WORKING PAPER

Liquidity at risk:
Joint stress testing of solvency and liquidity

NORGES BANK

Liquidity at Risk:
Joint Stress Testing of Solvency and Liquidity

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Abstract

The traditional approach to the stress testing of financial institutions focuses on capital adequacy and solvency. Liquidity stress tests are often applied in parallel to solvency stress tests, based on scenarios which may not be consistent with those used in solvency stress tests. We propose a structural framework for the joint stress testing of solvency and liquidity: our approach exploits the mechanisms underlying the solvency-liquidity nexus to derive relations between solvency shocks and liquidity shocks. These relations are then used to model liquidity and solvency risk in a coherent framework, involving external shocks to solvency and endogenous liquidity shocks. We introduce solvency-liquidity diagrams as a method for analysing the resilience of a balance sheet to the resulting combination of solvency shocks and endogenous liquidity shocks. Finally, we define the concept of 'Liquidity at Risk' which quantifies the liquidity resources required for a financial institution facing a stress scenario.

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1   Introduction

Stress testing of banks has become a pillar of bank supervision. Bank stress testing has mainly focused on solvency: a commonly used approach is to assess the exposure of bank portfolios to a macro-stress scenario and compare this exposure with the bank's capital in order to assess capital adequacy (Schuermann, 2014). This approach is in line with structural credit risk models which, following Merton (1974), has mainly emphasised solvency.

However it has become clear, especially in the wake of the 2008 financial crisis, that a typical route to failure for financial institutions may be a lack of *liquidity* triggered by a loss of short term funding (Duffie, 2010; Gorton, 2012). As noted in a famous letter of the SEC Chairman to the Basel Committee relating the events which led to the failure of Bear Stearns[1], the failure of Bear Stearns was triggered by a lack of liquidity resources, not capital. The failure of insurance giant AIG, which had a trillion dollar balance sheet, may be traced to a lack of liquidity resources to face margin calls triggered by its credit downgrade (McDonald and Paulson, 2015). These events has given rise to initiatives for the monitoring and regulation of bank liquidity, such as the Liquidity Coverage Ratio (LCR), the Net Stable Funding Ratio (NSFR) as well as liquidity stress testing to assess the adequacy of liquidity resources of banks.

Liquidity stress tests focus on a bank's ability to withstand hypothetical liquidity shocks. The usefulness of such stress tests hinges on the choice of the stress scenarios used for the liquidity shocks. While the Basel III framework emphasizes the need for a unified stress testing approach, the assessment of solvency and liquidity risk has remained largely fragmented. Calibration of liquidity shocks is based on supervisor experience rather than based on a forward assessment of market risk, notwithstanding the increased significance of margin requirements for derivatives under the new European (EMIR) and US (CFTC) rules (Cont, 2017). Current practice is to calibrate such scenarios based on stressed cash in-/out-flows and depositor runoffs in recent crisis episodes, using a backward looking approach (European Central Bank, 2019). Although the implementation of the LCR ratio has imposed more stringent liquidity requirements and strengthened banks' liquidity risk practices, its calibration is insensitive to the solvency position of the reporting bank and restricted to a prescribed scenario which may differ from the scenario that would deplete the bank's capital buffers.

Many theoretical and empirical studies have pointed to the importance of interactions between solvency and liquidity risk (Bernanke, 2013; Farag et al., 2013; Morris and Shin, 2016; Pierret, 2015; Rochet and Vives, 2004; Schmitz et al., 2019;

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[1]Letter to the Chairman of the Basel Committee on Banking Supervision on March 20, 2008 (`https://www.sec.gov/news/press/2008/2008-48.htm`).

Basel Committee on Banking Supervision, 2015). Interactions between solvency and liquidity are present in models of bank runs and debt roll-over coordination failures (Diamond and Rajan, 2005; Allen and Gale, 1998; Rochet and Vives, 2004). In a two-period model with short- and long-term liabilities, Morris and Shin (2016) identify two components of credit risk: the 'insolvency risk' associated to asset value realisation being below debt value, and the 'illiquidity risk' associated to a run by short-term creditors irrespective of the actual solvency state of the institution. Liang et al. (2013) present an extension of Morris and Shin (2016) approach to a multi-period dynamic bank run setting where a financial institution is financed through a mix of short-term and long-term debt. A noteworthy implication of this model is that total default risk increases in both rollover frequency and short-term debt ratio. Cont (2017) describes the role of margin requirements in the transformation of solvency risk into liquidity risk, thereby linking solvency and liquidity.

The importance of interplay between solvency and liquidity in the context of financial stability has been also evidenced in empirical studies (Cornett et al., 2011; Pierret, 2015; Du et al., 2015). Pierret (2015) shows that firms with increased solvency risk are more susceptible to liquidity problems and that availability of short-term funding decreases with solvency risk. Du et al. (2015) present empirical evidence that indicators of credit quality affect counterparty choice, with the consequence that creditworthiness affects the volume rather than the price of short-term funding. Schmitz et al. (2019) present evidence on the relationship between bank solvency and funding costs and show that neglecting the solvency-liquidity nexus leads to a significant underestimation of the impact of shocks on bank capital ratios.

Despite all the evidence on the close link between liquidity and solvency, liquidity stress tests have been often conducted separately from solvency stress tests (European Central Bank, 2019; Schuermann, 2014), and either fail to model the interaction of solvency and liquidity risk or include only a limited number of channels for such interactions. For example, in the Bank of Canada's stress test solvency risk affects roll-over risk, while in the Austrian Central Bank's stress test solvency risk limits the access of a financial institution to funding.

Our goal is to go beyond this and build a joint stress testing framework for solvency and liquidity which addresses the interrelations between them. Building on ideas introduced in Cont (2017), we introduce a model in which shocks to asset values generate endogenous liquidity shocks arising from multiple solvency-liquidity interactions channels, thus affecting both the solvency and liquidity of a financial institution.

Contribution We propose a structural framework for the joint stress testing of solvency and liquidity. Rather than modelling solvency and liquidity stress through separate channels, we focus on the mechanisms through which they interact and analyse the implications of these interactions for the dynamics of a balance sheet under stress. These mechanisms, summarised in Figure 1, lead to relations between solvency shocks and liquidity shocks. We exploit these relations to model liquidity and solvency risk in a coherent framework, involving external shocks to solvency and endogenous liquidity shocks.

We start from a stylised model of a balance sheet, distinguishing various components in terms of their interaction with the firm's liquidity. We then express the various mechanisms through which these balance sheet components may be affected in a stress scenario, described as a shock to asset values ('solvency shock'). Solvency shocks affect liquidity through margin requirements, via firm's ability to raise short-term funding and through the cost of this funding, leading to *endogenous liquidity shocks.*

Depending on the nature of a shock and firm's portfolio composition, financial institutions can become illiquid without being insolvent, or insolvent while remaining liquid, or – in the case of extreme shock – both illiquid and insolvent. The model shows how credit risk may be underestimated by models that do not account for the solvency-liquidity nexus.
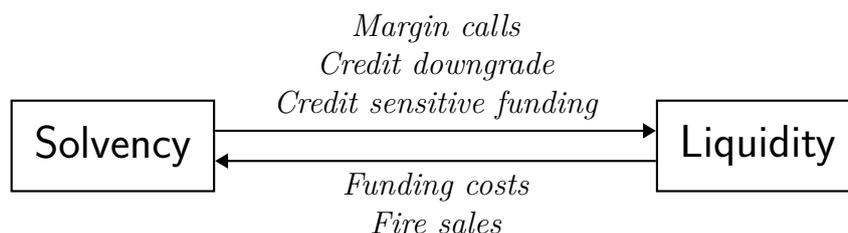
Figure 1: Mechanisms governing the solvency-liquidity nexus.

We introduce *solvency-liquidity diagrams* as a method for analysing the resilience of a balance sheet to the resulting combination of solvency shocks and endogenous liquidity shocks. Finally, we define the concept of 'Liquidity at Risk' which quantifies the liquidity resources required for a financial institution facing a stress scenario.

The stress testing methodology presented in this paper has been implemented in the form of an online application available at `http://r.kotlicki.pl/`.

Outline. Section 2 introduces the model and explains the various mechanisms through which solvency and liquidity interact. Section 3 discusses the mapping of balance sheet and regulatory data to the inputs required by the model. Section 4

introduces the concept of Liquidity at Risk and illustrates it with two examples: a synthetic balance sheet and the balance sheet of a global systemically important bank (G-SIB).

2 A framework for joint stress testing of solvency and liquidity

Figure 1 represents various mechanisms through which liquidity and solvency interact with each other. We introduce in this section a stress testing methodology that aims to capture these mechanisms.

2.1 Balance sheet representation

In order to model the mechanisms underlying solvency and liquidity of a balance sheet, we require a decomposition of the balance sheet into components based on their interactions with the solvency and liquidity of the balance sheet. On the asset side, we distinguish:

- *Liquid assets* category includes cash holdings, highly liquid assets easily convertible into cash and balances with central banks.

- *Marketable assets*, defined as assets not in the above category but available for repo or sale. In particular such assets need to be unencumbered by existing repurchase agreements. In the context of stress testing, it is conservative to assume that only (unencumbered) assets, mainly in the General Collateral (GC) category (subject to a low haircut under stress) would be available for repo in a stress scenario, which is what we shall assume in the examples below. Among these marketable assets we further distinguish:

  – Marketable assets subject to margin requirements;
  – Marketable assets not subject to margin requirements.

- *Illiquid assets* are defined as assets which are not 'marketable' in the above sense. In particular, encumbered assets shall be considered under this category. Among these assets we further distinguish:

  – Illiquid assets subject to margin requirements;
  – Illiquid assets not subject to margin requirements (typically loans).

On the liability side, we distinguish

- *Current liabilities*, payable in the short term (say, one week or 30 days).

- *Long term liabilities* maturing beyond this short-term horizon.

This leads to a stylised representation of the balance sheet, shown in Table 1. The difference between total assets and total liabilities is represented by the firm's equity E.

We further discuss in Section 3 the mapping of balance sheet data and regulatory data to the format presented in Table 1.

Assets	Liabilities and equity
Illiquid assets: (i) Subject to margin requirements, I	Current liabilities, S
(ii) Not subject to margin requirements, J	
Marketable assets: (i) Subject to margin requirements, M	Long-term liabilities, L
(ii) Not subject to margin requirements, N	
Liquid assets, C	Capital (equity), E

Table 1: Stylised balance sheet of a financial institution.

2.2   Dynamics of balance sheet components under stress

We now describe the dynamics of balance sheet components in a stress scenario. It is helpful to represent the sequence of transformations of balance sheet components as a two-period model, as in Figure 2.

Figure 2: Evolution of balance sheet components.

Consider a leveraged financial institution with a balance sheet as in Table 1. We denote by I_0, J_0, M_0, and N_0 the initial value of balance sheet components, the subscript 0 indicating their initial value at $t = 0$. The initial value of current liabilities S_0 represents the amount of liabilities maturing at $t = 2$, while L_0

represents the amount of liabilities maturing after $t = 2$. C_0 denotes the current level of cash reserves and balances with central banks.

We now consider the impact of an adverse market scenario on this balance sheet.

Stress scenarios Stress scenarios are typically defined in terms of shifts to risk factors such as real GDP, interest rates, credit spreads, equity prices, exchange rates, and other economic variables to which portfolio components are sensitive. Denoting by $X = (X_1, \ldots, X_d)$ these risk factors, each stress scenario may be described in terms of shocks $\Delta X = (\Delta X_1, \ldots, \Delta X_d)$ to risk factors.

Direct impact on solvency The reaction of portfolio components to such a stress scenario is evaluated using models calibrated to the risk structure of the portfolio. The models used to derive stress impacts differ across default shocks and market shocks. While the effect of default shocks on credit exposures may take time to materialise, market shocks immediately affect the fair valuation of market exposures. To produce an integrated risk modelling framework, we assume that firms assess the impact of default shocks on equity using a forward-looking approach (rather than an incurred loss method), and thus the horizon over which shocks hit P&L is the same across risk types. This view is consistent with the Basel III regulatory framework for internal-ratings based models, and the newly implemented accounting IFRS 9 provisions[2].

For credit shocks, defaults are considered in lending positions (in general valued according to accrual accounting), traded credit positions ('issuer default', positions measured at fair values) and counterparty exposures like OTC derivatives and Securities Financing Transactions. Impairment losses reduce the carrying amount of credit risk positions affecting the value of equity. Impairment charges can be computed as the impact of stressed credit risk parameters, i.e. probability of default (PD), loss given default (LGD) and exposure at default (EaD), on the initial value of the position. Shifts to PDs, LGDs, and EaDs can be expressed in terms of sensitivities to underlying risk factors.

For market shocks, the impact of the shocks on bank portfolios at partial or full fair valuation measurement, can be calculated either by revaluation of the positions in the portfolio under the stress scenario (full valuation method) as computed in firm internal stress tests and regulatory bottom-up stress tests or, as done frequently in top-down regulatory stress tests, by using a linear approximation of

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[2]To compute regulatory capital, banks using internal-ratings based models for credit risk take a forward-looking approach to determine capital ratios. From an accounting perspective, IFRS 9 requires loan allowances based on 12 month expected losses if the credit risk has not increased significantly, and expected lifetime losses for exposures that have deteriorated significantly.

the dependence of portfolio components with respect to risk factors, in terms of sensitivities to risk factors.

Denoting $\partial_k A$ the sensitivity of balance sheet component A to risk factor X_k, the change in the value of this balance sheet component in the risk scenario is then given by

$$\Delta M = \sum_{k=1}^{d} \partial_k M.\Delta X_k = \partial M.\Delta X, \tag{1}$$

where ∂M denotes the vector of sensitives of balance sheet component M. Similarly we may compute the changes in balance sheet items I, J, N as

$$\Delta I = \partial I.\Delta X, \qquad \Delta J = \partial J.\Delta X, \qquad \Delta N = \partial N.\Delta X. \tag{2}$$

These sensitivities may be computed using satellite models linking scenario shocks to credit risk parameters (default shocks), or calculating the impact of risk factors on fair-valued positions using the delta method (market shocks).[3]

Impact on liquidity   Liquidity risk arises from the uncertainty to meet payment obligations in a full and timely manner in a stressed environment. In the model, obligations coming due at $t = 2$ include four components.

1. *Unconditional liabilities*: these are liabilities maturing at $t = 2$. Their size corresponds to current liabilities and hence is denoted by S_0.

2. *Scheduled cash outflows*: these include contractual cash-flow obligations (e.g. interest payments on interest-bearing liabilities, coupons, operating costs), projected outflows from non-maturing liabilities (e.g. sight, operational deposits) and estimated drawdowns from undrawn credit and liquidity lines. Denoting these outflows by SCO, the stable component of short-term liabilities payable at $t = 2$ can then be expressed as

$$S_1 = S_0 + SCO. \tag{3}$$

3. *Contingent liquidity risks*:

   In a derivative transaction or securities financing transaction with no margin payments, although both sides may mark-to-market their position daily, there is no exchange of cash flows: any losses or gains purely affect the solvency of the institution. In this case, capital buffers are an adequate tool to address any risk externalities. On the other hand, if an asset is subject to margin requirements, this creates a liquidity outflow in the form of

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[3]See Section 3 for more details.

a variation margin payment. As a result, such shock not only affects the solvency of the institution but also its liquidity by drawing on the held cash reserves with an immediate effect (typically within a few days), since all payments are done in cash or liquid assets. Firms post and receive collateral to support or reduce the counterparty credit risk (CCR) relative to derivative transactions or to securities financing transactions, including transactions cleared through a central counterparty (CCP). Here we focus on liquidity needs from changes in the value of collateral posted by the bank (e.g. in repo transactions) rather than on collateral received (e.g. in reverse repos) to allow an integrated assessment of the solvency and liquidity risk of the firm from valuation shocks to the bank assets. For assets subject to variation margin, negative changes in asset values lead to margin calls that add to current liabilities, which we denote by

$$\Delta S = (\Delta I)^- + (\Delta M)^-, \tag{4}$$

whereas positive changes generates margin calls to the counterparty, which lead to cash inflows expected at $t = 2$, and which we denote by

$$\Delta C = (\Delta I)^+ + (\Delta M)^+, \tag{5}$$

where $(X)^+ = \max(0, X)$ denotes the positive part of a quantity X and $(X)^- = (-X)^+$. The interaction between solvency and liquidity risk through margin requirements and creditor runs may lead to a severe amplification of losses in a stressed environment.

4. *Credit downgrades and credit-sensitive funding*: The direct impact of the shocks described above on the firm's equity is given by

$$E_1 = E_0 + \Delta I + \Delta J + \Delta M + \Delta N + C_1 - S_1 + L_0. \tag{6}$$

If due to these losses the firm's equity falls below a threshold, then the firm may be subject to a *credit downgrade*. We assume such a downgrade occurs if the leverage ratio exceeds a level δ i.e.

$$\frac{I_1 + J_1 + M_1 + N_1 + C_1}{E_1} > \delta. \tag{7}$$

Such a downgrade may trigger the loss of credit sensitive funding, depositor runoffs, failure to roll over short-term debt or margin calls associated with a credit downgrade leading to a contingent cash outflow. We denote by S_D the increase in current liabilities resulting from a downgrade.

As a result, conditional on the stress scenario, current liabilities due at $t = 2$ increase to

$$S_2 = S_1 + \Delta S + S_D \mathbb{1}_{\text{downgrade}}. \tag{8}$$

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On the other hand, the reserve of liquid assets is increased by the scheduled cash inflows from contractual claims (e.g. interest payments) and maturing assets which are not reinvested (e.g. inflows from performing exposures and secured lending). Denoting this amount by SCI we have that

$$C_1 = C_0 + SCI \tag{9}$$

Mitigating actions At $t = 1$, if liquid assets are not enough to cover conditional cash outflows (expected and unexpected), the bank can undertake mitigating actions (from its contingency funding plan and recovery plan) to cover the liquidity shortfall λ which we define formally as

$$\lambda = (S_2 - \{C_1 + \Delta C\})^+. \tag{10}$$

In the short term, a financial institution has access to three sources of funding, stated in a usual order of preference:

1. Unsecuritised borrowing: we assume the financial institution to have access to short-term unsecuritised loans given at an exogenous market interest rate r_U. This access depends on the firm's creditworthiness: we assume that the firm's access to such funding ceases once it has been downgraded. Furthermore, the distance to downgrade leads to an upper bound on the volume of unsecuritised lending available to the firm:

$$v_U = (E_1\delta - \{I_1 + J_1 + M_1 + N_1 + C_1\})^+. \tag{11}$$

   In other words, we assume that the highly leveraged institutions are considered more risky on the market, and hence can access a smaller pool of liquidity than lesser leveraged firms. Subject to this constraint, the amount of money a financial institution will borrow through this channel can be expressed as

$$B_U = \min\{\lambda, v_U\}. \tag{12}$$

2. Repurchase agreements: in contrast to unsecuritised borrowing, the repo market requires the provision of liquid marketable (unencumbered) collateral as a form of security. The amount v_R of funding which may be raised

through this channel available is limited by the firm's pool of unencumbered marketable assets, discounted by the corresponding haircut parameter $h \in [0, 1)$, that is

$$v_R = (1 - h)(M_1 + N_1). \tag{13}$$

Consequently, the amount of cash that a financial institution will raise through the repo market is then given by

$$B_R = \min\{\lambda - B_U, v_R\},$$

with an associated borrowing cost given by the (exogenous) repo rate r_R.

3. Liquidation of assets (fire sales): we assume that in the short-term a liquidity-stressed financial institution can only sell a fraction $\theta \in [0, 1]$ of its illiquid assets in a fire sale with a price discount $\psi \in [0, 1)$. Note that only unencumbered illiquid assets (not subject to margin requirements) can be monetised in a fire sale. In other words, the maximum amount of liquidity that can be raised in a short-term can be expressed as

$$v_F = (1 - \psi)\theta J_1. \tag{14}$$

The fraction θ depends for example on the available market liquidity and the length of sales horizon. Consequently, we expect θ to be small in a stress test scenario. Similarly, we usually think of the associated fire sale discount to be large (in excess of 50%).

These mitigating actions increase the liquidity buffer of the bank at $t = 2$ to

$$C_2 = C_1 + \Delta C + B_U + B_R + \omega v_F, \tag{15}$$

where B_U represents the amount of new unsecuritised borrowing, similarly B_R is the amount borrowed on the repo market, and $\omega \in [0, 1]$ is an endogenous fraction of liquidated assets in a fire sale for a price discount of $\psi \in [0, 1)$ such that

$$\omega = \min\left\{\frac{(S_2 - (C_1 + \Delta C + B_U + B_R))^+}{(1 - \psi)\theta J_1}, 1\right\}. \tag{16}$$

The amount of long-term liabilities rises by the amount of new liabilities from unsecured and secured funding, and declines by the cash-flow amount due to credit risk sensitive funding, that is

$$L_2 = L_0 + (1 + r_U)B_U + (1 + r_R)B_R - S_D \mathbb{1}_{\text{downgrade}}. \tag{17}$$

As a consequence of these mitigating actions, the value of equity falls to

$$E_2 = E_1 - r_U B_U - r_R B_R - \omega\psi\theta J_1. \tag{18}$$

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Insolvency and illiquidity A financial institution is deemed insolvent when the equity falls below a certain threshold, here taken without loss of generality to be zero. That is, a firm fails due to insolvency when $E_2 < 0$. It is said to be illiquid when current liabilities exceed the firm's capacity to raise liquidity i.e. $C_2 < S_2$, where C_2 is the available liquidity, given by (15) and S_2 are the current liabilities due at $t = 2$, given by (8). It is possible for a firm to be illiquid without being insolvent, as it is possible to be insolvent without being illiquid.

The summary of dynamics of balance sheet components in our model is given by Figure 3.
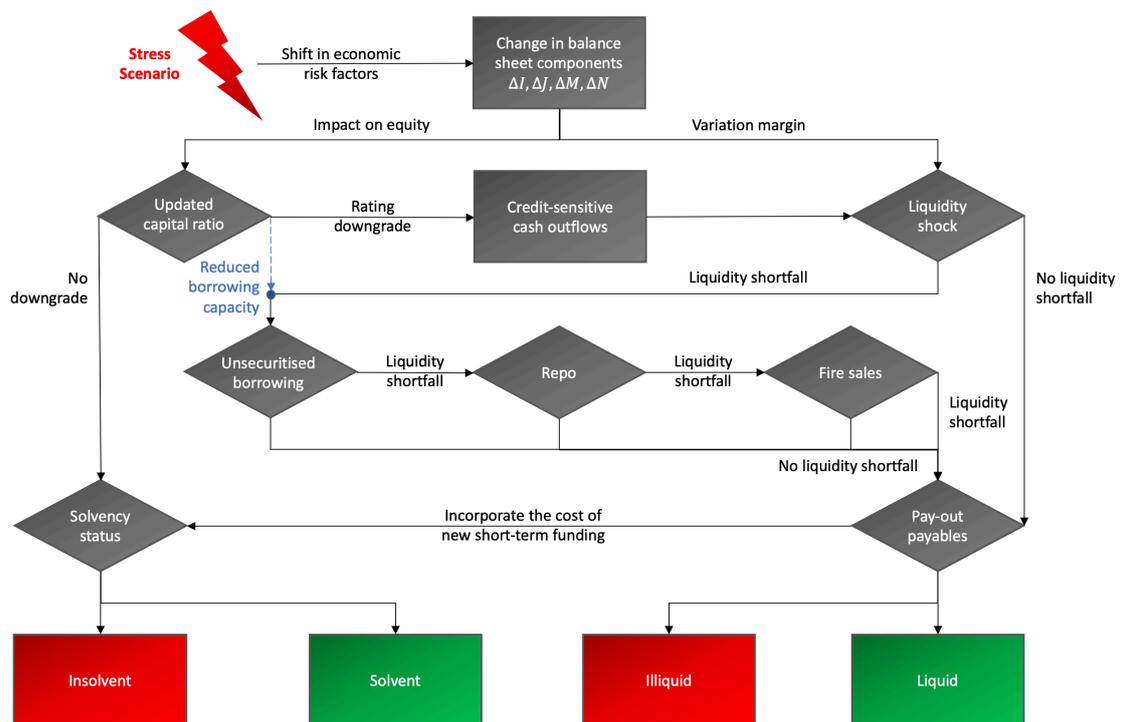

Figure 3: Joint stress test of solvency and liquidity.

2.3 Solvency-liquidity diagrams

The balance sheet dynamics in a stress scenario may be visualised in the form of a *solvency-liquidity diagram* in which the financial institution's equity is represented on the horizontal axis and its liquidity resources on the vertical axis (see Figure 4).
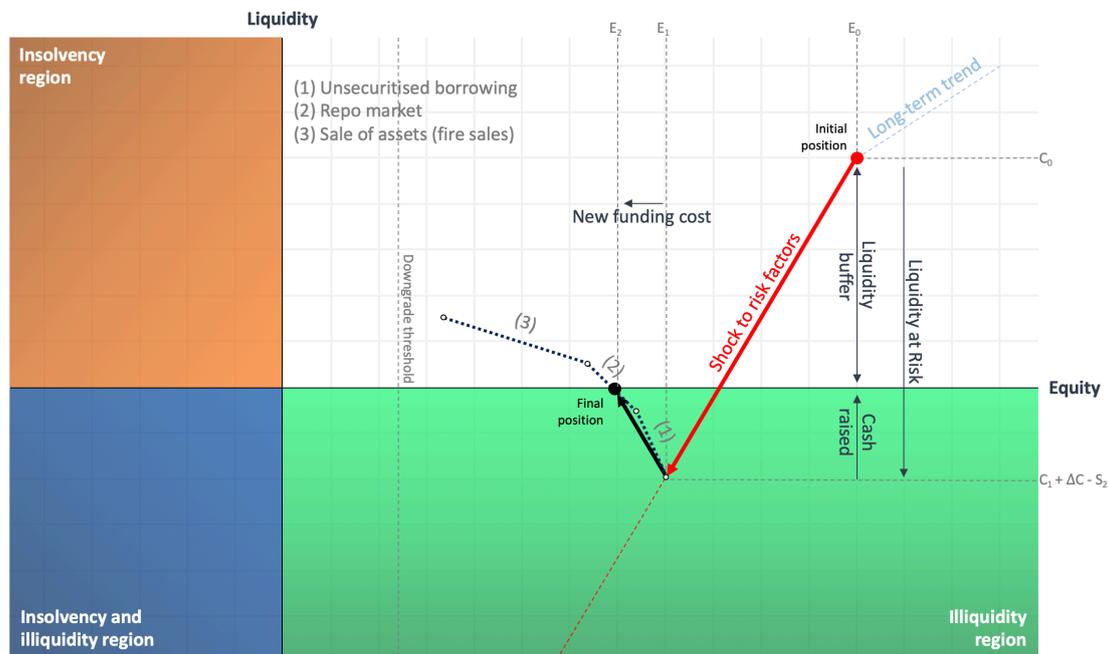

Figure 4: Solvency-liquidity diagram describing the behaviour of a balance sheet in a stress scenario.

A solvent and liquid institution corresponds to a point in the upper right quadrant (first quadrant). The vertical coordinate corresponds to its liquidity buffer while the horizontal coordinate correspond to the firm's equity.

A loss in asset values in a stress scenario moves this point to the left. Depending on the cash flows arising in the stress scenario, we will also have a vertical displacement upwards (if there is net incoming cash, for example due to variation margin and interest received) or downwards (if there are net outflows, for example from margin and interest payments).

Failure occurs when the institution exits this first quadrant. If it crosses the horizontal axis (see Figure 5a), this corresponds to an illiquidity induced default, while if it crosses the vertical axis (see Figure 5b) this corresponds to failure due to insolvency. The distance to the axes represents the capital and liquidity buffers (see Figure 4).

An adverse stress scenario leads in a 'south-west' shift on the diagram: the precise direction of the shift depends on balance sheet sensitivities, while the size

of the shift corresponds to the severity of the shock. A pure solvency shock draws on the capital buffer without affecting the firm's liquidity reserves, and hence corresponds to a horizontal shift on the solvency-liquidity diagram. On the other hand, a pure liquidity shock caused by a run of creditors or a failure to rollover short-term debt due to downgrade corresponds to a vertical shift on the diagram.

For a fixed adverse market scenario, the loss in equity due to the shock is independent of the balance sheet composition in terms of margin requirements. However, as the proportion of assets subject to variation margin increases, the reduction in the liquidity position of a financial institution (unencumbered assets) and its liquidity risk (sensitivity to market shocks) also increases. In that case, it becomes more likely that the firm becomes illiquid while still solvent as the shock severity increases.
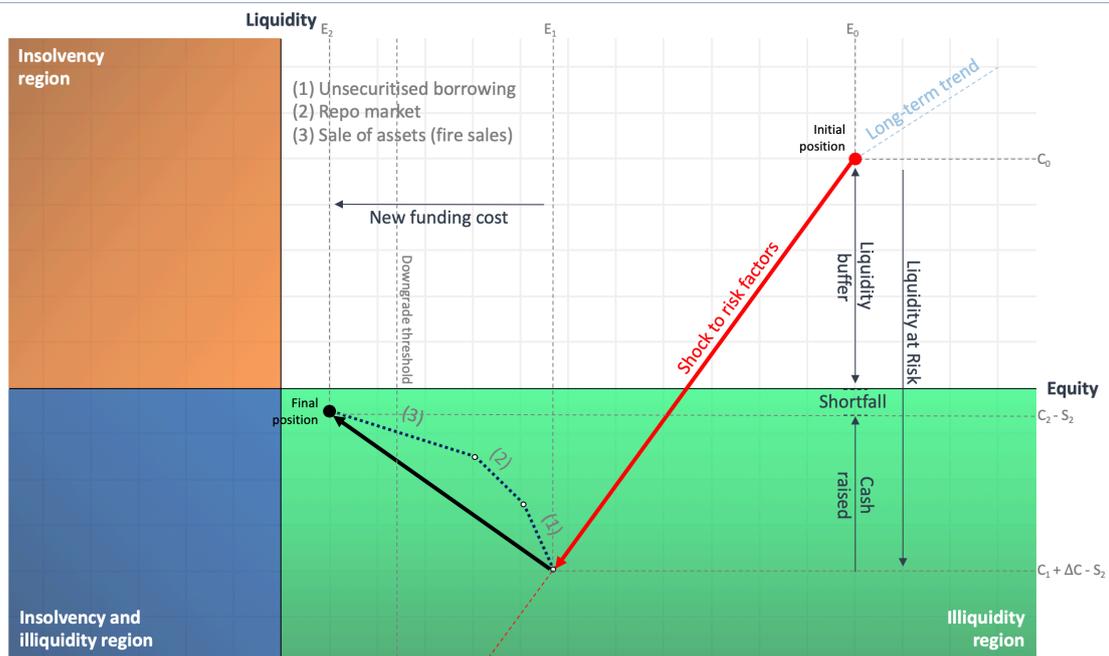
(a) Illiquidity induced default.
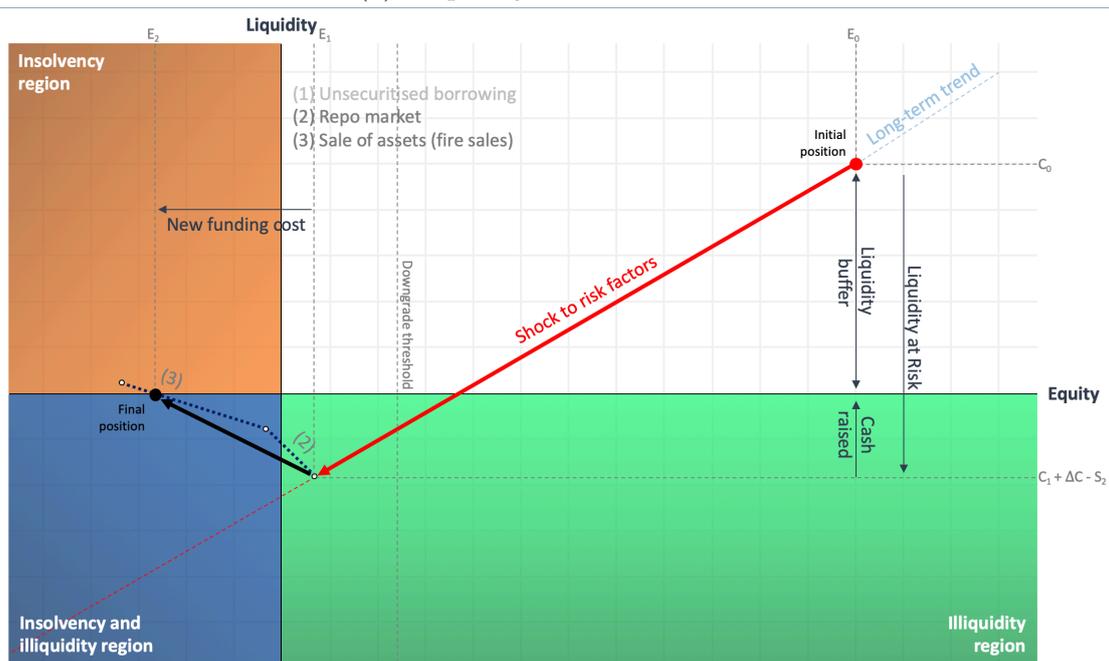
(b) Insolvency induced default.

Figure 5: Examples of scenario analysis using solvency-liquidity diagrams. (a) Stress scenario leading to illiquidity. (b) Stress scenario leading to insolvency.

3   Mapping of balance sheet variables and liquidity templates

The purpose of this section is to show how balance sheet information – especially in the format of templates available to regulators – may be mapped to the format shown in Table 1 used as an input for our stress testing approach. In this section we describe how to use various data sources to generate the inputs required in our framework. We then provide a numerical illustration using publicly available data for a global systemically important bank (G-SIB).

3.1   Data requirements

Our stress testing approach requires two types of inputs:

- Balance sheet data, with sufficient granularity in order to extract the categories displayed in Table 1.

- Risk parameters including credit scores, internal risk reports, and market risk sensitivities to be used for estimating the profit and loss (P&L) of various portfolio components in the stress scenario.

- Liquidity data to estimate the amount of available unencumbered assets, contractual maturity cash in-/out-flows, and the potential liquidity generation capacity of securities over different time horizons.

These requirements are not very different from the inputs of current solvency stress tests but require the data to be formatted in a slightly different way, as discussed in Section 2. Central banks and regulators typically have access to data on portfolio positions, risk parameters, pricing models and methodologies to assess sensitivities to stress. For instance, in the European reporting framework, financial data is collected in FINREP templates while risk data is submitted in COREP templates. The reporting requirements, defined by the European Banking Authority (EBA) via the implementation of technical standards or guidelines, are complemented with short-term exercise ad-hoc data requests which collect additional granular data on complex portfolios including sensitivities to moves in market risk factors. Our stress testing framework requires these data to be available at a sufficiently granular level to derive the above information for each component of the balance sheet.

Table 2 summarises the mapping of asset categories observed in regulatory and accounting templates to balance sheet components required in the model. Assets are classified as 'marketable' or 'illiquid'. Marketable refers to the availability of the

assets for raising short-term funding in a stress scenario, either through a repurchase agreement or sale. Such assets therefore need to be unencumbered by other repurchase agreements. Since we are interested in behaviour of the balance sheet under stress, we restrict marketable assets to those which can generate liquidity through monetization at stressed haircuts over the relevant time horizon. Illiquid assets that can be subject to fire sales include loans, investments in associates, and physical assets. Assets that are not available to raise funding and cannot be pledged for repo transactions include complex hard-to-value assets (Level 3 in the fair value hierarchy), goodwill, and deferred tax assets.

	Not subject to VM	Subject to VM
Illiquid assets	Loans Non-financial investments Physical assets	Non-standard OTC derivatives Encumbered assets
Marketable assets	Unencumbered General Collateral: (i) Assets held for trading (ii) Financial investments Equity	Exchange-traded derivatives Standardised OTC derivatives
Liquid assets	Cash (unencumbered) Reverse repos	

Table 2: Mapping of common asset classes to the model input format.

Once the balance sheet data have been mapped to the format shown in Table 2, the stress test requires estimating the variations in each component in the stress scenario considered. The estimation of P&L may be done either through full revaluation in a pricing model, which requires granular data on fixed-income and derivatives positions, or through a linear approximation, using sensitivities to risk factors. In the latter case one would only require sensitivities to risk factors aggregated at the level of the balance sheet components shown in Table 1.

Projection of losses in stress scenarios typically involves two types of risk: credit risk and market risk.

For credit risk assessment, the loss related to default events on lending positions, traded credit risk positions, and counterparty exposures like OTC derivatives needs to be projected. Impact on P&L through newly created adjustments for loan loss provisions can be estimated using satellite models based on internal ratings-based models or standardised approaches using stressed credit risk parameters. Under IFRS 9 accounting standards,[4] losses are generated from obligor grade

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[4] Under IFRS 9 implementation, credit risk is based on the categorisation of exposures in three stages: S1 (credit risk has not increased significantly since initial recognition, and provisions are based on a 12-month expected loss); S2 (credit risk has increased significantly, so the loss

migration using an expected loss, forward-looking approach.

To assess market risk we need to measure the impact of the shocks on the fair values of the underlying positions. Accounting data serve to classify exposures at fair value (mark-to-market) relative to exposures at amortised cost. While shocks to financial assets held for trading and financial assets designated at fair value through P&L impact directly, shocks to available-for-sale financial assets affect regulatory capital through Other Comprehensive Income (OCI). By contrast, shocks to held-to-maturity assets do not affect bank capital. The sensitivities with respect to the relevant (market) risk factors can be calculated by the stress tester using portfolio valuation models or can be requested to banks through regular regulatory submissions. These sensitivities report the impact of a risk factor move on the fair value of the position.

Basel Liquidity Monitoring Templates (Pohl, 2017) provide a granular decomposition of cash outflows and inflows by time horizon, which can be exploited to estimate liquidity needs arising from an adverse scenario over a defined time horizon. To populate the cash-flow equation, current liabilities can be extracted from maturing liabilities according to current contractual conditions from securities issued, unsecured funding by retail and wholesale counterparties, liabilities from secured funding, and additional outflows from derivative transactions and other contingent obligations.

Cash-outflows subject to uncertainty include flows associated to non-maturing liabilities (e.g. retail deposits, corporate deposits, financial institutions deposits, and deposits from other legal entities), undrawn committed credit and liquidity facilities, and other contractual obligations (e.g. interest payments, operational expenses). The stress tester needs to project cash-outflow estimates regarding the bank's anticipated funding needs (including from a creditor run), using banks' modelling assumptions on idiosyncratic shocks, relying on Basel LCR-prescribed scenario assumptions, or applying stressed run-off rates on credit sensitive contractual outflows (e.g. uninsured deposits, unsecured wholesale funding) linked to the solvency scenario.

Contingent liabilities from assets subject to margin requirements can be calculated by applying scenario shocks to risk factors on the value of collateral posted for counterparty credit risk exposure in derivative transactions and Securities Financing Transactions. These data are reported in the contractual mismatch and asset encumbrance submission of the Liquidity Monitoring Templates. While contingent outflows can be also triggered from changes in prices of financial instruments related to own securities issued, or unsecured funding instruments, these are typically not material.

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allowance should equal lifetime expected credit losses); and, S3 (exposure is considered credit-impaired with lifetime allowance and non-recognition of interest accrual).

3.2 Mapping example

We now give an example of such a mapping based on publicly available data for a European G-SIB at end 2017. Public data sources include the bank's annual report, Pillar 3 disclosures, and Fitch database. Balance sheet variables are mapped to the portfolio components of the balance sheet using portfolio data on credit risk and market risk positions.

Illiquid assets subject to margin requirements (asset class I) are mapped to encumbered assets pledged as collateral in derivative and securities financing transactions including trading portfolio assets[5], loans, and financial assets designated at fair value. These positions amount to a value of 64021 million EUR.

Illiquid assets not subject to margin requirements (asset class J) represent 514550 million EUR. This include three categories of assets:

- Encumbered assets, not pledged as collateral, but restricted and not available to secure funding: this category includes mainly financial assets for unit-linked investment contracts, and some lending positions. They reach 23573 million EUR.

- Assets that cannot be pledged as collateral, excluding derivative positions: this category covers some loans, cash collateral on securities borrowed, reverse repos, and other assets including cash collateral receivables, goodwill, and deferred tax assets. Assets in this category represent 167444 million EUR.

- Other realisable assets. These assets include most lending positions (i.e. loans in the banking book, due from banks, and financial assets designated at fair value), some trading portfolio assets, property investment, and investment in associates. The amount of realisable assets reaches 323532 million EUR.

Marketable assets subject to margin requirements (asset class M) denote the fair value of derivative transactions including Level 1 and Level 2 assets of the fair value hierarchy. These contrast with Level 3 instruments that do not have quoted prices in active markets and rely on valuation models where significant inputs are not based on observable market data (e.g. long-dated complex derivatives). The latter are considered non-marketable and cannot be monetised over a short time horizon. For the G-SIB considered in the example, derivative instruments include mainly interest rate and foreign exchange contracts, and to a lower extent equity contracts. Less significant are credit derivative and commodity contracts. The value of this category reaches 118227 million EUR.

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[5]This excludes financial assets for unit-linked investment contracts.

Finally, marketable assets not subject to margin requirements (asset class N) include unencumbered instruments available to secure funding. These marketable assets include financial assets at fair value for 45117 million EUR, trading portfolio assets for 68369 million EUR, financial assets available for sale for 8419 million EUR, and held-to-maturity instruments for 9166 million EUR. Overall, category N represents 131071 million EUR. To complete the mapping of balance sheet assets, liquid assets (asset class C), including unencumbered cash and balances with central banks, amount to 87775 million EUR.

The result of the mapping is shown in Table 3.

Assets	Liabilities and equity
Illiquid assets: (i) Subject to VM, $I_0 = 64021$ (ii) Not subject to VM, $J_0 = 514550$	Current liabilities, $S_0 = 598$
Marketable assets: (i) Subject to VM, $M_0 = 118227$ (ii) Not subject to VM, $N_0 = 131071$	Long-term liabilities, $L_0 = 863771$ (incl. deposits of 408999)
Liquid assets, $C_0 = 87775$	Equity, $E_0 = 51275$

Table 3: Simplified balance sheet of a European G-SIB for year 2017 (in millions of EUR).

4   Liquidity at Risk

The framework introduced above allows moving beyond a liquidity risk analysis purely based on exogenous *expected* cash flows and define a concept of liquidity stress *conditional* on a stress scenario, which we baptise *Liquidity at Risk*.

4.1   A conditional measure of liquidity risk

Definition (Liquidity at Risk)**.** *Consider a stress scenario defined in terms of shocks to asset values. We call* Liquidity at Risk *associated with this stress scenario the net liquidity outflows resulting from this stress scenario. In other words, Liquidity at Risk in a given stress scenario is given by*

Liquidity At Risk = Current Liabilities + Net Scheduled outflows
+ Net outflow of Variation Margin + Credit-contingent cash outflows

The liquidity shortfall in a stress scenario is thus given by the difference between the Liquidity at Risk associated with the stress scenario and the available liquid assets at the point where the scenario occurs.

The Liquidity at Risk is easy to read off from the solvency-liquidity diagrams introduced in Section 2.3: it corresponds to the vertical shift (i.e. the liquidity shock) induced by the stress scenario. In terms of the model variables defined in Section 2,

$$\text{Liquidity@Risk} = S_2 - (C_1 - C_0 + \Delta C). \tag{19}$$

We note that:

- Liquidity at Risk is a conditional concept: it quantifies the expected total draw on liquidity resources of the bank *conditional* on the stress scenario being considered. In particular, the evolution of liquid balances and current liabilities constitute a part of this measure.

- Liquidity at Risk measures a *net outflow* corresponding to the stress scenario considered. This can be compared to the liquidity resources potentially accessible to the bank in the stress scenario, including feasible mitigating actions, to assess the potential for default.

The concept of Liquidity at Risk does not refer to a specific stochastic/statistical model for generating risk scenarios. It may be applied to historical risk scenarios as well as hypothetical stress scenarios generated from a stochastic model for risk factors. In the case where one starts from such a statistical model for risk scenarios, one can define a corresponding notion of Liquidity At Risk given a certain confidence level (e.g 99% Liquidity at Risk), although in the present paper we will not use this approach.

4.2 Examples

We now illustrate the concept of Liquidity at Risk using two examples: a synthetic balance sheet and the balance sheet of a G-SIB.[6]

4.2.1 A synthetic bank balance sheet

We consider a synthetic example of a bank balance sheet given in Table 4. We study the effect of a typical stress scenario related to a shift in interest rates and equity market on the credit risk of the bank. Sensitivities given in Table 5 assume similar balance sheet composition as in the case of a European G-SIB (see Section 4.2.2), with the exception of a significantly increased sensitivity of illiquid assets not subject to variation margin to changes in interest rates. In other words, we consider a case of a bank that although is well capitalised with a leverage of 25%

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[6]These examples are given for the purpose of illustrating the stress testing methodology; some of the assumptions used in the examples may not be realistic.

and has sufficient liquidity to fully cover its current liabilities, it holds a portfolio of risky loans that are extremely sensitive to increase in interest rates. As a result, an increase in interest rates leads to a loss in the bank's equity without drawing much on its liquidity reserves: an interest rate shock is mostly a solvency shock[7]. On the other hand, most of the solvency impact due to an equity market shock is attributed to a large variation margin that results in strong liquidity pressure: in this case, an equity shock is mainly a liquidity shock.

Assets	Liabilities and equity
Illiquid assets: (i) Subject to VM, $I_0 = 200$	Current liabilities, $S_0 = 100$
(ii) Not subject to VM, $J_0 = 1300$	
Marketable assets: (i) Subject to VM, $M_0 = 300$	Long-term liabilities, $L_0 = 1400$
(ii) Not subject to VM, $N_0 = 90$	
Liquid assets, $C_0 = 110$	Capital (equity), $E_0 = 500$

Table 4: A synthetic example of balance sheet for a well-capitalised bank (in millions of EUR).

Risk factor	Shift	ΔI	ΔJ	ΔM	ΔN
Interest rates	+200 bps	8	80	16	24
Equity market	-500 bps	120	15	55	50

Table 5: Balance sheet sensitivities for the balance sheet shown in Table 4. Values represent a decrease in the value of balance sheet components (in millions) in response to a shift in the risk factor.

Let us assume that only 5% of unencumbered illiquid assets can be readily liquidated in a fire sale at a price discount of 50%. Furthermore, we assume a repo haircut of 25% with associated repo rate of 7%, and unsecuritised borrowing rate of 1% is available to the bank as long as its leverage ratio does not exceed $\delta = 11$. Here, we do not assume any sensitivity of funding costs to the bank's credit rating.

Consider a specific market stress scenario defined by an interest rate move of +200 bps and an equity market move of -500 bps. The Liquidity at Risk for this scenario is equal to 299 million EUR (of which 100 million EUR due to the current liabilities and 199 million EUR payable in a variation margin), which leads to a liquidity shortfall of 189 million EUR. In this case, this shortfall may be fully

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[7]As seen in Table 5, an increase of +200 bps in interest rates leads to a total loss of 128 million EUR, of which only 24 million EUR are payable in the form of a variation margin.

covered through unsecuritised borrowing available to the bank. The impact of this stress scenario on equity comprises of the reduction of 368 million EUR due to initial shock and further 2 million EUR in borrowing cost.

So far we have discussed Liquidity at Risk in a single stress scenario. If we assume a linear impact of risk factors on the portfolio components, one can then scale these shocks and estimate the impact of an extreme event on the portfolio components using the sensitivities given in Table 5. Figure 6 summarises the impact of a shock on interest rates and equity of up to 8% (under a linear impact assumption). Although the solvency impact of the move in interest rates is larger than that of a change in equity market[8], the shock size threshold at which we observe a bank failure is actually lower for the latter factor. This illustrates a crucial point: the interaction of solvency and liquidity risk matters to the credit default risk. Failure to incorporate it into a stress testing framework can significantly underestimate the total risk of a financial institution. An approach solely based on solvency risk would distinguish two regions in Figure 6: a region of sufficient capital buffer (no failure) and a region of failure where loss of equity in a shock scenario exceeds the available buffer. Liquidity stress tests focus on sufficient liquidity buffers and the bank's ability to access sources of short-term funding in order to withstand adverse liquidity shocks. Consequently, independently conducted solvency and liquidity stress tests will fail to identify the regions where failure arises through the interaction of solvency and liquidity rather than through one channel alone, and thus underestimate the risk of failure. These results are consistent with the observations in Schmitz et al. (2019) but push their conclusions further, showing that neglecting the liquidity-solvency nexus not only leads to underestimation of solvency risk but also of liquidity risk. The degree to which the credit risk is underestimated depends on the model parameters, balance sheet composition and sensitivities to risk factors.

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[8]A change of 100 bps in interest rates leads to equity loss of 64 million EUR, whereas only a 48 million EUR loss for the same equity market shock.
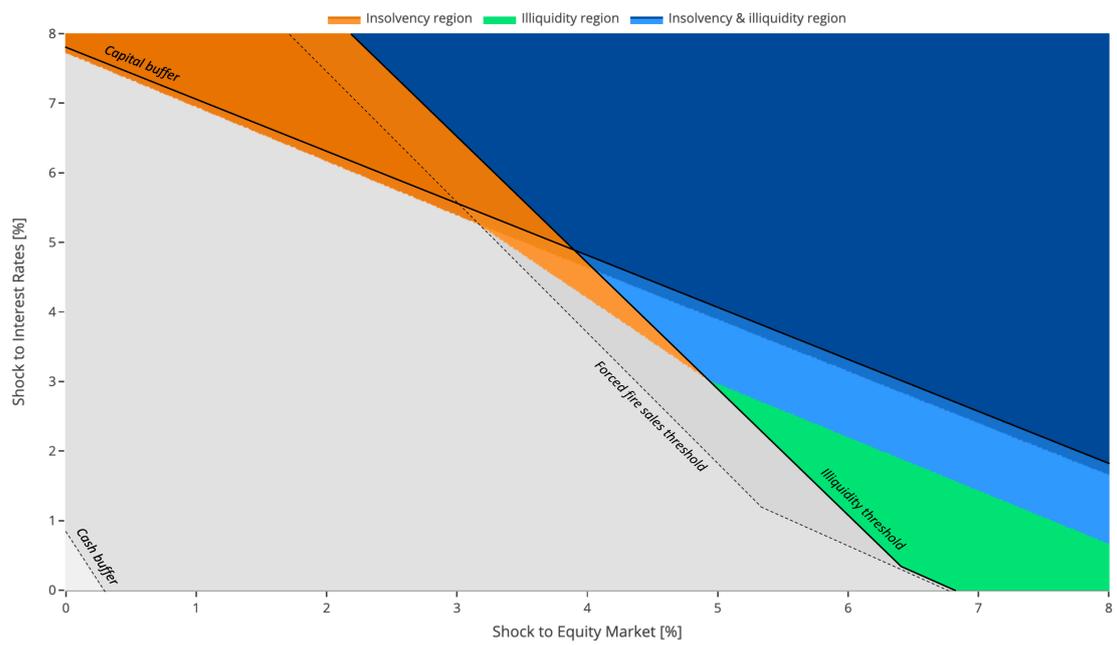
Figure 6: Insolvency and illiquidity regions for portfolio shown in Table 4, using a linear approximation based on sensitivities shown in Table 5.

4.2.2 A G-SIB example

Table 3 shows a synthetic view of the consolidated balance sheet data for a G-SIB; market sensitivities for balance sheet components are shown in Table 6. In the following, we assume that in a stress scenario only 5% of unencumbered illiquid assets can be sold in the short-term in a fire sale with an associated 50% discount. Funding through repo at a 5% rate requires a 32% haircut, while unsecuritised borrowing at 1% rate is available up to the downgrade threshold of $\delta = 20$.

Risk factor	Shift	ΔI	ΔJ	ΔM	ΔN
Interest rates	+200 bps	158	284	938	1582
Equity market	-500 bps	2554	2462	1968	2155

Table 6: Balance sheet sensitivities for the balance sheet shown in Table 3. Values represent a decrease in the value of balance sheet components (in millions EUR) in response to a shift in the risk factor.

We subject this balance sheet to a stress scenario defined by

- an interest rate move of +200 bps,

- an equity market move of -500 bps, and

- a 60% runoff of short-term funding and deposits in the downgrade scenario.

The impact of this specific stress scenario can be represented through a solvency-liquidity diagram, shown in Figure 7. Liquidity at Risk conditional on our market scenario equals to 261316 million EUR (5718 million EUR payable in a variation margin, 598 million EUR due to current liabilities and the remaining amount due to runoff on deposits), which exceeds the bank's liquidity buffer of 87775 million EUR, and results in a liquidity shortage of 173541 million EUR that needs to be covered through a mix of repo and fire sales.

Using the same linear impact assumptions as in the previous example, we can extrapolate this analysis to more extreme scenarios obtained by scaling the shocks to risk factors. The corresponding outcomes are represented in Figure 8. We observe that fire sales enter the picture already for moderate shocks (approximately beyond 2%) to equity and interest rates. This can lead to an adverse market impact and result in wide-spread of losses across the financial system. Under the severe depositor runoff assumption of 60%, we see that liquidity risk becomes a major component of the default risk.
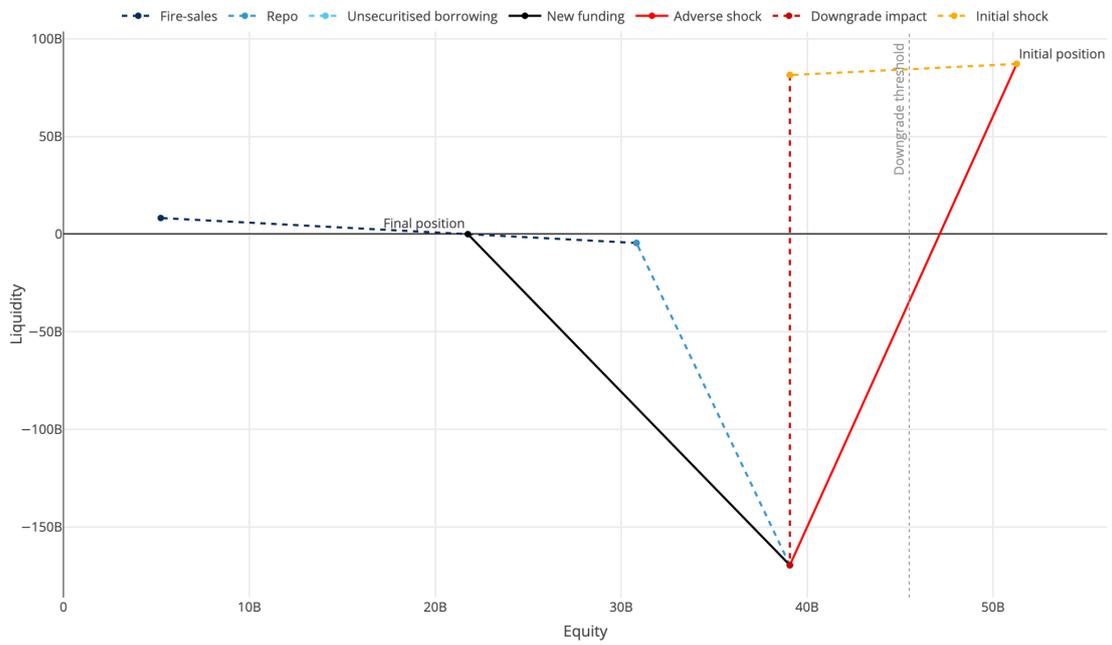
Figure 7: Solvency-liquidity diagram describing the behaviour of the balance sheet shown in Table 3 in a stress scenario with interest rate move of +200 bps, equity market move of -500 bps and a 60% runoff of deposits.
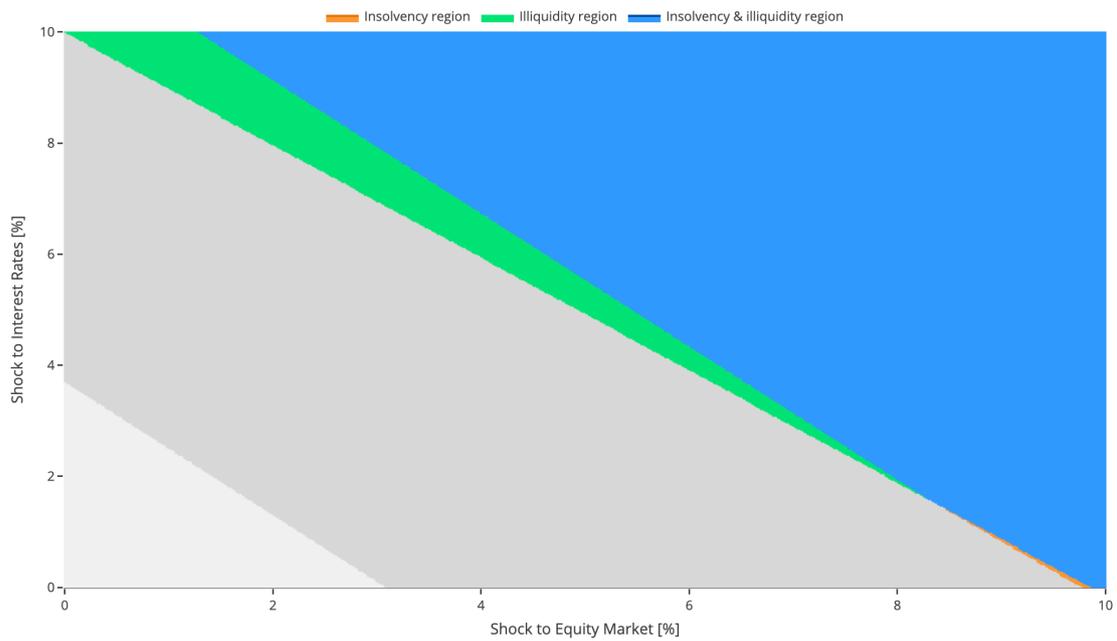
Figure 8: Insolvency and illiquidity regions for the balance sheet shown in Table 3 under a 60% runoff on deposits. Dark grey region corresponds to a market scenario in which the bank is forced to liquidate a fraction of its illiquid assets in a fire sale.

5　Concluding remarks

The global financial crisis confirmed that lack of liquidity is an inherent risk throughout the banking sector (Pohl, 2017). Liquidity and solvency are two inter-related dimensions of credit risk that cannot be modelled, or stressed, separately. Nonetheless, the interaction between liquidity and solvency tends to be omitted in stress testing practices. In response to calls from regulators to develop integrated liquidity and solvency stress tests (Basel Committee on Banking Supervision, 2015), we have developed a coherent framework for joint stress testing of solvency and liquidity risk.

In our framework, solvency shocks affect liquidity through margin requirements, via firm's ability to raise short-term funding, and through credit risk sensitive outflows, consequently leading to endogenous liquidity shocks. In turn, solvency stress is exacerbated through the cost of new funding resulting from a liquidity shortfall, and fire sales. We distinguish two types of failure: financial institutions can become illiquid without being insolvent, insolvent while remaining liquid, or – in the case of extreme shock – both illiquid and insolvent. The model illustrates the danger of underestimating credit risk by models that do not account for the solvency-liquidity nexus. As shown by our examples, balance sheet composition has a significant effect on the solvency-liquidity nexus. In particular, our insights show that structural solvency risk models are insufficient to capture this dependency and we advocate the use of a more granular balance sheet view by the regulators when conducting a stress test.

Our proposed framework provides a more realistic stress test framework which establishes coherence between design of solvency and liquidity stress tests. It also includes mitigating actions that can be extracted from the bank's contingency funding plan and recovery plan. By defining the concept of Liquidity at Risk, we provide a tool to quantify the total draw on liquidity resources of the bank conditional on the stress scenario defined directly in terms of an adverse shock to risk factors. Sudden liquidity stress can result in the inability to obtain sufficient funding in due time and can lead to insolvency.

The tool is calibrated using available regulatory templates on financial data, risk data, and liquidity monitoring templates. The model is amenable to reverse stress testing and naturally permits a range of sensitivity tests around crucial inputs including changes to the classification of fair valued instruments, the liquidity generation capacity of unencumbered securities, the evolution of market haircuts and funding costs, and fluctuations in creditors' risk appetite framework.

The model yields useful policy implications for central banks and supervisory authorities. It helps supervisors to identify whether managerial options to fend off liquidity risk are helpful to avoid breaching regulatory solvency/liquidity ratios, given the scenario. It reveals sources of systemic spillovers, i.e. shocks to risk

factors that can become a conduit of systemic risk propagation through fire sales and threaten financial stability. Crucially, it helps authorities to form judgment around the provision of central bank emergency liquidity assistance to 'illiquid but solvent' financial institutions. Ultimately, it serves to quantify the amount of funding for resolution which remain perhaps the key likely impediment in banking resolution.

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