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GAUTI B. EGGERTSSON,
RAGNAR E. JUELSRUD,
LAWRENCE H. SUMMERS
AND ELLA GETZ WOLD

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Negative nominal interest rates and the bank lending channel*

Gauti B. Eggertsson† Ragnar E. Juelsrud‡

Lawrence H. Summers§ Ella Getz Wold¶

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Abstract

Following the crisis of 2008, several central banks engaged in a new experiment by setting negative policy rates. Using aggregate and bank level data, we document that deposit rates stopped responding to policy rates once they went negative and that bank lending rates in some cases increased rather than decreased in response to policy rate cuts. Based on the empirical evidence, we construct a macro-model with a banking sector that links together policy rates, deposit rates and lending rates. Once the policy rate turns negative, the usual transmission mechanism of monetary policy through the bank sector breaks down. Moreover, because a negative policy rate reduces bank profits, the total effect on aggregate output can be contractionary. A calibration which matches Swedish bank level data suggests that a policy rate of - 0.50 percent increases borrowing rates by 15 basis points and reduces output by 7 basis points.

*This working paper should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. This paper replaces an earlier draft titled *Are Negative Nominal Interest Rates Expansionary?* We are grateful to compricer.se and Christina Soderberg for providing bank level interest rate data. We are also grateful to seminar and conference participants at Bundesbanken, Brown University, CEF 2018, the European Central Bank, the University of Maryland, Norges Bank, Bank of Portugal, The Riksbank and Martin Flodén, Artashes Karapetyan, John Shea, Dominik Thaler and Michael Woodford for discussion. We thank INET for financial support.

†Brown University. E-mail: gauti.eggertsson@brown.edu

‡Norges Bank. E-mail: ragnar@juelsrud.no

§Harvard University. E-mail: lawrence_summers@harvard.edu

¶Brown University. E-mail: ella_wold@brown.edu

1 Introduction

Between 2012 and 2016, a handful of central banks reduced their policy rates below zero for the first time in history. While *real* interest rates have been negative on several occasions, *nominal* rates have not. The recent experience implies that negative policy rates have become part of the central banker's toolbox, and calls into question the relevance of the zero lower bound (ZLB). However, the impact of negative policy rates on the macroeconomy remains unknown. The goal of this paper is to contribute to filling this gap, by analyzing the effectiveness of negative policy rates in stimulating the economy through the bank lending channel.

Understanding how negative nominal interest rates affect the economy is important in preparing for the next economic downturn. Interest rates have been declining steadily since the early 1980s, resulting in worries about secular stagnation (see e.g. Summers 2014, Eggertsson and Mehrotra 2014 and Caballero and Farhi 2017). In a recent paper, Kiley and Roberts (2017) estimate that the ZLB will bind 30-40 percent of the time going forward. In Figure 1 we report interest rate cuts during previous recessions in the US and the Euro Area since 1970. On average, nominal interest rates are reduced by 5.9 and 5.5 percentage points respectively (see Table 4 in Appendix A for more details). With record low interest rates, policy rate cuts of this magnitude may be difficult to achieve in the future - without rates going negative.

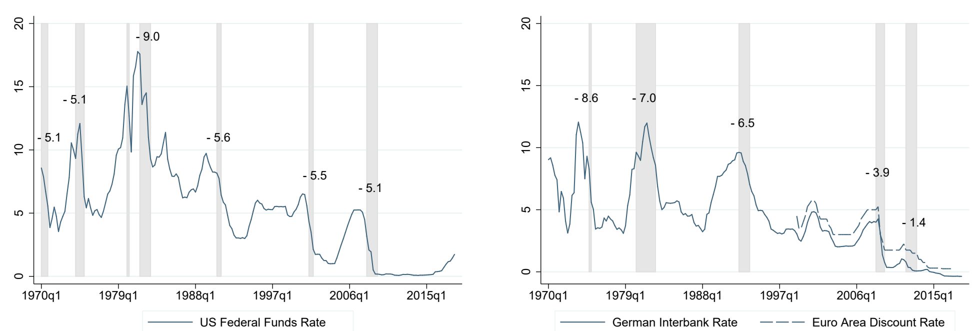


Figure 1: Interest rates for the US and the Euro Area. Source: St. Louis FRED.

An alternative to negative interest rates is unconventional monetary policy measures, such as credit easing, quantitative easing and forward guidance. There are several reasons, however, why it is important to consider policy measures beyond these tools. Some of the credit policies used by the the Federal Reserve, the FDIC and the Treasury were severely constrained by Congress following the crisis, as stressed by Bernanke, Geithner, and Paulson (2018). Hence, these options are no longer available without legislative change. Moreover, there remains little, if any, consensus among economists on how effective quantitative easing and forward guidance is. Plausible estimates range from considerable effects to none (see e.g. Greenlaw, Hamilton, Harris, and West (2018) for a somewhat skeptical review, Swanson (2017) for a more upbeat assessment, and Greenwood, Hanson, Rudolph, and Summers (2014) for a discussion of debt management at the zero lower bound). Accordingly, understanding the effectiveness of negative interest rates should be high on the research agenda.

Central banks which implemented negative rates argued that there is nothing special about zero. When announcing a negative policy rate, the Swedish Riksbank wrote in their monetary policy report that "*Cutting the repo rate below zero, at least if the cuts are in total not very large, is expected to have similar effects to repo-rate cuts when the repo rate is positive, as all channels in the transmission mechanism can be expected to be active"* (The Riksbank, 2015). Similarly, the Swiss National Bank declared that *"the laws of economics do not change significantly when interest rates turn negative*" (Jordan, 2016). Many were skeptical however. For instance, Mark Carney of the Bank of England was "*... not a fan of negative interest rates"* and argued that *"we see the negative consequences of them through the financial system*" (Carney, 2016). One such consequence is a reduction in bank profitability, which has caused concern in the Euro Area (Financial Times, 2016). Consistent with this view, Waller (2016) coined the policy a "*tax in sheep's clothing*", arguing that negative interest rates act as any other tax on the banking system and thus reduces credit growth.

In this paper we investigate the impact of negative rates on the macroeconomy, both from an empirical and theoretical perspective.[1] The first main contribution of the paper is to use a combination of aggregate and bank level data to examine the pass-through of negative rates via the banking system. We focus primarily on Sweden, which is an interesting starting point for multiple reasons. First and most importantly, we have unique daily bank level data for Swedish banks, which allows us to make inference about the pass-through. Second, the Swedish Riksbank reduced the policy rate multiple times in negative territory, providing more variation to work with than in the other countries. Third, there are important features of the Swedish economy which suggests that negative rates should work relatively well in Sweden. Not only do Swedish households have limited cash use, but banks also have low deposit shares relative to banks in the Euro Area (both considerations will turn out to be important in understanding the transmission of negative policy rates). Hence, if negative policy rates were not transmitted to lower bank rates in the Swedish banking system it is unlikely that this will happen in other countries.

[1]Note that we do not attempt to evaluate the impact of other monetary policy measures which occurred simultaneously with negative interest rates. That is, we focus exclusively on the effect of negative interest rates, and do not attempt to address the effectiveness of asset purchase programs or programs intended to provide banks with cheap financing (such as the TLTRO program initiated by the ECB).

We document that negative policy rates have had limited pass-through to deposit rates, which are bounded close to zero. This implies that policy rate cuts to negative levels are not transmitted to the main funding source of banks. What about bank lending rates? Using daily bank level data, we document that once the deposit rate becomes bounded by zero, interest rate cuts into negative territory lead to an increase rather than a decrease in lending rates. We document that this holds across a range of different loan contracts. In addition to a significant reduction in pass-through to lending rates, there is also a substantial increase in dispersion. We show that the rise in dispersion can be linked to banks financing structures. Banks that rely more heavily on deposit financing are less likely to reduce their lending rates once the policy rate goes negative. Focusing on bank level lending *volumes*, we show that Swedish banks which rely more heavily on deposit financing also have lower credit growth in the post-zero period. This is consistent with similar findings for the Euro Area (Heider, Saidi, and Schepens, 2016).

Motivated by these empirical results, the second main contribution of the paper is methodological. We construct a model, building on several papers from the existing literature, that allows us to address how changes in the policy rate filters through the banking system to various other interest rates, and ultimately determines aggregate output. The framework has four main elements. First, we introduce paper currency, along with money storage costs, to capture the role of money as a store of value and illustrate how this generates a bound on bank deposit rates. Second, we incorporate a banking sector and nominal frictions along the lines of Benigno, Eggertsson, and Romei (2014), which delivers well defined deposit and lending rates. Third, we incorporate demand for central bank reserves as in Curdia and Woodford (2011) in order to obtain a policy rate which can potentially differ from the commercial bank deposit rate. Fourth, we allow for the possibility that the cost of bank intermediation depends on banks' net worth as in Gertler and Kiyotaki (2010).

The central bank determines the interest rate on reserves and can set a negative policy rate as banks are willing to pay for the transaction services provided by reserves. Since money is a store of value however, the deposit rate faced by commercial bank depositors is bounded at some level (possibly negative), in line with our empirical findings. The bound arises because the bank's customers will choose to store their wealth in terms of paper currency if charged too much by the bank.[2] Away from the lower bound on the deposit rate, the central bank can stimulate the economy by lowering the policy rate. This reduces both the deposit rate and the rate at which households can borrow, thereby increasing demand. Once

[2] There are other reasons why there might be a lower bound on the deposit rate, which we do not explore in this paper. Rather, we choose to introduce a lower bound as a consequence of the combination of money as a store of value and storage costs, motivated by both the existing literature and survey evidence suggesting that households would withdraw cash had they faced a negative interest rate, see Figure 21 in Appendix A.

the deposit rate reaches its effective lower bound however, reducing the policy rate further is no longer expansionary. As the central bank loses its ability to influence the deposit rate, it cannot stimulate the demand of savers via the traditional intertemporal substitution channel. Furthermore, as banks' funding costs (via deposits) are no longer responsive to the policy rate, the bank lending channel of monetary policy breaks down. Using our bank level evidence from Sweden to match the observed increase in the interest rate spread in response to a negative policy rate, our model suggests that a policy rate of -0.5 percent increases borrowing rates by approximately 15 basis points and reduces output by about 7 basis points.

We do not analyze other parts of the monetary policy transmission mechanism, and thus cannot exclude the possibility that negative interest rates has an effect through other channels. Examples include any expansionary effects working through the exchange rate or asset prices. The main take-away of the paper is that the bank lending channel - traditionally considered one of the most important transmission mechanisms of monetary policy - collapses once the deposit rate becomes bounded, thus substantially reducing the overall effectiveness of monetary policy (see e.g. Drechsler, Savov, and Schnabl (2017) for evidence on the importance of deposit collection for bank funding in the US).

Literature review Jackson (2015) and Bech and Malkhozov (2016) document the limited pass-through of negative policy rates to aggregate deposit rates, but do not evaluate the effects on the macroeconomy. Heider, Saidi, and Schepens (2016) and Basten and Mariathasan (2018) document that negative policy rates have not lead to negative deposit rates in the Euro Area and Switzerland, respectively. While Basten and Mariathasan (2018) find that Swiss banks primarily reduce reserves in response to negative rates, Heider, Saidi, and Schepens (2016) find that banks with higher deposit shares have lower lending growth in the post-zero environment. We contribute to the empirical literature on the pass-through of negative rates by exploiting a unique dataset on daily bank level lending rates to provide novel micro evidence on the decoupling of lending rates from the policy rate. Furthermore, we show how the lack of pass-through to lending rates can be explained by cross-sectional variation in the reliance on deposit financing.

Given the radical nature of the policy experiment pursued by several central banks, the theoretical literature is perhaps surprisingly silent on the expected effects of this policy.[3][4] The study which is perhaps most related to our theoretical analysis is Brunnermeier and

[3] There is however a large literature on the effects of the zero lower bound. See for example Krugman (1998) and Eggertsson and Woodford (2006) for two early contributions.

[4] Our paper is also related to an empirical literature on the connection between interest rate levels and bank profits (Borio and Gambacorta 2017, Kerbl and Sigmund 2017), as well as a theoretical literature linking credit supply to banks net worth (Holmstrom and Tirole 1997, Gertler and Kiyotaki 2010).

Koby (2017), who contemplate a reversal rate in which further interest rate cuts become contractionary. The mechanism in their paper is different from ours, however, and not motivated by the zero lower bound that is generated by the existence of cash giving rise to a bound on deposit rates. The reversal rate they analyze depends on maturity mismatch on the bank's balance sheet and net interest margin on new business, making the reversal rate time varying and dependent on market structure and balance sheet characteristics, as well as whether interest rate changes are anticipated or not. The lower bound on the deposit rate, which is the key mechanism in our analysis, does not feature into their model.[5] Moreover, the deposit bound is independent of the features considered in Brunnermeier and Koby (2017) (such as maturity mismatch, market structure etc.). The deposit bound has strong empirical support, and we derive it theoretically from the households' portfolio allocation problem. In our model, as soon as the deposit rate reaches the lower bound, further interest rate cuts are no longer expansionary – in line with the data.

Rognlie (2015) also analyses the impact of negative policy rates theoretically. However, in his model households face only one interest rate, and the central bank can control this interest rate directly. Thus, the model does not allow for a separate bound on deposit rates which is critical for our analysis.

There exists an older literature, dating at least back to the work of Silvio Gesell more than a hundred years ago, which contemplates more radical monetary policy regime changes than we do here (Gesell, 1916). In our model, the storage cost of money, and hence the lower bound, is treated as fixed. However, policy reforms could change this cost and thus change the lower bound directly. An example of such policies is a direct tax on paper currency, as proposed first by Gesell and discussed in detail by Goodfriend (2000) and Buiter and Panigirtzoglou (2003) or actions that increase the storage cost of money, such as eliminating high denomination bills. Another possibility is abolishing paper currency altogether. These policies are discussed in, among others, Agarwal and Kimball (2015), Rogoff (2017a) and Rogoff (2017b), who also suggest more elaborate policy regimes to circumvent the ZLB. The results presented here do not contradict these ideas. Rather, they suggest that given the current institutional framework, negative interest rates are not an effective way to stimulate aggregate demand via the bank lending channel.

[5] In an updated version of the paper, they acknowledge that if there is a lower bound on the deposit rate this can be an additional factor that can influences the reversal rate. In their calibrated model, however, they find a reversal rate of -1 %. This reversal rate implies that the negative rates which have been implemented so far (the lowest being -0.75% in Switzerland), should be expansionary, which is at odds with our empirical findings for Sweden.

2 Negative Interest Rates in Practice

In this section, we investigate the pass-through of negative interest rates to deposit and lending rates. We focus on Sweden, for which we have daily bank level data on lending rates.

2.1 Bank Financing Costs

Most accounts of expansionary monetary policy focus on how a cut in policy rates will lower lending rates, and thus stimulate aggregate demand. The usual transmission mechanism works through a reduction in deposit rates, which lowers the financing cost of banks. We start by exploring the first stage of this transmission process.

In Sweden, the policy rate essentially refers to the interest rate banks receive for holding transaction balances at the Riksbank.[6] The policy rate does not apply to anything on the banks liability side, but rather is the return on an asset. The policy rate then gets transmitted via arbitrage to the interbank rate, and through the interbank rate to other bank funding sources. Figure 2 shows the decomposition of liabilities for Swedish banks as of September 2015.[7] The most important funding source is deposits, accounting for about half of bank liabilities. We start by considering deposit financing, before moving on to other financing sources.

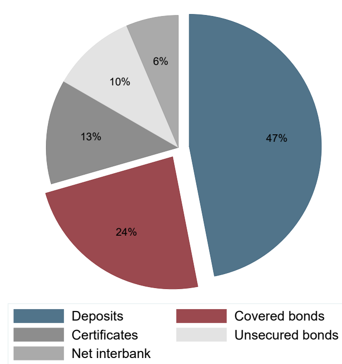


Figure 2: Decomposition of liabilities (as of September 2015) for large Swedish banks. Source: The Riksbank

[6]The exact implementation of negative rates differ across the countries which have implemented them, see Bech and Malkhozov (2016) for an overview. In the case of Sweden, the Riksbank operates a corridor system. The policy rate refers to the repo rate. Banks can borrow from the Riksbank at 75 basis points above the policy rate and central bank reserves earn an interest rate 75 basis points below the policy rate. Consider for example a policy rate of - 0.5 %. In order to implement this rate, the Riksbank sells certificates in repo transactions that pay - 0.5 %. As the banks are obtaining -1.25 % on their reserves, they will use the reserves to purchase these certificates. In this sense the repo rate is essentially equivalent to the Riksbank directly paying - 0.5 % on bank reserves.

[7]Note that net interbank lending need not equal zero as not only traditional banks have access to the interbank market.

2.1.1 Bank Deposits

Figure 3 depicts aggregate deposit rates in Sweden.[8] Prior to the policy rate becoming negative, the aggregate deposit rate is below the policy rate and moves closely with the policy rate. As the policy rate turns negative this relationship breaks down. Instead of following the policy rate into negative territory, the deposit appears bounded at some level close to zero. In the right panel of Figure 3, we depict a counterfactual deposit rate, constructed by assuming that the markdown from the repo rate is constant and equal to the pre-zero average. As seen from the graph, this counterfactual deposit rate is roughly a percentage point lower than the actual deposit rate.

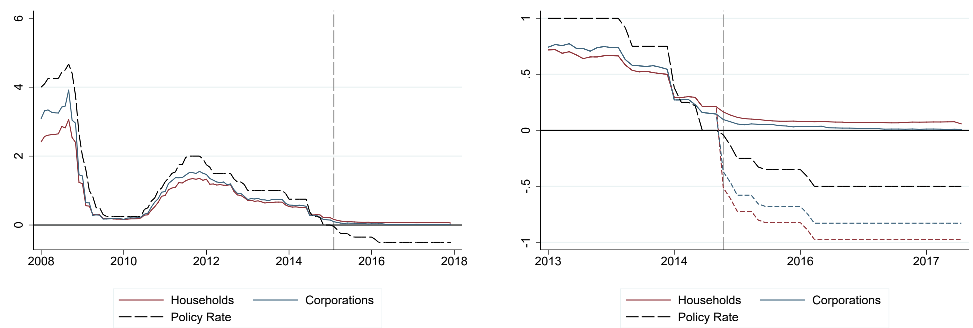


Figure 3: Aggregate deposit rates in Sweden. The policy rate is defined as the repo rate. Right panel: The red and blue dashed lines capture counterfactual lending rates calculated under the assumption that the markup to the repo rate was constant and equal to the average markup in the period 2008m1-2015m1. Source: The Riksbank, Statistics Sweden.

In Section 2.2 we move to daily data and the sample then covers the final six interest rate cuts made between 2014 and 2016. For future reference it is useful to study the aggregate deposit rates for these final six cuts. This is done in Figure 4, where we calculate the change in the deposit rate relative to the change in the repo rate. The first bar captures the average relative change in deposit rates prior to 2014. In this case, the aggregate deposit rate changed by on average 60 percent as much as the repo rate. For the post-2014 data, the relative change in the deposit rate is somewhat lower. For the policy rate cuts in positive territory, the deposit rate falls by approximately 40 percent as much as the repo rate. For the first two cuts in negative territory, i.e. to -0.1 percent and to -0.25 percent, the pass-through remains relatively unchanged. For the final two interest rate cuts however, the pass-through collapses to roughly zero. As the deposit rate has reached its lower bound, reducing the

[8]The aggregate deposit rate is a weighted average of the interest rate on different deposit accounts. It thus includes both highly liquid checking accounts, as well as less liquid fixed deposit accounts with minimum deposit amounts.

policy rate deeper into negative territory does not lead to further reductions in the deposit rate. This will be important when we consider the transmission to lending rates.

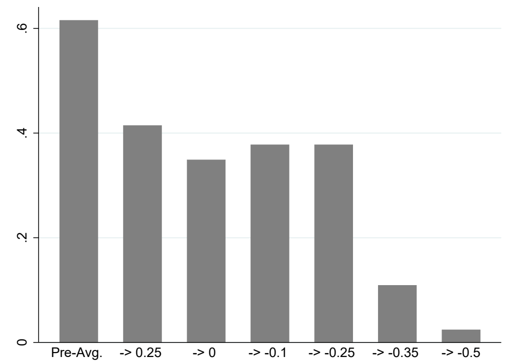


Figure 4: Change in the aggregate deposit rate for households relative to the change in the repo rate - at times of changes to the repo rate. Source: The Riksbank, Statistics Sweden.

The reluctance of deposit rates to fall below zero is not isolated to the Swedish case. The same holds for Switzerland, Japan, Denmark, Germany and the Euro Area as a whole, as shown in Figure 17 in Appendix A. Even though policy rates go negative, bank deposit rates remain above zero.

What is causing deposit rates to be bounded? In the model in Section 3, the lower bound arises because people have the alternative of holding cash. One Swedish krona today will still be worth one krona tomorrow, thus yielding a zero interest rate. Hence, a negative deposit rate would be inconsistent with people holding deposits. An alternative to this hypothesis, which is also consistent with the model, is that people view negative bank deposit rates as "unfair". In any case, negative interest rates would cause households to substitute away from deposits. Consistent with this, survey evidence from ING (2015) shows that 76 percent of consumers would withdraw money from their savings accounts if rates turned negative (see Figure 21 in Appendix A).

Even with nominal deposit rates being bounded, an increase in fees could decrease the effective deposit rate.[9] Given the importance of deposit financing however, the increase in fees would need to be substantial. A simple calculation based on the average deposit share and the pre-zero relationship between the deposit rate and the policy rate, suggests that commission income as a share of assets would have to increase by roughly 75 percent (see Figure 18 in Appendix A). However, the data suggests that the income generated from fees, if anything, *declined* after the Riksbank introduced negative rates in 2015. Also note that, if

[9]Conceptually however, if the bound on deposit rates arises from the existence of cash or notions of fairness, one would expect the effective deposit rate to be subject to the same bound.

there was full pass-through to effective deposit rates via fees, this should imply that the pass-through to lending rates would be unaffected by negative policy rates. Section 2.2 documents that the pass-through to lending rates also collapses, consistent with the empirical evidence that fees did not have a material impact on effective deposit rates in Sweden.

2.1.2 Other Financing Sources

About half of Swedish bank liabilities come in other forms than deposits, as shown in Figure 2. The largest component is covered bond issuance. Figure 5 compares the interest rate on covered bonds to the policy rate. As with deposit rates, the correlation between the policy rate and covered bond rates is weaker once the policy rate turns negative. This is especially true for covered bonds with longer maturities. We have limited information on unsecured bonds and certificates, which make up a smaller share of bank liabilities.

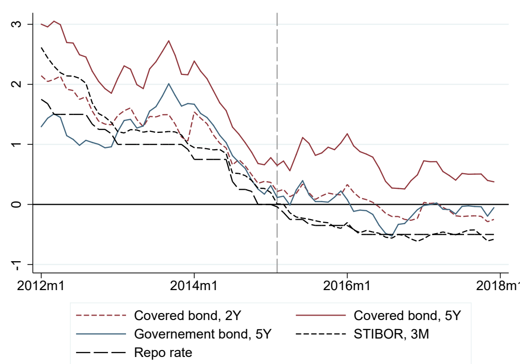


Figure 5: Interest rates. Sweden. Source: The Riksbank

Even if the pass-through to covered bond rates is weaker, we see from Figure 5 that the interest rate on covered bonds with shorter maturities eventually becomes negative, suggesting a stronger pass-through than for deposit rates. If banks respond to negative policy rates by shifting away from deposit financing, they would therefore reduce their marginal financing costs. However, Figure 6 shows that this is not the case. There is no noticeable increase in bonds issuance as rates goes negative, and the deposit share actually *increases*. There are at least three possible explanations for why banks did not shift away from deposit financing: i) maintaining a base of depositors creates some synergies which other financing sources do not, ii) the room for new issuances of covered bonds may be limited by the availability of bank assets to use for collateral, and iii) Basel III regulation makes deposit financing more attractive in terms of satisfying new requirements. In any case, the empirical evidence suggests that deposit rates is the most important component of not only average, but also marginal funding costs in Sweden during this period.

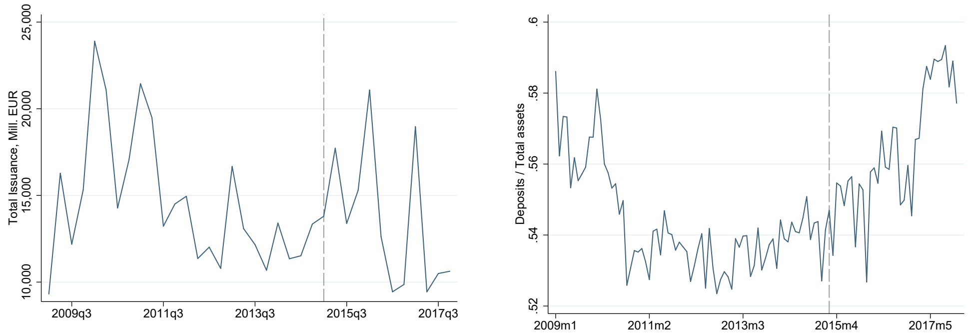


Figure 6: Left panel: Issuance of covered bonds, Swedish banks. Right panel: Deposit share, Swedish banks. Vertical lines correspond to the date negative interest rates were implemented. Source: Association of Swedish Covered Bond Issuers, The Riksbank and Statistics Sweden

An estimate of financing costs The balance sheet composition illustrated in Figure 2 can be used to proxy banks' funding costs. One such estimate is depicted in Figure 7. The estimated time series is a relatively conservative estimate in the sense that it does not incorporate the increase in deposit reliance. Moreover, the most beneficial (lowest) interest rate is assigned to the funding sources for which interest rate data is lacking. As the solid line in Figure 7 indicates, the estimate of the banks funding cost follows the policy rate less closely as the policy rate falls below zero.

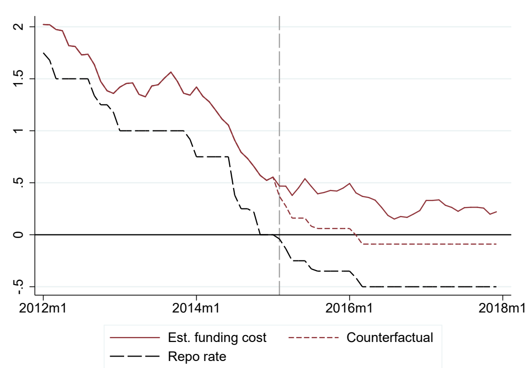


Figure 7: Estimated average funding costs. The estimated average funding cost is computed by taking the weighted average of the assumed interest rate of the different funding sources of the bank. Certificates are assumed to have the same interest rate as 2Y covered bonds, while unsecured debt are assumed to have the same interest rate as 2Y covered bonds plus a 2 percent constant risk-premium. The counterfactual series correspond to the case when the spread between the repo rate and the estimated funding cost remain fixed at pre-negative levels. Weights based on the liability structure of large Swedish banks, see Figure 2. Source: The Riksbank

How much lower would total funding costs be if the correlation with the repo rate was unchanged? The dashed line is a counterfactual funding cost estimate generated by assuming that the markup of the funding cost over the repo rate is equal to the pre-zero markup. The estimate suggests that total funding costs would have been roughly 0.25 percentage points lower if there had been no reduction in pass-through.

If policy rate cuts in negative territory do not lead to meaningful reductions in bank funding costs, this raises the fundamental question of whether they can be expected to lower lending rates. The next section addresses this question.

2.2 Bank Lending

This section considers the effect of negative rates on the banks asset side, i.e. how it affects lending rates. Figure 8 depicts aggregate lending rates in Sweden and suggests that the transmission of policy rates to lending rates is weakened as the policy rate becomes negative.[10] This insight will be confirmed by the bank level data in the next section. A simple calculation shows that if the markup over the repo rate had stayed constant and equal to the average markup in the pre-zero environment, aggregate lending rates for both households and corporations would have been approximately 0.3 percentage points lower. This is illustrated in the right panel of Figure 8.

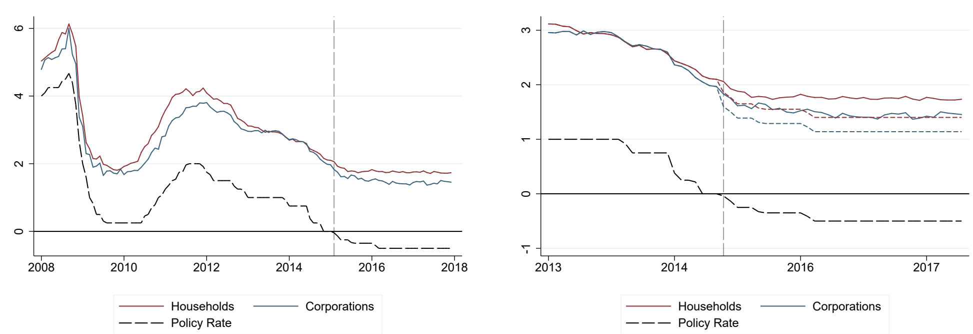


Figure 8: Aggregate lending rates in Sweden. The policy rate is defined as the repo rate. Right panel: The red and blue dashed lines capture counterfactual lending rates calculated under the assumption that the markup to the repo rate was constant and equal to the average markup in the period 2008m1-2015m1. Source: The Riksbank, Statistics Sweden.

Aggregate lending rates for Switzerland, Japan, Denmark, Germany and the Euro Area are depicted in Figure 19 in Appendix A. In the absence of bank level data it is difficult to draw inference from this aggregate data, even if in the case of Switzerland and Denmark it seems particularly clear that there is little, if any, action in the aggregate time series. A key difficulty in drawing inference for the Euro area is that negative reserve rates were associated

[10]Aggregate lending rates are weighted averages over different loan contracts, including loans with and without collateral, with fixed and floating interest rate periods etc.

with the European Central Bank directly offering credit at the negative policy rate, unlike in the case of Sweden. Furthermore, because deposit rates are higher in the Euro Area, they have more room to fall before reaching the lower bound. Hence, we would expect a larger impact on lending rates for Euro Area banks.

We proceed by using two bank level datasets for Swedish banks. First, we have daily bank level data on a rich set of mortgage rates for the largest Swedish banks, provided by the price comparison site *compricer.se*. We exploit the high frequency of the data to evaluate the causal effect of reductions in the policy rate, and compare the monetary policy transmission to lending rates across positive and negative territory. Second, we complement our analysis by using bank level data on monthly lending volumes from Statistics Sweden.

2.2.1 Bank Level Lending Rates

Figure 9 plots daily 5 year fixed-rate mortgage rates for the largest Swedish banks from 2014 to 2016.[11] The vertical lines denote days when the policy rate was cut, with the repo rate level reported on the x-axis. The first two lines capture repo rate cuts in positive territory. For both cuts there is an immediate and homogeneous decline in bank lending rates. The third line marks the day the repo rate turned negative and the three proceeding lines capture further repo rate cuts. The response in bank lending rates to these interest rate cuts is fundamentally different. While there is some initial reduction in lending rates, most of the rates increase again shortly thereafter. As a result, the total impact on lending rates is limited.

Figure 9 includes the correlation between the repo rate and the aggregate deposit rate, as illustrated by the black x'es measured on the right y-axis. The x'es correspond to the bar chart in Figure 4. When the deposit rate is still responsive, lending rates fall in response to policy rate cuts. Once the deposit rate has reached its lower bound, i.e. the two last policy rate cuts, lending rates no longer fall. This highlights an important point: the pass-through to lending rates is smaller once the deposit rate is unresponsive. For the two last repo rate cuts there is a complete breakdown in the transmission of policy rates to both aggregate deposit rates and to bank level mortgage rates.

[11] Figure 24 in Appendix A shows that the bank level data aggregates well to match official data.

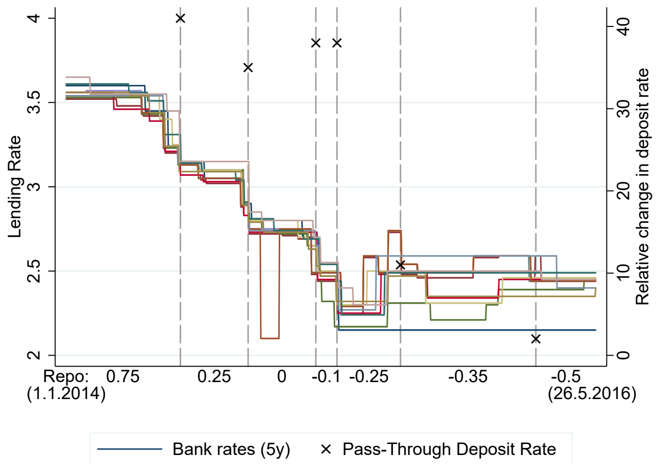


Figure 9: Bank level lending rates in Sweden. Interest rate on mortgages with five-year fixed interest period. The red vertical lines mark days in which the repo rate was lowered. The label on the x-axis shows the value of the repo rate. Small x'es denote the change in the deposit rate relative to the change in the policy rate (%), measured on the right y-axis. Source: Compricer.se

The reduction in pass-through holds across a wide range of loan types. Figure 10 plots bank-level lending rates across three different contracts, a floating rate mortgage (3m), a mortgage with a 1 year fixed-rate period (1y) and a mortgage with a 3 year fixed-rate period (3y). In all three cases, we see that the interest rate cuts in negative territory have very limited pass-through to bank lending rates.

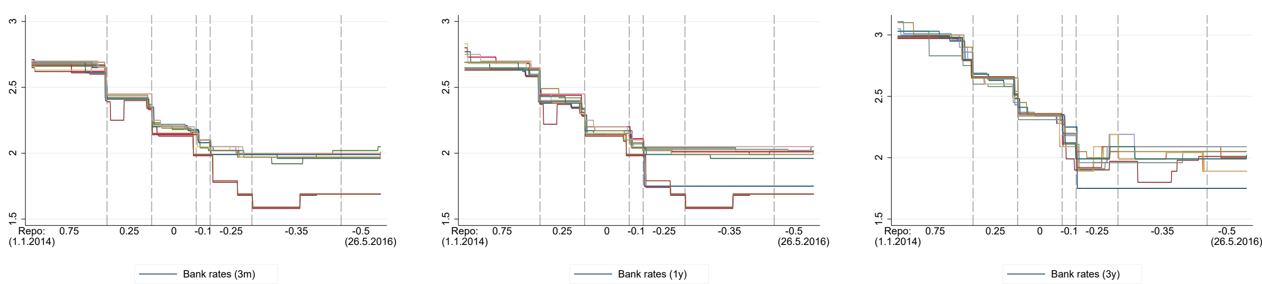


Figure 10: Bank level lending rates with a floating interest rate (3m) (left panel) and a fixed interest rate period of 1y (mid panel) and a fixed interest rate period of 3y (right panel). The red solid line capture days with repo rate reductions. Source: Compricer.se

Figure 11 depicts box plots of bank level correlations between lending rates and the policy rate. The blue box depicts the empirical distribution of correlations prior to the Riksbank going negative, in which case the median correlation is roughly 0.75. The black box corresponds to the empirical distribution for the full period of negative rates, in which case the median correlation is slightly lower. Finally, the red box corresponds to the empirical distribution of correlations after the deposit rate becomes unresponsive to changes in the repo rate (i.e. the last two policy rate cuts). Consistent with the previous figure, once the deposit rate is bounded there is a substantial drop in correlations, with the median correlation becoming negative. There is furthermore a large increase in dispersion, as correlations range from roughly negative 0.5 to positive 0.5.

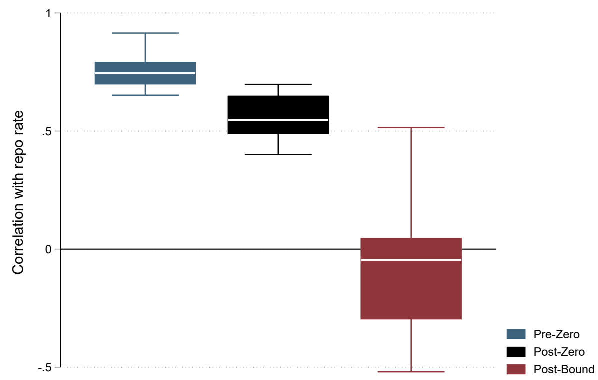


Figure 11: The distribution of bank level correlations between changes in lending rates and the repo-rate when the repo rate is positive ("Pre-zero"), the repo rate is negative ("Post-zero") and the repo rate is negative and the deposit rate is non-responsive ("Post-Bound"). 5-year fixed interest rate period. Source: compricer.se and own calculations.

Figure 9 and 11 suggest that bank behavior in the post-zero period is relatively heterogeneous. That is, some banks continue to have a positive co-movement between their lending rate and the repo rate, while the sign is reversed for others. What is causing this increase in dispersion? One theory is that differences in the reliance on deposit financing means that banks are being differentially affected by negative interest rates. Given that there are frictions in raising different forms of financing - and some sources of financing are more responsive to monetary policy changes than others - cross-sectional variation in balance-sheet components can induce variation in how monetary policy affects banks (Kashyap and Stein, 2000). Figure 12 investigates whether banks' funding structures affect their willingness to lower lending rates, by plotting the bank level correlation between lending rates and the repo rate after the deposit rate became bounded, as a function of banks' deposit shares. The figure confirms a negative relationship between the deposit share and the correlation with the repo rate. Banks with higher deposit shares are less responsive to policy rate cuts in negative territory. Weighting observations by market shares, this relationship is statistically significant at the one percent level. The regression line reported in the figure indicates that a ten percentage points increase in the deposit share is associated with a reduction in the correlation of approximately 0.18 correlation points.[12]

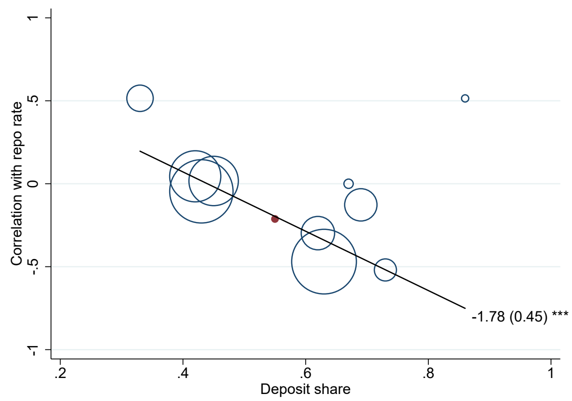


Figure 12: Correlation between lending rate and repo rate after the repo rate turned negative and the deposit rate reached its lower bound, as a function of the banks' deposit share. Size of circles indicate market share. Gray square indicates Ålandsbanken, for which we do not have the market share. Regression coefficient (standard.error) also reported. $***$ indicates $p < 0.01$. Swedish banks. Interest rate on 5 year fixed-rate mortgages. Source: compricer.se, Statistics Sweden and own calculations.

We conclude this section with regression evidence that is useful for the model calibration in Section 3. The regression is outlined in equation (1), with the dependent variable being the monthly change in lending rates for bank i, $\Delta i^b_{i,t}$. On the right hand side is the change in the repo rate, Δi^r_t, and the change in the repo rate interacted with a dummy variable $I_t^{post\,bound} = 1$ if $t > 2015m4$, i.e. the period in which the deposit rate is bounded.

$$\Delta i^b_{i,t} = \alpha + \beta \Delta i^r_t + \gamma \Delta i^r_t \times I_t^{post\,bound} + \epsilon_{i,t} \tag{1}$$

The regression results are reported in Table 1. In normal times, a one percentage point decrease in the repo rate reduces bank lending rates by on average 0.53 to 0.69 percentage points. Once the deposit rate becomes bounded however, this relationship flips. A one percentage point reduction in the repo rate, now *increases* bank lending rates by 0.03 to 0.31 percentage points. This reversal in sign holds across all loan contracts.

[12] Although average correlations drop across all fixed interest-rate periods, the increase in dispersion is most prevalent across longer fixed-rate periods. Hence, for shorter fixed-rate periods the relation with deposit shares is not statistically significant.

	(1) 3 months	(2) 1 year	(3) 3 years	(4) 5 years
Δi_t^r	0.579***	0.533***	0.640***	0.686***
	(34.35)	(28.56)	(16.74)	(13.72)
$\Delta i_t^r \times I_t^{post\,bound}$	-0.606***	-0.623***	-0.926***	-0.994***
	(-10.27)	(-9.54)	(-6.92)	(-5.68)
Constant	-0.00480**	-0.00718***	-0.0162***	-0.0193***
	(-2.94)	(-3.98)	(-4.37)	(-3.99)
N	308	308	308	308

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1: Regression results from estimating equation (1). Dependent variable is $\Delta i_{i,t}^b$ at the monthly frequency. Observations are weighted according to bank size.

2.2.2 Bank Level Lending Volumes

So far we have investigated the effect of negative policy rates on bank interest rates. Here we present evidence on bank lending volumes. Motivated by the cross-sectional relationship between deposit shares and lack of pass-through shown in Figure 12, we now investigate whether banks with high deposit shares also have lower growth in lending volumes. The difference in difference regression is specified in equation (2).

$$\Delta \log(\text{Lending}_{i,t}) = \alpha + \beta \left(I_t^{post\,zero} \times \text{Deposit share}_i \right) + \delta_i + \sum_k \delta_k \mathbf{1}_{t=k} + \epsilon_{i,t} \tag{2}$$

For comparison, we keep our analysis the same as that in Heider, Saidi, and Schepens (2016), who investigate the impact of negative policy rates in the Euro Area.[13] The dependent variable is the percentage 3-month growth in bank level lending. $I_t^{post\,zero}$ is an indicator variable equal to one after the policy rate became negative, while Deposit share$_i$ is the deposit share of bank i in year 2013. As an alternative specification, we replace Deposit share$_i$ with an indicator $\mathbf{1}_{\text{High deposit,i}}$ for whether bank i has a deposit share above the median in 2013. We include bank fixed effects δ_i to absorb time-invariant bank characteristics, and month-year fixed effects δ_k to absorb shocks common to all banks. Standard errors are clustered at the bank level. We restrict our sample to start in 2014, thus choosing a relatively short time period around the event date. The coefficient of interest is the interaction coefficient β. If

[13]We have also tried substituting $I_t^{post\,zero}$ with $I_t^{post\,bound}$, and the results are similar.

banks with high deposit shares have lower credit growth than banks with low deposit shares after the policy rate breaches the zero lower bound, we expect to find $\hat{\beta} < 0$.

The regression results are reported in Table 2. Focusing on column (1) first, the interaction coefficient is negative and significant at the five percent level. An increase in the deposit share is associated with a reduction in credit growth in the post-zero environment. The effect is economically significant - a one standard deviation increase in the deposit share decreases lending growth by approximately 0.18 standard deviations.

In column (2) we consider average credit growth for banks with above and below median deposit shares. While we lose some precision by using only an indicator variable, the coefficient is still negative and statistically significant at the ten percent level. On average, banks with high deposit shares had four percentage points lower growth in credit compared to banks with low deposit shares. We thus conclude that, due to the lower bound on the deposit rate, banks which rely heavily on deposit financing are less responsive to policy rate cuts in negative territory. The cross-sectional evidence presented here is consistent with the results in Heider, Saidi, and Schepens (2016) and the survey evidence in Figure 20 in Appendix A, where the vast majority of European banks report that they have not increased lending volumes in response to negative policy rates.

Dependent variable:	$\Delta \log (Lending)_{i,t}$	
	(1)	(2)
$I_t^{\text{post}} \times$ Deposit share$_i$	-0.09^{**}	
	(-2.09)	
$I_t^{\text{post}} \times \mathbf{1}_{\text{High deposit},i}$		-0.04^{*}
		(-1.85)
Clusters	40	40
Bank FE	Yes	Yes
Month-Year FE	Yes	Yes
Observations	$1,113$	$1,113$

Table 2: Regression results from estimating equation (2). Dependent variable: $\Delta \log (\text{Lending})_{i,t} \equiv \log (Lending)_{i,t} - \log (\text{Lending})_{i,t-3}$. Monthly bank level data from Sweden.

3 Negative Interest Rates in Theory

Motivated by the empirical evidence in the previous section, we now develop a formal framework to understand the impact of negative policy rates on lending rates and lending volumes. Section 3.1 builds a partial equilibrium banking model that is then embedded in a general equilibrium framework in section 3.2, nesting the standard New Keynesian model.

3.1 Negative Interest Rates in a Partial Equilibrium Model of Banking

The goal of this section is to illustrate how changes in policy rates normally affect deposit and lending rates, and how this changes once the deposit rate becomes bounded. For now we directly impose a bound on the deposit rate, formally derived in the full model in the next section.

A bank decides how much deposits to collect, d_t, how many loans to extend, l_t, how much reserves to hold at the central bank, R_t, as well as how much physical cash to hold m_t. Denote interest on reserves i^r, interest on deposits i^s, and interest on loans i^b. In making its choices, the bank takes these interest rates as given. Cash pays no interest, but carries a proportional storage costs $S(M_t) = \gamma M_t$ for some $\gamma \geq 0$. The price level is normalized so that $P_t = 1$, but will be endogenous in the next section.

A bank is modeled as in Curdia and Woodford (2011). All profits z_t are paid out to the owner at time t. The bank thus only holds enough assets on its balance sheet to pay off depositors in the next period so that

$$(1+i_t^s)d_t = (1+i_t^b)l_t + (1+i_t^r)R_t + m_t - S(m_t) \tag{3}$$

The bank faces an intermediation cost function $\Gamma(l_t, R_{t,} m_t, z_t)$. Reserves lower intermediation costs for the bank up to some point $\bar{R}$, i.e. $\Gamma_R < 0$ for $R < \bar{R}$ and $\Gamma_R = 0$ for $R \geq \bar{R}$. Similarly, $\Gamma_m < 0$ for $m < \bar{m}$ and $\Gamma_m = 0$ for $m \geq \bar{m}$. Bank intermediation costs are increasing in lending due to for example unmodeled default, i.e $\Gamma_l > 0$. Finally, higher bank profits weakly reduce the marginal cost of lending, i.e. $\Gamma_{lz} \leq 0$. This assumption is discussed further below.

Using equation (3), bank profits can be expressed in a static way as

$$z_t = \frac{i_t^b - i_t^s}{1+i_t^s} l_t - \frac{i_t^s - i_t^r}{1+i_t^s} R_t - \frac{i_t^s + \gamma}{1+i_t^s} m_t - \Gamma\left(l_t, R_t, m_t, z_t\right) \tag{4}$$

A partial banking equilibrium is defined by exogenous (i_t^s, i_t^b, i_t^r) taken as given by banks and values for R_t, l_t, m_t, z_t solving equation (4) and the first order conditions (5) - (7):

$$R_t: \quad \frac{i_t^s - i_t^r}{1+i_t^s} = -\Gamma_R(l_{t,} R_t, m, z_t): \tag{5}$$

$$m_t: \quad \frac{i_t^s + \gamma}{1 + i_t^s} = -\Gamma_m(l_t, R_t, m_t, z_t) \tag{6}$$

$$l_t: \quad \frac{i_t^b - i_t^s}{1 + i_t^s} = \Gamma_l(l_t, R_t, m_t, z_t) \tag{7}$$

Figure 13 depicts the demand for reserves D given by equation (5), with R on the x-axis and i^s on the y-axis. The interest on reserves i^r is treated as fixed for now, and could for example correspond to 0 as prior to 2008 in the US. The lower the deposit rate, the more reserves are demanded by banks. We have chosen a simple specification for the function Γ for the purposes of the figure.[14]

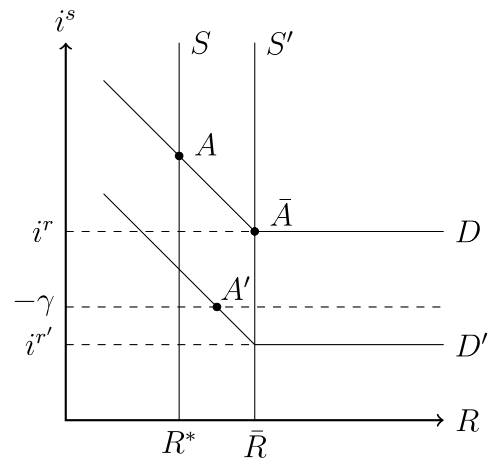


Figure 13: Reserves - Demand and Supply.

Letting the bank be a representative bank, one way of thinking about how the central bank determines the risk-free interest rate i^s is with open market operations in government bonds (purchased by reserves). Open market operations directly set the supply of reserves R^*, which pins down i^s at point A in Figure 13. This closely resembles how policy was conducted prior to 2008. An increase in reserves by the central bank would then lower i^s until it reaches the point $\bar{R}$. At that point, banks are fully satiated in reserves and the deposit rate and the reserve rate are equal, $i^s = i^r$.

Alternatively, the central bank could keep banks satiated in reserves by choosing $R \geq \bar{R}$, implying $i^r = i^s$. Changes in the reserve rate would then directly change the deposit rate as well. Such an equilibrium is illustrated at point $\bar{A}$. This implementation of policy better captures the current policy regime in the US and in Sweden. Following this policy arrangement, we will refer to i^r as the policy rate.

[14] It is simply linear in R until the satiation point $\bar{R}$ is reached, at which point $\Gamma_R = 0$. More generally we assume that $\lim_{R\to 0} \Gamma_R = \infty$ which implies that there is no zero lower bound on interest on reserves.

In addition to reserves, banks also demand money, as given by equation (6) and depicted in Figure 14 with i^s on the y-axis. With i^s determined by the central bank's choice of reserves and interest on reserves, the central bank elastically supplies paper currency to satisfy whatever money is demanded at that rate. As in the case of reserves, we assume banks (and households) hold money because it is useful to facilitate transactions - up until some point. Typically, the monetary satiation is assumed to occur at 0 and hence the interest rate on deposits cannot fall below 0. In the next section, we show how storage costs of money can imply a bound below 0. Here, we take the bound as exogenously given at $-\gamma$.

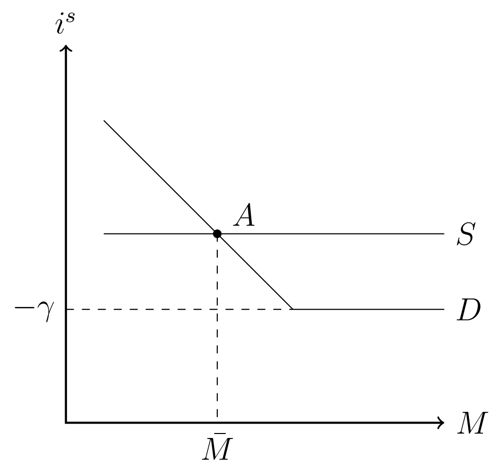


Figure 14: Money - Demand and Supply.

The fact that i^s cannot fall below $-\gamma$ also has implications for the relationship between reserves, the interest on reserves and the deposit rate. Consider again Figure 13. What happens if the central bank changes the interest rate on reserve to some $i^{r\prime} < -\gamma$, while at the same time setting reserves so that $R \geq \bar{R}$? The reduction in the reserve rate shifts the demand curve down to D'. Because the deposit rate is bounded at $-\gamma$, a new equilibrium arises at point A'. Observe that an equilibrium cannot take place at $\bar{R}$. At this point the marginal benefit of holding reserves is zero (due to satiation), yet the marginal cost is higher, i.e. $-i^{r\prime}$. Banks will then prefer holding money, and so reserves will flow into vault cash.

The first order condition for lending in equation (7) governs what happens to bank lending when the reserve rate is lowered. First consider the case in which the bound on the deposit rate is non-binding. In this case, the deposit rate also falls, thereby lowering bank financing costs and increasing loan supply. If the deposit rate is constrained by the lower bound however, there is no reduction in financing costs and so no increase in loan supply. Moreover, when the reserve rate is lowered without a reduction in the deposit rate, bank profits fall. This is simply because banks receive a lower interest rate on one of their assets, without having to pay a lower interest rate on their liabilities. This will in turn increase the cost of bank intermediation through the function $\Gamma(l, R, m, z)$. As a result, the supply of loans is reduced.

The effect on lending can be shown formally by solving the partial equilibrium using a linear approximation. Expression (8) captures the increase in lending at a given borrowing rate i^b when the interest rate on reserves is reduced and $i^r = i^s$. In this case $\frac{\partial \hat{l}_t}{\partial \hat{i}_t^r} < 0$, as $\Gamma_{lz} < 0$ and $|\Gamma_z| < 1$.[15] Expression (9) captures the decrease in lending when i^s is fixed and there is only a reduction in i_t^r, which corresponds closer to what we have seen in the data. In this case $\frac{\partial \hat{l}_t}{\partial \hat{i}_t^r} > 0$, so that a reduction in the reserve rate leads to a *reduction* in lending volumes.[16]

$$\frac{\partial \hat{l}_t}{\partial \hat{i}_t^r} = -\left[\frac{1+\Gamma_l}{l\Gamma_{ll}} - \frac{z\Gamma_{lz}}{l\Gamma_{ll}}\frac{z+l+m+\Gamma}{z(1+\Gamma_z)}\right] < 0 \tag{8}$$

$$\left.\frac{\partial \hat{l}_t}{\partial \hat{i}_t^r}\right|_{i^s \text{fixed}} = -\frac{z\Gamma_{lz}}{l\Gamma_{ll}}\frac{\bar{R}}{z(1+\Gamma_z)} > 0 \tag{9}$$

The reduction in lending given by (9) relies fundamentally on the negative value of the partial derivative Γ_{zl}. This assumption captures, in a reduced form manner, the established link between banks' net worth and their operational costs - assuming there is a one-to-one mapping between net worth and profits. We do not make an attempt to microfound this assumption, which is explicitly done in among others Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010), as well as documented empirically in for instance Jiménez, Ongena, Peydró, and Saurina (2012). If $\Gamma_{zl} = 0$, there is no feedback effect from bank profits to credit supply. Importantly, however, a negative policy rate does still not increase lending.

This partial equilibrium analysis already hints to very different effects of policy rate cuts in negative territory. If the policy rate cut does not lead to a reduction in deposit rates, there is no reduction in bank funding costs. The reduction in the reserve rate then implies lower bank profits as long as banks hold reserves in positive supply at the central bank. Hence, as the critics have stated, a negative reserve rate essentially works as a tax in the partial equilibrium banking model. To the extent that banks are constrained in their lending by their net worth, this will suppress credit supply.

The argument put forward by the proponents of negative interest rates however, is that there should be a reduction in the borrowing *rate* faced by borrowers. This in turn could stimulate spending. In order to evaluate this claim we move on to a general equilibrium framework, in which i^b is no longer held fixed.

[15]We have checked that $|\Gamma_z| < 1$ holds in all our numerical results.

[16]Throughout the paper we let $\hat{x}_t$ denote the deviation of x_t from its steady state value x.

3.2 Negative Policy Rates in General Equilibrium

We now embed the banking model in a general equilibrium model, in which the borrowing rate is endogenously determined by loan supply and demand. In this case the choices of the bank feed into aggregate demand, which in turn affects borrowing and lending rates in general equilibrium. Our main finding will be that the borrowing rate is predicted to increase, rather than decrease, when the policy rate becomes negative. The full model is relegated to Appendix C, with key elements outlined in the main text and the log-linear equilibrium conditions needed to close the model summarized in Table 3.

We first highlight how the bound on the deposit rate is derived. Household $j \in \{s, b\}$ consumes, holds money, saves/borrows and supplies labor. Households of type b are borrowers and make up a fraction χ of the population, while households of type s are savers and make up the remaining share $1-\chi$. Saver households can store their wealth either by depositing their savings in banks, thereby earning an interest rate of i_t^s, or by holding money which is the unit of account.

Let $\Omega\left(\frac{M_t^j}{P_t}\right)$ be the utility from holding real money balances with $\Omega' \geq 0$ and $\Omega'\left(\frac{M_t^j}{P_t}\right) = 0$ for $\frac{M_t^j}{P_t} \geq \bar{m}$. Letting $U'\left(C_t^j\right)$ be the marginal utility of consumption and i_t^j the interest rate faced by a type j agent, optimal money holdings satisfy

$$\frac{\Omega'\left(\frac{M_t^j}{P_t}\right)}{U'\left(C_t^j\right)} = \frac{i_t^j + S'\left(M_t^j\right)}{1+i_t^j} \tag{10}$$

The lower bound on the deposit rate $\underline{i}^s$ is the lowest value of i_t^s satisfying equation (10). The lower bound therefore depends crucially on the marginal storage cost, which is typically assumed to be zero, hence the zero lower bound. We instead assume proportional storage cost $S\left(M_t^s\right) = \gamma M_t^s$ This implies a lower bound $\underline{i}^s = -\gamma$, so that the bound can be negative if $\gamma > 0$.

In order to generate a recession, we consider a preference shock ζ, which reduces current consumption. This type of shock is standard in the ZLB literature. In Appendix D, we report results from a debt deleveraging shock, which has similar implications for the effectiveness of negative interest rates.[17]

Table 3 reports a log-linear approximation of the equilibrium conditions. First, total output $\hat{Y}_t$ is given by the consumption of the two agents, $\hat{C}_t^b$ and $\hat{C}_t^s$, as shown in equation (11). The consumption of each agent in turn, is determined by their respective Euler equations,

[17] To keep the current model exposition simple, we only include the necessary notation for the debt deleveraging shock in Appendix D.

(12) and (13), where $\hat{\pi}_t$ denotes the deviation of inflation from its steady state level. These equations, together with the budget constraint of the borrower in equation (14), where $\hat{b}_t^b$ denotes the real value of the borrowers nominal debt, determine both the demand for credit and the supply of savings that is generated from the saver households. The production structure, which assumes monopolistically competitive firms that face price rigidities in the form of Calvo pricing, is borrowed from Benigno, Eggertsson, and Romei (2014) and can be summarized by the standard New Keynesian Phillips curve shown in equation (15).

The partial equilibrium banking model outlined in the previous section is directly incorporated into the model. Recall that there we treated (i_t^b, i_t^s, i_t^r) as exogenous. Now they are determined in equilibrium by the first order conditions of banks and households, along with policy. Equations (16) and (17) are log-linear approximations of the first order condition for lending in equation (7). This condition no longer just determines loan supply for given values of i_t^s and i_t^b, rather it specifies a general equilibrium interest rate spread $\hat{\omega}_t$ associated with a particular level of bank lending. Equation (18) is an expression for bank profits, where $\hat{z}_t$ denotes profits, while equation (19) is the banking sectors demand for reserves.[18]

Equation (20) defines the natural rate of interest $\hat{r}_t^n$, which depends on the exogenous preference shock $\hat{\zeta}$, as well as the endogenous interest rate spread. The model is closed by monetary and fiscal policy. The only government liabilities in the model are that of the central bank (currency plus reserves). Any seignorage revenues or losses are rebated to the representative saver, so that no fiscal variables enter directly into the equilibrium determination. Equation (21) is a Taylor rule that is formulated in terms of a policy rate that corresponds to the interest on reserves. We follow the recent literature by allowing for time variation in the intercept of the rule, $\hat{r}_t^n$, corresponding to the natural rate of interest. There is no lower bound on interest on reserves. As discussed in the previous section, we assume a policy regime in which the central bank satiates the banking sector in reserves whenever it can so that $\hat{i}_t^s = \hat{i}_t^r$. Equation (22) recognizes however, that the deposit rate is bounded in line with the data.

[18]As in the case of the households, we simplify the exposition of the model by omitting the banks demand for currency, as this plays a trivial role, see Appendix C for full model.

$$\hat{Y}_t = \frac{\chi C^b}{Y}\hat{C}^b_t + \frac{(1-\chi)C^s}{Y}\hat{C}^s_t \tag{11}$$

$$\hat{C}^b_t = \mathbb{E}_t\hat{C}^b_{t+1} - \sigma\left(\hat{i}^b_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{\zeta}_t + \mathbb{E}_t\hat{\zeta}_{t+1}\right) \tag{12}$$

$$\hat{C}^s_t = \mathbb{E}_t\hat{C}^s_{t+1} - \sigma\left(\hat{i}^s_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{\zeta}_t + \mathbb{E}_t\hat{\zeta}_{t+1}\right) \tag{13}$$

$$\hat{b}^b_t = \frac{\hat{b}^b_t}{\pi\beta^b} + \frac{\hat{i}^b_{t-1} - \hat{\pi}_t}{\pi\beta^b} + \frac{c^b}{b^b}\hat{C}^b_t - \chi\frac{y}{b^b}\hat{Y}_t \tag{14}$$

$$\hat{\pi}_t = \kappa\hat{Y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{15}$$

$$\hat{i}^b_t = \hat{i}^s_t + \hat{\omega}_t \tag{16}$$

$$\hat{\omega}_t = \omega(\nu-1)\hat{b}^b_t - \iota\omega\hat{z}_t \tag{17}$$

$$\hat{z}_t = \frac{\chi b^b(1+\omega)}{\omega\chi b^b + (1-\iota)\Gamma}\hat{i}^b_t - \frac{d}{\omega\chi b^b + (1-\iota)\Gamma}\hat{i}^s_t + \frac{R}{\omega\chi b^b + (1-\iota)\Gamma}\hat{i}^r_t \tag{18}$$

$$\hat{i}^s_t = \hat{i}^r_t - R\hat{R}_t \tag{19}$$

$$\hat{r}^n_t = \hat{\zeta}_t - \mathbb{E}_t\hat{\zeta}_{t+1} - \chi\hat{\omega}_t \tag{20}$$

$$\hat{i}^r_t = \hat{r}^n_t + \phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t \tag{21}$$

$$\hat{i}^s_t = \max\left\{i^s_{bound}, \hat{i}^r_t\right\} \tag{22}$$

Here we assume an exponential utility function $1 - exp(-qC_t) + \frac{L_t^{1+\eta}}{1+\eta}$, where $q > 0$ and $\eta > 0$, and assume the bank intermediation cost is given by $\Gamma(l_t, R_t, z_t) = l_t^\nu z_t^{-\iota} + \frac{1}{2}\left(R_t - \overline{R}\right)^2$ if $R_t < \overline{R}$ and $\Gamma(l_t, R_t, z_t) = l_t^\nu z_t^{-\iota}$ otherwise. We define $1 > \beta \equiv \chi\beta^b + (1-\chi)\beta^s > 0$, $\sigma \equiv \frac{1}{qY} > 0$, $\omega \equiv \frac{\beta^s}{\beta^b} - 1 > 0$, $\kappa \equiv (1-\alpha)(1-\alpha\beta)(\eta + \sigma^{-1})/\alpha > 0$ and $i^s_{bound} \equiv (1-\gamma)\beta^s - 1$.

Table 3: Summary of log linearized equilibrium conditions.

Given the policy rule (21) and absent a bound on deposit rates, variations in $\hat{\zeta}_t$ have no effect on either output or inflation and $\hat{i}^r_t = \hat{i}^s_t = \hat{r}^n_t$ always. However, this result only holds as long as the natural rate of interest is not so negative that the lower bound on the deposit rate becomes binding. The key question we are interested in answering is what happens when the deposit rate is constrained at the lower bound, and the central bank reduces the policy rate further into negative territory.

3.3 A Numerical Example

We now parameterize the model to assess the effect of negative rates. The parameters of the numerical example are summarized in Table 6 in Appendix D, and all the standard parameters are chosen from the literature. One notable exception is the parameter ι - which is specific to our model and governs the feedback effect from bank profits to credit supply.

We consider several different values of ι, based on the empirical estimates using Swedish bank level data in Table 1. As a lower bound, we let $\iota = 0$. In this case, bank profits do not affect credit supply. In our model, a negative reserve rate should then not lead to any changes in lending rates, consistent with the behavior of some Swedish banks. For intermediate values of ι we use the average coefficient estimates for all loan contracts in Sweden, and the coefficient estimate for the 5 year fixed-interest rate period contracts. These estimates correspond to $\iota = 0.66$ and $\iota = 0.88$ respectively, and we pick the latter as our baseline estimate. Finally, as a higher bound we consider the behavior of the banks that increased their lending rates the most. To arrive at an estimate for these banks we use the coefficient for the 5 year loan contracts, and subtract two standard errors from the estimated coefficient. Given a normal distribution, this captures the behavior of banks with lending rate increases in the 95th percentile. In this case the coefficient estimate is - 0.75, implying $\iota = 1.235$.[19]

The result of our numerical experiment is reported in Figure 15. It shows the dynamic evolution of interest rates, output and inflation following a negative preference shock. The dashed black line depicts the case in which there is no bound on any interest rate (*No bound*). In this case, $i_t^r = i_t^s$ is always feasible. The preference shock reduces the natural rate of interest through equation (20). Absent policy interventions, i.e. cuts in the reserve rate, this would lead to a demand recession. With our specification of policy however, the reduction in the natural rate of interest triggers a reduction in the central bank reserve rate. When the central bank lowers the reserve rate in absence of any bounds, this lowers the deposit rate one-to-one. The reduction in the deposit rate stimulates the consumption of saver households. In addition, lowering the deposit rate reduces the banks financing costs. This increases the banks willingness to lend, which decreases the borrowing rate, thereby stimulating the consumption of borrower households. Hence, the reduction in the reserve rate passes through to the other interest rates in the economy, thereby stimulating aggregate demand and leaving output and inflation unaffected by the shock.

[19]Note that, since we calibrate ι to match the reduced-form relationship between policy rates and lending rates, the results in this section is invariant to the assumptions we make about banks funding sources. Adding a second funding source, without a zero lower bound but with imperfect substitutability with respect to deposits, would not change the results but rather yield a larger value of ι.

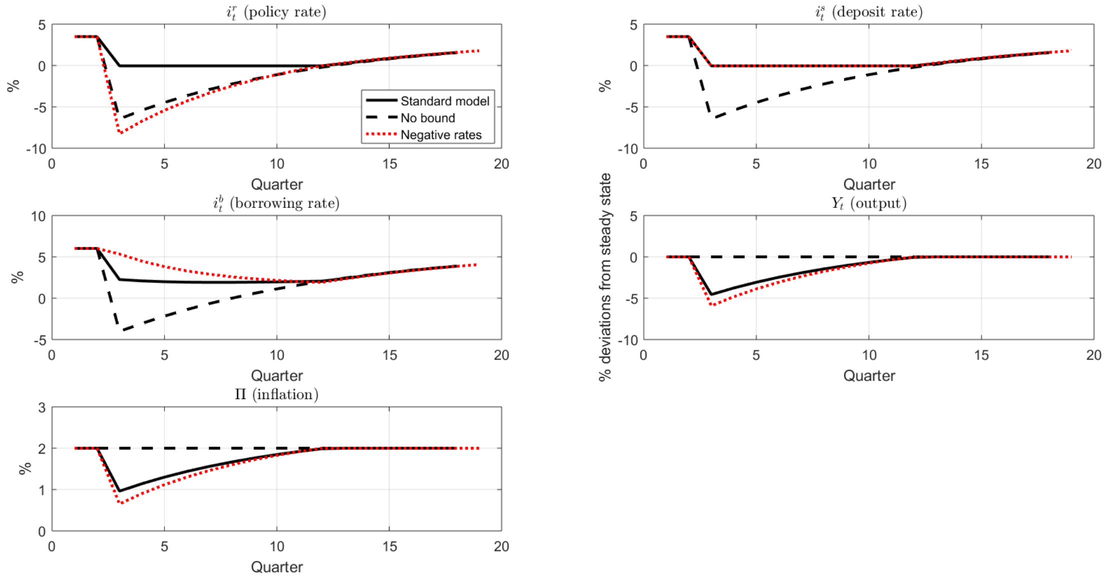


Figure 15: Impulse responses from a preference shock with $\iota = 0.88$.

Next, consider what happens if the deposit rate is bounded and the central bank chooses *not* to go negative, a case corresponding to the behavior of several central banks during the crisis, such as the Federal Reserve. This case is depicted by the solid black line in Figure 15 (*Standard model*). In this case, the inability of the central bank to cut rates results in an output fall of about 4.5 percent and a 1 percent drop in inflation - picked to match the data. We now move on to asking our main question - what happens is the deposit rate is bounded and the central bank still chooses to set a negative reserve rate? The result of this experiment is captured by the dashed red line (*Negative rates*).

As seen from the red line in Figure 15, a negative reserve rate is not expansionary when the deposit rate is bounded.[20] As the negative policy rate is not transmitted to deposit rates, there is no reduction in bank financing costs, and so no reduction in lending rates. Further, the reduction in bank profits leads to an increase in intermediation costs for $\iota > 0$. This increases the interest rate spread. Accordingly, output falls by an additional percentage point when the central bank goes negative.

For the shock studied, having the central bank follow a Taylor rule implies a large negative reserve rate. However, the countries which have implemented negative rates have only ventured modestly below zero. We now explore what happens if the central bank sets a reserve rate equal to - 0.5 percent, to mimic the Swedish case.

[20]Here, so as not to exaggerate the negative effect, we assume that the Taylor rule is such that the central bank targets the natural rate but does not respond to output and inflation gaps when the bound is binding.

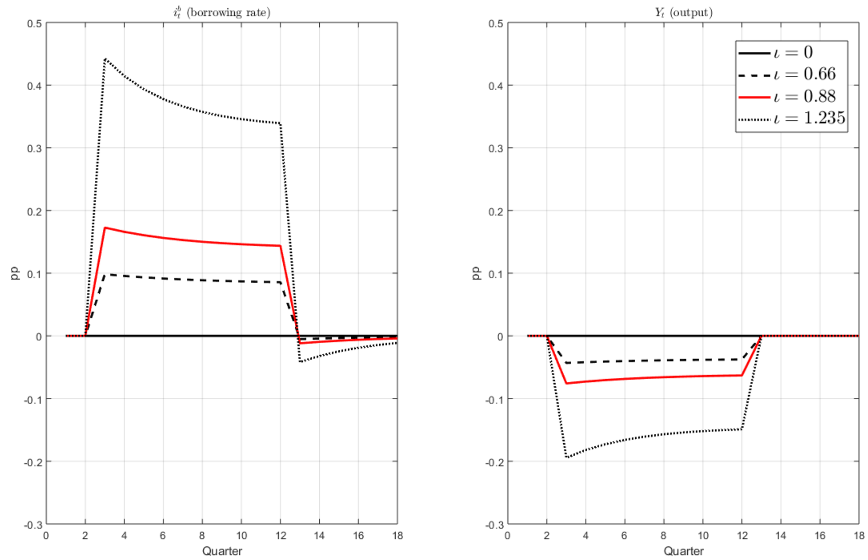


Figure 16: Difference in IRFs for $\hat{i}^b_t$ and $\hat{y}_t$ between negative rates model and standard model for different values of ι.

In Figure 16 we plot the difference in the borrowing rate and output between the standard model and the negative rates model. That is, we compare the outcomes when the central bank reduces the reserve rate below the bound on the deposit rate to the outcomes when the central bank does not push below this bound. As seen from the figure, a negative reserve rate will tend to increase the borrowing rate and reduce output. How strong this effect is depends crucially on the ι-parameter, i.e. on the feedback from bank profits to intermediation costs. In our baseline case with $\iota = 0.88$, the borrowing rate is approximately 15 basis points higher if the central bank goes negative, and output is approximately 7 basis points lower, a modest albeit economically significant effect.

4 Discussion and Extensions

We have focused on the transmission of monetary policy through the bank sector. It is possible that negative interest rates stimulate aggregate demand through other channels, for example through wealth effects or through the exchange rate. The point we want to make is that the pass-through to bank interest rates - traditionally the most important channel of monetary policy - is weakened with negative policy rates.

Even if lending volumes do not respond positively to negative policy rates, there could potentially be an effect on the composition of borrowers. It has been suggested that banks may respond to negative interest rates by increasing risk taking. Heider, Saidi, and Schepens

(2016) find support for increased risk taking in the Euro Area, using volatility in the return-to-asset ratio as a proxy for risk taking. According to their results, banks in the Euro Area responded to the negative policy rate by increasing return volatility. This is certainly not the traditional transmission mechanism of monetary policy, and it is unclear whether such an outcome is desirable.

Another mechanism through which negative interest rates could have an effect is through signaling about future interest rates. While it is possible that such a signaling mechanism played a role, it is unclear why it could not be achieved via direct announcements of future policy rates. It is also worth noting that government borrowing rates might have been reduced due to negative policy rates. To the extent that this stimulated fiscal expansions, that would be an additional way through which negative rates had an effect.

Finally, we have assumed that the banking sector is perfectly competitive. As shown by Drechsler, Savov, and Schnabl (2017), however, there is compelling evidence that banks have considerable market power and thus are able to pay deposit rates below the risk-free interest rate. Our model could be extended to include market power in the bank sector, but as long as there is a bound on the deposit rate, our result would still apply. In that setting, the bound on the deposit rate would have a negative effect of the banks balance sheet, independently of negative policy rates, as it would prevent banks from benefiting from the interest rate spread between the deposit rate and the risk-free rate. This could possibly amplify the contractionary effect of negative policy rates (depending on the curvature of $\Gamma_{zl} = 0$), thus strengthening the results.

5 Conclusion

Since 2014, several countries have experimented with negative policy rates. In this paper, we have documented that negative central bank rates have not been transmitted to aggregate deposit rates, which remain stuck at levels close to zero. As a result, aggregate lending rates remain elevated as well. Using bank level data from Sweden, we documented a disconnect between the policy rate and lending rates, once the policy rate fell below zero. We further showed that this disconnect is partially explained by reliance on deposit financing. Consistent with this, we found that Swedish banks with high deposit shares cut back on lending relative to other banks - once the policy rate turned negative.

Motivated by our empirical findings, we developed a New Keynesian model with savers, borrowers, and a bank sector. By including money storage costs and central bank reserves, we captured the disconnect between the policy rate and the deposit rate at the lower bound. In this framework, we showed that a negative policy rate was at best irrelevant, but could potentially be contractionary due to a negative effect on bank profits.

A key limitation of our analysis is that the long run effects of negative interest rates might differ from the short run effects. This could either weaken or strengthen our results. On one hand, banks may become more willing to pass negative rates onto their depositors over time, either via directly lowering deposit rates or by increasing fees. On the other hand, consumers and firms may adopt alternative strategies to circumvent negative rates, such as investing in money storage facilities. If the central bank charges very negative interest rates for holding the transaction balances of commercial banks, it is possible that banks will adopt alternative payment technologies to avoid having to pay the negative rates.

Given the long-term decline in interest rates, the need for unconventional monetary policy is likely to remain high in the future. Our findings suggest that negative interest rates are not a substitute for regular interest rate cuts in positive territory, at least to the extent that these cuts are expected to work via the bank lending channel. The question remains, however, what is? Alternative monetary policy measures include quantitative easing, forward guidance and credit subsidies such as the TLTRO program implemented by the ECB. While the existing literature has made progress in evaluating these measures, the question of how monetary policy should optimally be implemented in a low interest rate environment remains a question which should be high on the research agenda.

References

Agarwal, R., and M. Kimball (2015): *Breaking through the zero lower bound.* International Monetary Fund.

Basten, C., and M. Mariathasan (2018): "How Banks Respond to Negative Interest Rates: Evidence from the Swiss Exemption Threshold," Discussion paper.

Bech, M. L., and A. Malkhozov (2016): "How have central banks implemented negative policy rates?," *Working paper.*

Benigno, P., G. B. Eggertsson, and F. Romei (2014): "Dynamic debt deleveraging and optimal monetary policy," Discussion paper, National Bureau of Economic Research.

Bernanke, B., T. Geithner, and H. Paulson (2018): "What We Need to Fight the Next Financial Crisis," Discussion paper, The New York Times.

Borio, C., and L. Gambacorta (2017): "Monetary policy and bank lending in a low interest rate environment: diminishing effectiveness?," *Journal of Macroeconomics.*

Brunnermeier, M. K., and Y. Koby (2017): "The Reversal Interest Rate: An Effective Lower Bound on Monetary Policy," *Preparation). Print.*

Buiter, W. H., and N. Panigirtzoglou (2003): "Overcoming the zero bound on nominal interest rates with negative interest on currency: Gesell's solution," *The Economic Journal*, 113(490), 723–746.

Caballero, R. J., and E. Farhi (2017): "The safety trap," *The Review of Economic Studies*, p. rdx013.

Carney, M. (2016): "Bank of England Inflation Report Q and A," *Bank of England Inflation Report.*

Curdia, V., and M. Woodford (2011): "The central-bank balance sheet as an instrument of monetary policy," *Journal of Monetary Economics*, 58(1), 54–79.

Drechsler, I., A. Savov, and P. Schnabl (2017): "The deposits channel of monetary policy," *The Quarterly Journal of Economics*, 132(4), 1819–1876.

Eggertsson, G. B., and P. Krugman (2012): "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach," *The Quarterly Journal of Economics*, 127(3), 1469–1513.

Eggertsson, G. B., and N. R. Mehrotra (2014): "A model of secular stagnation," Discussion paper, National Bureau of Economic Research.

Eggertsson, G. B., and M. Woodford (2006): "Optimal monetary and fiscal policy in a liquidity trap," in *NBER International Seminar on Macroeconomics 2004*, pp. 75–144. The MIT Press.

Financial Times (2016): "Negative rates still hurting eurozone banks," *Financial Times*.

Gali, J. (2008): "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework," .

Gerali, A., S. Neri, L. Sessa, and F. M. Signoretti (2010): "Credit and Banking in a DSGE Model of the Euro Area," *Journal of Money, Credit and Banking*, 42(s1), 107–141.

Gertler, M., and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of monetary economics*, 3(3), 547–599.

Gesell, S. (1916): "The Natural Economic Order, translated by Philip Pye, 2002," .

Goodfriend, M. (2000): "Overcoming the zero bound on interest rate policy," *Journal of Money, Credit and Banking*, pp. 1007–1035.

Greenlaw, D., J. D. Hamilton, E. Harris, and K. D. West (2018): "A Skeptical View of the Impact of the Fed Balance Sheet," Discussion paper, National Bureau of Economic Research.

Greenwood, R., S. Hanson, J. S. Rudolph, and L. H. Summers (2014): *Government debt management at the zero lower bound*. Hutchins Center on Fiscal & Monetary Policy at Brookings Institution.

Guerrieri, L., and M. Iacoviello (2015): "OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily," *Journal of Monetary Economics*, 70, 22–38.

Heider, F., F. Saidi, and G. Schepens (2016): "Life Below Zero: Bank Lending Under Negative Policy Rates," *Working paper*.

Holmstrom, B., and J. Tirole (1997): "Financial intermediation, loanable funds, and the real sector," *the Quarterly Journal of economics*, 112(3), 663–691.

ING (2015): "Negative rates, negative reactions," Discussion paper, ING Economic and Financial Analysis.

Jackson, H. (2015): “The international experience with negative policy rates,” Discussion paper, Bank of Canada Staff Discussion Paper.

Jiménez, G., S. Ongena, J.-L. Peydró, and J. Saurina (2012): “Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications,” *The American Economic Review*, 102(5), 2301–2326.

Jordan, T. (2016): “Monetary policy using negative interest rates: a status report,” Discussion paper, Swiss National Bank.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015): “Household leveraging and deleveraging,” *Review of Economic Dynamics*, 18(1), 3–20.

Kashyap, A. K., and J. C. Stein (2000): “What do a million observations on banks say about the transmission of monetary policy?,” *American Economic Review*, pp. 407–428.

Kerbl, S., and M. Sigmund (2017): “Negative Interest Rates: Forecasting Banks Profitability in a New Environment,” *Working Paper*.

Kiley, M. T., and J. M. Roberts (2017): “Monetary policy in a low interest rate world,” *Brookings Paper on Economic Activity*.

Krugman, P. R. (1998): “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 1998(2), 137–205.

Rognlie, M. (2015): “What lower bound? Monetary policy with negative interest rates,” *Unpublished manuscript, Department of Economics, Harvard University (November 23)*.

Rogoff, K. (2017a): “Dealing with Monetary Paralysis at the Zero Bound,” *The Journal of Economic Perspectives*, 31(3), 47–66.

Rogoff, K. S. (2017b): *The Curse of Cash: How Large-Denomination Bills Aid Crime and Tax Evasion and Constrain Monetary Policy*. Princeton University Press.

Rotemberg, J. J., and M. Woodford (1997): “An optimization-based econometric framework for the evaluation of monetary policy,” *NBER macroeconomics annual*, 12, 297–346.

Summers, L. H. (2014): “US economic prospects: Secular stagnation, hysteresis, and the zero lower bound,” *Business Economics*, 49(2), 65–73.

Swanson, E. T. (2017): "Measuring the effects of Federal Reserve forward guidance and asset purchases on financial markets," Discussion paper, National Bureau of Economic Research.

The Riksbank (2015): "Monetary Policy Report February 2015," Discussion paper, Sveriges Riksbank.

Waller, C. (2016): "Negative interest rates: a tax in sheep's clothing," Discussion paper.

Woodford, M. (2003): "Interest and prices," .

A Additional Figures and Tables

	USA		Euro Area		Sweden		Denmark		Switzerland	
Nominal Rates	Initial	Easing	Initial	Easing	Initial	Easing	Initial	Easing	Initial	Easing
1970	8.9	5.1	-	-	-	-	-	-	-	-
1975	11	5.1	12	8.6	8.0	1.5	-	-	7.0	6.5
1981	18	9.0	12	7.0	12	3.5	-	-	7.5	5.7
1990	8.6	5.6	-	-	-	-	-	-	-	-
1992	-	-	9.6	6.5	12	7.5	15	11	8.8	8.0
2001	6.5	5.5	-	-	-	-	-	-	-	-
2008	5.3	5.1	4.3	3.9	4.8	4.5	4.5	4.0	2.4	2.4
2011	-	-	1.0	1.4	2.1	2.5	1.1	1.6	0.07	2.0
Average		5.9		5.5		3.9		5.5		4.9
(pre-zero)		(5.9)		(6.5)		(4.3)		(7.5)		(5.7)
	USA		Euro Area		Sweden		Denmark		Switzerland	
Real Rates	Initial	Easing	Initial	Easing	Initial	Easing	Initial	Easing	Initial	Easing
1970	3.9	5.1	-	-	-	-	-	-	-	-
1975	4.5	9.1	5.1	7.7	-1.0	3.9	-	-	-2.8	3.0
1981	8.2	3.6	5.9	4.2	1.8	2.2	-	-	1.0	4.9
1990	5.5	4.9	-	-	-	-	-	-	-	-
1992	-	-	5.6	3.5	7.7	7.4	14	9.7	4.7	3.5
2001	4.1	5.8	-	-	-	-	-	-	-	-
2008	2.5	4.5	2.1	3.3	1.9	3.3	2.5	4.6	1.6	3.9
2011	-	-	-0.08	2.2	1.1	2.5	-0.1	0	0.72	2.2
Average		5.5		4.2		3.9		4.8		3.5
(pre-zero)		(5.5)		(4.7)		(4.2)		(7.2)		(3.8)

Table 4: Nominal and real interest rates at start of recession (%) and interest rate cut in response to recession (pp). Recession dates for the US are NBER recession dates. Recession rates for the Euro Area are CEPR recession dates. Recession dates for Sweden, Denmark and Switzerland are defined as the overlap between the country specific OECD recession indicators and the CEPR recession dates for the Euro Area. The interest rates are the federal funds rate for the US and short term interbank rates for the remaining countries (the German interbank rate is used for the Euro Area) Source: St. Louis FRED and own calculations.

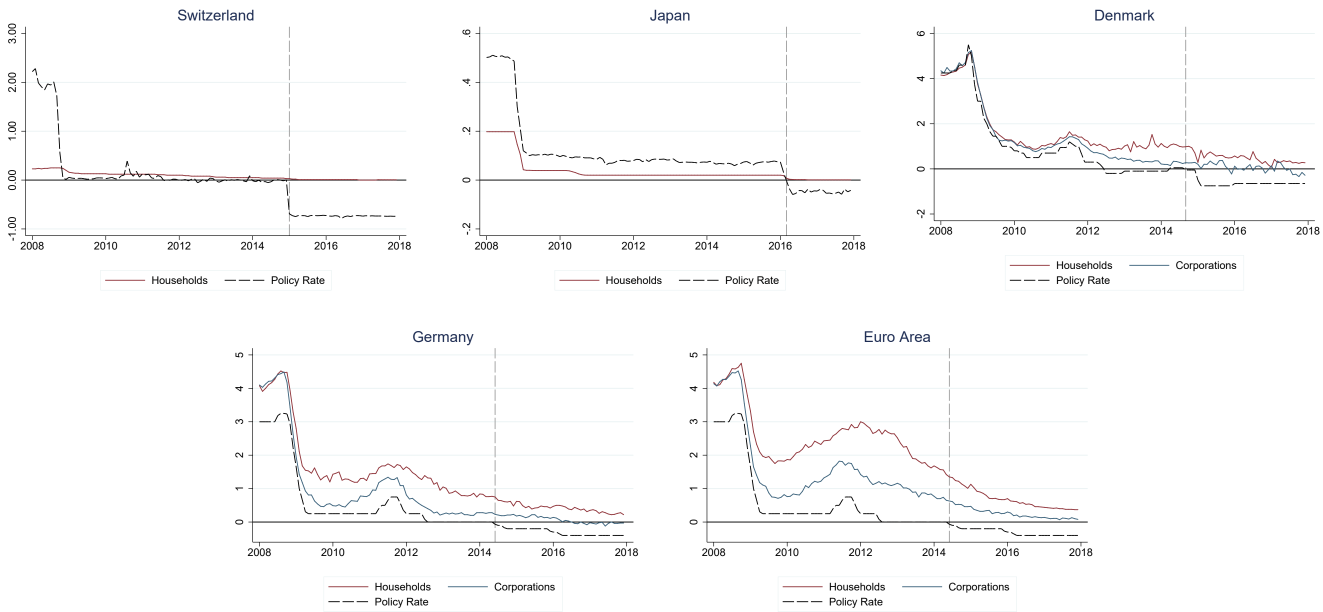


Figure 17: Aggregate deposit rates for Switzerland, Japan, Denmark, the Euro Area and Germany. The policy rates are defined as SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japan), the Certificates of Deposit Rate (Denmark) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: the Swiss National Bank (SNB), Bank of Japan, the Danish National Bank (DNB), and the European Central Bank (ECB).

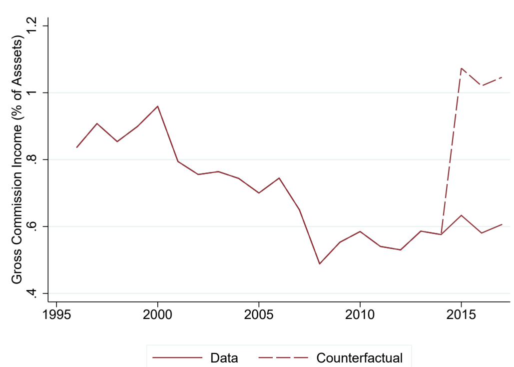


Figure 18: Actual and Counterfactual Commission Income as a Share of Assets (%). The counterfactual commission income is calculated as the amount of commission income that would be necessary to make up for the bound on the nominal deposit rate, all else equal. The counterfactual commission income is given by actual commission income plus $\frac{Deposits_t}{Assets_t}\left(i_t - i_t^{cf}\right) = 0.47\,(0.03 - (-0.9)) = 0.44$, where i_t is the average aggregate deposit rate and i_t^{cf} is a counterfactual deposit rate calculated under the assumption that the markdown to the repo rate is constant and equal to the pre-zero markdown. Source: Statistics Sweden and own calculations.

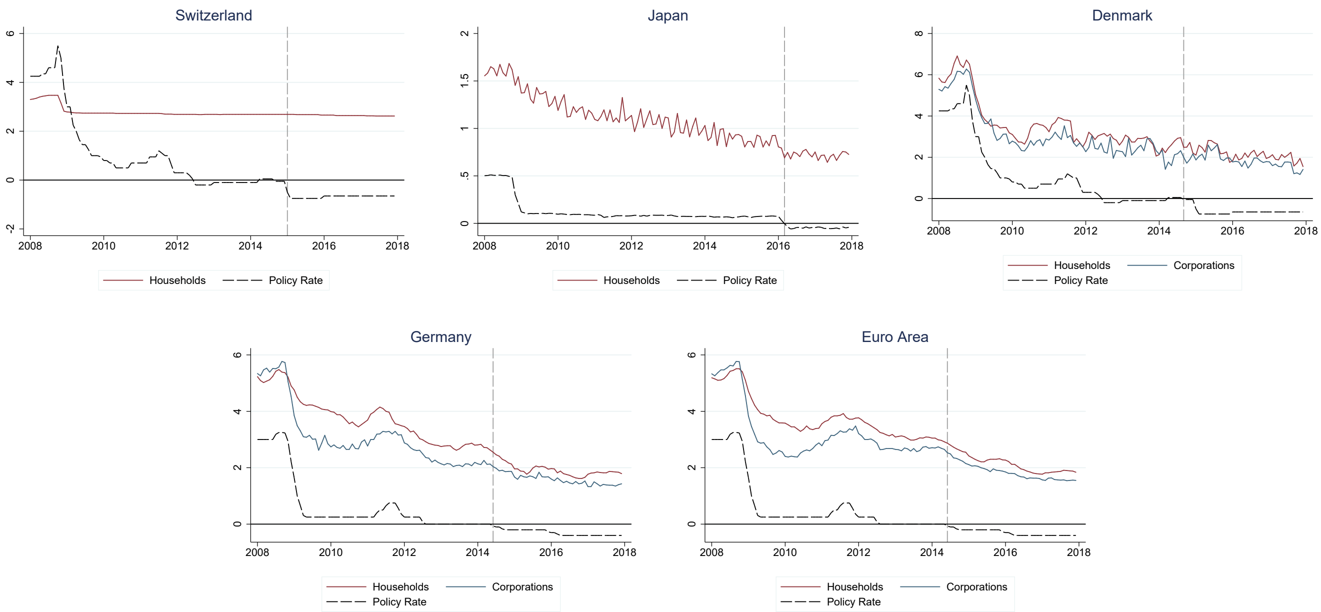


Figure 19: Aggregate lending rates for Switzerland, Japan, Denmark, the Euro Area and Germany. The policy rates are defined as SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japan), the Certificates of Deposit Rate (Denmark) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: the Swiss National Bank (SNB), Bank of Japan, the Danish National Bank (DNB), and the European Central Bank (ECB).

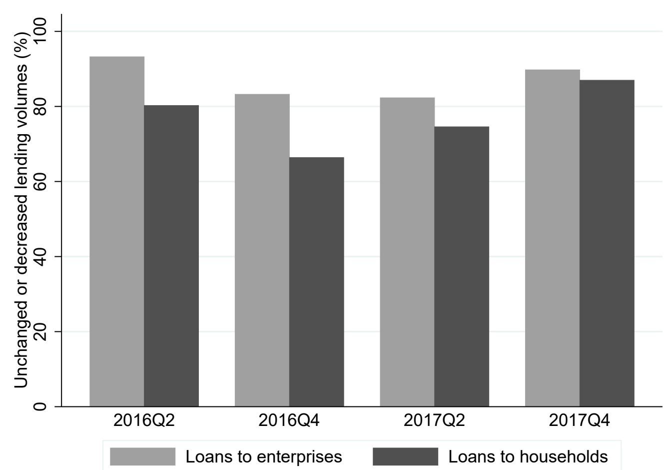


Figure 20: Share of banks answering that the negative ECB policy rate has had a negative or neutral effect on their lending volume in the past six months. Household loans only include loans to house purchases. Source: ECB bank lending survey.

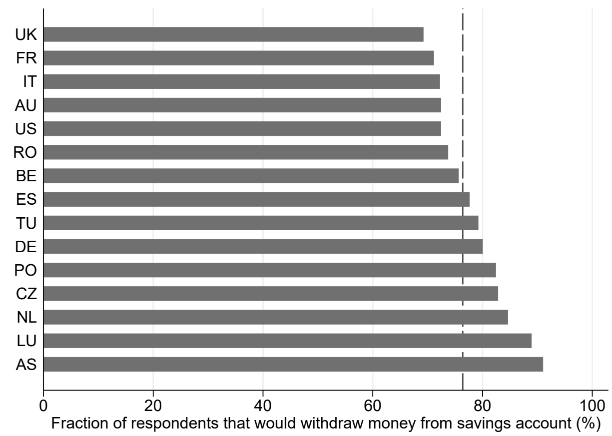


Figure 21: Fraction of households who would withdraw money from their savings account if they were levied a negative interest rate. Solid line represent unweighted average of 76.4 %. Source: ING (2015)

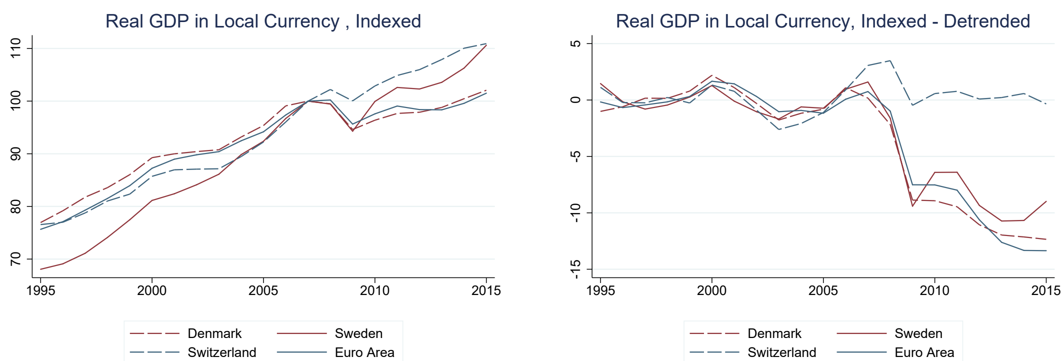


Figure 22: Gross domestic product in constant prices. Local currency. Indexed so that GDP^{2008}=100. The right panel shows the detrended series using a linear time trend based on the 1995-2007 period. Source: St. Louis FRED and own calculations.

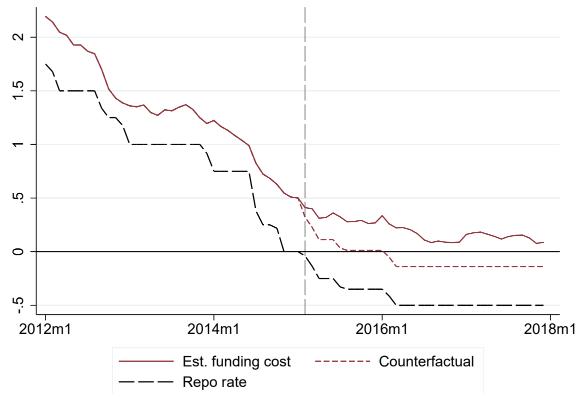
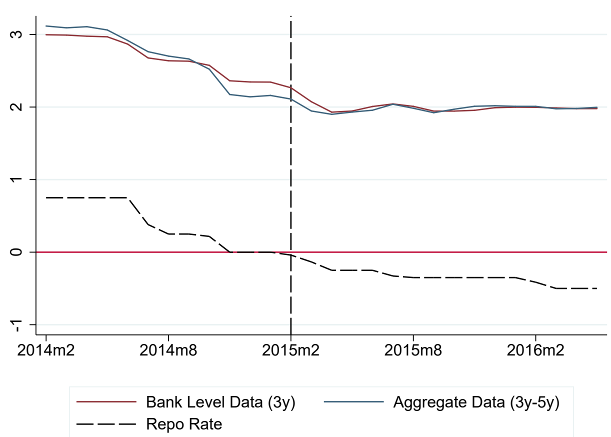


Figure 23: Estimated average funding costs. The estimated average funding cost is computed by taking the weighted average of the assumed interest rate of the different funding sources of the bank. Certificates are assumed to have the same interest rate as STIBOR 3M, while unsecured debt are assumed to have the same interest rate as STIBOR 3M plus a 2 percent constant risk-premium. The "counterfactual" series correspond to the case when the spread between the repo rate and the estimated funding cost remain fixed at pre-negative levels. Weights from Figure 2 used. Source: The Riksbank and own calculations.



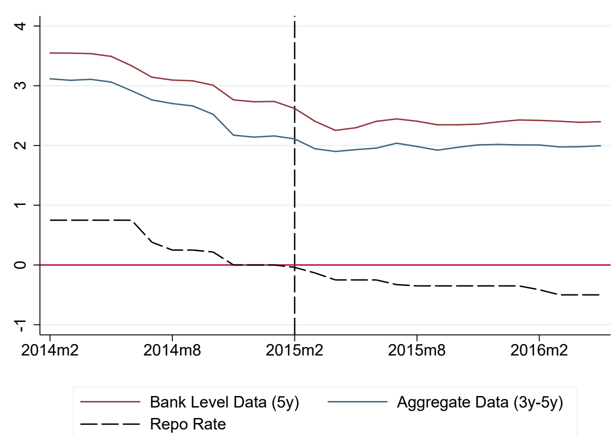


Figure 24: Comparing bank level mortgage rates to aggregate data. We aggregate the bank level mortgage rates using market shares, supplemented with data on lending volumes. The blue line (Aggregate Data) depicts the official average mortgage interest rate for loans with a fixed interest rate period of 3-5 years. Source: Swedish Banker's Association (market shares), Statistics Sweden (lending volumes, aggregate rates).

B Marginal and Average Rate on Reserves

In our model, central bank reserves earn a single interest rate i^r. In reality, central banks can adopt exemption thresholds and tiered remuneration schemes so that not all reserves earn the same interest rate. Hence, even though the key policy rate is negative, not all central bank reserves necessarily earn a negative interest rate. Here we provide a short overview of the different remuneration schedules implemented in the Euro Area, Denmark, Japan, Sweden and Switzerland. For a more detailed analysis see Bech and Malkhozov (2016).

In the Euro Area, required reserves earn the main financing operations rate - currently set at 0.00 percent. Excess reserves on the other hand, earn the central bank deposit rate - currently set at -0.40 percent. Hence, only reserves in excess of the required level earn a negative interest rate. A similar remuneration scheme is in place in Denmark. Banks can deposit funds at the Danish central bank at the current account rate of 0.00 percent. However, there are (bank-specific) limits on the amount of funds that banks can deposit at the current account rate. Funds in excess of these limits earn the interest rate on one-week certificates of deposits - currently set at -0.65 percent.

The Riksbank issues one-week debt certificates, which currently earn an interest rate of -0.50 percent. While there is no reserve requirement, the Swedish central bank undertakes fine-tuning operations to drain the bank sector of remaining reserves each day. These fine-tuning operations earn an interest rate of -0.60 percent.[21] The Swiss central bank has the lowest key policy rate at -0.75 percent. However, due to high exemption thresholds the majority of reserves earn a zero interest rate. The Bank of Japan adopted a three-tiered remuneration schedule when the key policy rate turned negative. As a result, central bank reserves earn an interest rate of either 0.10, 0.00 or -0.10 percent.

Due to the tiered remuneration system, there is a gap between the average and the marginal reserve rate. Bech and Malkhozov (2016) calculate this gap as of February 2016, as illustrated in Figure 25.

[21] Any residual reserves earn the deposit rate of -1.25 percent.

Central bank remuneration schedules (mid-February 2016) — Table 1

	European Central Bank	Sveriges Riksbank	Swiss National Bank	Danmarks Nationalbank
Exemption threshold	Minimum reserve requirement	.	Individual exemption	Current account limit
Aggregate amounts	Local currency, in billions			
Overnight deposits (reserves)				
Below threshold	113	–50[1]	303	29
Above threshold	650		170	.[3]
Term (one-week)	.	187	.	119
Policy rates	Basis points			
Overnight deposits (reserves)				
Below threshold	5	–60[2]	0	0
Above threshold	–30		–75	.[3]
Term (one-week)	.	–50	.	–65
Weighted average rate	–25	–52	–27	–52
Marginal minus average rate[4]	–5	–8	–48	–13

[1] Amount of fine-tuning operations. In addition, overnight deposits with central bank represent SEK 0.01 billion. [2] Rate applied to fine-tuning operations. Overnight deposits with central bank earn –125 basis points. [3] Amounts above the aggregate current account limit are converted into one-week certificates of deposit (Box 2). [4] Marginal rate is the rate on overnight deposits with central bank above exemption threshold.

Sources: Central banks; authors' calculations.

Figure 25: Reserve rates - Source: Bech and Malkhozov (2016).

C Details of the model

Households

We consider a closed economy, populated by a unit-measure continuum of households. Households are of two types, either patient (indexed by superscript s) or impatient (indexed by superscript b). Patient households have a higher discount factor than impatient agents, i.e. $\beta^s > \beta^b$. The total mass of patient households is $1-\chi$, while the total mass of impatient households is χ. In equilibrium, impatient households will borrow from patient households via the banking system, which we specify below. We therefore refer to the impatient households as "borrowers" and the patient households as "savers".

Households consume, supply labor, borrow/save and hold real money balances. At any time t, the optimal choice of consumption, labor, borrowing/saving and money holdings for a household $j \in \{s, b\}$ maximizes the present value of the sum of utilities

$$\mathcal{U}_t^j = \mathbb{E}_t \sum_{T=t}^{\infty} \left(\beta^j\right)^{T-t} \left[U\left(C_T^j\right) + \Omega\left(\frac{M_T^j}{P_T}\right) - \ V\left(N_T^j\right)\right] \zeta_t \tag{23}$$

where ζ_t is a random variable following some stochastic process and acts as a preference shock.[22] C_t^j and N_t^j denote consumption and labor for type j respectively, and the utility function satisfies standard assumptions clarified below.

Households consume a bundle of consumption goods. Specifically, there is a continuum of goods indexed by i, and each household j has preferences over the consumption index

$$C_t^j = \left(\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}} \tag{24}$$

where $\theta > 1$ measures the elasticity of substitution between goods.

Agents maximize lifetime utility (equation (23)) subject to the following flow budget constraint:

$$M_t^j + B_{t-1}^j \left(1 + i_{t-1}^j\right) = W_t^j N_t^j + B_t^j + M_{t-1}^j - P_t C_t^j - S\left(M_{t-1}^j\right) + \Psi_t^j + \psi_t^j - T_t^j \tag{25}$$

B_t^j denotes one period risk-free debt of type j ($B_t^s < 0$ and $B_t^b > 0$). For the saver, B_t^s consists of bank deposits and government bonds, both remunerated at the same interest rate i_t^s by arbitrage. Borrower households borrow from the bank sector only, at the banks' lending rate i_t^b. $S\left(M_{t-1}^j\right)$ denotes the storage cost of holding money. Ψ_t^j is type j's share of firm profits, and ψ_t^j is type j's share of bank profits. Let Z_t^{firm} denote firm profits, and Z_t denote bank profits. We assume that firm profits are distributed to both household types based on their population shares, i.e. $\Psi_t^b = \chi Z_t^{firm}$ and $\Psi_t^s = (1-\chi) Z_t^{firm}$. Bank profits on the other hand are only distributed to savers, which own the deposits by which banks finance themselves.[23] Hence, we have that $\psi_t^b = 0$ and $\psi_t^s = Z_t$.

The optimal consumption path for an individual of type j has to satisfy the standard Euler-equation

$$U'\left(C_t^j\right)\zeta_t = \beta^j \left(1 + i_t^j\right) \mathbb{E}_t \left(\Pi_{t+1}^{-1} U'\left(C_{t+1}^j\right) \zeta_{t+1}\right) \tag{26}$$

Optimal labor supply has to satisfy the intratemporal trade-off between consumption and labor[24]

[22] We introduce the preference shock as a parsimonious way of engineering a recession.

[23] Distributing bank profits to both household types would make negative interest rates even more contractionary. The reduction in bank profits would reduce the transfer income of borrower households, causing them to reduce consumption. We believe this effect to be of second order significance, and so we abstract from it here.

[24] We assume that the function V is increasing in N and convex with well defined first and second derivatives.

$$\frac{V'\left(N_t^j\right)}{U'\left(C_t^j\right)} = \frac{W_t^j}{P_t} \tag{27}$$

Finally, optimal holdings of money have to satisfy[25]

$$\frac{\Omega'\left(\dfrac{M_t^j}{P_t}\right)}{U'\left(C_t^j\right)} = \frac{i_t^j + S'\left(M_t^j\right)}{1+i_t^j} \tag{28}$$

The lower bound on the deposit rate $\underline{i}^s$ is typically defined as the lowest value of i_t^s satisfying equation (28). The lower bound therefore depends crucially on the marginal storage cost. With the existence of a satiation point in real money balances, zero (or constant) storage costs imply $S'\left(M_t^s\right) = 0$ and $\underline{i}^s = 0$. That is, the deposit rate is bounded at exactly zero. With a non-zero marginal storage cost however, this is no longer the case. If storage cost are convex, for instance, the marginal storage cost is increasing in M_t^s. In this case, there is no lower bound. Based on the data from Section 2, a reasonable assumption is that the deposit rate is bounded at some value close to zero. This is consistent with a proportional storage cost $S\left(M_t^s\right) = \gamma M_t^s$, with a small $\gamma > 0$. We therefore assume proportional storage costs for the rest of the paper, in which the lower bound on deposit rates is given by $\underline{i}^s = -\gamma$.[26]

We assume that households have exponential preferences over consumption, i.e. $U(C_t^j) = 1 - exp\{-qC_t^j\}$ for some $q > 0$. The assumption of exponential utility is made for simplicity, as it facilitates aggregation across agents. Under these assumptions, the labor-consumption trade-off can easily be aggregated into an economy-wide labor market condition[27]

$$\frac{V'\left(N_t\right)}{U'\left(C_t\right)} = \frac{W_t}{P_t} \tag{29}$$

Letting G_t denote government spending,[28] aggregate demand is given by

$$Y_t = \chi C_t^b + (1-\chi)\, C_t^s + G_t \tag{30}$$

[25]We assume a satiation point for money. That is, at some level $\bar{m}^j$ households become satiated in real money balances, and so $\Omega'(\bar{m}^j) = 0$.

[26]This nests the case of no storage costs, in which case $\gamma = 0$.

[27]To see this, just take the weighted average of equation (27) using the population shares χ and $1-\chi$ as the respective weights.

[28]Government policies are explained below.

Firms

Each good i is produced by a firm i. Production is linear in labor, i.e.

$$Y_t(i) = N_t(i) \tag{31}$$

where $N_t(i)$ is a Cobb-Douglas composite of labor from borrowers and savers respectively, i.e. $N_t(i) = \left(N_t^b(i)\right)^{\chi} \left(N_t^s(i)\right)^{1-\chi}$, as in Benigno, Eggertsson, and Romei (2014). This ensures that each type of labor receives a total compensation equal to a fixed share of total labor expenses. That is,

$$\begin{aligned} W_t^b N_t^b &= \chi W_t N_t & (32) \\ W_t^s N_t^s &= (1-\chi) W_t N_t & (33) \end{aligned}$$

where $W_t = \left(W_t^b\right)^{\chi} \left(W_t^s\right)^{1-\chi}$ and $N_t = \int_0^1 N_t(i)\, di$.

Given preferences, firms face a downward-sloping demand function

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t \tag{34}$$

We introduce nominal rigidities by assuming Calvo-pricing. That is, in each period, a fraction α of firms are not able to reset their price. Thus, the likelihood that a price set in period t applies in period $T > t$ is α^{T-t}. Prices are assumed to be indexed to the inflation target Π.

A firm that is allowed to reset their price in period t sets the price to maximize the present value of discounted profits in the event that the price remains fixed. That is, each firm i choose $P_t(i)$ to maximize

$$\mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[\Pi^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - \frac{W_T}{P_T} Y_T(i)\right] \tag{35}$$

where $\lambda_T \equiv q\left(\chi \exp\left\{-qC_T^b\right\} + (1-\chi)\exp\left\{-qC_T^s\right\}\right)$, which is the weighted marginal utility of consumption and $\beta \equiv \chi\beta^b + (1-\chi)\beta^s$.[29]

Denoting the markup as $\mu \equiv \dfrac{\theta}{\theta-1}$, firms set the price as a markup over the average of expected marginal costs during the periods the price is expected to remain in place. That is,

[29] Recall that the firm is owned by both types of households according to their respective population shares.

the first-order condition for the optimal price $P(i)_t^*$ for firm i is

$$\frac{P(i)_t^*}{P_t} = \mu \frac{\mathbb{E}_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T\left(\frac{P_T}{P_t}\frac{1}{\Pi^{T-t}}\right)^{\theta}\frac{W_T}{P_T}Y_T\right\}}{\mathbb{E}_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T\left(\frac{P_T}{P_t}\frac{1}{\Pi^{T-t}}\right)^{\theta-1}\frac{W_T}{P_T}Y_T\right\}} \tag{36}$$

This implies a law of motion for the aggregate price level

$$P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\Pi^{1-\theta} \tag{37}$$

where P_t^* is the optimal price from equation (36), taking into account that in equilibrium $P_t^*(i)$ is identical for all i. We denote this price P_t^*.

Since prices are sticky, there exists price dispersion which we denote by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di \tag{38}$$

with the law of motion

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}} \tag{39}$$

We assume that the disutility of labor takes the form $V(N_t^j) = \frac{(N^j)^{1+\eta}}{1+\eta}$. We can then combine equations (36) - (39), together with the aggregate labor-consumption trade-off (equation (29)) to get an aggregate Phillips curve of the following form:

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{40}$$

where

$$F_t = \lambda_t Y_t + \alpha\beta\mathbb{E}_t\left\{F_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1}\right\} \tag{41}$$

and

$$K_t = \mu\frac{\lambda_t\Delta_t^{\eta}Y_t^{1+\eta}}{z\exp\{-zY_t\}} + \alpha\beta\mathbb{E}_t\left\{K_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta}\right\} \tag{42}$$

and

$$\lambda_T \equiv z\left(\chi \exp\left\{-qC_T^b\right\} + (1-\chi)\exp\left\{-qC_T^s\right\}\right) \tag{43}$$

Since every firm faces demand $Y(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$ and $Y_t(i) = N_t(i)$, we can integrate over all firms to get that

$$N_t = \Delta_t Y_t \tag{44}$$

Banks

Our banking sector is made up of identical, perfectly competitive banks. Bank assets consist of one-period real loans l_t. In addition to loans, banks hold real reserves $R_t \geq 0$ and real money balances $m_t = \frac{M_t}{P_t} \geq 0$, both issued by the central bank.[30] Bank liabilities consist of real deposits d_t. Reserves are remunerated at the interest rate i_t^r , which is set by the central bank. Loans earn a return i_t^b. The cost of funds, i.e. the deposit rate, is denoted i_t^s. Banks take all of these interest-rates as given.

Financial intermediation takes up real resources. Therefore, in equilibrium there is a spread between the deposit rate i_t^s and the lending rate i_t^b. We assume that banks' intermediation costs are given by a function $\Gamma(l_t, R_t, m_t, z_t)$, in which $z_t = \frac{Z_t}{P_t}$ is real bank profit.

We assume that the intermediation costs are increasing and convex in the amount of real loans provided. That is, $\Gamma_l > 0$ and $\Gamma_{ll} \geq 0$. Central bank currency plays a key role in reducing intermediation costs.[31] The marginal cost reductions from holding reserves and money are captured by $\Gamma_R \leq 0$ and $\Gamma_m \leq 0$ respectively. We assume that the bank becomes satiated in reserves for some level $\bar{R}$. That is, $\Gamma_R = 0$ for $R \geq \bar{R}$. Similarly, banks become satiated in money at some level $\bar{m}$, so that $\Gamma_m = 0$ for $m \geq \bar{m}$. Banks can thus reduce their intermediation costs by holding reserves and/or cash, but the opportunity for cost reduction can be exhausted. Finally, we assume that higher profits (weakly) reduce the marginal cost of lending. That is, we assume $\Gamma_{lz} \leq 0$. We discuss this assumption below.

Following Curdia and Woodford (2011) and Benigno, Eggertsson, and Romei (2014) we assume that any real profits from the bank's asset holdings are distributed to their owners in period t and that the bank holds exactly enough assets at the end of the period to pay

[30] Because we treat the bank problem as static - as outlined below - we can express the maximization problem in real terms.

[31] For example, we can think about this as capturing in a reduced form way the liquidity risk that banks face. When banks provide loans, they take on costly liquidity risk because the deposits created when the loans are made have a stochastic point of withdrawal. More reserves helps reduce this expected cost.

off the depositors in period $t+1$.[32] Furthermore, we assume that storage costs of money are proportional and given by $S(M) = \gamma M$. Under these assumptions, real bank profits can be implicitly expressed as:

$$z_t = \frac{i_t^b - i_t^s}{1+i_t^s} l_t - \frac{i_t^s - i_t^r}{1+i_t^s} R_t - \frac{i_t^s + \gamma}{1+i_t^s} m_t - \Gamma(l_t, R_t, m_t, z_t) \tag{45}$$

Any interior l_t, R_t and m_t have to satisfy the respective first-order conditions from the bank's optimization problem[33]

$$l_t \ : \ \frac{i_t^b - i_t^s}{1+i_t^s} = \Gamma_l(l_t, R_t, m_t, z_t) \tag{46}$$

$$R_t \ : \ -\Gamma_R(l_t, R_t, m_t, z_t) = \frac{i_t^s - i_t^r}{1+i_t^s} \tag{47}$$

$$m_t \ : \ -\Gamma_m(l_t, R_t, m_t, z_t) = \frac{i_t^s + \gamma}{1+i_t^s} \tag{48}$$

The first-order condition for real loans says that the banks trade off the marginal profits from lending with the marginal increase in intermediation costs. The next two first-order conditions describe banks demand for reserves and cash. We assume that reserves and money are not perfect substitutes, and so minimizing the intermediation cost implies holding both reserves and money. This is not important for our main result.[34]

The first-order condition for loans pins down the equilibrium credit spread ω_t defined as

$$\omega_t \equiv \frac{1+i_t^b}{1+i_t^s} - 1 = \frac{i_t^b - i_t^s}{1+i_t^s} \tag{49}$$

Specifically, it says that

$$\omega_t = \Gamma_l\left(b_t^b, R_t, m_t, z_t\right) \tag{50}$$

where we have used the market clearing condition in equation (51) to express the spread as a function of the borrowers real debt holdings b_t^b.

[32] The latter is equivalent to assuming that $\left(1+i_t^b\right) l_t + \left(1+i_t^r\right) R_t + m_t - S(m_t) = \left(1+i_t^s\right) d_t$.

[33] Assuming that $\Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t\right)$ is such that there exists a unique z solving equation (45).

[34] The assumption that banks always wants to hold some reserves is however important for the effect of negative interest rates on bank profitability. If we instead assume that the sum of money holdings and reserves enters the banks cost function as one argument, the bank would hold only money once $i^r < -\gamma$. Hence, reducing the interest rate on reserves further would not affect bank profits. However, such a collapse in central bank reserves is not consistent with data, suggesting that banks want to hold some (excess) reserves.

$$l_t = \chi b_t^b \tag{51}$$

That is, the difference between the borrowing rate and the deposit rate is an increasing function of the aggregate relative debt level, and a decreasing function of banks' net worth.

Why do bank profits affect intermediation costs? We have assumed that the marginal cost of extending loans (weakly) decreases with bank profits. That is, $\Gamma_{lz} \leq 0$. This assumption captures, in a reduced form manner, the established link between banks' net worth and their operational costs. We do not make an attempt to microfound this assumption here, which is explicitly done in among others Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010).[35]

In Gertler and Kiyotaki (2010) bank managers may divert funds, which means that banks must satisfy an incentive compatibility constraint in order to obtain external financing. This constraint limits the amount of outside funding the bank can obtain based on the banks net worth. Because credit supply is determined by the total amount of internal and external funding, this means that bank lending depends on bank profits. In an early contribution, Holmstrom and Tirole (1997) achieve a similar link between credit supply and bank net worth by giving banks the opportunity to engage (or not engage) in costly monitoring of its non-financial borrowers. For recent empirical evidence on the relevance of bank net worth in explaining credit supply, see for example Jiménez, Ongena, Peydró, and Saurina (2012).

Importantly, our main result is that negative interest rates are *not expansionary*. This does not depend on profits affecting intermediation costs. However, the link between profits and the intermediation cost is the driving force behind negative interest rates being *contractionary*. If we turn off this mechanism, negative interest rates still reduce bank profits, but this does not feed back into aggregate demand.[36]

Government

The consolidated government budget constraint is given by

$$B_t^g + M_t^{tot} + P_t R_t = (1 + i_{t-1}^g) B_{t-1}^g + M_{t-1}^{tot} + (1 + i_{t-1}^r) P_t R_{t-1} + G_t - T_t \tag{52}$$

[35] Another way to interpret the implied link between bank profits and credit supply is to include a capital requirement. In Gerali, Neri, Sessa, and Signoretti (2010) a reduction in bank profits reduces the banks' capital ratio. In order to recapitalize the bank lowers credit supply.

[36] Alternatively, we could assume that bank profits do not affect intermediation costs, but that bank profits are distributed to all households. A reduction in bank profits would then reduce aggregate demand through the borrowers budget constraint.

where B_t^g is one period government debt, $M_t^{tot} = M_t + M_t^s + M_t^b$ is total money supply - which is the sum of money held by banks and each household type, i_t^g is the one period risk-free rate on government debt, G_t is government spending, and $T_t = \chi T_t^b + (1 - \chi) T_t^s$ is the weighted sum of taxes on the two household types.

The conventional way of defining monetary and fiscal policy, abstracting from reserves and the banking sector (see e.g. Woodford 2003), is to say that fiscal policy is the determination of end of period government liabilities, i.e. $B_t^g + M_t^{tot}$, via the fiscal policy choice of G_t and T_t. Monetary policy on the other hand, determines the *split* of end of period government liabilities B_t^g and M_t^{tot}, via open market operations. This in turn determines the risk-free nominal interest rate i_t^g through the money demand equations of the agents in the economy. The traditional assumption then, is that the one period risk-free rate on government debt corresponds to the policy rate which the monetary authority controls via the supply of money through the money demand equation.

We define monetary and fiscal policy in a similar way here. Fiscal policy is the choice of fiscal spending G_t and taxes T_t. This choice determines total government liabilities at the end of period t – the left hand side of equation (52). Total government liabilities are now composed of public debt and the money holdings of each agent, as well as reserves held at the central bank. Again, monetary policy is defined by how total government liabilities is split between government bonds B_t^g, and the overall supply of central bank issuance. In addition, we assume that the central bank sets the interest rate on reserves i_t^r. The supply of central bank currency is then given by

$$CBC_t = P_t R_t + M_t + M_t^s + M_t^b \tag{53}$$

Given these assumptions, the financial sector itself determines the allocation between reserves and money. That is, the split between the money holdings of different agents and reserves held by banks is an endogenous market outcome determined by the first order conditions of banks and households.

In order to clarify the discussion, it is helpful to review two policy regimes observed in the US at different times. Consider first the institutional arrangement in the US prior to the crisis, when the Federal Reserve paid no interest on reserves, so that $i_t^r = 0$. As seen from equation (47), this implies that banks were not satiated in reserves. The policy maker then chose CBC_t so as to ensure that the risk-free rate was equal to its target. In this more general model, the policy rate is simply the risk-free nominal interest rate, which is equal to the deposit rate and, assuming that depositors can also hold government bonds, the interest rate paid on one period government bonds, i.e. $i_t^s = i_t^g$.

Consider now an alternative institutional arrangement, in which paying interest on reserves is a policy tool. Such a regime seems like a good description of the post-crisis monetary policy operations, both in the US and elsewhere. The central bank now sets the interest rate on reserves equal to the risk-free rate, i.e. $i_t^r = i_t^s = i_t^g$, and chooses CBC_t to implement its desired target. From the first order condition for reserves (47), we see that $i_t^s = i_t^r$ implies that $\Gamma_R = 0$. Hence, as long as banks are satiated in reserves, the central bank implicitly controls i_t^s via i_t^r. A key point, however, is that $\Gamma_R = 0$ is not always feasible due to the lower bound on the deposit rate. If the deposit rate is bounded at $\underline{i}^s = -\gamma$, and the central bank lowers i_t^r below $-\gamma$, then $i_t^s > i_t^r$. The first order condition then implies $\Gamma_R > 0$. Intuitively, it is not possible to keep banks satiated in reserves when they are being charged for their reserve holdings. More explicitly, we assume that the interest rate on reserves follows a Taylor rule given by equation (54). Because of the reserve management policy outlined above, the deposit rate in equilibrium is either equal to the reserve rate or to the lower bound, as specified in equation (55).

$$i_t^r = r_t^n \Pi_t^{\phi_\pi} Y_t^{\phi_Y} \tag{54}$$

$$i_t^s = \max\left\{\underline{i}^s, i_t^r\right\} \tag{55}$$

Before closing this section it is worth pointing out that it seems exceedingly likely that there also exists a lower bound on the reserve rate. Reserves are useful for banks because they are used to settle cash-balances between banks at the end of each day. However, banks could in principle settle these balances outside of the central bank, for example by ferrying currency from one bank to another (or more realistically trade with a privately owned clearing house where the commercial banks can store cash balances). Hence, because banks have the option to exchange their reserves for cash, there is a limit to how negative i_t^r can become. We do not model this bound here, as it does not appear to have been breached (yet) in practice. Instead we focus on the bound on deposit rates - which is observable in the data.

Equilibrium

Non-linear Equilibrium Conditions

Definition 1. The non-linear equilibrium is defined as a sequence of 17 endogenous quantities $\{C_t^b,\ C_t^s,\ b_t^b,\ m_t^b,\ m_t^s,\ \tau_t^s,\ cbc_t,\ Y_t,\ \Pi_t,\ F_t,\ K_t,\ \Delta_t,\ \lambda_t,\ l_t,\ R_t,\ m_t,\ z_t\}_{t=0}^{\infty}$ and three prices $\left\{i_t^s, i_t^b, i_t^r\right\}_{t=0}^{\infty}$ which satisfy equations (56)- (75), for given initial conditions Δ_0, b_0^b and a sequence of shocks $\{\zeta_t\}_{t=0}^{\infty}$.

$$\exp\left\{-qC_t^b\right\}\zeta_t = \beta^b\left(1+i_t^b\right)\mathbb{E}_t\left(\Pi_{t+1}^{-1}\exp\left\{-qC_{t+1}^b\right\}\right)\zeta_{t+1} \tag{56}$$

$$\exp\left\{-qC_t^s\right\}\zeta_t = \beta^s\left(1+i_t^s\right)\mathbb{E}_t\left(\Pi_{t+1}^{-1}\exp\left\{-qC_{t+1}^s\right\}\right)\zeta_{t+1} \tag{57}$$

$$C_t^b + m_t^b + \frac{1+i_{t-1}^b}{\Pi_t}b_{t-1}^b = \chi Y_t + \frac{1-\gamma}{\Pi_t}m_{t-1}^b + b_t^b \tag{58}$$

$$\frac{\Omega'\left(m_t^b\right)}{U'\left(C_t^b\right)} = \frac{i_t^b+\gamma}{1+i_t^b} \tag{59}$$

$$\frac{\Omega'\left(m_t^s\right)}{U'\left(C_t^s\right)} = \frac{i_t^s+\gamma}{1+i_t^s} \tag{60}$$

$$\Pi_t cbc_t = cbc_{t-1} + i_{t-1}^r R_{t-1} - \Pi_t \tau_t^s \tag{61}$$

$$cbc_t = R_t + m_t + m_t^s + m_t^b \tag{62}$$

$$Y_t = \chi C_t^b + (1-\chi)C_t^s \tag{63}$$

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{64}$$

$$F_t = \lambda_t Y_t + \alpha\beta\mathbb{E}_t\left\{F_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1}\right\} \tag{65}$$

$$K_t = \mu\frac{\lambda_t\Delta_t^\eta Y_t^{1+\eta}}{q\exp\left\{-qY_t\right\}} + \alpha\beta\mathbb{E}_t\left\{K_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta}\right\} \tag{66}$$

$$\lambda_t = q\left(\chi\exp\left\{-qC_t^b\right\} + (1-\chi)\exp\left\{-qC_t^s\right\}\right) \tag{67}$$

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^\theta\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}} \tag{68}$$

$$z_t = \frac{i_t^b - i_t^s}{1+i_t^s}l_t - \frac{i_t^s - i_t^r}{1+i_t^s}R_t - \frac{i_t^s+\gamma}{1+i_t^s}m_t - \Gamma\left(l_t, R_t, m_t, z_t\right) \tag{69}$$

$$\frac{i_t^b - i_t^s}{1+i_t^s} = \Gamma_l\left(l_t, R_t, m_t, z_t\right) \tag{70}$$

$$-\Gamma_R\left(l_t, R_t, m_t, z_t\right) = \frac{i_t^s - i_t^r}{1+i_t^s} \tag{71}$$

$$-\Gamma_m\left(l_t, R_t, m_t, z_t\right) = \frac{i_t^s+\gamma}{1+i_t^s} \tag{72}$$

$$i_t^r = r_t^n \Pi_t^{\phi_\pi} Y_t^{\phi_Y} \tag{73}$$

$$i_t^s = \max\left\{-\gamma, i_t^r\right\} \tag{74}$$

$$l_t = \chi b_t^b \tag{75}$$

Steady state

We denote the steady-state value of a variable X_t as X.

First, observe that in steady-state inflation is at the inflation target Π . As a result, there is no price dispersion ($\Delta = 1$).

Combining this with the Phillips curve, we have that steady-state output is pinned down by the following equation

$$\mu \frac{Y^{\eta}}{q \exp\{-qY\}} = 1 \tag{76}$$

From the Euler equation of a household of type j we have that

$$1 + i^j = \frac{\Pi}{\beta^j} \tag{77}$$

Using the steady-state interest rates, we can jointly solve for all bank-variables. Notice that in steady-state banks are satiated in reserves, and so $R = \overline{R}$ by assumption. Furthermore, if the intermediation cost function is additive between money and the other arguments (which we assume, see below), the steady-state level of money holdings for banks is independent of other bank variables. Therefore, only bank profits and bank lending have to be solved jointly.

Given total debt and interest rates, the borrowers budget constraint and money demand can be solved for steady state consumption and money holdings:

$$C^b = \chi Y + \frac{\Pi - 1 - i^b}{\Pi} b^b - \frac{\Pi - 1 + \gamma}{\Pi} m^b \tag{78}$$

$$\Omega'(m^b) = \frac{i^b + \gamma}{1 + i^b} U'\left(C^b\right) \tag{79}$$

Then, using the aggregate resource constraint we have that

$$C^s = \frac{1 - \chi^2}{1 - \chi} Y + \frac{\chi}{1 - \chi} \left(\frac{\Pi - 1 - i^b}{\Pi} b^b - \frac{\Pi - 1 + \gamma}{\Pi} m^b \right) \tag{80}$$

The savers money demand follows from

$$\Omega'(m^s) = \frac{i^s + \gamma}{1 + i^s} U'\left(C^s\right) \tag{81}$$

Finally, given the steady-state holdings of reserves and real money balances we can use the total money supply equation and the consolidated government budget constraint to solve for the remaining variables.

Log-linearized equilibrium conditions

We log linearize the non-linear equilibrium conditions around steady state, and define $\hat{x} \equiv \frac{X_t - X}{X}$. For the intermediation cost function we assume the following functional form

$$\Gamma\left(l_t, R_t, m_t, z_t\right) = \begin{cases} l_t^{\nu} z_t^{-\iota} + \dfrac{1}{2}\left(R_t - \overline{R}\right)^2 + \dfrac{1}{2}\left(m_t - \overline{m}\right)^2 & \text{if } R_t < \overline{R} \text{ and } m_t < \overline{m} \\ l_t^{\nu} z_t^{-\iota} & \text{if } R_t \geq \overline{R} \text{ and } m_t \geq \overline{m} \end{cases} \tag{82}$$

The system of non-linear equations can be simplified by aggregating the production side and inserting firm and bank profits into the respective budget constraints, in addition to the government budget constraint. Log-linearizing around the unique steady states leaves us with the final approximate equilibrium defined below.

Definition 2. The approximate equilibrium is defined as a sequence of 10 endogenous quantities $\left\{\hat{C}_t^b,\, \hat{C}_t^s,\, \hat{Y}_t,\, \hat{b}_t^b,\, \hat{\pi}_t,\, \hat{\omega}_t,\, \hat{m}_t,\, \hat{z}_t,\, \hat{r}_t^n,\, \hat{R}_t\right\}$ and three prices $\left\{\hat{i}_t^b,\, \hat{i}_t^s,\, \hat{i}_t^r\right\}_{t=0}^{\infty}$ which satisfy equations in Table (5), for a given sequence of shocks $\left\{\hat{\zeta}_t\right\}_{t=0}^{\infty}$.

$$\hat{Y}_t = \frac{\chi C^b}{Y}\hat{C}^b_t + \frac{(1-\chi)C^s}{Y}\hat{C}^s_t \tag{83}$$

$$\hat{C}^b_t = \mathbb{E}_t\hat{C}^b_{t+1} - \sigma\left(\hat{i}^b_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{\zeta}_t + \mathbb{E}_t\hat{\zeta}_{t+1}\right) \tag{84}$$

$$\hat{C}^s_t = \mathbb{E}_t\hat{C}^s_{t+1} - \sigma\left(\hat{i}^s_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{\zeta}_t + \mathbb{E}_t\hat{\zeta}_{t+1}\right) \tag{85}$$

$$\hat{b}^b_t = \frac{\hat{b}^b_t}{\pi\beta^b} + \frac{\hat{i}^b_{t-1} - \hat{\pi}_t}{\pi\beta^b} + \frac{c^b}{b^b}\hat{C}^b_t - \chi\frac{y}{b^b}\hat{Y}_t \tag{86}$$

$$\hat{\pi}_t = \kappa\hat{Y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{87}$$

$$\hat{i}^b_t = \hat{i}^s_t + \hat{\omega}_t \tag{88}$$

$$\hat{\omega}_t = \omega(\nu-1)\hat{b}^b_t - \iota\omega\hat{z}_t \tag{89}$$

$$\hat{m}_t = \frac{m-\overline{m}}{m}\frac{1-i^s-\gamma}{i^s+\gamma} \tag{90}$$

$$\hat{z}_t = \frac{\chi b^b(1+\omega)}{\omega\chi b^b + (1-\iota)\Gamma}\hat{i}^b_t - \frac{d}{\omega\chi b^b + (1-\iota)\Gamma}\hat{i}^s_t + \frac{R}{\omega\chi b^b + (1-\iota)\Gamma}\hat{i}^r_t \tag{91}$$

$$\hat{i}^s_t = \hat{i}^r_t - R\hat{R}_t \tag{92}$$

$$\hat{r}^n_t = \hat{\zeta}_t - \mathbb{E}_t\hat{\zeta}_{t+1} - \chi\hat{\omega}_t \tag{93}$$

$$\hat{i}^r_t = \hat{r}^n_t + \phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t \tag{94}$$

$$\hat{i}^s_t = \max\left\{i^s_{bound}, \hat{i}^r_t\right\} \tag{95}$$

Table 5: Summary of log linearized equilibrium conditions.

D Calibration and numerical simulation of an alternative shock

Calibration

We pick the size of the preference shock to generate an approximately 4.5 percent drop in output on impact. This reduction in output is chosen to roughly mimic the average reduction in real GDP in Sweden, Denmark, Switzerland and the Euro Area in the aftermath of the financial crisis, as illustrated in Figure 22.[37] The drop in output in the US was of similar

[37] Detrended real GDP fell sharply from 2008 to 2009, before partially recovering in 2010 and 2011. The partial recovery was sufficiently strong to induce an interest rate increase. We focus on the second period of falling real GDP (which occurred after 2011), as negative interest rates were not implemented until 2014-2015. Targeting a reduction in real GDP of 4.5 percent is especially appropriate for the Euro Area and Sweden. Real GDP fell by somewhat less in Denmark, and considerably less in Switzerland. This is consistent with the central banks in the Euro Area and Sweden implementing negative rates because of weak economic activity, and the central banks in Denmark and Switzerland implementing negative rates to stabilize their exchange

order. The persistence of the preference shock is set to generate a duration of the lower bound of approximately 12 quarters. We choose parameters from the existing literature whenever possible. We target a real borrowing rate of 4 %[38] and a real deposit rate of 1.5 %, yielding a steady state credit spread of 2.5 %. The preference parameter q is set to 0.75, which generates an intertemporal elasticity of substitution of approximately 2.75, in line with Curdia and Woodford (2011). We set the proportional storage cost to 0.01, yielding an effective lower bound of - 0.01 %. This is consistent with the deposit rate being bounded at zero for most types of deposits, with the exception of slightly negative rates on corporate deposits in some countries. We set $\overline{R} = 0.07$, which yields steady-state reserve holdings in line with average excess reserves relative to total assets for commercial banks from January 2010 and until April 2017.[39] We set $\overline{m} = 0.01$, implying that currency held by banks in steady state accounts for approximately 1.5 percent of total assets. This currency amount corresponds to the difference between total cash assets reported at US banks and total excess reserves from January 2010 until April 2017.

The parameter ν measures the sensitivity of the credit spread to private debt. We set ν so that a 1 % increase in private debt increases the credit spread by 0.12 %, as in Benigno, Eggertsson, and Romei (2014). The final parameter is ι. In our baseline scenario we set $\iota = 0.88$ which would generate an increase in average borrowing rates in negative territory consistent with Figure 9 in the main text. While ι is not important for our main result that negative interest rates are not expansionary, it is important for determining the feedback effect from bank profits to aggregate demand.

All parameter values are summarized in Table 6. Due to the occasionally binding constraint on i_t^s, we solve the model using OccBin (Guerrieri and Iacoviello, 2015) for the preference shock. For simplicity, we consider a cashless limit for the household's problem.[40]

Debt deleveraging shock

Here we show the dynamic transition of our model to an alternative shock, a debt deleveraging shock. To introduce this shock, we augment the intermediation cost function so that it becomes $\Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t\right)$,where $\bar{l}_t$ is a stochastic cost shifter we can interpret as a "debt limit". Introducing this in the model alters the bank problem minimally. The most important

rates.

[38]This is consistent with the average fixed-rate mortgage rate from 2010-2017. Series MORTGAGE30US in the St.Louis Fed's FRED database.

[39]We use series EXCSRESNS for excess reserves and TLAACBW027SBOG for total assets from commercial banks, both in the St.Louis Fed's FRED database.

[40]There are no additional insights provided by allowing households to hold money in the numerical experiments, even if this feature of the model was essential in deriving the bound on deposits.

Parameter	Value	Source/Target
Inverse of Frisch elasticity of labor supply	$\eta = 1$	Justiniano et.al (2015)
Preference parameter	$q = 0.75$	Yields IES of 2.75(Curdia and Woodford, 2011)
Share of borrowers	$\chi = 0.61$	Justiniano et.al (2015)
Steady-state gross inflation rate	$\Pi = 1.005$	Match annual inflation target of 2%
Discount factor, saver	$\beta^s = 0.9901$	Annual real savings rate of 1.5 %
Discount factor, borrower	$\beta^b = 0.9963$	Annual real borrowing rate of 4 %
Probability of resetting price	$\alpha = 2/3$	Gali (2008)
Taylor coefficient on inflation gap	$\phi_\Pi = 1.5$	Gali (2008)
Taylor coefficient on output gap	$\phi_Y = 0.5/4$	Gali (2008)
Elasticity of substitution among varieties of goods	$\theta = 7.88$	Rotemberg and Woodford (1997)
Proportional storage cost of cash	$\gamma = 0.01$ %	Effective lower bound $\underline{i}_t^s = -0.01\%$
Reserve satiation point	$\overline{R} = 0.07$	Target steady-state reserves/total assets of 13 %
Money satiation points	$\overline{m} = 0.01$	Target steady-state cash/total assets of 1.5 %
Marginal intermediation cost parameters	$\nu = 6$	Benigno, Eggertsson, and Romei (2014)
Level of safe debt	$\bar{l} = 1.3$	Target debt/GDP ratio of 95 %
Link between profits and intermediation costs	$\iota = 0.88$	1 % increase in profits $\approx$ 0.01 % reduction in credit spread

Shock	Value	Source/Target
Preference shock	2.5 % temporary decrease in ζ_t	Generate a 4.5 % drop in output on impact
Persistence of preference shock	$\rho = 0.9$	Duration of lower bound of 12 quarters
Debt deleveraging shock	50 % permanent reduction in $\bar{l}$	Generate a 4.5 % drop in output on impact

Table 6: Parameter values

change is that the first-order condition for lending is now

$$l_t \quad : \quad \frac{i_t^b - i_t^s}{1 + i_t^s} = \frac{1}{\bar{l}_t}\Gamma_l\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t\right) \tag{96}$$

The equilibrium credit spread will therefore be a function of $\bar{l}_t$, in addition to l_t and z_t. The debt deleveraging shock we consider is a permanent reduction in the debt limit $\bar{l}_t$, a shock often referred to as a "Minsky Moment" (Eggertsson and Krugman, 2012).[41] We pick the size of the permanent reduction in $\bar{l}$ to yield similar initial drop in output and inflation as for the preference shock.

The dynamic transition paths are shown in Figure 26 A permanent reduction in the debt limit directly increases the interest rate spread, causing the borrowing rate to increase. The initial increase in the borrowing rate is substantial, due to the shock's impact on bank profits and the feedback effect via ι. In the frictionless case, the central bank can perfectly counteract this by reducing the reserve rate below zero. Given the bound on the deposit rate however, the central bank looses its ability to bring the economy out of a recession. Any attempt at doing so, by reducing the reserve rate below zero, only lowers bank profits and aggregate demand further.

In some respects this shock – with the associated rise in the borrowing rate – resembles more the onset of the financial crisis, when borrowing rates (in some countries) increased. The preference shock considered in the main body of the text is more consistent with the situation further into the crisis, when both deposit and lending rates were at historical low levels (perhaps reflecting slower moving factors such as those associated with secular stagnation, see Eggertsson and Mehrotra (2014)). From the point of view of this paper however, it makes no difference which shock is considered in terms of the prediction it has for the effect of negative central bank rates. In both cases, the policy is neutral when there is no feedback from bank profits, and contractionary when there is such a feedback.

[41] In terms of calibration, we note that, conditional on the steady-state credit spread, $\bar{l}$ pins down the steady-state level of private debt. We choose $\bar{l}$ to target a steady state private debt-to-GDP ratio of approximately 95 percent, roughly in line with private debt in the period 2005 - 2015 (Benigno, Eggertsson, and Romei, 2014). We consider a permanent reduction in $\bar{l}$ which generates an initial reduction in output and inflation similar to the preference shock.

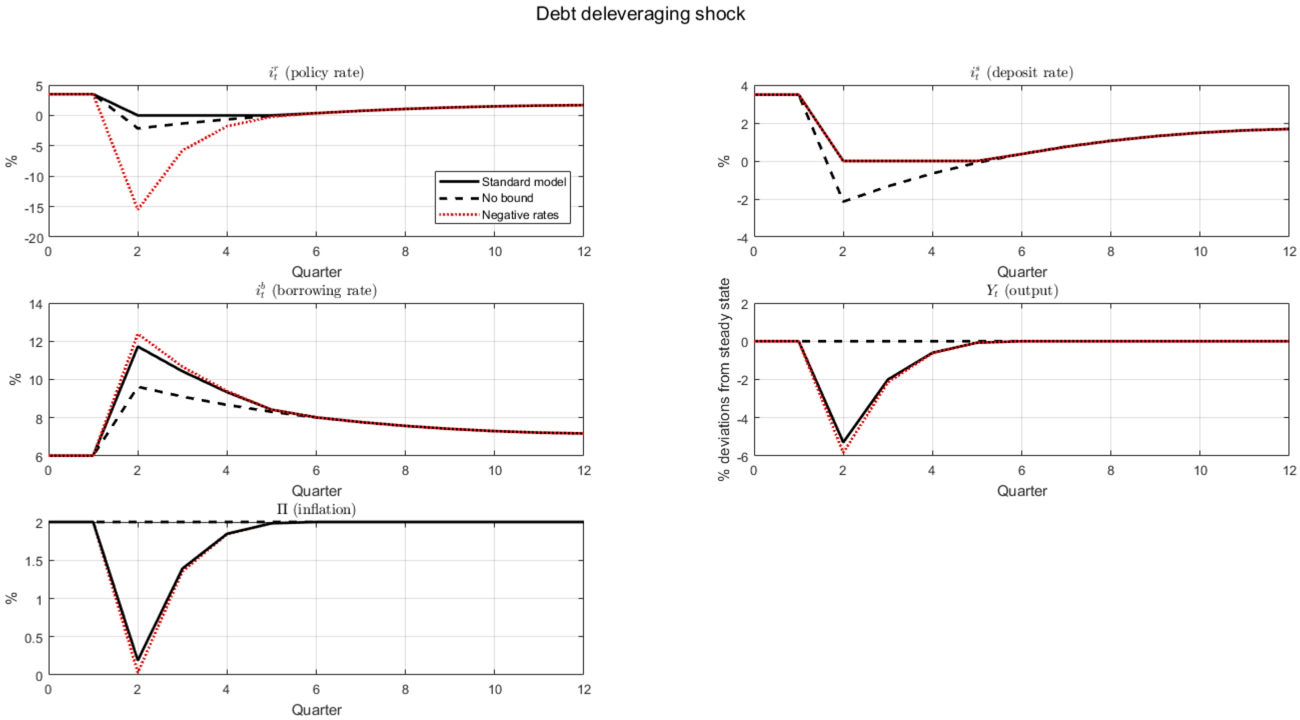


Figure 26: Impulse response functions following a debt deleveraging shock (a permanent reduction in $\bar{l}_t$), under three different models. S*tandard model* refers to the case where there is an effective lower bound on both deposit rates and the central bank's policy rate. *No bound* refers to the case where there is no effective lower bound on any interest-rate. *Negative rates* refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.