STAFF MEMO

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Abstract

This paper documents a set of models used by Norges Bank in estimating the output gap. The models take into account developments in key cyclical indicators such as GDP[1], unemployment, inflation, wage growth, investment, house prices and credit growth. As the output gap cannot be observed, there is no direct way of evaluating the estimated output gap. Criteria for a good estimate of the output gap can, however, be the extent to which the output gap estimates provide information about future developments in GDP growth, inflation and unemployment. Measured this way, the models have good forecasting properties compared with simple trend estimations solely based on GDP data. The forecasting properties for an average of the models are shown to be better than for each individual model. The models' estimates of the output gap also have relatively good real-time properties.

1 Introduction

Norges Bank bases its assessments of the output gap, i.e. the difference between actual and potential output, on a broad set of indicators and models that are revised and expanded over time. This paper documents key models and indicators used in estimating the output gap.

The output gap is not observable, and there is no widely agreed best method for estimating the output gap. No method is perfect, and all methods involve the use of judgement. The academic literature has developed many methods based on different statistical methods and various economic theories about cyclical drivers. As the output gap is unobservable, it is also challenging to evaluate the different methods for estimating potential output.

*The views and conclusions in this publication are those of the authors and do not necessarily reflect those of Norges Bank and should therefore not be reported as representing the views of Norges Bank. We would like to thank Ida Wolden Bache, Tord Krogh, Per Espen Lilleås and Ingrid Solberg for helpful comments.

[1]GDP Mainland Norway

A good measure of the output gap should nevertheless satisfy certain criteria. The estimate of the output gap should have good real-time properties, i.e. the historical estimates of the output gap should show little change as a result of new information. Moreover, a common interpretation of potential output is output consistent with stable price and wage inflation. In periods when capacity utilisation is high and employment is growing rapidly relative to the labour force, price and wage pressures tend to increase. A good measure of the output gap should therefore provide information about future developments in inflation and wage growth. The output gap's ability to forecast GDP and unemployment has also been cited as evaluation criteria for the output gap (see Armstrong (2015) and Kamber et al. (2017)).

Many methods can be used to measure the output gap (see e.g. Hjelm and Jonsson (2010) for a good overview). The most widely used methods are simple univariate methods (statistical filters). These methods are simple in practice, and characteristically only use GDP data. The so-called Hodrick-Prescott (HP) filter is an example of a univariate method. The HP filter yields potential GDP by minimising the difference between actual and potential GDP given a limitation on how much potential GDP growth can vary over time. With sufficient information about the future, the HP filter probably provides a good picture of capacity utilisation, but it is well known that the HP filter has problems in estimating the output gap in real time (see Hamilton (2017) for a comprehensive discussion of the HP filter).

Norges Bank uses multivariate models which, in addition to GDP data, use data on other variables such as unemployment, wage growth, inflation, investment, credit growth and house prices. Such models have much better real-time properties and also have better real-time forecasting properties compared with simple univariate methods (HP filter). On the whole, an average of the models features better forecasting properties than the individual models.

The paper is structured as follows. Section 2 presents the different models used. Section 3 evaluates the output gap estimates derived from the different models, while Section 4 shows the output gap estimates from various indicators that are not included in the model system. Section 5 provides a summary.

2 Output gap models

To estimate the output gap, Norges Bank uses a set of models. The models use data on both real and nominal variables. The models are based on two different multivariate methods, and within each group of methods various models are estimated. The models within each group of models differ in terms of frequency and the data used. The different models used are presented in the following sections.

2.1 Unobserved component (UC) models

Unobserved component (UC) models is one class of models used to estimate the output gap. A UC model posits that GDP (y_t) can be decomposed into an output gap ($\hat{y}_t$) and potential GDP ($\bar{y}_t$) which are both unobservable:

$$y_t = \hat{y}_t + \bar{y}_t \tag{1}$$

In addition, the model specifies how the unobserved variables evolve over time:

$$\hat{y}_t = \lambda_y \hat{y}_{t-1} + \epsilon_t \tag{2}$$

$$\Delta \bar{y}_t = G_t + \eta_t \tag{3}$$

The output gap (equation (2)) depends on the output gap in the previous period plus a shock (ϵ_t) that is phased out over time. The change in potential GDP (equation (3)) depends on potential growth (G_t) and a shock to the level of potential GDP (η_t). Potential growth is also allowed to vary over time:[2]

$$G_t = C_G + \lambda_G (G_{t-1} - C_G) + \psi_t \tag{4}$$

In equation 4, ψ_t represents a shock to potential growth that is gradually phased out over time given by the parameter λ_G. C_G is a constant. An estimation of the four equations above is sufficient to obtain an estimate of the output gap. If we only use GDP data, this would be an example of a univariate model, and the HP filter can be derived as special case of the model above. Developments in variables other than GDP can, however, be useful in identifying the three GDP shocks above. Kuttner (1994), for example, uses the relationship between the output gap and inflation (Phillips curve) to inform the output gap. Apel and Jansson (1999) use in addition a relationship between the output gap and unemployment (Okun's law), and Domenech and Gomez (2006) emphasise the importance of the investment share in the economy as a key variable for identifying cyclical developments. Furthermore, Borio et al. (2013) argue that it is important to take into account data on financial variables when assessing potential output. The following UC models are estimated:

1. UC 1: GDP for mainland Norway, real wage growth[3] and registered unemployment (Norwegian Labour and Welfare Administration (NAV)). Annual data (1990-2017)
2. UC 2: GDP for mainland Norway, real wage growth[3] and unemployment (LFS). Annual data (1990-2017)
3. UC 3: GDP for mainland Norway, real wage growth[3], registered unemployment (NAV) and business investment as a percentage of GDP for mainland Norway. Annual data (1990-2017)
4. UC 4: GDP for mainland Norway and change in domestic inflation. Quarterly data (1990Q1 - 2017Q3)
5. UC 5: GDP for mainland Norway, registered unemployment (NAV) and domestic inflation. Quarterly data (1990Q1 - 2017Q3)

[2]Potential growth can also be assumed to be deterministic (constant) or a random walk (λ_G =1).
[3] Annual wage (TBU)/CPI

6. UC 6: GDP for mainland Norway and four-quarter growth in credit. Quarterly data (1990Q1 - 2017Q3)

7. UC 7: GDP for mainland Norway and four-quarter growth in house prices. Quarterly data (1990Q1 - 2017Q3)

The models differ in terms of estimation frequency, the data used, estimation period and modelling of potential growth. All of the models are estimated using Bayesian methods.[4] The models that are estimated on annual data are converted to quarterly data using the Denton algorithm (Denton (1971)). Missing data within a year are extrapolated using simple AR forecasts.

To illustrate the method in further detail, UC 1 is used, which is inspired by Blagrave et al. (2015). A Phillips curve for real wages and a relationship between unemployment and the output gap are specfied (Okun's law):

$$\hat{W}_t = \lambda_W \hat{W}_{t-1} + \gamma \hat{y}_t + \upsilon_t \tag{5}$$

$$\hat{u}_t = \lambda_u \hat{u}_{t-1} + \beta \hat{y}_t + \omega_t \tag{6}$$

Both the wage gap ($\hat{W}_t$) and the unemployment gap ($\hat{u}_t$) depend on the output gap ($\hat{y}_t$).[5]

Chart 1: Estimated output gap from an unobserved component model. Effect of including unemployment and real wage growth data. Percent. 1990-2016

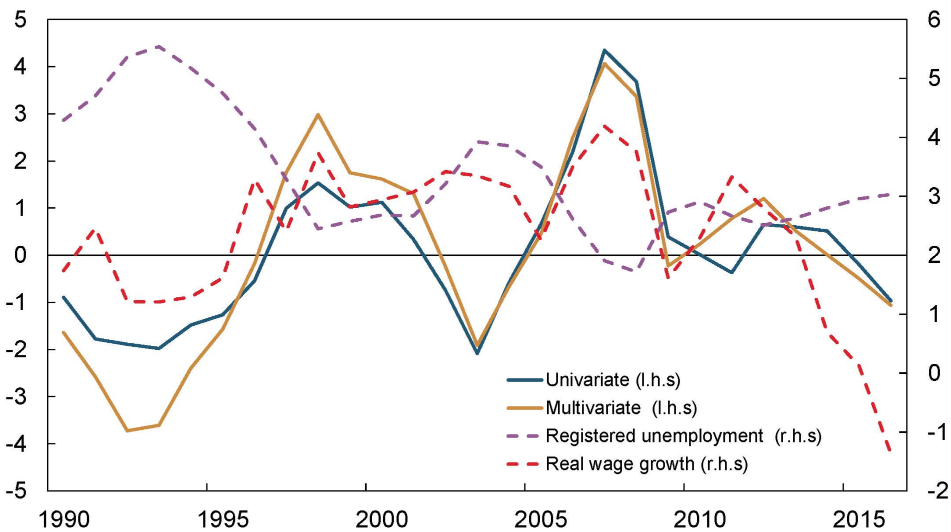


Chart 1 shows an estimate of the output gap when the model described above is estimated. The blue line shows an estimate of the output gap when only GDP data are used (i.e. when only the equations 1 to 4 are used). The broken lines in the chart show

[4]The models and estimation results are described further in the Appendix.

[5]Both wage and employment a time-varying trend are estimated, which is specified in the same way as for GDP (equation (3)), but they are omitted in the description here for the sake of simplicity. The full model specification is described in the Appendix.

unemployment and real wage developments, and the yellow line shows estimated output gap when the additional data are used (equation 5 and 6 are included). At the beginning of the 1990s, unemployment was high and real wage growth was low. If those data are taken into account in the estimation of the output gap, the gap is shown to be more negative at the beginning of the 1990s compared with an estimate that only uses GDP data. The multivariate model also indicates higher capacity utilisation at the end of the 1990s, while the estimation of the output gap in the period preceding the financial crisis is approximately the same. In the years following the financial crisis, the multivariate model indicates somewhat higher capacity utilisation, both as a result of higher real wage growth and lower unemployment. In the wake of the fall in oil prices in 2014, the model indicates somewhat lower capacity utilisation compared with the univariate model, primarily due to a sharp fall in real wage growth.

2.2 Structural VAR models

Another model class of models used to estimate the output gap is structural VAR models. Like the UC models, structural VAR models use data from a number of variables to estimate the output gap. The starting point is to estimate a VAR model given by:

$$y_t = \mu_0 + A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \tag{7}$$

where y_t is a vector of endogenous variables, μ_0 is a vector of constant terms and u_t is a vector of error terms. A_l for $l \in [1, p]$ is a matrix with coefficients for p lag of y.

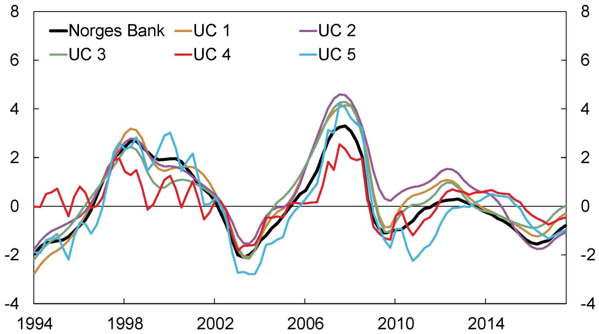
Two VAR models are estimated inspired by Blanchard and Quah (1989) and Cerra and Saxena (2000). The model inspired by Blanchard and Quah (1989) (SVAR 1) uses GDP growth for mainland Norway and unemployment (NAV), while the model based on Cerra and Saxena (2000) (SVAR 2) also includes domestic inflation. To estimate the output gap, the models must be identified so that GDP growth can be decomposed into structural shocks. In Blanchard and Quah (1989), it is assumed that GDP growth is driven by two types of shock: demand shocks and (permanent) supply-side shocks. Demand shocks are identified as shocks that do not affect GDP in the long term. Potential output is thereafter defined as GDP in the absence of the identified demand shocks. Cerra and Saxena (2000) identify in addition a temporary supply-side shock that does not have long-term effects on domestic inflation or GDP. Like Blanchard and Quah (1989), potential output is given by the part of GDP that is only explained by the permanent supply-side shocks.

2.3 Output gap estimates from the different models

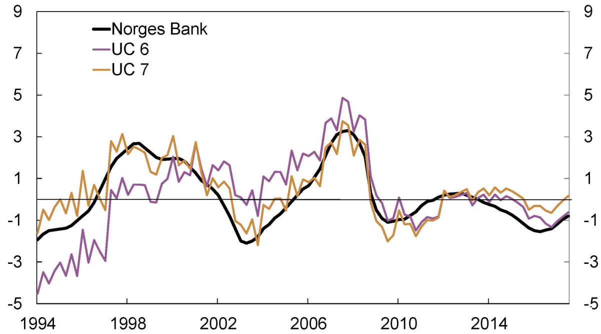
Chart 2 shows output gap estimates from the different models together with Norges Bank's assessments of the output gap as presented in the Monetary Policy Report 4/17 (Norges Bank (2017)). Panel (a) shows the estimates based on the first five unobserved component models. Norges Bank's estimates of the output gap are closely in line with the estimates from the different models. However, in retrospect, some of the models indicate that capacity utilisation was somewhat higher in the pre-crisis period than in Norges

Bank's assessment. In the period following the financial crisis, the different models have a somewhat different view of the path of the output gap, in particular the model that uses LFS unemployment data (UC 2). This model indicates that the output gap was positive until the fall in oil prices in summer 2014. Following the fall in oil prices, this model also indicates that the output gap has been substantially more negative compared with the other models.

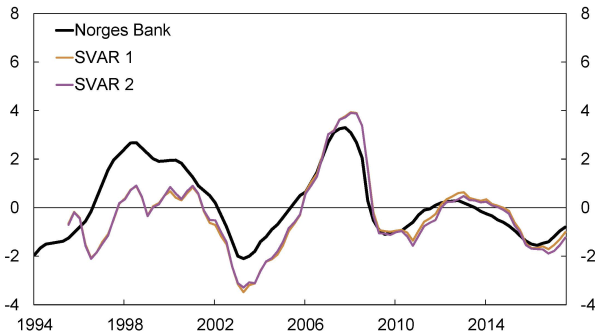
Chart 2: *Output gap estimates from the different models and output gap estimates weighted together. 1994Q1 - 2017Q3*



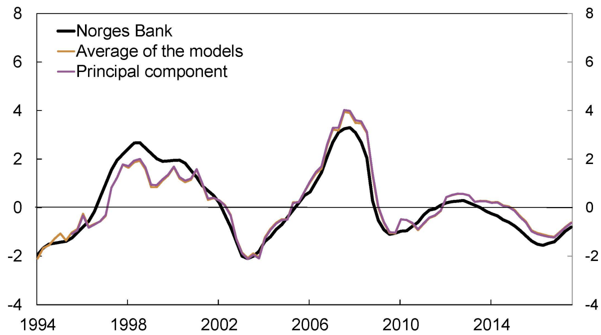
(a) UC models



(b) Financial UC models



(c) SVAR models



(d) Weighted estimates

The output gap estimates from the financial UC models (panel (b)) show somewhat different paths. The model where credit growth is used to provide information on the output gap indicates that the output gap was substantially lower in the mid-1990s. This is because credit growth was low in this period compared with its historical average, and the growth potential of the economy is therefore assessed to be higher. In the 2000s, credit growth accelerated. This may indicate that financial imbalances were building up and that potential growth was therefore lower than output consistent with stable inflation. The model therefore indicates that the output gap was substantially higher during the downturn in 2003-2004 and in the period preceding the financial crisis. In the period following the financial crisis, the model is closely in line with Norges Bank's assessments, but indicates that the potential growth has been somewhat lower in recent years. The model that includes house price inflation does not present the same picture as the gap based on credit growth, probably because house price developments correlate to a greater extent with GDP developments, and that house price developments therefore give little information beyond the information provided by GDP growth.

The two structural VAR models (panel (c)) largely show the same output gap path, and are closely in line with Norges Bank's assessments of the output gap, especially in the period following the financial crisis. The VAR models do not indicate the same positive output gap at the end of the 1990s compared with the UC models and the assessments of Norges Banks. The structural VAR models therefore indicate that the growth potential of the economy was higher at the end of the 1990s and at the beginning of the 2000s. Thus, this also indicates that the output gap was more negative during the downturn in 2003-2004.

Overall, the different models are closely in line with Norges Bank's output gap estimates over time. Panel (d) shows different ways to weight together the different estimates of the output gap. The chart shows a simple unweighted average and a principal component (a common factor).[6] The two ways to weight together the different estimates mainly show the same picture. The models indicate that potential growth has been somewhat lower in recent years compared with Norges Bank's assessments.

3 Evaluation of the models

Since the output gap is not observable, there is no direct way of evaluating the model estimates. Evaluating output gap estimates is therefore demanding. Nevertheless, there are some criteria that good estimates of the output gap should satisfy:

1. Inform future inflation, unemployment and GDP, both "in-sample" and "out-of-sample"
2. Ex-post revisions should not be too extensive, i.e. that the estimate of the output gap does not change substantially when more information about the future emerges.

To evaluate point 1, the following equations are estimated where the output gap estimates from the different models are included as explanatory variables[7]:

$$u_{t+1} - u^*_{t+1} = \alpha + \beta \hat{y}_t + \varepsilon_t \tag{8}$$

$$y_{t+h} - y_t = \alpha + \beta \hat{y}_t + \varepsilon_t \tag{9}$$

$$\pi_{t+1} = \alpha + \sum_{s=0}^{3} \lambda_s \pi_{t-s} + \sum_{s=0}^{3} \beta_s \hat{y}_{t-s} + \varepsilon_t \tag{10}$$

$$cpi_{t+h} - cpi_t = \alpha + \beta \hat{y}_t + \varepsilon_t \tag{11}$$

where u_t is registered unemployment (NAV) and u^*_t is estimated steady state unemployment rate[8] from the Special Feature in Monetary Policy Report 2/17. y_t is the logarithm

[6]Principal component analysis is a method for transforming a set of (potentially) correlated variables into a set of uncorrelated variables called principal components. Panel (d) shows the first principal component, which is the component that explains most of the variation in the estimated output gaps.

[7]The evaluation equations are obtained from Bjørnland et al. (2005), Armstrong (2015) and Kamber et al. (2017)

[8]Unemployment that is consistent with normal capacity utilization.

of mainland GDP, π_t is four-quarter growth in domestic inflation (CPI-ATED) an cpi_t is the logarithm of domestic prices (CPI-ATED).

Equation 8 is based on Okun's law, which states that there is a close and stable relationship between fluctations in the output gap and unemployment over time. As illustrated in Monetary Policy Report 1/15 (Norges Bank (2015)) unemployment reacts with a lag of one to two quarters to variations in output. Therefore a good measure of the output gap should inform unemployment one quarter ahead. Equation 9 tests how accurately the output gap today predicts GDP further out in time. If the output gap is positive, GDP growth should slow further out, and the opposite should occur if the output gap is negative. How accurately the output gap predicts future GDP developments is therefore an important evaluation criterion. Equation 10 is a simple Phillips curve, where inflation (given by the four-quarter growth in domestic inflation) depends on own dynamics and the output gap in preceding periods. The last equation tests how accurately the output gap predicts changes in CPI along different time horizons.

3.1 Results

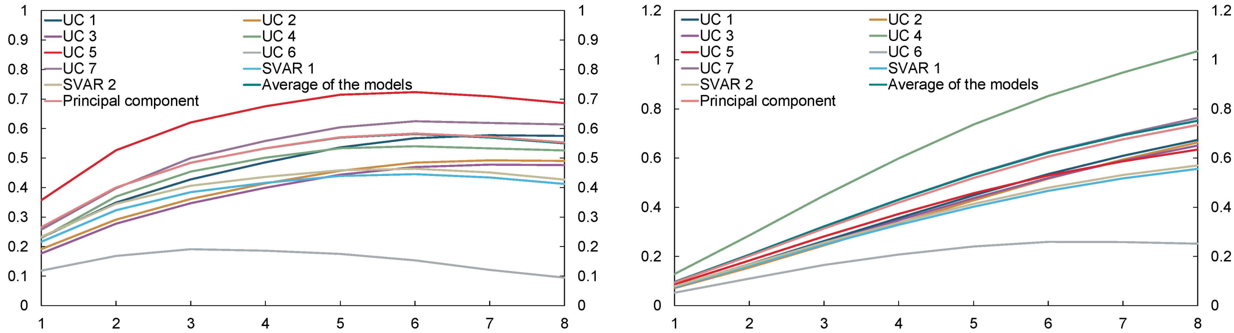
Table 1 shows the evaluation based on equations (8)-(11) "in-sample". Here, the equations are estimated when all available data are used, and it is tested for whether the output gap is significant ($Sign$), the sign and size of the coefficient (β) and the degree of the variation in the different variables that can be explained (R^2). The first three columns in Table 1 show the results for domestic inflation. The sum of the coefficients of the output gap is positive in all of the models, and is of the same size (column 1). Column 2 shows the results from a so-called likelihood ratio test, which tests whether the output gap provides significant explanatory power beyond what is already explained by the own dynamics of inflation. This is the case for all of the models, and the output gap explains, together with own dynamics, approximately 90 percent of the variation in domestic inflation. All of the output gap models also help to explain the future changes in the level of CPI. Chart 3 shows the size of the output gap coefficient in equation (11) and R^2 at different horizons. Most of the models indicate that an increase in the output gap of one percentage point results in an increase in CPI between 0.6 and 0.8 percentage point after two years. The explanatory power of the output gap (given by R^2) appears to be strongest for the four to six quarter horizons. The output gap from the different models is negative and significant for unemployment. The estimated output gaps also explain much of the variation in unemployment. The different output gap estimates also explain changes in GDP, especially at longer horizons (two to eight quarters). The output gap estimates are negative in all of the models, but most of the models have on average (over time horizons where the output gap is significant) a relatively low explanatory power. However, both the size of the output gap coefficient and its explanatory power increase with the time horizon, see Chart 4. A simple HP filter ($\lambda = 40000$) also explain much of the variation in inflation, GDP and price changes at different horizons and unemployment "in-sample", which indicates that the HP filter provides a good picture of the output gap when there is sufficient data about the future is used when the output gap is estimated.

Table 1: Evaluation of the output gap models "in-sample".

	Inflation			Change in CPI			Unemployment			Change in GDP		
	π_{t+1}			$cpi_{t+h} - cpi_t$			$u_{t+1} - u*_{t+1}$			$y_{t+h} - y_t$		
	β^1	*Sign.*[2]	R^2	β^4	*Sign.*[3]	R^{2^4}	β	*Sign.*[3]	R^2	β^4	*Sign.*[3]	R^{2^4}
HP 40 000	0.21	Yes	0.89	0.30	h=1-8	0.45	-0.26	Yes	0.84	-0.42	h=1-8	0.18
UC 1	0.21	Yes	0.90	0.39	h=1-8	0.47	-0.34	Yes	0.94	-0.38	h=4-8	0.11
UC 2	0.22	Yes	0.89	0.38	h=1-8	0.40	-0.33	Yes	0.92	-0.43	h=3-8	0.15
UC 3	0.20	Yes	0.89	0.38	h=1-8	0.38	-0.36	Yes	0.89	-0.41	h=4-8	0.10
UC 4	0.20	Yes	0.90	0.63	h=1-8	0.46	-0.45	Yes	0.62	-0.69	h=2-8	0.11
UC 5	0.22	Yes	0.90	0.39	h=1-8	0.63	-0.28	Yes	0.80	-0.57	h=2-8	0.23
UC 6	0.21	Yes	0.88	0.19	h=1-8	0.15	-0.20	Yes	0.39	-0.37	h=1-8	0.14
UC 7	0.21	Yes	0.90	0.46	h=1-8	0.52	-0.30	Yes	0.57	-0.36	h=1-8	0.08
SVAR 1	0.21	Yes	0.89	0.34	h=1-8	0.38	-0.32	Yes	0.83	-0.77	h=2-8	0.29
SVAR 2	0.18	Yes	0.89	0.35	h=1-8	0.40	-0.32	Yes	0.83	-0.78	h=2-8	0.29
Average of the models	0.21	Yes	0.89	0.46	h=1-8	0.49	-0.38	Yes	0.90	-0.60	h=1-8	0.19
Principal component	0.21	Yes	0.89	0.45	h=1-8	0.50	-0.37	Yes	0.91	-0.69	h=2-8	0.20

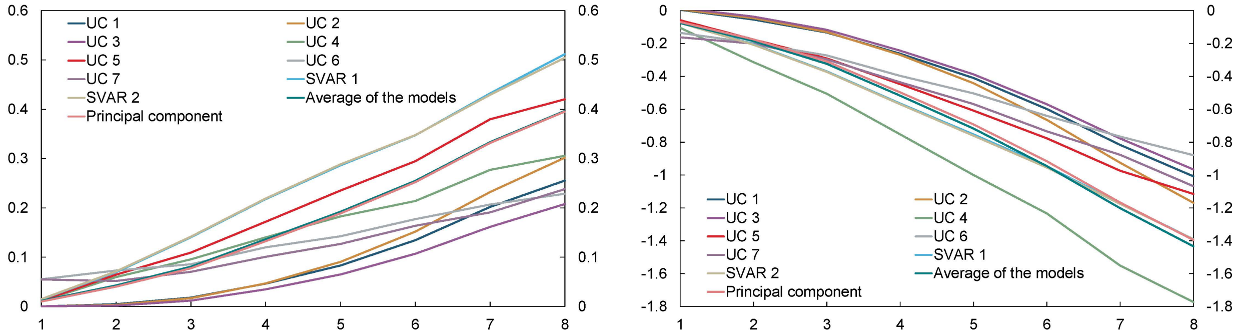
1) The sum of the coefficients of the output gap in equation 10 ($\sum_{s=0}^{3} \beta_s$).
2) Likelihood ratio test of whether the output gap has significant explanatory power on a 5 percent significance level beyond what is explained by own dynamics of inflation.
3) t-test with a 5 percent significance level for the output gap coefficient. For GDP growth and change in the CPI, the horizons where the output gap is significant are specified.
4) Average over horizons where the output gap is significant.

Chart 3: *"In-sample" evaluation of changes in the CPI at different horizons*



(a) R^2 (b) β

Chart 4: *"In-sample" evaluation of changes in GDP at different horizons.*



(a) R^2 (b) β

Table 2 shows the results when evaluating the output gap's ability to predict inflation, the change in CPI and GDP at different horizons and unemployment "out-of-sample". In this exercise, we estimate the output gaps and equations (8)-(11) recursively, and then project inflation and unemployment the quarter after the estimation is stopped.[9] For the change in CPI and GDP we make projections one to eight quarters ahead. The models are evaluated by comparing the squared deviations between actual observations of inflation, the change in CPI and GDP and unemployment with the forecasts from the models (MSFE). Standardisation is carried out by examining the different output gaps' forecast errors relative to the forecast errors from an output gap estimated using a simple HP filter ($\lambda = 40000$). As shown in the table, the average of the multivariate models have relatively low forecast errors for both GDP, unemployment and CPI. With the exception of the ability to project inflation one quarter ahead, most of the models perform better than the HP filter (values over (below) 1 mean that the model has a larger (smaller) average forecast error than a HP filter). This applies in particular to GDP and unemployment.

[9] First, the output gaps and equations (11)-(14) are estimated using data up to 2004 Q4 and then estimates are given for 2005 Q1. Thereafter the estimation period is extended by one quarter at a time up to 2017 Q2.

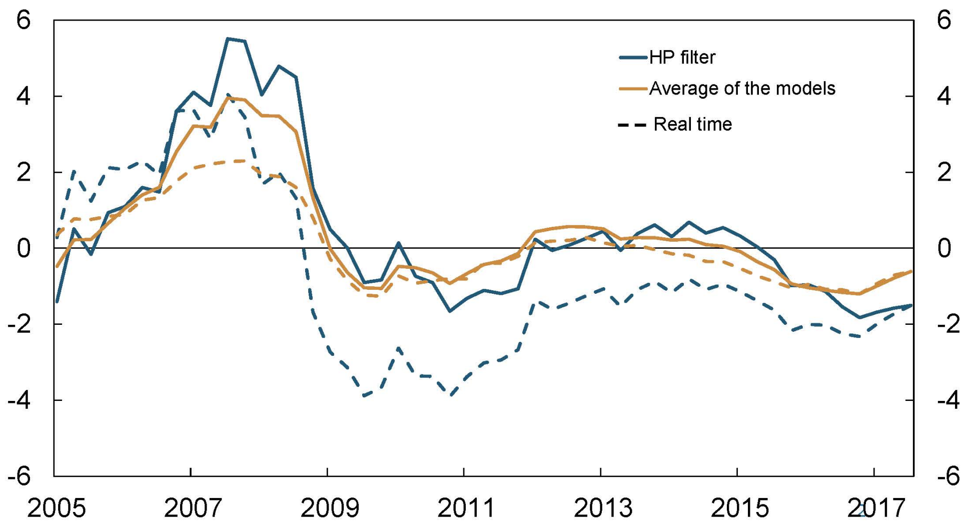
Table 2: Evaluation of the output gap models. Forecast error (MSFE) relative to a HP filter ($\lambda = 40000$)

Model	Change in GDP $y_{t+h} - y_t$	Unemployment $u_{t+1} - u*_{t+1}$	Inflation π_{t+1}	Change in CPI $cpi_{t+h} - cpi_t$
UC 1	0.72	0.15	1.28	1.04
UC 2	0.75	0.16	1.19	0.90
UC 3	0.77	0.32	1.38	1.26
UC 4	0.68	0.62	1.01	0.63
UC 5	0.78	1.12	0.99	0.87
UC 6	0.91	1.94	1.49	1.03
UC 7	1.34	1.64	0.99	1.46
SVAR 1	0.66	0.20	1.1	1.23
SVAR 2	0.63	0.21	1.03	0.96
Average of the models	0.74	0.27	1.06	0.85
Principal component	0.75	0.29	1.05	0.82

The evaluation uses equations (8)-(11). For GDP and the change in CPI, MSFE is based on the average one to eight quarters ahead.

The second criterion for a good estimate of the output gap is that the output gap, as measured in real time, does not change substantially when more data become available. Chart 5 shows output gap estimates from a simple HP filter and the average of the models estimated in this paper, both when the entire data set is used and in real time. The difference between the "final" output gap estimate and the real-time estimate is considerably smaller for the multivariate models compared with a simple HP filter (Chart 5). However, the difference in output gap estimates from the two methods is reduced when the HP filter uses information about future GDP developments.

Chart 5: Real-time properties of the multivariate models and a HP filter ($\lambda = 40000$).



Overall, the evaluation suggests that multivariate models perform better than univariate models based on the evaluation criteria defined above. In addition, an unweighted average of the multivariate models has good forecasting properties for GDP, unemployment and CPI. The average will probably also be more robust over time than the individual models since a number of data sources and different model specifications are weighted together.

4 Labour market indicators

In the assessment of the output gap, significant weight is put on developments in the labour market. For a substantial part of important indicators from the labour market we have insufficient historical data, and they can for this reason not be included in the multivariate models. An indicator system for the output gap, including indicators mainly from the labour marked, have therefor been constructed. Registered unemployment and LFS unemployment is included both in the models and in the indicator system. Unemployment is a central variable in the assessment of the labour market and Norges Banks assessment of the steady state unemployment rate may deviate from the model estimates. Norges Banks assessment of the unemployment gaps are therefore included in the set of indicators.

The labour market indicators are, together with the model estimates, a part of the overall assessment of the output gap. Table 3 provides an overview of the labour market indicators, how they are constructed, and whether the indicators lead/lag the output gap. All of the indicators are standardised trough a simple regression between the output gap (as Norges Bank has assessed it historically) and the indicators. The following equations are estimated for each of the indicators:

$$\hat{y}_t^{NB} = \alpha + \beta x_s \tag{12}$$

where $\hat{y}^{NB}$ is the output gap, as assessed by Norges Bank historically, and where x the relevant indicator. s specifies which time shift indicator currently has the highest correlation with the output gap. The different indicators' estimates of the output gap are given by $\hat{y}_t = \hat{\alpha} + \hat{\beta} x_s$

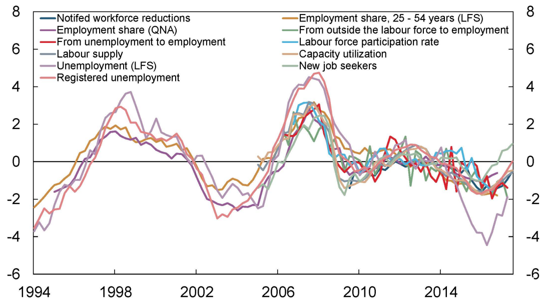
Table 3: Labour market indicators

Indicator	Source	Construction	Lag[1]
Labour supply	Reg. network.	Direct indicator	- 1
Capacity utilization	Reg. network.	Direct indicator	- 1
Labour force participation rate, 15-74 years	LFS	Trend from 2013, demographically adjusted	+ 3
Employment share, 25 - 54 years	LFS	Deviation from average	+ 3
Employment share	QNA	Trend from 2013, demographically adjusted	+ 3
Registered unemployment	NAV	Trend calculated as in MPR 2/17	+ 1
Unemployment	LFS	Trend calculated as in MPR 2/17	+ 1
Job vacancies	Statistics Norway	Share of employed	- 3
Notified workforce reductions	NAV	Share of employed	- 2
New job seekers	NAV	Share of employed	0
From unemployment to employment	AKU	Share of unemployed	+ 1
From outside the labour force to employment	AKU	Share of persons outside the labour force	+ 1

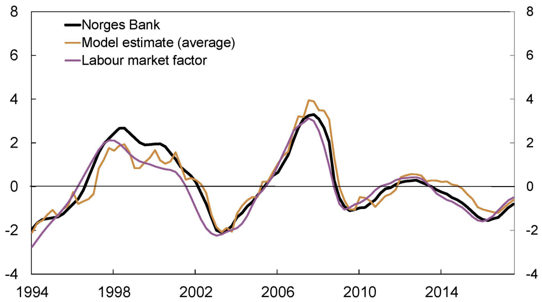
1) Denotes whether the indicator leads the output gap or vice versa. -/+ denotes that the indicator leads/lags the output gap. 0 indicates contemporaneous correlation.

Chart 6a shows the individual indicators and Chart 6b shows the output gap estimates of an average of the models and a common factor[10] for the labour market indicators and the output gap as they were presented in Monetary Policy 4/17. Norges Bank's output gap estimates are closely in line with the labour market indicators, but they give some indication that the growth potential of the economy has been somewhat lower recently, particularly compared with the model estimate. As more historical data are accumulated for the different indicators and new insight is gained, these indicators can be incorporated into the models.

Chart 6: *Labour market indicators*



(a) Labour market indicators



(b) Labour market factor, model estimates and Norges Banks output gap

5 Summary

This article has presented the models currently used by Norges Bank as a basis for assessing the output gap.[11] The models use data from a number of key cyclical indiactors, such as GDP, unemployment, inflation, wage growth, investment and financial variables. The models satsify the criteria for a good measure of the output gap. Compared with simple trend estimates, which only use GDP data, the models have good real-time properties, i.e. the historical estimates of the output gap show little change when new information emerges. Most of the models have good forecasting properties compared with a simple HP-filter. The results suggest that an (unweighted) average of the models generally has better forecasting properties than individual models.

In the assessment of the output gap, there are a number of key indicators that have not yet been included in the models, partly owing to insufficient historical data for some of these indicators. Overall, these indicators are closely in line with the estimates from the models, but suggests to a certain extent that potential growth has been lower in recent years. As longer historical data on the indicators are obtained and more insight is gained, these can be incorporated into the model framework.

[10]This common factor is estimated using a Kalman Filter. The Kalman Filter estimates the common factor by imposing that the output gap cannot change substantially from one quarter to the next, at the same time as the output gap must weight together the indicators over time. This method for weighting together the indicators is particularly well suited in situations with unbalanced data sets.

[11]There is no mechanical relationship between the model estimates and Norges banks' estimate of the output gap.

6 Appendix

6.1 UC model 1 og 2: GDP mainland Norway, unemployment (NAV/LFS) og real wage growth

Definition of GDP:

$$y_t = \hat{y}_t + \bar{y}_t \tag{13}$$

Process for the output gap:

$$\hat{y}_t = \lambda_y \hat{y}_{t-1} + \epsilon_t \tag{14}$$

Process for the growth in potential GDP:

$$\Delta \bar{y}_t = G_t + \eta_t \tag{15}$$

Process for potential growth:

$$G_t = C_G + \lambda_G (G_{t-1} - C_G) + \psi_t \tag{16}$$

Process for the real wage gap:

$$\hat{W}_t = \lambda_W \hat{W}_{t-1} + \gamma \hat{y}_t + \upsilon_t \tag{17}$$

Process for the trend in real wages:

$$\Delta \bar{W}_t = C_W + \lambda_{\bar{W}} (\Delta \bar{W}_{t-1} - C_W) + \mu_t \tag{18}$$

Process for the unemployment gap:

$$\hat{u}_t = \lambda_u \hat{u}_{t-1} + \beta \hat{y}_t + \omega_t \tag{19}$$

Process for the change in unemployment trend (NAIRU):

$$\Delta \bar{u}_t = \lambda_{\bar{u}} \Delta \bar{u}_{t-1} + \upsilon_t \tag{20}$$

Table 4: UC 1. Estimated and calibrated parameters. Annual data. Estimation period: 1990-2017

Parameter	Prior	Prior distribution	Posterior
λ_y	0.7(0.2)	Gamma	0.78(0.12)
λ_G	0.9(0.2)	Gamma	0.79(0.12)
λ_W	0.75(0.25)	Gamma	0.89(0.14)
$\lambda_{\bar{W}}$	0.6(0.2)	Gamma	0.60(0.13)
λ_u	0.5(0.2)	Gamma	0.43(0.08)
$\lambda_{\bar{u}}$	0.9(0.1)	Gamma	0.87(0.08)
γ	0.29	Calibrated	
β	-0.29	Calibrated	
σ_ϵ	2(10)	Inverse Gamma	1.24(0.18)
σ_η	2(10)	Inverse Gamma	0.74(0.24)
σ_ψ	1(10)	Inverse Gamma	0.35(0.17)
σ_υ	0.5(10)	Inverse Gamma	0.16(0.09)
σ_μ	2(10)	Inverse Gamma	0.77(0.11)
σ_ω	0.4(10)	Inverse Gamma	0.09(0.03)
σ_v	0.2(10)	Inverse Gamma	0.06(0.03)

Standard deviation in parantheses.

Table 5: UC 2. Estimated and calibrated parameters. Annual data. Estimation period: 1990-2017

Parameter	Prior	Prior distribution	Posterior
λ_y	0.7(0.2)	Gamma	0.80(0.16)
λ_G	0.9(0.2)	Gamma	0.77(0.13)
λ_W	0.75(0.25)	Gamma	0.88(0.12)
$\lambda_{\bar{W}}$	0.6(0.2)	Gamma	0.57(0.13)
λ_u	0.5(0.2)	Gamma	0.50(0.07)
$\lambda_{\bar{u}}$	0.9(0.1)	Gamma	0.85(0.09)
γ	0.29	Calibrated	
β	-0.29	Calibrated	
σ_ϵ	2(10)	Inverse Gamma	1.15(0.27)
σ_η	2(10)	Inverse Gamma	0.83(0.30)
σ_ψ	1(10)	Inverse Gamma	0.31(0.17)
σ_υ	0.5(10)	Inverse Gamma	0.17(0.10)
σ_μ	2(10)	Inverse Gamma	0.78(0.12)
σ_ω	0.4(10)	Inverse Gamma	0.23(0.07)
σ_v	0.2(10)	Inverse Gamma	0.05(0.03)

Standard deviation in parantheses.

6.2 UC model 3: GDP for mainland Norway, unemployment (NAV), real wage growth and investment share in mainland Norway

In addition to equations (13 til 20)), a correlation is included between investment share (business investment as share of GDP for mainland Norway) and the output gap, inspired by Domenech and Gomez (2006).

$$x_t = \lambda_x x_{t-1} + (1 - \lambda_x)\bar{x}_t + \gamma_x \hat{y}_t + e_t \tag{21}$$

$$\Delta \bar{x}_t = z_t \tag{22}$$

where x_t is the investment share and $\bar{x}_t$ is the trend component of the investment share. e_t is a demand shock to the investment share, while z_t represents a shock to the level of potential investment.

Table 6: UC 3. Estimated and calibrated parameters. Annual data. Estimation period: 1990-2017

Parameter	Prior	Prior distribution	Posterior
λ_y	0.7(0.2)	Gamma	0.73(0.15)
λ_G	0.9(0.2)	Gamma	0.79(0.12)
λ_W	0.75(0.25)	Gamma	0.88(0.13)
$\lambda_{\bar{W}}$	0.6(0.2)	Gamma	0.61(0.13)
λ_u	0.5(0.2)	Gamma	0.48(0.08)
$\lambda_{\bar{u}}$	0.9(0.1)	Gamma	0.85(0.1)
λ_x	0.7(0.2)	Gamma	0.4(0.07)
γ	0.29	Calibrated	
β	-0.29	Calibrated	
γ_x	0.5	Calibrated	
σ_ϵ	2(10)	Inverse Gamma	1.23(0.22)
σ_η	2(10)	Inverse Gamma	0.59(0.23)
σ_ψ	1(10)	Inverse Gamma	0.43(0.24)
σ_υ	0.5(10)	Inverse Gamma	0.17(0.1)
σ_μ	2(10)	Inverse Gamma	0.80(0.13)
σ_ω	0.4(10)	Inverse Gamma	0.11(0.04)
σ_v	0.2(10)	Inverse Gamma	0.08(0.06)
σ_e	1(10)	Inverse Gamma	0.05(0.03)
σ_z	1(10)	Inverse Gamma	0.05(0.03)

Standard deviation in parentheses.

6.3 UC modell 4: GDP mainland Norway and change in domestic inflation

Definition of GDP:

$$y_t = \hat{y}_t + \bar{y}_t \tag{23}$$

Process for the output gap:

$$\hat{y}_t = \lambda_y \hat{y}_{t-1} + \epsilon_t \tag{24}$$

Process for the change in potential GDP:

$$\Delta\bar{y}_t = \Delta\bar{y}_{t-1} + \eta_t \tag{25}$$

Process for the change in domestic inflation:

$$\Delta\pi_t = \gamma\hat{y}_t + \upsilon_t + \delta\upsilon_{t-1} \tag{26}$$

$$\upsilon_t = \eta_t \tag{27}$$

The model above is based on Kuttner (1994) (see also Hjelm and Jonsson (2010)). The model relates the output gap ($\hat{y}$) to the change in domestic inflation ($\Delta\pi_t = \pi_t - \pi_{t-1}$). Modelling inflation in first differences means that potential GDP is given as the GDP level that is consistent with constant inflation.

Table 7: UC 4. Estimated and calibrated parameters. Quarterly data. Estimation period: 1990Q1-2017Q3

Parameter	Prior	Prior distribution	Posterior
λ_y	0.9(0.2)	Gamma	0.9(0.06)
γ	0.1[0,1]	Uniform	0.03(0.07)
δ	0.3(0.2)	Gamma	0.1(0.07)
σ_ϵ	0.7(10)	Inverse Gamma	0.48(0.04)
σ_η	0.1(10)	Inverse Gamma	0.12(0.04)
σ_υ	0.2(10)	Inverse Gamma	0.1(0.01)

Standard deviation in parentheses.

6.4 UC model 5 :GDP mainland Norway, domestic inflation and unemployment (NAV)

Definition of GDP:

$$y_t = \hat{y}_t + \bar{y}_t \tag{28}$$

Process for the output gap:

$$\hat{y}_t = \hat{y}_{t-1} + \beta\hat{u}_t + \epsilon_t \tag{29}$$

Process for the growth in potential GDP:

$$\Delta\bar{y}_t = \Delta\bar{y}_{t-1} + \eta_t \tag{30}$$

Process for the unemployment gap:

$$\hat{u}_t = \lambda_u \hat{u}_{t-1} + \omega_t \tag{31}$$

Prcess for the trend in unemployment (NAIRU):

$$\Delta\bar{u}_t = \Delta\bar{u}_{t-1} + v_t \tag{32}$$

Process for domestic inflation:

$$\pi_t = \lambda_\pi \pi_{t-1} + \gamma \hat{u}_{t-2} + \upsilon_t \tag{33}$$

Table 8: UC 5. Estimated and calibrated parameters. Quarterly data. Estimation period: 1990Q1-2017Q3

Parameter	Prior	Prior distribution	Posterior
λ_y	0.8(0.1)	Gamma	0.9(0.06)
β	-3.45	Calibrated	
λ_u	0.9(0.2)	Gamma	0.9(0.04)
γ	-0.4[-1,0]	Uniform	0.49(0.22)
λ_π	0.5(0.2)	Gamma	0.39(0.12)
σ_ϵ	0.7(10)	Inverse Gamma	0.41(0.04)
σ_η	0.1(10)	Inverse Gamma	0.04(0.04)
σ_ω	0.2(10)	Inverse Gamma	0.1(0.01)
σ_v	0.1(10)	Inverse Gamma	0.07(0.01)
σ_υ	0.2(10)	Inverse Gamma	0.21(0.02)

Standard deviation in parentheses.

6.5 UC modell 6 og 7: GDP mainland Norway and growth in credit/house prices

Definition of GDP:

$$y_t = \hat{y}_t + \bar{y}_t \tag{34}$$

Process for the output gap:

$$\hat{y}_t = \lambda_y \hat{y}_{t-1} + \gamma x_t + \epsilon_t \tag{35}$$

Process for the change in potential GDP:

$$\Delta \bar{y}_t = \Delta \bar{y}_{t-1} + \eta_t \tag{36}$$

Process for the financial variable (x):

$$x_t = x_{t-1} + \upsilon_t \tag{37}$$

Two models are estimated where x is total four-quarter credit growth (C2 households and enterprises) and house prices less their historical averages. In the models, the output gap's standard deviation (ϵ) is set equal to the standard deviation of change in the output gap estimated using an HP filter with λ equal to 1600, i.e. $\sigma_\epsilon = std(\Delta \bar{y}_t^{HP})$ where $\bar{y}^{HP}$ is the output gap using a simple HP filter. The standard deviation of potential output growth is then scaled up by a factor z z to ensure that the relative output gap variation is the same as for a normal HP filter, i.e. $\sigma_\eta = (1/z)\sigma_\epsilon$.

Table 9: UC 6. Estimated and calibrated parameters. Quarterly data. Estimation period: 1990Q1-2017Q3

Parameter	Prior	Prior distribution	Posterior
z	4.44	Calibrated	
λ_y	0.8(0.2)	Gamma	0.71(0.11)
γ	0.1(0.2)	Gamma	0.15(0.06)
σ_v	1.2(10)	Inverse Gamma	1.17(0.07)

Standard deviation in parentheses.

Table 10: UC 7. Estimated and calibrated parameters. Quarterly data. Estimation period: 1990Q1-2017Q3

Parameter	Prior	Prior distribution	Posterior
z	7.2	Calibrated	
λ_y	0.8(0.2)	Gamma	0.80(0.13)
γ	0.1(0.2)	Gamma	0.05(0.012)
σ_v	3(10)	Inverse Gamma	3.1(0.18)

Standard deviation in parentheses.

6.6 SVAR 1: Mainland GDP and unemployment (NAV)

VAR model for quarterly mainland GDP growth and unemployment (NAV) (Blanchard and Quah (1989)). The model includes two lags of the endogenous variables. A demand shock and a supply-side shock are identified. The demand shock is identified as a shock that has no long-term effects on GDP.

6.7 SVAR 2: Mainland GDP, unemployment (NAV) and domestic inflation

VAR model for quarterly mainland GDP growth, unemployment (NAV) and domestic inflation (CPI-ATED) (Cerra and Saxena (2000)). The model includes two lags of the endogenous variables. A demand shock, a temporary supply-side shock and a permanent supply-side shock are identified. The demand shock is identified as a shock that has no long-term effects on GDP, and the temporary supply-side shock is identified as a shock that has no long-term effects on domestic prices.

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