
$$r^{**} = g + n + \rho$$

The long-term equilibrium real interest rate (r^{**}) is determined by growth potential, i.e. the sum of productivity growth (g) and population growth (n) in addition to the household rate of time preference (ρ). The more weight households place on consumption today relative to future consumption, the higher the time preference rate is.⁶

According to the Ramsey model, the real interest rate and potential growth move more or less in tandem. It is assumed that households prefer to smooth consumption over time. Higher potential growth and hence higher expected income therefore increase the propensity to consume and reduce the propensity to save. This implies a higher real interest rate. The more households prefer to consume today relative to the future, i.e. the more impatient they are, the lower the propensity to save and the higher the real interest rate.

Higher potential growth can also lead to a higher long-term equilibrium real interest rate via higher demand for investment. When productivity growth increases, for example, this will increase the marginal return on capital. A marginal return that is higher than the real interest rate increases the propensity to invest. Investment demand and the equilibrium real interest rate will accordingly rise. This is consistent with Wicksell (1907) who maintained that: "... the upward movement of prices, whether great or small in the first instance, can never cease so long as the rate of interest is kept lower than its normal rate, i.e. the rate consistent with the then existing marginal productivity of real capital."

The relationship between investment and saving is illustrated in Chart 1. Investment demand (I⁰) is negatively dependent on the real interest rate, because a lower real interest rate makes fixed investment more profitable. The saving curve (S⁰) is rising because households are assumed to reduce current consumption relative to future consumption when the real interest rate increases. It is important to distinguish between preferred quantities ex ante and actual quantities ex post. Preferred saving ex ante may be different from preferred investment.



It is then up to the real interest rate to achieve a balance so that these are equal ex post (point A on the chart). Globally – or in a closed economy – saving is always equal to investment ex post.

Changes in potential growth and the household rate of time preference lead to permanent changes in saving and investment behaviour and hence to changes in the long-term equilibrium real interest rate. A higher investment preference shifts the demand curve outwards in Chart 1 (from I⁰ to I¹). The new and higher real interest rate level generates more saving, so that the increase in investment demand is covered. A new adjustment takes place at point B. One way of looking at this is that when investment demand increases, the economy needs a higher real interest rate in order not to overheat, and it can take the higher real interest rate without dampening the activity level. A higher saving preference shifts the saving supply outwards (from S⁰ til S¹). A lower real interest rate leads to higher investment, which accordingly absorbs the increase in the saving supply. A new adjustment takes place at point C. When the saving supply increases, the economy can take a lower real interest rate without overheating, and it needs a lower real interest rate to prevent a dampening of the activity level.

The Ramsey model is stylised and most useful as a starting point for assessing long-term developments in the real interest rate. The model indicates a long-term relationship between potential growth and the real interest rate.

3 A closer look at the neutral real interest rate

Definition

The concept "neutral real interest rate" is generally associated with the real interest rate level which implies that monetary policy is neither expansionary nor contractionary. There is no definitive definition of the neutral

⁶ In the Ramsey model, the saving ratio is determined by consumers maximising their utility. The expression in equation (1) is based on a simplified assumption that the utility function is logarithmic. This simplification makes the discussion somewhat simpler without losing the central points of the model. Blanchard and Fisher (1989) and Romer (2001) provide a more in-depth discussion of this question and the Ramsey model in general. Hammerstrøm and Lønning (2000) also provide a somewhat more detailed discussion.

⁷ Given the assumptions in footnote 6 it can be shown that $MPC - v = r^{**} = g + n + \rho$, where MPC is the marginal productivity of capital (gross) and v is the depreciation rate of capital (Romer, 2001). If, for the sake of simplicity, we assume that households' rate of time preference is zero, the net marginal productivity of capital (MPC - v) must be equal to the real interest rate, which in turn must be equal to potential economic growth. The expression can be interpreted as an equilibrium condition. Suppose, for example, that the marginal return on capital increases as a result of technological advances. The marginal return on capital is then higher than the real interest rate, which provides an incentive for increased investment. As a result, investment demand increases, and the real interest rate rises.

real interest rate, and there are a number of approaches to it in the literature.

Yellen (2005), president of the San Francisco Federal Reserve, states: "Conceptually, policy can be deemed "neutral" when the federal funds rate reaches a level consistent with full employment of labor and capital resources over the medium run."

We accordingly define the neutral real interest rate as the real interest rate level, which in the medium term is consistent with a closed output gap. The output gap is defined as the difference between actual and potential output, which is the output level that is consistent with stable inflation over time. Chart 2 illustrates a hypothetical path for the real interest rate and the output gap. The central bank sets the interest rate such that the monetary policy objectives are expected to be achieved. In the medium term, the output gap is expected to stabilise at around zero.⁸

The neutral real interest rate can change over time. Yellen describes this as follows: "The value of [the neutral rate] depends on the strength of spending – that is, the aggregate demand for U S produced goods and services. Aggregate demand, in turn, depends on a number of factors. These include fiscal policy; the pace of growth in our main trading partners; movements in assets prices, such as stocks and housing, that influence the propensity of households to save and spend; the slope of the yield curve, which determines the level of long-term interest rates associated with any given value of the federal funds rate; and the pace of technological change, which influences spending..."

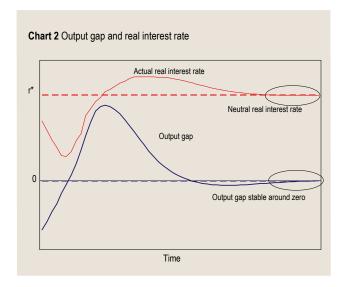
Yellen is referring here to different disturbances to the economy that may lead to changes in the neutral real interest rate. Disturbances to the economy may influence the prospects of closing the output gap in the medium term. Positive demand shocks of a certain duration tend to widen the output gap. To counteract this, and ensure that the output gap stabilises at around zero in the medium term, the real interest rate must increase. This means that the neutral real interest rate has increased. Similarly, negative demand shocks of a certain duration will tend to reduce the output gap. To counteract this, and stabilise the output gap at around zero in the medium term, the real interest rate must be reduced. This means that the neutral real interest rate has fallen.

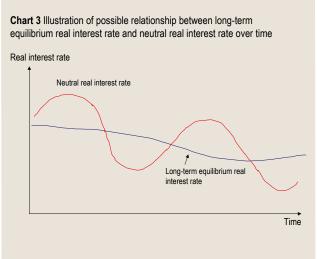
The relationship between the long-term equilibrium real interest rate and the neutral real interest rate

Whereas the long-term equilibrium real interest rate is determined by factors such as productivity, population growth and long-term saving preferences, the neutral real interest rate is *additionally* influenced by various disturbances that influence the economy in the medium term. Examples are temporary changes in fiscal policy and in consumer and investment demand. The relationship between the long-term equilibrium real interest rate and the neutral real interest rate is illustrated in Chart 3. The neutral real interest rate can be envisaged as moving around and towards the long-term equilibrium real interest rate over time (in the absence of new shocks).9

The relationship between the neutral and the actual real interest rate

In the event of stickiness of wage and price formation, the central bank can influence the real interest rate and economic developments by changing the policy rate. The real interest rate may therefore deviate from the neutral level, depending on how the central bank seeks to orient monetary policy. This in turn depends on the central bank's trade-off between different objectives, such as stable inflation on the one hand, and stable output and employment on the other.





^{8 &}quot;Medium term" is not clearly defined at the outset. To provide some idea of the time perspective, the medium term can probably be thought of as a horizon of from 1–2 years and up to 5–6 years "Medium term" may therefore be different from the central bank's horizon for achieving the monetary policy objectives, such as that inflation shall be at a particular level.

⁹ New-Keynesian theory can be used to shed more light on this relationship. In these models, the neutral real interest rate is interpreted as the real interest rate that would have prevailed if wages and prices had been flexible also in the short to medium term. In general, the neutral real interest rate will depend on all disturbances that influence the supply and demand side of the economy (see Appendix 1 for a more detailed discussion).

In summary, the three real interest rate concepts are related as follows:

- Long-term equilibrium real interest rate: Determined by economic fundamentals such as long-term saving behaviour, productivity and population growth.
- Neutral real interest rate: Determined by all the
 disturbances to the economy that influence the prospect of closing the output gap in the medium term.
 These include the fundamentals that determine the
 long-term equilibrium real interest rate, but also disturbances of a more temporary nature.
- Actual real interest rate: Determined by the central bank's desire to conduct an expansionary or contractionary monetary policy. When economic disturbances occur, the central bank sets the real interest rate lower or higher than the neutral level with a view to stabilising the economy so that monetary policy objectives are achieved.

4 The neutral real interest rate in a small open economy

The definition of the neutral real interest rate – "the real interest rate level, which in the medium term is consistent with a closed output gap" – also holds for a small open economy. However, a small open economy is heavily influenced by global factors. One possible point of departure for discussing interest rates in a small open economy is risk-adjusted uncovered interest rate parity:

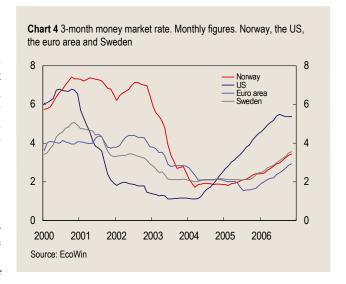
(2)
$$i_D = i_G + (e^e - e) + rp$$

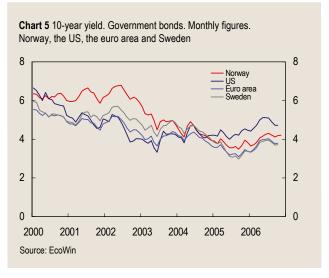
In this equation, i_D is the domestic interest rate, i_G is the global interest rate, e is the exchange rate, e^e is the expected future exchange rate and rp is a risk premium. The exchange rate is defined as the number of units of the domestic currency that must be paid for one unit of the foreign currency. When the price of a foreign currency is expected to rise, the domestic currency is expected to depreciate, i.e., $(e^e - e) > 0.10$

When the risk premium is zero, uncovered interest rate parity holds. The expected return on investing globally (measured in domestic currency) is then equal to the return on investment in the home country. If the expected return on global investment differs from the return on domestic investment, investors will shift toward investments yielding the highest returns. Suppose, for example, that the global interest rate falls. Domestic fixed-income securities will then be more attractive to both domestic and foreign investors. Demand for them will increase, leading to both lower domestic interest rates and an appreciation of the domestic currency.

The risk premium does not have to be zero.¹¹ As a result of factors relating to the risk premium, exchange rate and expected exchange rate, global and domestic interest rates do not necessarily move entirely in pace with one another. Nevertheless, interest rate parity provides a reasonable explanation for why domestic interest rates are influenced by global interest rates: If finical market participants anticipate large differences in expected returns in different countries, they will tend to make portfolio changes that reduce the difference in expected return.

Normally the relationship between global and domestic interest rates will be stronger for long-term rates than for short-term rates (see Charts 4 and 5). Long-term interest rates are largely determined by expected growth and by inflation expectations, which do not necessarily differ substantially across countries. Short-term rates are largely determined by a country's monetary policy, which may differ depending on the cyclical phase of the country's economy at the time.





¹⁰ In equation (2) the exchange rate is expressed in logarithmic form.

¹¹ If the risk premium is not zero, it means that investors are willing to hold both domestic and foreign fixed-income securities, even if the expected return on the two is different.

Just as global nominal interest rates may influence domestic nominal interest rates, global saving and investment behaviour and the global neutral real interest rate may influence the neutral real interest rate in a small, open economy. There is no simple relationship between the global neutral real interest rate and the neutral real interest rate in a small, open economy. The relationship will depend on how the economies function, and the disturbances to which they are exposed. Global disturbances may have ripple effects for the demand and supply sides of a small, open economy, and thereby contribute to output deviating from potential output. Disturbances arising in a small, open economy will not normally affect economic developments in the rest of the world. A detailed analysis of these relationships will require a model of the global economy and the domestic economy. We will confine ourselves here to pointing to some mechanisms which may contribute to an understanding of how the neutral real interest rate in a small, open economy can be influenced by global factors.

Our starting point is a stylised relationship between demand for fixed investment and the supply of real saving globally and at home, assuming unrestricted and cost-free trading of goods and services. Movements of capital between countries are disregarded in order to highlight some central points which will also apply in a pure barter economy. The analysis is then expanded to include movements of capital between countries (a portfolio theory approach).

Chart 6 shows demand for real investment and the supply of real savings globally and domestically. The small country cannot influence the global interest rate (r*), and must take it as a given. This means that all investment and saving in the small country take place at the global real interest rate. It is initially assumed that saving is equal to investment, both globally and domestically (point A). This means that the balance of trade is zero for both "countries". 12 It is further assumed that the real interest rate is the same as the neutral rate both at home and abroad.

Chart 6 Effect of lower global neutral real interest rate. Real economic approach

Global Small, open economy

Real interest rate

S'0

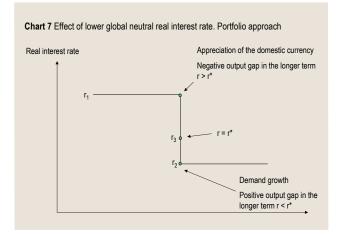
S'1

Saving (S), Investment (I) Import surplus

A higher global saving preference shifts the global saving supply curve outwards (from S*0 to S*1). This pushes global real interest rates down (from r*0 to r*1) and increases global investment demand, which absorbs the increase in the global saving supply. The domestic real interest rate will then fall, providing an incentive to reduce saving (point B') and increase fixed investment (point B"). The difference between investment and saving is equal to the trade deficit. Output remains equal to potential output in the small country because the increase in investment demand is covered by higher imports.¹³

To provide a better understanding of the dynamics in a small open economy when there is a preference to increase global saving, the analysis is broadened to include capital movements (see Chart 7). Initially, the neutral global real interest rate is equal to the neutral domestic real interest rate (r_1) . The global neutral real interest rate is assumed to fall to r_2 .

- If the domestic interest rate remains unchanged at r₁, the difference against the global rate will increase. This will contribute to an appreciation of the domestic currency. The appreciation will dampen demand and reduce the output gap in the medium term in the home country. An unchanged real interest rate can therefore not be an equilibrium: the neutral real interest rate must have fallen. The question is, by how much.
- If the domestic real interest rate is reduced as much as the global real interest rate (r₂), the interest rate differential between them will remain unchanged. It is then reasonable to assume that the nominal and the real exchange rate will also remain unchanged. However, a lower domestic real interest rate will have an expansionary effect. Unless the entire increase in demand is covered by imports, the output gap will increase in the medium term. The export and import pattern will change slowly over time, while interest and exchange rates will adapt rapidly



¹² From economic theory and national accounts we know that R=C+I+(A-B), where the letters stand for production, consumption, investment, exports and imports, in that order. Moreover, S=R-C, where S is saving. It follows from this that S=I+(A-B), i.e. that a country can save through fixed investment or by having a balance of trade surplus. When saving is equal to investment, the balance of trade is zero (for the sake of simplicity we do not distinguish here between the trade balance and the current account).

¹³ In practice some frictions arise, as a result of which the domestic real interest rate will be different from the global rate. For example, a small open economy can probably not accumulate a trade deficit without having to pay a higher risk premium. It is commonly assumed that the risk premium – and accordingly the real interest rate – increases with a country's debt.

to a new equilibrium in a world with well developed capital markets. It therefore appears more realistic to assume that a combination of a lower real interest rate and a stronger real exchange rate is what is required to stabilise the output gap in the medium term in a world with free capital movements.

• It therefore appears reasonable that the new level for the domestic neutral real interest rate should lie somewhere between the old global level (r₁) and the new global level (r₂), for example r₃. A domestic real interest rate fall from r₁ to r₃ will have an expansionary effect and contribute to a larger output gap. The fact that the interest rate differential is positive (r₃ > r₂) contributes to strengthening the real exchange rate and reducing the output gap. It is conceivable that these effects are offsetting so that that the overall monetary policy stance remains unchanged and consistent with a closed output gap in the medium term.

5 Estimation of the long-term equilibrium real interest rate and neutral real interest rate

Potential growth and long-term equilibrium real interest rate

Potential growth may be of importance to both the long-term equilibrium real interest rate and the neutral real interest rate. Table 1 shows average growth and the average real interest rate from 1986 and 1994 for the G7 countries and Norway. The general picture is that average growth lies in a range from just under 2.5 per cent to just over 3.0 per cent. The interval for the real interest rate is somewhat larger.

Table 1 Growth and short-term real interest rate for the G7 countries and Norway*

	G7		Norway	
	Growth	Real interest rate	Growth	Real interest rate
1986–2006	2,6	2,5	2,4	4,6
1994–2006	2,5	1,6	3,1	3,0

* Growth is measured as average four-quarter growth over the period in question. The real interest rate is a short-term nominal interest rate deflated by consumer prices. The G7 countries are Canada, France, Germany, Italy, Japan, the UK and the US.

Sources: EcoWin and Norges Bank

The European Central Bank (ECB) estimates potential growth in the euro area to lie in the lower end of the range, 2–2½ per cent¾, while it is widely believed that the growth potential in the US is somewhat higher, at about 3 per cent.¹5 In Norway, potential growth is estimated at about 2½ per cent.¹6 The estimates for potential growth and the long-term equilibrium real interest

rate are highly uncertain. The overall impression is that for both Norway and the G7 countries, the long-term equilibrium real interest rate normally appears to be in a range around $2^{1/2}$ – $3^{1/2}$ per cent. Assigning a more precise estimate would be to over-rate the methods and possibilities available for estimating the long-term equilibrium real interest rate.

Methods for estimating the neutral real interest rate

There are a number of methods for assessing the neutral real interest rate (see Giammarioli and Valla (2004) for an overview). One possible estimate of the neutral real interest rate is the average of historical real interest rates. If the neutral real interest rate is constant over time, an average of historical real interest rates over an entire business cycle will provide an indication of the level of the neutral real interest rate. The problem with the method is that the neutral real interest rate cannot be assumed to be constant over time. It can also be difficult to decide when a business cycle starts and ends.

Other methods attempt to measure market participants' expectations regarding future short-term real interest rates. This is done by means of real return bonds, market surveys (for example by Consensus Forecasts) and by estimating market participants' future interest rate expectations via market rates (implied rates¹⁷). The shortcoming of these methods is, first, that they do not necessarily capture market participants' actual interest rate expectations, and second that market participants' future interest rate expectations may deviate from the neutral real interest rate.

One commonly used method for estimating the neutral real interest rate is to specify an econometric model, combine actual data and a priori assumptions about developments in the unobservable variables (often other unobservable variables, such as potential output and equilibrium unemployment, are also included), and to use the Kalman filter to estimate the neutral real interest rate. The problem with the method is that the model that forms the basis for the calculations is often highly simplified compared with reality. The estimates are generally sensitive to a number of technical choices in the estimation process, and are therefore shrouded in uncertainty.

The neutral real interest rate can also be estimated using dynamic stochastic general equilibrium (DSGE) models, which are often based on New-Keynesian theory. In these models, the participants are forward-looking, while the central bank sets the interest rate with a view to stabilising inflation and output over time. Wages and prices are sticky in the short term, but flexible in the long term. If the assumption about sticky nominal wages and prices is relaxed, the flexible price version of

¹⁴ See ECB (2005) and Trichet (2005).

 $^{15~\}mbox{See}$ for example $\mbox{\it Financial Times}$ (2006a, 2006b) and the IMF (2006).

¹⁶ This is Norges Bank's estimate of potential mainland growth in *Inflation Report* 3/06.

¹⁷ For estimation and interpretation of implied rates, see Kloster (2000) and Myklebust (2005).

the model emerges, i.e. the developments in economic variables that would have occurred if all prices had been flexible. In these models, the neutral real interest rate is interpreted as the real interest rate that applies in the "flexible price" version (see Appendix 1). This method of estimating the neutral real interest rate is on the one hand theoretically appealing, as there is a relationship between the neutral real interest rate and other variables, like the output gap, which is consistent with theory. This is not necessarily the case with the other more "traditional" methods described above. On the other hand, a model with quantified coefficients is required. The model does not necessarily have to be true to reality. The estimate of the neutral real interest rate is therefore sensitive to the choice of model and the estimation and calibration of the model's parameters. For further details, see Gali (2002) and Giammarioli and Valla (2004). Amato (2005) discusses some differences in the "flexible price" solution for the neutral real interest rate and more traditional empirical methods.

It is clear from the above that there is no simple method for estimating the neutral real interest rate. A number of methods exist, and there is uncertainty attached to all of them. Nevertheless, the literature, in which a broad range of different methods are used, can generally contribute to providing an overall picture of the magnitude of the neutral real interest rate.

Estimates of the global neutral real interest rate

The ECB (2004) points out that many estimates of the neutral real interest rate in the euro area lie in the interval 2–3 per cent, but also refers to the substantial uncertainty associated with the estimates. The ECB argues that the neutral real interest rate in the euro area may have fallen in the past 10–15 years as a result of lower productivity and population growth in the euro area, the elimination of exchange rate risk within the euro area after the introduction of a common currency, improved public finances prior to the implementation of the common currency and a fall in the inflation risk premium due to a fall in inflation expectations to a stable, low level.¹⁸

Giammarioli and Valla (2003) present arguments for a gradual fall in the neutral real interest rate in the euro area, from about 4 per cent in the mid-1990s to around 3 per cent in 2000. Cuaresma, Gnan and Ritzberger-Gruenwald (2003) indicate that the neutral real interest rate in the euro area has fallen somewhat since 2000, and propose a level of around 2 per cent at the end of 2002. Garnier and Wilhelmsen (2004) also find that the

neutral real interest rate has fallen in recent years, both in the euro area and in Germany. Goldman Sachs (2004) maintains that the neutral real interest rate in the euro area has fallen over the past 15 years, and estimates it at around 2 per cent in October 2004.

Laubach and Williams (2003) estimate the neutral real interest rate in the US from the early 1960s up to 2002. They find that the neutral real interest rate has fallen gradually over time. A possible explanation for this trend may be a fall in the inflation risk premium. Aside from the general fall in the long-term trend, Laubach and Williams find that the neutral real interest rate was temporarily low in the mid-1990s, but rose in the latter half of this decade. A widely accepted explanation for the latter is the high productivity growth (new economy wave) of the latter half of the 1990s. In the first few years of this century, the neutral real interest rate in the US fell, which can be explained by the sharp fall in equity prices and slower growth in these years. Laubach and Williams estimate the neutral real interest rate in the US at about 3 per cent in mid-2002. The OECD (2004) updates the Laubach and Williams study, and finds that the neutral real interest rate in the US may be just over 2 per cent at the end of 2004.¹⁹

In a speech given in October 2004, Roger W. Ferguson refers to a fall in US interest rates from 2001 to 2004, and points out that even though short-term real interest rates fell substantially, the neutral real interest rate fell at the same time. Factors contributing to the fall in the neutral real interest rate included: "... an unusual hesitancy on the part of businesses to hire and spend emerged in 2001 after the collapse of equity prices ... and ... the restraint imposed on domestic consumers from an increase in the cost of energy." ²⁰

Manrique and Manuel Marques (2004) estimate the neutral real interest rate in the US and Germany from the mid-1960s to the end of 2001. Their results for the US are comparable with those of Laubach and Williams. Whereas the neutral real interest rate rose somewhat in the latter half of the 1990s, it fell in the years immediately after the turn of the century. Towards the end of 2001 it was estimated at about 2½ per cent. Amato (2005) argues that the neutral real interest rate in both the US and the euro area may be in the range $2\frac{1}{2}-2\frac{3}{4}$ per cent, which is consistent with the estimates of the BIS (2005). Goldman Sachs (2005) estimates the neutral real interest rate in the US at about 2.5 per cent. Wu (2005) argues that the neutral real interest rate in the US has varied between 4 and 2 per cent since the 1960s and that it was around 2½ per cent in early 2005.

The neutral real interest rate is also mentioned from

¹⁸ Uncertain future inflation may lead to an inflation risk premium and higher real interest rate. The nominal interest rate can be expressed as $i = r^e + \pi^e + rp_\pi + rp_{term}$, where i is the nominal interest rate, r^e is the expected real interest rate, π^e is expected inflation, rp_π is an inflation risk premium and rp_{term} is the term premium. While the term premium reflects the extra expected return investors require for investing in fixed-income securities with a long maturity, the inflation risk premium reflects the extra expected return they require because future inflation is uncertain. Uncertain future inflation makes the real value of investments uncertain. Investors may require extra compensation – a risk premium – for this. As inflation fell in the 1980s and 1990s, so that inflation expectations became entrenched at a low and stable level, it is reasonable to believe that the inflation risk premium also fell, which contributes to a lower real interest rate.

¹⁹ Laubach and Williams (2003) are the first to use the Kalman filter to estimate the neutral real interest rate, and the article is one of the most widely quoted works in the empirical literature on the neutral real interest rate. A number of subsequent studies for both the US and other countries are based on the "Laubach and Williams method". The estimates are very uncertain and sensitive to a number of choices associated with the method. Ferguson (2004) therefore maintains, with reference to Laubach and Williams' estimates, that "... clearly, this estimate is not measured sufficiently precisely to be a useful guide to policy ...".

²⁰ See Ferguson (2004). Ferguson was Vice Chairman of the US Federal Reserve Board from 1999–2006.

time to time in the press. The Financial Times (2005) refers to a neutral nominal policy rate (the federal funds rate): "... generally seen as a range centred around 41/4 per cent ..., and ... the central bank's presumed 1–2 per cent comfort range based on the core personal consumption expenditure measure ...". This implies a neutral real interest rate of around 2.75 per cent. In an article of 12 July 2004, the same newspaper refers to Robert Parry, former president of the San Francisco Federal Reserve, who is of the opinion that an estimate of the neutral real interest rate may be: "... the average for the real federal funds since the 1960s of 2.5–3.5 per cent." If we take account of the widespread view that the neutral real interest rate has fallen gradually during this period, Parry's lower limit may be a reasonable estimate.

There are also studies for other countries. Björksten and Karagedikli (2003) and Lam and Tkacz (2004) present arguments for a fall in the neutral real interest rate in New Zealand and Canada, respectively. Brzoza-Brzezina (2006) finds that the neutral real interest rate is somewhat higher in Poland than in the US and the euro area. Sveriges Riksbank (2006) finds that $3\frac{1}{2}$ –5 per cent may be a reasonable range for the neutral nominal key rate in Sweden.

Estimates of the neutral real interest rate in Norway

We shall look more closely at the neutral real interest rate in Norway. Chart 8 shows developments in inflation, measured by changes in consumer prices and the short-term real interest rate since 1987. The chart also shows an estimate of long-term inflation expectations since the early 1990s. It is reasonable to believe that, as inflation became entrenched at a low level in the 1990s, long-term inflation expectations became similarly entrenched. The long-term inflation expectations are measured by average inflation up to the time when the inflation target was introduced in March 2001 (about 2 per cent), thereafter by the inflation target of 2.5 per cent. Low and stable inflation has probably contributed to a permanent fall in the inflation risk premium and accordingly the neutral level. In the past 10–12 years, the real interest rate has largely ranged from just under 1 per cent to just over 6 per cent. High values for the real interest rate indicate that it has been higher than the neutral real interest rate, while low values indicate that it has been lower than the neutral level.

Chart 9 shows implied long-term forward rates deflated by long-term inflation expectations. The starting point for the calculation is nominal implied five-year rates five years ahead, which is an estimate of market participants' expectations regarding the future nominal interest rate. To the extent that implied rates are unaffected by cyclical factors, they may reflect the expected interest rate level when the output gap is closed in the future. This measure of market participants' expected

Chart 8 Inflation measured by the consumer price index, long-term inflation expectations and short-term real interest rate*. Norway. Quarterly figures 12 12 9 Real interest rate 9 6 6 3 3 ٥ 0 -3 -3 1986 1989 1992 1995 1998 2001 2004 *3-month money market rate less annual inflation measured by the consumer price index Source: EcoWin and Norges Bank



real interest rate five years ahead has ranged from just over 1 per cent to about 4 per cent in the last 7–8 years. In recent years it has fallen, and is now about 2 per cent. As discussed above, implied interest rates do not necessarily provide a reliable estimate of market participants' interest rate expectations, and their expectations regarding the future real interest rate may differ from the neutral real interest rate. Implied interest rates, in particular, may partly reflect cyclical factors and as a result not be entirely in line with the interest rate level that is consistent with a closed output gap in the medium term.²¹

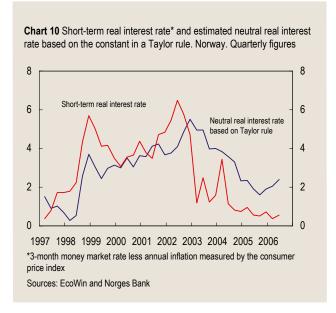
A Taylor rule can also be used as the starting point for estimating the neutral real interest rate. A rule of this kind says something about how the interest rate should be set, depending on the size of the inflation gap (inflation less the inflation target) and the output gap. When both gaps are zero, the interest rate should be set at the neutral rate. The constant in the Taylor rule can therefore be interpreted as the neutral nominal interest rate. We have estimated a Taylor rule for Norway for the

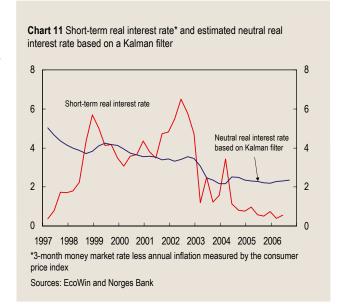
²¹ This is also pointed out by First Securities (2006).

period 1997-2006, in which the estimate of the neutral nominal 3-month interest rate is just under 6 per cent. When the inflation target of 2.5 per cent is subtracted, this implies an estimated neutral real interest rate of just over 3 per cent on average over the whole period. Alternatively, the Taylor rule can be solved for the constant for given values of the coefficients of inflation and the output gap.²² Measured in this way, the neutral real interest rate has ranged in the last couple of years from just under 2 per cent to just over 3 per cent (see Chart 10).²³ There is considerable uncertainty associated with these methods. Central banks never set the interest rate solely on the basis of a Taylor rule. In consequence, mechanical calculation of the constant will not necessarily produce a reliable estimate of the neutral real interest rate. In Chart 10, for example, the estimated neutral real interest rate around the peak in 2002/2003 is clearly too high, and reflects the actual interest rate setting rather than the level of the neutral real interest rate.

Chart 11 shows an estimate of the neutral real interest rate in Norway which has been arrived at by specifying a very simple econometric model and estimating the neutral real interest rate by means of the Kalman filter. The chart indicates that the neutral real interest rate may now be less than 2½ per cent. For a more detailed discussion of the method, see Appendix 2.

The methods used above do not provide an exact estimate of the neutral real interest rate in Norway, which we estimate will normally lie in the interval $2\frac{1}{2}-3\frac{1}{2}$ per cent. In recent times, with low real interest rates globally, we cannot exclude the possibility that it may be even lower. In recent years, historical real interest rates have moved around this range. Moreover, the methods based on implied interest rates, the Taylor rule and the





Kalman filter are consistent with this level.

The estimates of the neutral real interest rate in Norway have been reduced over time. On the basis of historical data, Hammerstrøm and Lønning (2000) find that 3-4 per cent may be a reasonable range for the neutral real interest rate in Norway. In view of developments in estimates for the global neutral real interest rate and different estimates for the neutral real interest rate in Norway, it appears reasonable to revise this somewhat downward. A lower inflation risk premium, in particular, may have contributed to this (see footnote 18). After a period of falling inflation in the 1980s, it took some years before inflation became entrenched at a low and stable level (see Chart 8). From the mid-1990s, it is reasonable to believe that the inflation risk premium has been considerably lower than in the 1980s and early 1990s. This points towards a lower neutral real interest rate.

6 Conclusions

Whereas the long-term real interest rate is determined by economic fundamentals such as potential growth and private saving behaviour, the neutral real interest rate is additionally affected by disturbances of a more temporary nature which influence the supply and demand sides of the economy.

The neutral real interest rate can be defined as "the real interest rate level which in the medium term is consistent with a closed output gap". Protracted disturbances to the economy may affect the prospects of closing the output gap in the medium term. For example, expansionary shocks will tend to widen the output gap. This means that

²² This method is used by Sveriges Riksbank (2006).

²³ The estimated Taylor rule is given by $i_{3M} = 5.7 + 2.2$ ($\pi - \pi^*$) + 0.3 (Y-Y*), where i_{3M} , ($\pi - \pi^*$) and (Y-Y*) are the three-month nominal money market rate, the inflation gap and the output gap, respectively. In order to provide a sufficiently long period for estimating the equation, we have used quarterly data since 1997, i.e. before inflation targeting was introduced in March 2001. The starting point was chosen partly because it was "from this point in time [January 1997] that daily quotations and month-to-month variations in the exchange rate show that the krone is floating." (Gjedrem, 2000). The output gap coefficient is not significantly different from zero, and sensitive to the estimation period that has been chosen. The other coefficients are significantly different from zero. The magnitudes of the coefficients appear reasonable and are in line with estimates for other countries. In the calculations upon which Chart 10 is based, the inflation gap coefficient is 1.5, while the output gap coefficient is 0.5. These are the same coefficients as used by Taylor (1993).

the neutral real interest rate has increased. The neutral real interest rate may deviate from the long-term equilibrium real interest rate, but will vary and, in the absence of new shocks, move towards the long-term equilibrium real interest rate over time.

Because of free movements of capital between countries, the interest rates in a small open economy, including the neutral rate, are dependent on global interest rates. However, there is no simple relationship between the global neutral real interest rate and the neutral real interest rate in a small, open economy. The relationship will depend on how the economies function, and the shocks to which they are exposed. Global shocks may have ripple effects for the demand and supply sides of a small, open economy – which may affect the prospects of closing the output gap in the medium term.

In a small, open economy, exchange rate factors may influence the neutral real interest rate. It is the overall orientation of monetary policy – the combination of the real interest rate and the real exchange rate – which is decisive for economic activities and hence for the prospects of closing the output gap in the medium term. In isolation, a stronger exchange rate will dampen economic activity. The prospects of closing the output gap in the medium term must therefore be assessed in the light of the effect that assumed interest rate movements abroad and in Norway have on the exchange rate.

There are several methods for estimating the neutral real interest rate, but there is substantial uncertainty attached to all of them. Nevertheless, a broad spectrum of methods can provide a picture of the range in which the neutral real interest rate lies. An overall evaluation implies that a range of around $2\frac{1}{2}-3\frac{1}{2}$ per cent may normally be regarded as covering both the long-term equilibrium real interest rate and the neutral real interest rate in Norway. In recent times, with low real interest rates globally, we cannot exclude the possibility that the neutral real interest rate in Norway may be even lower. The neutral real interest rate, both globally and in Norway, has probably fallen compared with the 1980s and the first half of the 1990s. One reason for this is probably lower inflation risk premia as inflation and inflation expectations have become entrenched at a low and stable level.

Appendix 1. New-Keynesian theory on the neutral real interest rate

In New-Keynesian models, the output gap is interpreted as the difference between overall output and the level of output that is consistent with flexible wages and prices (hereafter called potential output).²⁴ The neutral real interest rate can thus be interpreted as the real interest rate that applies when wages and prices are flexible. A strength of this definition is that there is a theoretically consistent relationship between the neutral real interest

rate and other variables in the economy, such as the output gap. A weakness is that the neutral real interest rate in such models is sensitive to the model specification. Woodford (2003) has pointed out that it may be optimal in terms of welfare to use monetary policy to steer the economy towards equilibrium with flexible prices.²⁵

Developments in the economy based on a New-Keynesian model can be expressed by two equations, one for the output gap, x_t (the IS curve), and one for inflation (the Phillips curve), π_t , see equations (1) and (2) respectively.

(1)
$$x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1} - r_t^*)$$

(2)
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

The IS curve is based on the Euler equation for optimal adaptation of private consumption over time, where i_t is the short-term nominal interest rate and E_t π_{t+1} is expected inflation in the next period. The coefficient (σ) expresses the intertemporal substitution elasticity, i.e. how much consumers are willing to postpone consumption if the real interest rate increases by one percentage point. The difference $(i_t - E_t \pi_{t+1})$ expresses the short-term real interest rate (ex ante), while r_t^* is the neutral real interest rate. Output depends one-to-one on expected output, because households want to smooth consumption over time. When the real interest rate is higher than the neutral real interest rate or is expected to be in the future, this will contribute to reduced consumption and a smaller output gap. The Phillips curve is based on optimal wage and price setting. The coefficient β can be interpreted as enterprises' discount factor, which is normally assumed to be close to 1. When the output gap increases, it adds to pressure on wages and prices because wage earners demand higher real wages for working more (because $\kappa > 0$), and enterprises will increase prices because production costs are assumed to increase at the margin.

In the short and medium term, monetary policy can be used to stabilise developments in output and prices. As a result of the implied and explicit costs associated with changes in prices and wages, it may take time before economic disturbances feed fully through to prices and wages. By adjusting the nominal interest rate (i_t) and having a rule for how the interest rate should be adjusted in the future, the central bank can influence the real interest rate and market participants' expectations. If, however, wages and prices are fully flexible, the central bank has no part to play in stabilisation policy. The reason is that a change made by the central bank in the nominal interest rate will lead to an equivalent change in expected inflation, so that the real interest rate is not affected. The real interest rate will thus always be equal to the neutral real interest rate when prices and wages are flexible.

In this model, disturbances to the supply and demand sides of the economy lead to changes in the neutral real interest rate. Thus disturbances that lead to an increase in the neutral real interest rate imply that monetary policy may be perceived as expansionary. It may therefore be necessary to increase the policy rate in order to avoid pressure on wages and prices. The neutral real interest rate can be written (see Gali, 2002)²⁶:

(3)
$$r_t^* = \rho + \rho_a \Delta a_t + (1 - \psi_g)(1 - \rho_g)g_t$$

The neutral real interest rate can be divided into three components:

- Household demand/discount factor (p): If households place a high value on consumption now, relative to consumption in the future, this discount factor will be higher. A higher real interest rate will then be necessary to ensure that overall demand is not higher than potential output.
- Productivity growth (Δa_t): When productivity *growth* increases, so does the neutral real interest rate. The persistence of a productivity growth shock depends on ρ_a (0< ρ_a <1). Over time, increased productivity means increased output and income. Since the households in this model prefer to smooth their consumption over time, they will increase consumption immediately when their lifetime income increases. The real interest rate will therefore have to rise to ensure that demand does not increase more than potential output. The more persistent the productivity growth shock is $(\rho_a$ is higher), the more the neutral real interest rate will increase. It follows from this that a one-off change in the productivity level $(\rho_a = 0)$ will not change the neutral real interest rate.

Empirically, however, there is little to indicate that consumption changes suddenly as a result of, for example, a change in productivity growth. The model above can be expanded to include an assumption of habit persistence in consumption.²⁷ This means that consumption and the neutral real interest rate will increase less in connection with a higher lifetime income than without such a condition. If productivity growth increases while habit formation remains strong, the neutral real interest rate may fall to motivate households to increase their demand as much as potential output. Otherwise there will be idle economic resources.

Demand shock (g_t) : When public authorities increase their demand, this also contributes to a rise in the neutral real interest rate. If the demand shock is permanent (ρ_g approaches 1), the real interest rate increases less. The reason is a condition in this model relating to balanced government budgets. Permanently increased public sector demand means higher taxation, lower household lifetime income, and hence lower private consumption.

If the labour market is not very flexible (ψ_g is low), the labour supply and hence potential output will only increase slightly when public sector demand increases. The real interest rate must accordingly increase more to prevent total demand from being higher than potential output.

The neutral real interest rate will therefore depend on both short-term and long-term disturbances that influence the supply and demand side of the economy. The above model can be expanded to include fixed investment and capital, but this will not have any significant impact on the qualitative results.

Appendix 2. Estimating the neutral real interest rate with a Kalman filter

The neutral real interest rate cannot be observed. A method frequently used to estimate unobservable variables is the Kalman filter. By combining actual data and a priori assumptions about developments in unobservable variables, the Kalman filter provides estimates of the latter. The neutral real interest rate is defined as the real interest rate level which is consistent in the medium term with a closed output gap. According to economic theory, the neutral real interest rate will depend on unobservable variables such as time preferences and growth in potential output. Empirical studies seek to use these relationships to estimate the neutral real interest rate at the same time as other unobservable variables (see for example Laubach and Williams (2003), Garnier and Wilhelmsen (2004) and Larsen and McKeown (2004)).

In this article, we use the Kalman filter to estimate the neutral real interest rate with a highly simplified economic model as the starting point. First, we assume that the real interest rate (r_t) can be split into a trend component (r_t^*) and a cyclical component (e_t) (see equation (1)):

(1)
$$r_t = r_t^* + e_t$$
 where $e_t \sim N(0, \sigma_e^2)$

The trend component is defined here as the neutral real interest rate. We furthermore assume that the neutral real interest rate depends on a constant (µ), which can be interpreted as a long-term equilibrium real interest rate, and disturbances (z_t) which cause the neutral real interest rate to deviate from the long-term equilibrium real interest rate. The disturbances are assumed to follow an AR1 process, i.e. they depend on the disturbances in the previous period and any new shocks in the current period (ε_t).

(2)
$$r_t^* = \mu + z_t$$

(2)
$$r_t^* = \mu + z_t$$

(3) $z_t = \rho z_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$

These three equations can be used alone to estimate the neutral real interest rate and can thus be regarded as a

²⁶ For the sake of simplicity, we assume here that $\sigma = 1$ in equation (1), which is in line with empirical calculations. Population growth is not taken into account.

²⁷ See inter alia Fuhrer (2000) for a brief overview of empirical and theoretical studies of habit formation in consumption, and for a presentation of the consequences for the Euler equation of expanding the household utility function to include this habit formation.

one-sided Hodrick-Prescott filter, because the filter does not use information about the real interest rate ahead in time to estimate the real interest rate today. However, we also want to use information about a possible economic relationship to estimate the neutral real interest rate and assume that the output gap (x_t) depends on the real interest rate gap $(r_t - r_t^*)$ as follows:

(4)
$$x_t = \alpha x_{t-1} - \beta (r_{t-1} - r^*_{t-1}) + \eta_t$$
 where $\eta_t \sim N(0, \sigma_n^2)$

This equation is a simplified version of the IS curve in Norges Bank's macroeconomic model (see Husebø et al. (2004)). By setting up these equations in "state-space" format, we can estimate the parameters (ρ , α and β), the standard deviation of the shocks e_t , ϵ_t and η_t and calculate the neutral real interest rate.²⁸ We take the output gap from Inflation Report 3/06 as given, and therefore do not estimate this variable. We place restrictions on the variance of the shocks (σ_{η}^2 =2.5, σ_{e}^2 =0.02 and σ_{ϵ}^2 =0.02) to ensure a relatively smooth path for the neutral real interest rate. These restrictions also contribute to the neutral real interest rate being influenced by both actual developments in the real interest rate and the economic relationship (4). Without these restrictions, the neutral real interest rate has a tendency to be estimated as equal to the actual real interest rate or the long-term equilibrium real interest rate. This is because the model (1)–(4) is very simple. The long-term equilibrium real interest rate (µ) is assumed here to be constant over time and is set at 2.5, and can be regarded as a measure of underlying productivity growth in the Norwegian economy.

Table 1 Parameter estimates

Parameter	Estimate	Standard deviation		
α	0.97	0.006		
β	0.17	0.005		
ρ	0.99	0.001		

The parameter estimates for the period 1981 Q1 – 2006 Q3 are shown in Table 1. They show that the output gap is strongly dependent on the output gap in the previous period, and that the real interest rate gap has a negative impact on the output gap. The coefficient ρ is approximately equal to 1, which means that developments in the neutral real interest rate can be regarded as a "random walk", i.e. that changes in the neutral real interest rate are permanent. However, this may be because the model that has been used is too simple. The estimates of the neutral real interest rate are shown in Chart 11.

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