STAFF MEMO

# Handling structural break points in NEMO

NO 2 | 2020

ERLING MOTZFELDT KRAVIK AND KENNETH SÆTERHAGEN PAULSEN

NORGES BANK

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ISSN 1504-2596 (online)
ISBN 978-82-8379-137-2 (online)

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HANDLING STRUCTURAL BREAK POINTS IN NEMO

# Handling structural break points in NEMO*

Erling Motzfeldt Kravik† Kenneth Sæterhagen Paulsen‡

February 7, 2020

**Abstract**

This paper documents a new feature in Norges Bank's policy model NEMO, namely the ability to handle structural break points, i.e. shifts in one or more parameter values at a specific point in time. This property is introduced to enable the model to answer new policy-relevant questions, such as the effect of changes in the inflation target and the effect of a sudden drop in the expected long-term oil price. We document the theoretical solution technique and illustrate its usage through a practical example. Additionally, we present a procedure for estimating break points. Our results indicate that including structural shifts is important when interpreting data. Neglecting structural shifts can lead to wrong interpretations of history, which, potentially, could also affect forecast performance.

*This staff memo should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. We would like to thank Karsten R. Gerdrup, Ørjan Robstad and our colleagues at the Monetary Policy Department of Norges Bank for their helpful comments and suggestions. The usual disclaimer applies.

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# Contents

# 1 Introduction

Standard solution methods for linear rational expectations models deal with the case where the parameters of the structural model, such as individual preferences, are constant (Kulish and Pagan, 2017). In many circumstances, however, this does not give a satisfactory description of the economy. To accurately model a change in some underlying preference or policy, one would need to change one or more parameters at a specific point in time. Usually, this relates to steady-state parameters which cause permanent changes in the model.[1] For instance, if the inflation target changes or there is a sudden drop in the expected long-term oil price, this should be accounted for in the model by a shift in one or more parameter values at time $t$, i.e. we should include *structural break points*[2] in the model.

Since Norges Bank's inflation target was changed in March 2018, the Bank has had the ability to include structural break points in their main model for economic and monetary policy analysis, the Norwegian Economy Model (NEMO). As will be explained below, the method has already been used in the Bank's conduct of monetary policy. We are not aware of other central banks that utilize this procedure in large scale DSGE models on a regular basis.

This staff memo documents the theoretical solution technique of break points in NEMO, illustrate its usage through examples and show possible implications of not incorporating structural break points properly.

The document is organized as follows: Section 2 gives a short introduction to NEMO, whereas section 3 discusses the solution technique and challenges that break points can accommodate. We demonstrate the importance of including break points in NEMO through an example in Section 4. Section 5 describes a procedure to estimate the magnitude and timing of break points based on Bayesian techniques. Lastly, Section 6 concludes.

# 2 Norges Bank's policy model NEMO

The forecasting and policy analysis system in Norges Bank is organized around NEMO. NEMO is a large-scale DSGE open-economy New Keynesian model for monetary policy analysis and forecasting. The model was launched in 2006 and has continuously been updated and extended along several dimensions.

NEMO consists of households, domestic (traditional) firms, an oil sector, a government sector and the monetary authority. In addition, there are separate production sectors for housing and non-housing capital goods as well as a banking sector. All agents have rational, or model-consistent, expectations with respect to all prices and quantities, with households' house price expectations being an important exception.

The central bank conducts optimal monetary policy, i.e. sets the interest rate to minimize a loss function. Alternatively the model can be solved under a Taylor type rule.

A schematic illustration of NEMO along with some more explanatory text is placed in Appendix B.1. In 2019, the model was re-estimated and thoroughly documented in Kravik and Mimir (2019).

[1]This need not be the case, however. One could also have shifts in dynamic parameters that do not affect the steady state.

[2]In the document, we use the terms (structural) break points, (structural) breaks and parameter shifts interchangeably.

# 3 Break points in NEMO

Traditionally, when solving DSGE models, parameters of the model have been assumed to stay constant. Up until recently, that was also the case with NEMO. However, new solution techniques have been developed making it possible to include structural break points without large computational costs.[3]

The benefits of including structural break points in DSGE policy models such as NEMO, should be obvious. For instance, if data display abrupt and long-lasting changes in particular observable variables, indicating a structural break, a proper way of handling such breaks would be to change one or more structural parameters of the model that can be mapped back to the observable(s) at the time of the break. Another useful case is when structural breaks appear in observed policy targets or regulations that are embedded into the model as parameters, such as the inflation target or bank capital requirements. In most cases, the structural parameters in question also change the steady state of the model (as in the examples given below), but the procedure also makes it possible to alter parameters that only change the dynamics of the model.[4] The change in parameters after the break makes the model interpret historical data differently, draw other shocks and therefore give different forecasts.

In recent history, there have been two episodes in Norway where Norges Bank has decided to include break points in NEMO:

- First, on 2 March 2018 Norges Bank's inflation target was lowered from 2.5 to 2 percent. This induced a corresponding change in the steady-state inflation rate in NEMO that would otherwise be hard to implement without the new procedure.[5]

- Second, in *Monetary Policy Report* 1/19 Norges Bank wrote that they "now apply the assumption that the depreciation of the equilibrium exchange rate after the oil price fall was slightly more pronounced than assumed earlier." This is likely to mean that a permanent decline in the real oil price in NEMO was introduced, which, given the structure of the model, resulted in a weaker steady-state real exchange rate and a lower real wage.

In both of these cases, introducing the break points led to shifts in the steady state of the model. However, in the former case, the steady states of *real* variables remained unchanged.

Subsections 3.2 and 3.3 below illustrate the effects of a change in the inflation target and a change in the steady-state real oil price in NEMO, respectively.

[3]The model parameters used for the examples in Section 3 and 4 deviate somewhat from the ones in Norges Bank's MPR framework and in Kravik and Mimir (2019).

[4]For instance, one can imagine that there are periods when rigidities are higher than "normal" times, due to, say, political turbulence. This could then be mapped back to the model by a change in one or more adjustment cost parameters.

[5]In NEMO, the new inflation target was gradually introduced as the Bank assumed some lag in expectations formations (see *Special Feature: Monetary policy implications of a new inflation target* in Norges Bank's *Monetary Policy Report* (MPR) 1/18). All MPRs are available at `https://www.norges-bank.no/en/topics/Monetary-policy/monetary-policy-report/`.

## 3.1 The solution technique

Constant parameter DSGE models can be represented in the following form

$$E_t[F(X_{t+1}, X_t, X_{t-1}, U_t, \theta)] = 0, \tag{1}$$

where $F$ is the set of possibly non-linear equations of the model, $X_t$ is a $Q \times 1$ vector of endogenous variables, $U_t$ is a $N \times 1$ vector of exogenous variables, and $\theta$ is the parameter vector of the model. Introducing break points entails that we let the parameters become regime-dependent:

$$E_t[F(X_{t+1}, X_t, X_{t-1}, U_t, \theta_r)] = 0, \tag{2}$$

where $\theta_r$ is the vector of parameters in regime $r \in \{1, ..., s\}$. As long as the structural break points are unanticipated, i.e. the parameter changes are not known or expected by the agents before they occur, solving equation (2) is reduced to finding the solution for each regime separately. In general, where regime $r$ depends on time $t$. See Appendix A.1 for details.

The procedure outlined above gives a simple and flexible way of applying break-points to a model, with the only computational cost being to solve the model once per regime. These properties of the technique are important because of the large size of NEMO and the heavy reliance on the model in the MPR process.

Note that this approach is different from what is often referred to as regime-switching DSGE models (RS-DSGE models), in which the regime in period $t$ is (either endogenously or exogenously) determined by a Markov process and the agents have the capability to form expectations over the regimes.[6] Also within the RS-DSGE framework one can have absorbing regimes by proper calibration of the Markov chain, but in general, this procedure is computationally more demanding.[7]

Kulish and Pagan (2017) show how a similar method to the one used in NEMO can be extended to the case where the structural changes are foreseen. Implementing this feature in NEMO is left for future work.

## 3.2 A change in the inflation target

Altering structural parameters can be interpreted as permanent shocks. When a structural break is introduced, potentially all endogenous variables will transition from the previous to the new steady state (similar to impulses responses from a temporary shock). See Appendix A.2 for details.

Figure 1 shows impulse responses from a sudden (and unanticipated) negative shock of 0.5 percentage points to the (annualized) inflation target in NEMO. Variables are shown in percent deviation from the original steady state. Flow variables are in fixed prices on a quarterly basis. The dotted lines indicate the new steady state. The model is solved under optimal policy.

[6] Strictly speaking, both the Markov switching framework and the break point framework deal with switching regimes. However, the term regime-switching model is usually applied to the former case.

[7] For a number of years there has been a growing literature on how to deal with structural breaks in RS-DSGE models, and technically, implementing regime switching is now relatively easy. For instance, Maih (2015) has developed a flexible and easy-to-use object-oriented toolbox, named *RISE*, that is able to solve RS-DSGE models, allowing for endogenous transition probabilities and for agents to react to anticipated events (freely available). See reference for a comprehensive review of the literature on Markov switching models up until 2015.

Although agents in the model have rational expectations (except with respect to house prices), the change in inflation target leads to a relatively slow transition to the new steady state due to rigidities operating in the model. The new inflation level is reached after about 20 quarters. The positive inflation gap that immediately opens causes the central bank to hike the policy rate which increases the real policy rate by about 20 basis points (at maximum) on an annual basis. The contractionary monetary policy is transmitted to the real economy through the banking sector via a gradual pass-through from the policy rate to the household and corporate lending rates. This suppresses demand components, such as output, investment and consumption. The real wage level also falls. The rise in the real policy rate contributes to a stronger real exchange rate, generating a reduction in exports.

Note that a change in the inflation target causes a trade-off as it opens up a positive inflation gap and a negative output gap. As a result, the monetary authority chooses a smooth transition to the new inflation target.

The real effects of a change in the inflation rate in the long-run are negligible.

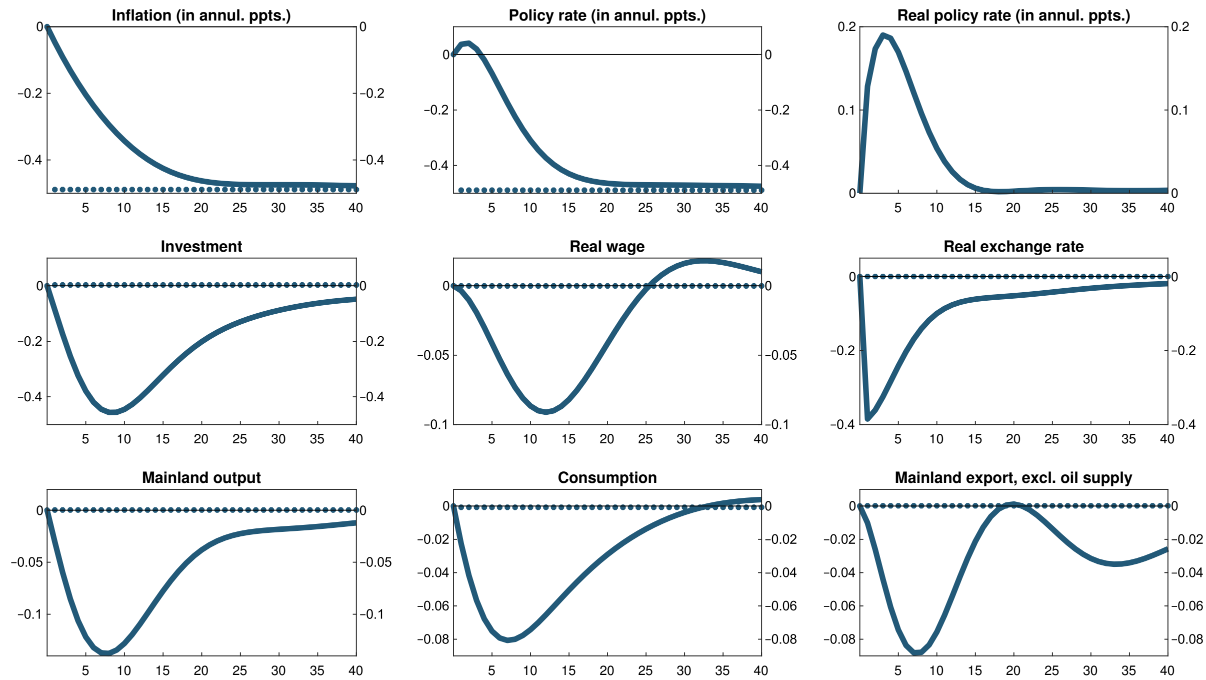


Figure 1: A drop in the inflation target in NEMO. Flow variables are in fixed prices on quarterly terms in percent deviation from the original steady state. The real wage, real exchange rate and real policy rate are deflated by the inflation rate. The dotted lines indicate the new steady states.

## 3.3 A change in the steady-state oil price

Oil and gas production have traditionally made up about 45 to 50 percent of total export value and between 20 to 30 percent of total GDP in Norway since 2000 (Statistics Norway, 2016). Permanent changes in the oil price can therefore potentially have large effects the long-term growth path and real wages in Norway.

The oil sector in NEMO builds on Bergholt *et al.* (2019). The sector consists of supply firms as well as a domestic and a foreign extraction firm. A decrease in real oil prices (caused by an increase in supply of oil internationally) reduces demand for supply goods both from the foreign and domestic extraction firms (as oil production and oil investments are discouraged). This will affect the rest of the economy as the oil supply sector uses domestic and imported factor inputs in production. Lower oil prices also has a positive effect on trading partners' production and export demand of *traditional* goods (as foreign marginal costs are decreased).[8]

Figure 2 shows impulse responses to a 10 percent permanent reduction in international oil prices. Starting with the new steady state levels (the dotted lines in the figure), NEMO shows that the oil price drop necessitates a 6 percent depreciation of the real exchange rate in order to keep the country's net foreign asset position unchanged in the steady state. This causes a shift in the production structure, leading to a 6.6 percent *increase* in traditional (non oil-related) exports and a 1.6 percent *decrease* in oil supply export goods. Oil investments (which is part of mainland Norway) contracts about 7 percent. Imports are reduced by 9 percent. All numbers are in fixed prices.

The real wage level as well as consumption and investment fall by around 4 percent in the long run. Given the reduction in imports and the increase in exports, mainland output is reduced less than 1 percent.

Turning to the dynamics of the model, the UIP and the rational expectation properties of the model make the real exchange rate quickly adjust to (and overshoot) the new steady state. This increases traditional export and reduces import. Export of oil supply goods is reduced due to lower foreign demand.

Consumption, investment and the real wage are adjusted quite slowly to their new steady state, whereas oil investment under-shoots its new steady state for a rather long time because of the necessary downsizing of the capital level in the oil sector. The policy rate is increased less than inflation in the first quarters (creating a reduction in the real interest rate) to avoid mainland output from falling further below its new steady state. At maximum, the real interest rate is increased by 0.3 percent (on an annualized level).

[8]Currently, the foreign block in NEMO is modelled in gap-terms only. As a consequence, permanent oil price changes do not lead to permanent changes in economic developments for trading partners, and hence no permanent change in export demand. As the oil price gap is always zero in the exercise depicted in this particular exercise, there will also be no effects from export demand in the dynamics of the model.

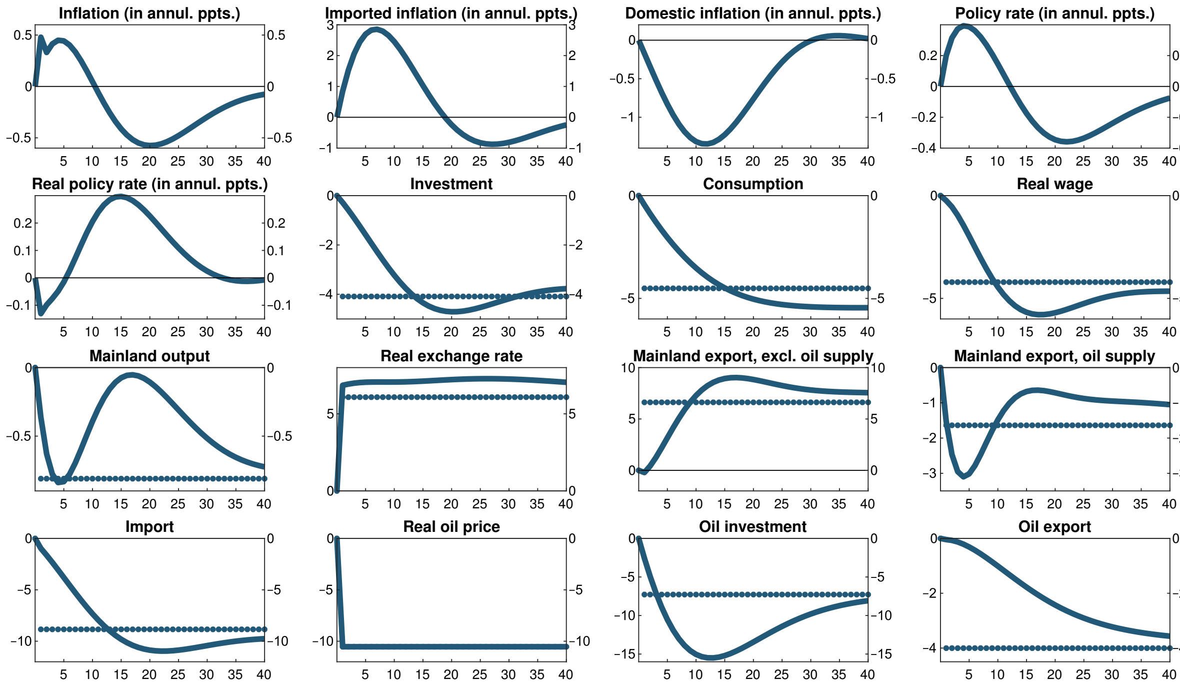


Figure 2: A 10 percent permanent reduction in international oil prices in NEMO. Economic development for trading partners is assumed to be unchanged. Flow variables are in fixed prices on quarterly terms in percent deviation from the original steady state. The real wage, real exchange rate, real oil price and real policy rate are deflated by the inflation rate. The dotted lines indicate the new steady states.

# 4 A practical illustration of the importance of break points

How will the ability to include break points affect our understanding of the workings of the economy? In this section, we highlight the importance and advantages of the new solution technique by comparing it to an example where data is filtered using a constant-parameter model.

We first simulate NEMO for 100 periods starting at period -50 (see Appendix A.3 for technicalities). In the simulation, we include a 10 percent drop in the steady-state oil price in period 0 and onwards – we refer to this model as the *True Model (TM)*. Then we construct what we label the *Naive Model (NM)*: a model that is identical to the TM except that there is no steady-state shifts in the oil price.

To make the experiment policy relevant, we then save down 26 level variables from the simulations of the TM, corresponding to the observables in NEMO, and create gaps out of these variables.[9] For real variables, we take logs and run a two-sided HP-filter (with

[9]The 26 variables comprise the following: **Real domestic variables:** Output, consumption, exports (traditional and oil supply), imports, government expenditures, investment (traditional, housing and oil related), and hours worked. **Financial variables:** Household and corporate credit. **Price variables:** Wages, consumer prices (CPI and imported), house prices, lending rates (households and corporations), money market interest rates, and the policy rate. **International variables:** The exchange rate, the

$\lambda = 1600$), whereas for prices and interest rates, we take logs and subtract the mean.

Figure 3 shows the true levels, the true trends (i.e. the steady state of the variables) and the constructed trends for output, output growth, the oil price, the real exchange rate, the policy rate and inflation around the time of the oil price shift in period 0. The corresponding gaps are shown in Figure 4. As is clear, the HP-filter struggles to capture the structural break and, hence, the estimated gaps for mainland output, the real exchange rate and the oil price differ quite a lot from the true gaps. This highlights the importance of handling structural breaks correctly, but also sheds light on the larger issue of pre-filtering data itself.[10]

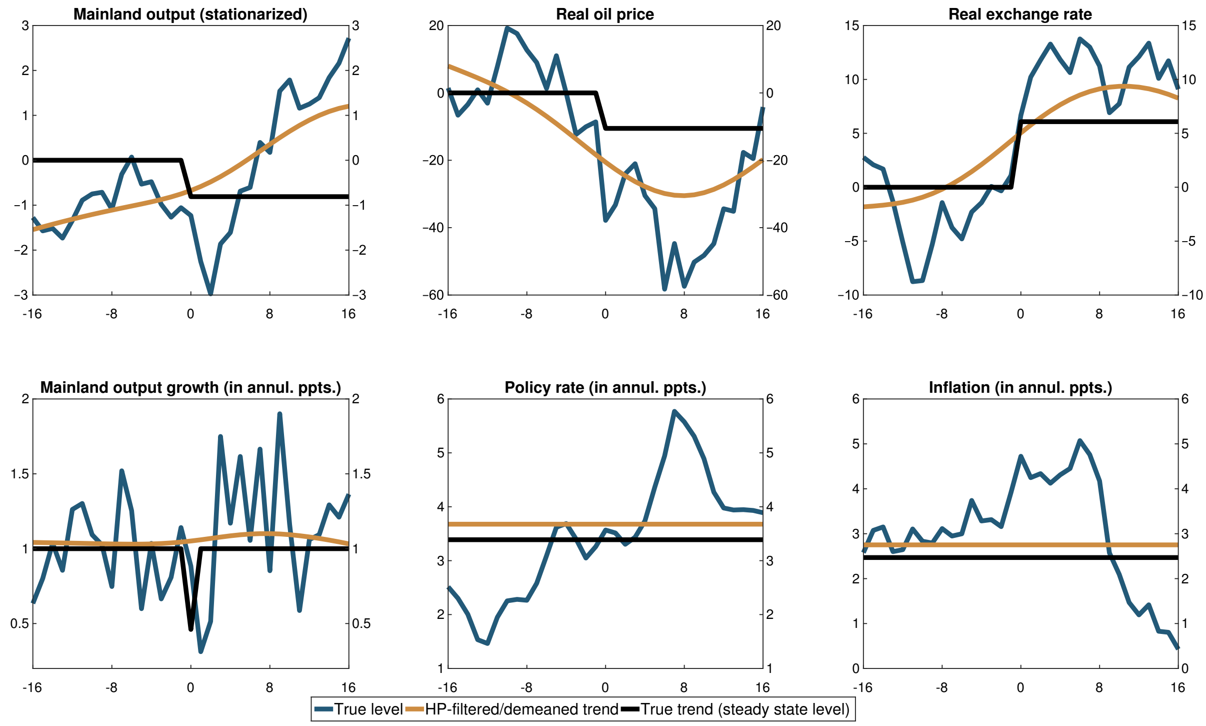


Figure 3: The blue lines show the true (simulated) levels, whereas the yellow lines show the HP-filtered/demeaned trends based on the level variables. Black lines indicate the true steady state. For the top three figures, the y-axis indicates deviation from the original steady state.

international oil price and foreign GDP (trading partners and global), money market rates, and inflation.

[10]See Canova and Ferroni (2011) for a discussion.

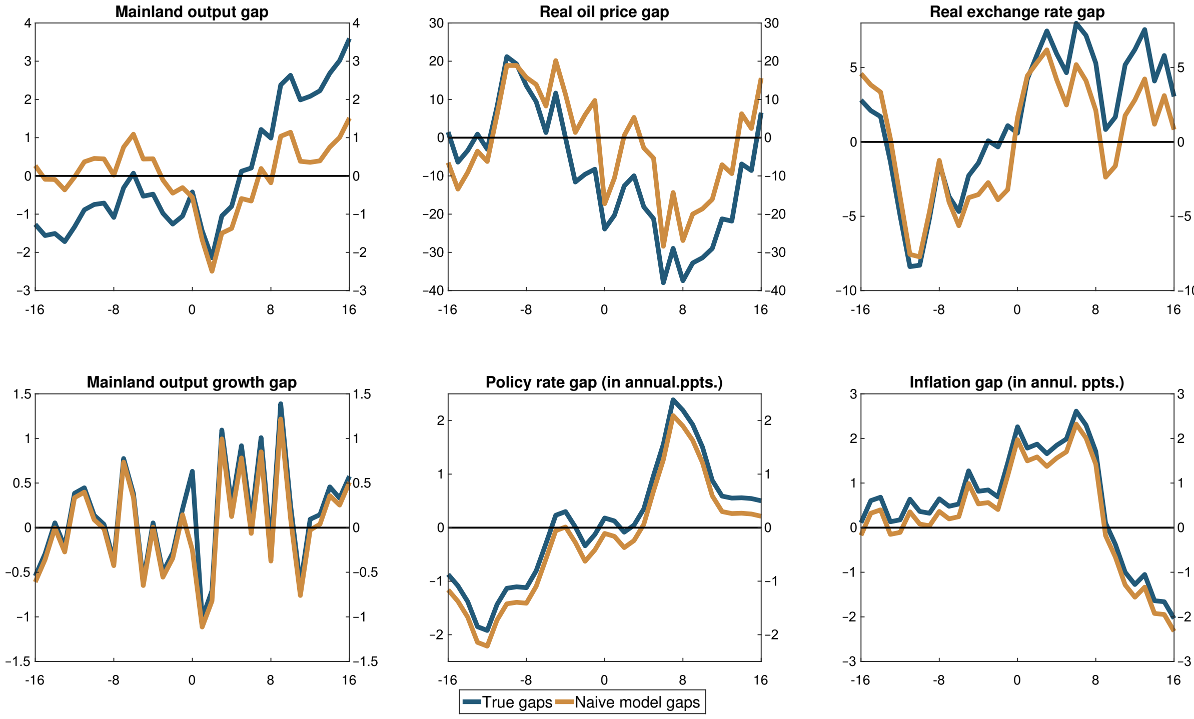


Figure 4: The blue lines indicate the true (simulated) gaps, whereas the yellow lines are the constructed gaps based on true level variables. Gaps are defined as percentage deviation from the steady state.

Next, we treat the gaps as observables and use a Kalman smoother approach to filter out shocks in the two models, respectively, as described in Appendix A.4. We then create shock decompositions of the output gaps and output growth gaps based on the two different models, as depicted in Figure 5 and 6. The 26 shocks are grouped in accordance with Table 2 in Appendix B.2.

The ability to include structural breaks allows the TM to filter the data with permanent shocks instead of attributing structural changes to a set of temporary shocks. Therefore, the story is quite different across the two models for how they interpret the observed output gap and the output growth gap.

Looking at Figure 5, the TM correctly recognizes that there has been a shift in the steady-state oil price level from period 0. The shift contributes to a drop in output growth for the first periods after time 0 (grey bars). Furthermore, as it takes time before mainland output reaches the new lower steady-state level (see the behaviour of mainland output in Figure 2), some of the positive output gap in the periods after the break is also correctly attributed to the steady-state shift in oil prices.

Turning to the NM and Figure 6, the misinterpretation of data is clear. In the NM, there is no steady-state shift and hence the model instead is forced to explain the constructed output gap by using temporary shocks. In general, the NM draws larger supply shocks and lower demand shocks, foreign shocks and oil sector shocks.

The NM also draws more monetary policy shocks compared to the TM. Before the structural break, the NM interprets the output gap to be close to zero or positive (whereas it in fact is negative in the TM), and hence uses negative monetary policy shocks to partly explain the negative policy rate gap and the near-zero output gap. After the break, however, this is reversed. The negative output gap in the NM would, all else equal, lead to a stronger reduction of the interest rate. The NM therefore needs to draw positive monetary policy shocks to explain the observed policy rate gap. The positive monetary policy shocks partly explain the negative output gap in the NM, but due to the persistence of the shocks before the structural break, it takes time before the net effect of monetary policy shocks on the output gap becomes negative.

The difference between the shock decompositions are placed in Appendix B.3.

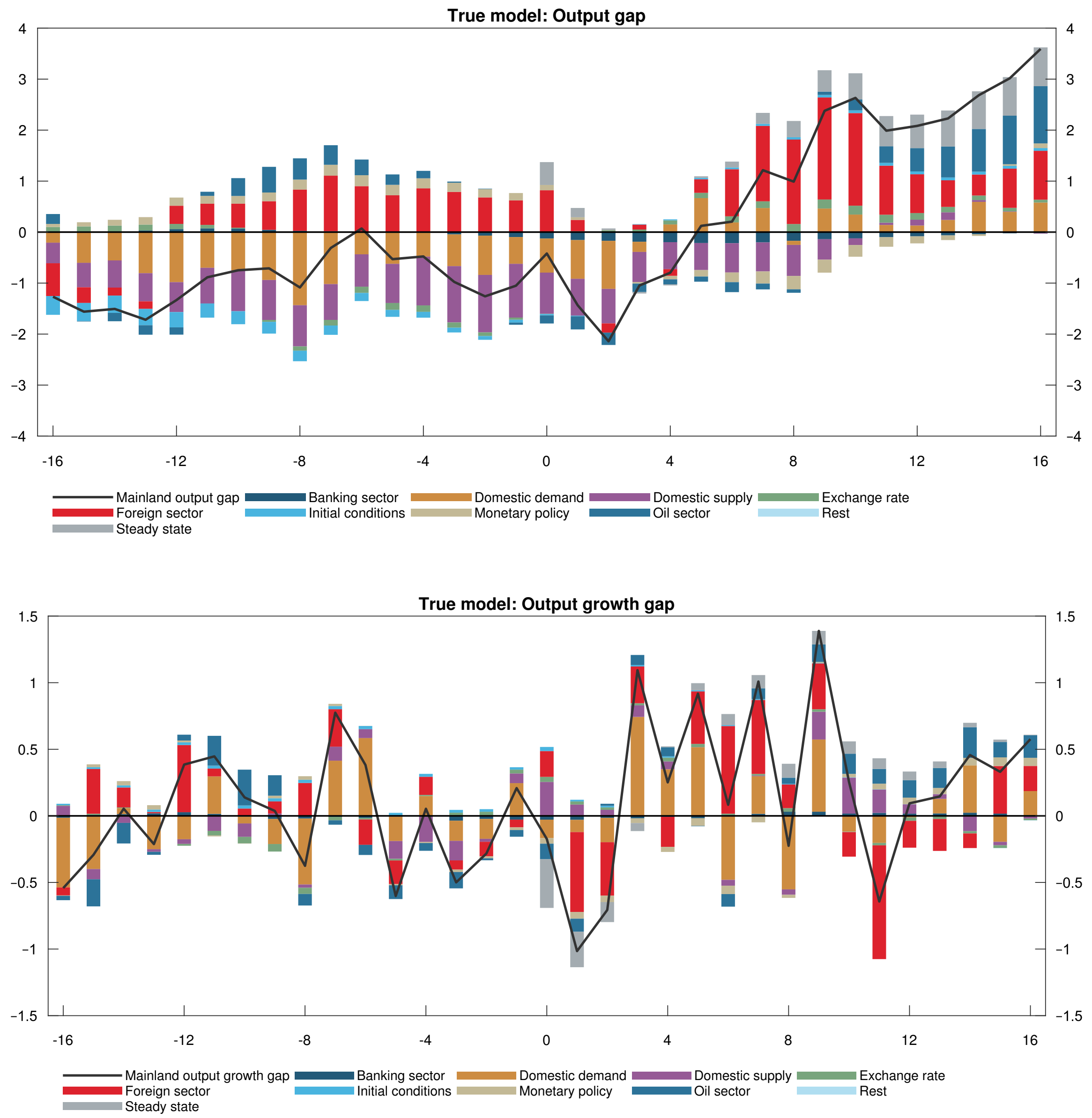


Figure 5: Historical shock decomposition of the true (simulated) output gap and the output growth gap using the True Model. A 10 percent oil price drop takes place at period 0. Shocks are grouped according to Table 2 in Appendix B.2.

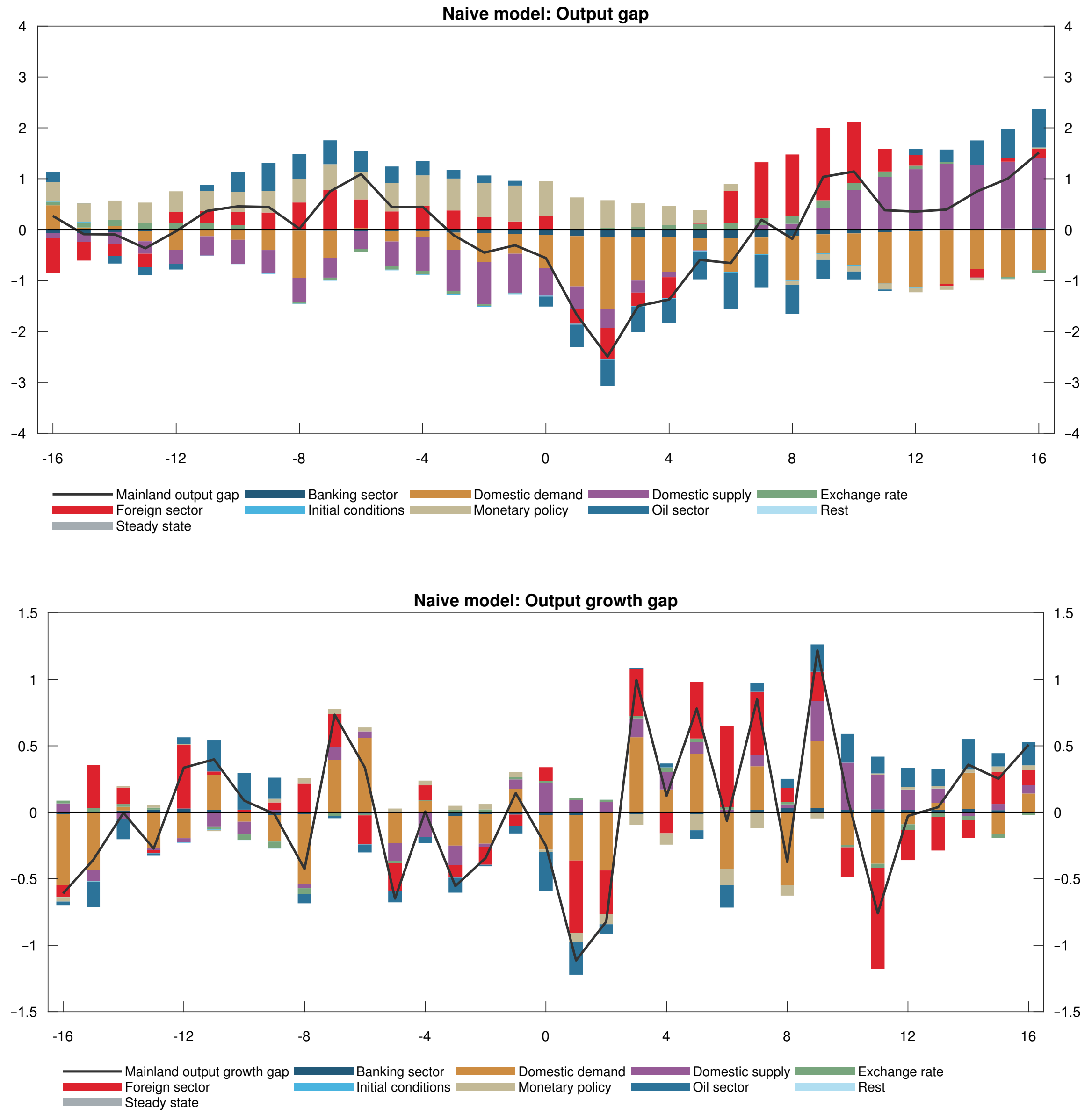


Figure 6: Historical shock decomposition of the constructed output gap and the output growth gap using the Naive Model. Shocks are grouped according to Table 2 in Appendix B.2.

# 5 Estimation of break points

In the last section we assumed that both the magnitude and the timing of the decline in the steady-state oil price was known prior to filtering. In a real life application, this information is not necessarily known, and we need to estimate it based on the data at hand. In this section we do this by applying a Bayesian approach, where we maximize the posterior distribution given the observed data and the model.

First, we formulate a prior on the magnitude and the timing of the break. For the magnitude, we choose a normal prior with mean of no break, and standard deviation of 0.1. For the timing of the break, we use a uniform discrete prior on the interval -4 to 4. To evaluate the log-likelihood given the parameters we use the Kalman filter described in Appendix A.4. For more details on the estimation procedure, see Appendix A.5.

We estimate the model over the same simulated sample as in the previous section. The estimation results are provided in Table 1. We can see the estimation is able to detect the timing and magnitude of the decline in the oil price quite well: Results indicate the timing of the break at period 0 and a decline in the steady-state oil price of 9.8 percent.

Table 1: Estimation results

| Parameter | Prior distribution | Prior mean | Prior st. dev. | Posterior mode |
|---|---|---|---|---|
| Magnitude of break | Normal | 1 | 0.1 | 0.902 |

| Parameter | Prior distribution | Prior mean | Prior bounds | Posterior mode |
|---|---|---|---|---|
| Timing of break | Discrete Uniform | 0 | [-4,4] | 0 |

In Figure 7 we plot the curvatures of the log-posterior distributions, log-likelihood and log-prior distributions around the prior means. As we see from the figure, the prior distributions do not steer the estimation to the conclusion, as the shape of the log-posterior distributions and log-likelihoods are almost identical. Based on this, we can conclude that we are able to identify the decline in the oil price well.

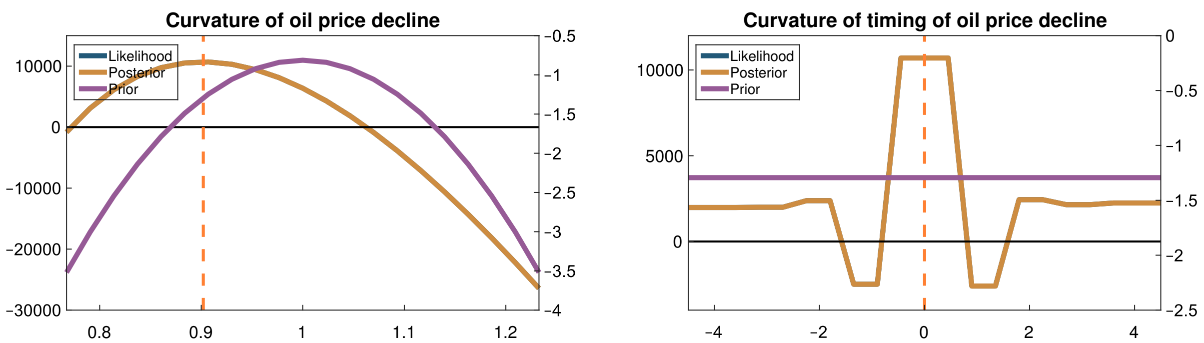


Figure 7: The blue lines (hidden behind the yellow lines) indicate the curvatures of the log-likelihoods, the yellow lines are the curvatures of the log-posterior distributions,[a] whereas the purple lines are the log-prior distributions of the magnitude of the oil price decline and timing of the oil price decline, respectively. The figures are centered around the prior means. The orange dashed lines indicate the posterior modes. The true value is 0.9 for the decline in the oil price, while it is 0 for the timing of the decline in the oil price.

[a] As defined by the last term in equation (33) in Appendix A.5.

Although the estimation routine was successful, a weakness of this exercise is that the data are simulated from the model, and that the sample is quite long. As noted by Kulish and Pagan (2017), in practice, it is usually the magnitude of changes in the properties of observable variables that is used to help define subsamples for which a time-invariant structure is assumed to be valid. This is also the procedure followed by Norges Bank in the examples mentioned in Section 3. Investigating the estimation procedures on actual data is left for future research.

# 6 Concluding remarks

Standard solution methods for rational expectations models deal with the case where the parameters of the model are time-invariant. However, to accurately model a change in some underlying preference or policy, one would need to change one or more parameters at a specific point in time. Since Norges Bank's inflation target was changed in March 2018, Norges Bank has had the ability to include simple structural break points in their main model for economic and monetary policy analysis, the Norwegian Economy Model (NEMO). This property was introduced to enable the model to answer new policy-relevant questions, such as the effect of changes in the inflation target and the effect of a sudden drop in the expected long-term oil price.

This staff memo has documented the theoretical solution technique of break points, illustrated its usage through examples and showed possible implications of not incorporating structural break points properly. Additionally, we have presented a procedure for estimating break points. Our results indicate that including structural shifts is important when interpreting data. Neglecting structural shifts can lead to wrong interpretations of history, which, potentially, could also affect forecast performance.

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# Appendices

## A Algorithm for handling break points in DSGE models

### A.1 The solution technique

Dynamic stochastic general equilibrium (DSGE) models can be put on the form

$$E_t[F(X_{t+1}, X_t, X_{t-1}, U_t, \theta_{S(t)})] = 0, \tag{3}$$

where $F$ is a function that represents the set of possibly non-linear equations of the model, $X_t$ are the endogenous variables. $U_t$ are the exogenous variables, which cannot appear with any order of lead or lag, with size $N$. Let the number of equations be given by $M$, which must also be the number of endogenous variables $(= Q)$.[11] $\theta_{S(t)}$ is the parameter vector of the model that may change unexpectedly over time. In what follows we assume that $\theta_{S(t)} \in \mathbb{R}^p$, and that there are countable number of different regimes, i.e. $\theta_{S(t)} \in \{\theta_1, ..., \theta_s\}$ for some finite $s \in \mathbb{N}^+$, and that the function $S(t)$ is a indicator function for which regime we are in at the time $t$. $p \in \mathbb{N}^+$ will be the number of parameters of the model.

To be able to solve the non-linear system in equation (3) we need to do an approximation. The first step is to solve the multivariate non-linear system

$$F(X_r^{ss}, X_r^{ss}, X_r^{ss}, 0, \theta_r) = 0 \; \forall r \in [1, s], \tag{4}$$

where the found solution $X_r^{ss}$ is the steady state in regime $r$. Then we do a 1st order Taylor expansion around the steady state in each regime to get

$$A^+_{\theta_r}(X_{t+1} - X_r^{ss}) + A^0_{\theta_r}(X_t - X_r^{ss}) + A^-_{\theta_r}(X_{t-1} - X_r^{ss}) + B_{\theta_r} U_t = 0. \tag{5}$$

In the case of optimal monetary policy we optimize a loss function (where $W$ are the weights in the loss function)

$$\sum_{t=0}^{\infty} \beta^t X_t' W X_t, \tag{6}$$

s.t. equations in (5). In the monetary policy rule case the rule is already part of the equations in (5).

We are looking for a solution on the form

$$X_t = X_{S(t)}^{ss} + A_{S(t)}(X_{t-1} - X_{S(t)}^{ss}) + C_{S(t)} U_t, \tag{7}$$

As noted in section 3.1.1 of Kulish and Pagan (2017) we can then find the solution to the problem by finding the solution for each regime separately. Which means that we can use one of the standard solution methods suggested in the literature. We use the generalized Shur form suggested by Klein (2000) when monetary policy is specified using a rule, and the loose commitment algorithm suggested by Debortoli *et al.* (2010) if monetary policy is specified using a loss function.

[11] Except if the model is solved with optimal monetary policy. In this case $M = Q - 1$.

## A.2 Impulse response functions (IRFs)

For break point models the break points are set to precise dates, but since IRFs do not relate to any date, we need to use another way of defining the regimes. In the case where we have added $B$ break points we have $R = B + 1$ regimes. In the case of $B = 1$ we have the first regime is going from the beginning of time until, but not including the given break period, and the second regime is going from the break period to the end of all time. By classifying the regimes in this way we can produce IRFs by condition on the regimes for all periods of the IRFs, call this path of regimes $\rho(t)\ \forall\ t \in \{0, ..., T\}$, where $T$ is the total number of periods of the IRF. $\rho(0)$ is the regime we start out from. An IRF for the structural shock $j$ that hits at period 1 is then given by

$$E\left[\frac{(\partial X_t - X^{ss}_{\rho(t)})}{\partial U_{j,1}}\right] = A_{\rho(t)} E\left[\frac{\partial (X_{t-1} - X^{ss}_{\rho(t)})}{\partial U_{j,1}}\right]\ \forall t \in \{1, ..., T\}, \tag{8}$$

where $E\left[\frac{\partial (X_0 - X^{ss}_{\rho(1)})}{\partial U_{j,1}}\right] = C_{\rho(1)} e_{j,1} + A_{\rho(1)}(X^{ss}_{\rho(0)} - X^{ss}_{\rho(1)})$. $e_{j,1}$ is a vector of zeros except at the element $j$ where it is 1, and $X^{ss}_{\rho(0)}$ is the starting values of the endogenous variables, e.g. the steady state in the regime we start out from. Equation (8) will then give you the IRF of the variables as deviation from the steady state. To get the IRF in levels we need to add the steady state at each period $(X^{ss}_{\rho(t)})$ to $E\left[\frac{(\partial X_t - X^{ss}_{\rho(t)})}{\partial U_{j,0}}\right]$. In the special case that $\rho_t = r\ \forall t \in \{1, ..., T\}$ we get the definition of IRF of a shock in a specific regime $r$.

We can also specify a IRF to a change from one regime $r$ to regime $s$. It is given by

$$E\left[X_t - X^{ss}_{\rho(t)}\right] = A_{\rho(t)} E\left[X_{t-1} - X^{ss}_{\rho(t)}\right]\ \forall t \in \{1, ..., T\}, \tag{9}$$

where $X_0 = X^{ss}_{\rho(0)}$, where $\rho(0) = r$ and $\rho(t) = s\ \forall t > 0$.

## A.3 Simulating the model

To simulate a model with unexpected structural break points one can use the solution found in equation (7) to get

$$Y_{t+h} = X^{ss}_{S(t+h)} + A_{S(t+h)}(Y_{t-1+h} - X^{ss}_{S(t+h)}) + C_{S(t+h)} U_{t+h}. \tag{10}$$

In equation (10) we first remove the steady-state at time $t + h$ from the initial value $Y_{t-1+h}$, before we make a simulation of the deviation from the steady-state by the full second term. Then the contribution from the simulated exogenous shocks may be added, before we re-add the steady-state values at time $t + h$ again. To simulate a series with length $H$, one iterates equation (10) $H$ periods, and $U_{t+h} \sim N(0, I)$.

## A.4 Filtering and smoothing with break points

We can rewrite the model in equation (7) into a state-space representation. The measurement equation can be posted as

$$Y_t = H X_t + v_t. \tag{11}$$

$Y_t$ are the observable variables with size $O \times 1$, $H$ is the observation matrix with size $O \times M$, and $v_t$ are the measurement errors with size $O \times 1$. The measurement error is assumed to be normally distributed with covariance matrix $R$, where $R$ has size $O \times O$,[12] i.e.

$$v_t \sim N(0, R). \tag{12}$$

The state equation linking the current state of the state variables with its own lags and some exogenous disturbances is given by (7). The disturbances ($U_t$) is assumed to be normally distributed with covariance matrix I,[13] i.e.

$$U_t \sim N(0, I). \tag{13}$$

See Hamilton and Press (1994), Section 13.1, for a more thorough description of the state-space representation.

### A.4.1 Kalman filter

The piecewise linear Kalman filter can be used to get estimates of state variables, or the unobservable variables, given the observable variables and the parameter values of the model. This filter assumes that the model is stationary in each regime.

Let us start out with some definitions. $X_{t|t-1} = E_{t-1}[X_t]$ is the expectation of $X_t$ given information on the observed variables up until time $t-1$, while $X_{t|t} = E_t[X_t]$ is the expectation of $X_t$ given information on the observed variables up until time $t$.

First we can use equation (11) to predict $Y_t$ using $X_{t|t-1}$

$$Y_{t|t-1} = HX_{t|t-1}, \tag{14}$$

as $E_{t-1}[v_t] = 0$. To get a measure of the forecast error variance we can use equations (11) and (14) to get

$$\begin{aligned} F_t &= E[(Y_t - Y_{t|t-1})(Y_t - Y_{t|t-1})'] \\ &= E[H(X_t - X_{t|t-1})(X_t - X_{t|t-1})'H'] + E[v_t v_t'] \\ &= HP_{t|t-1}H' + R, \end{aligned} \tag{15}$$

where $P_{t|t-1}$ is the variance in the error when forecasting $X_t$ given information on the observed variables up until time $t-1$. We need $F_t$ as we want to update the projection of $X_{t|t-1}$ given the new information on $Y_t$

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + E[(X_t - X_{t|t-1})(Y_t - Y_{t|t-1})']F_t^{-1}(Y_t - Y_{t|t-1}) \\ &= X_{t|t-1} + P_{t|t-1}H'F_t^{-1}(Y_t - HX_{t|t-1}), \end{aligned} \tag{16}$$

where we in line 2 have used equation (14) and

$$\begin{aligned} E[(X_t - X_{t|t-1})(Y_t - Y_{t|t-1})'] &= E[(X_t - X_{t|t-1})(H(X_t - X_{t|t-1}) + v_t)'] \\ &= P_{t|t-1}H', \end{aligned} \tag{17}$$

The associated variance of $X_{t|t}$ is

[12] It is assumed that the measurement error is uncorrelated across time. In the examples in Section 4, we assume $R = 0$.

[13] It is assumed that the exogenous disturbances is uncorrelated across time.

$$\begin{aligned}P_{t|t} &= E[(X_t - X_{t|t})(X_t - X_{t|t})'] \\ &= P_{t|t-1} - P_{t|t-1}H'F_t^{-1}HP_{t|t-1}.\end{aligned} \tag{18}$$

But we are interested in $X_{t+1|t}$ and $P_{t+1|t}$. These we can find by first noting that

$$X_{t+1|t} = X^{ss}_{S(t+1)} + A_{S(t+1)}(X_{t|t} - X^{ss}_{S(t+1)}). \tag{19}$$

By substituting equation (16) into equation (19) we get

$$X_{t+1|t} = X^{ss}_{S(t+1)} + A_{S(t+1)}(X_{t|t-1} - X^{ss}_{S(t+1)}) + K_t\nu_t, \tag{20}$$

where we have defined $\nu_t = Y_t - HX_{t|t-1}$ and $K_t = A_{S(t+1)}P_{t|t-1}H'F_t^{-1}$. With the associated variance

$$\begin{aligned}P_{t+1|t} &= A_{S(t+1)}P_{t|t}A'_{S(t+1)} + C_{S(t+1)}C_{S(t+1)} \\ &= A_{S(t+1)}(P_{t|t-1} - P_{t|t-1}H'F_t^{-1}HP_{t|t-1})A'_{S(t+1)} + C_{S(t+1)}C'_{S(t+1)} \\ &= A_{S(t+1)}P_{t|t-1}(A'_{S(t+1)} - H'K'_t) + C_{S(t+1)}C'_{S(t+1)}.\end{aligned} \tag{21}$$

The starting values of the filter is set to the unconditional mean of $X_1$ and $P_1$

$$X_{1|0} = E[X_1] = X_1^{ss} \tag{22}$$

$$vec(P_{1|0}) = E[X_1X_1'] = (I_{M^2} - A_1 \otimes A_1)^{-1}vec(C_1C_1'). \tag{23}$$

To evaluate the log likelihood we can at each step of the filtering calculate

$$\ell_t = -log(|F_t|) - \nu_t'F_t^{-1}\nu_t. \tag{24}$$

Which means that the full log likelihood over T periods can be calculated as

$$\mathcal{L} = -\frac{T \cdot O \cdot log(2\pi)}{2} + \frac{\sum_{t=1}^{T}\ell_t}{2}. \tag{25}$$

### A.4.2 Kalman smoother

In contrast to the piecewise linear Kalman filter the piecewise linear Kalman smoother uses all the information in the observable variables to estimate the unobservable variables. The first part of the smoother is to run through the filter. Then by a backward recursion on the following equation you can get the smoothed estimates

$$X_{t-1|T} = X_{t-1|t-1} + J_{t-1}(X_{t|T} - X_{t|t-1}) \text{ for } t = T, \ldots, 2 \tag{26}$$

$$J_{t-1} = P_{t-1|t-1}A_t'P^{-1}_{t|t-1}. \tag{27}$$

See Hamilton and Press (1994), Section 13.6, for a more thorough description of the Kalman smoother and its properties. The algorithm implemented in the code used in this paper follows the smoothing steps of Koopman and Durbin (1998):

$$R_t = A'_{S(t)}R_{t+1} + H'F_t^{-1}\nu_t - H'K_t'R_{t+1}, \tag{28}$$

$$X_{t|T} = X_{t|t-1} + P_{t|t-1}R_t, \tag{29}$$

where $R_{T+1} = 0$. Smoothed estimate of $U_t$ can be found using

$$u_{t|T} = C_{S(t)}^{-1}(X_{t|T} - X_{S(t)}^{ss} - A_{S(t)}(X_{t-1|T} - X_{S(t)}^{ss})). \tag{30}$$

for $t > 1$, while for $t = 1$ we get

$$u_{1|T} = C_{S(t)}^{-1}(X_{1|T} - X_{1|0}). \tag{31}$$

## A.5 Estimating break points

We are interested in estimating the parameters $\theta_i$ $\forall$ $i$ and potentially the timing of the structural breaks, i.e. the $S(t)$ function. When estimating $S(t)$ we assume that the number of structural breaks $s$ is known prior to estimation. Let the timing of the $s$ breaks be collected into the vector $\theta^b$, then we can collect all parameters of the model as $\theta = [\theta_1, \cdots, \theta_s, \theta_b]$. $\theta$ will have size $k = (p+1)s$.[14]

As is normal in this literature, we first formulate a set of marginal independent priors

$$p(\theta|\mathbb{M}) = p(\theta^1) \times \cdots \times p(\theta^k). \tag{32}$$

A Bayesian approach constitute of estimating the parameters $\theta$ by maximization of the posterior distribution given by

$$p(\theta|Y, \mathbb{M}) = \frac{\mathcal{L}(Y|\theta, \mathbb{M}) \times p(\theta|\mathbb{M})}{p(Y|\mathbb{M})} \propto \mathcal{L}(Y|\theta, \mathbb{M}) \times p(\theta|\mathbb{M}). \tag{33}$$

As the prior and posterior distributions of the timing of break points are discrete we cannot use a derivative based maximization methods to estimate the model. Instead we use the artificial bee colony (ABC) algorithm. See for example the survey of Karaboga *et al.* (2012) for an introduction to this method.

[14]Estimation can of course be limited to a subset of $\theta$ if wanted.

# B Non-technical Appendix

## B.1 A short description of NEMO

NEMO consists of households, intermediate goods and final goods producing firms, an oil sector, a government sector and the monetary authority. In addition, there are separate production sectors for housing and non-housing capital goods as well as a banking sector. All agents have rational, or model-consistent, expectations with respect to all prices and quantities, with households' house price expectations being an important exception.

The model is thoroughly documented in Kravik and Mimir (2019). A technical documentation of all derivations, first-order conditions, the full steady-state solution and the stationarization of the model can be found in Kravik *et al.* (n.d.).[15]

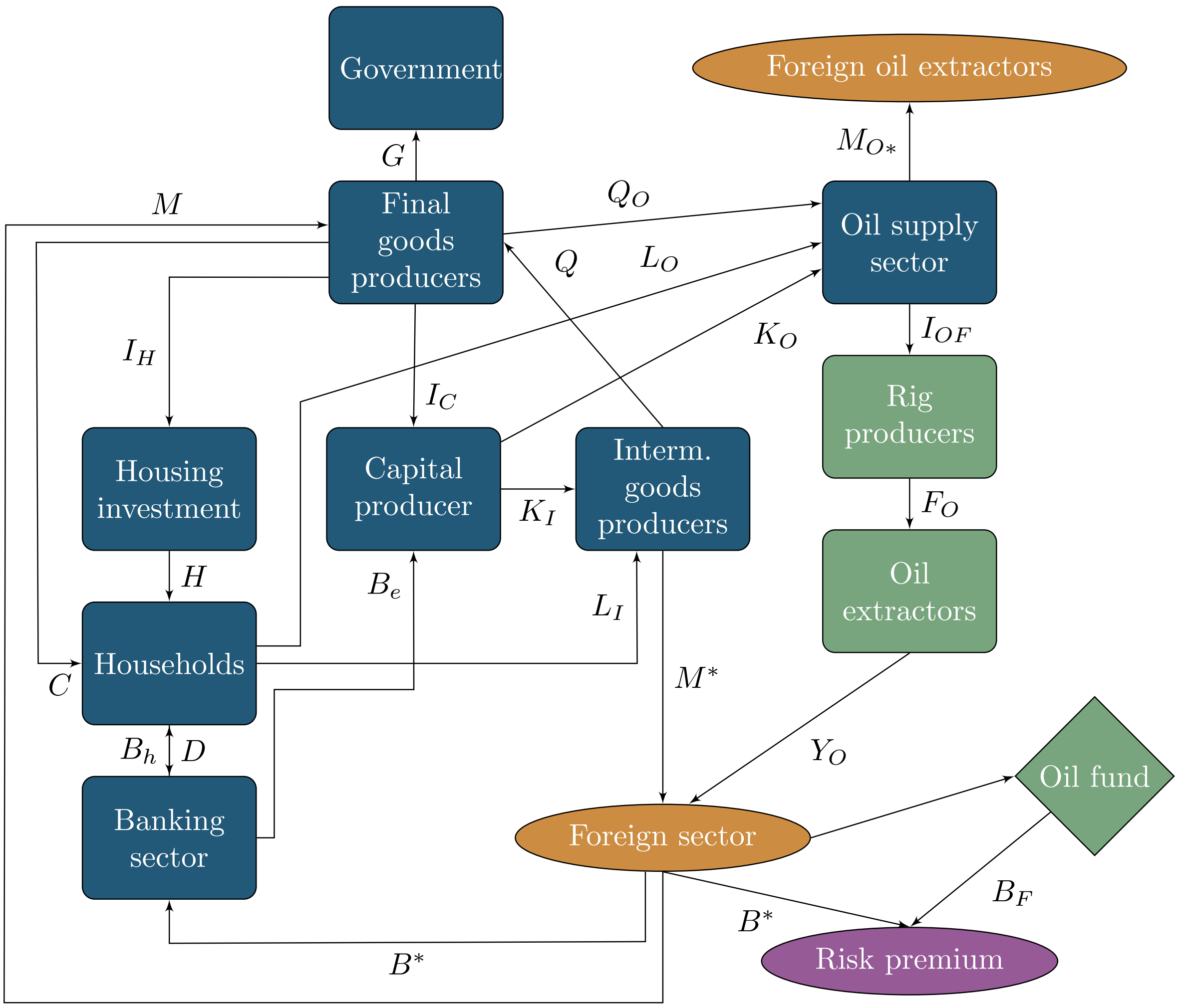


Figure 8: A bird's eye view of NEMO

Figure 8 provides a schematic illustration of the model and displays how the different sectors and agents are linked to each other. The numeraire good of the model, *the final good*, is shown near the top of the figure. This is produced by combining inputs from the

[15] Kravik *et al.* (n.d.) is a live document. See the reference list for a link to the latest version.

domestic firms ($Q$), labeled intermediate goods producers in the figure, and imports ($M$). The final goods are converted into household consumption ($C$), corporate investment ($I_C$), housing investment ($I_H$), government expenditures ($G$) and used as inputs in the oil sector ($Q_O$). The intermediate goods producers employ labor supplied by households ($L_I$), rent capital from entrepreneurs ($K_I$) and sell their goods to the final goods producers ($Q$) and as export ($M^*$). The oil sector uses labor ($L_O$), capital ($K_O$) and final goods ($Q_O$) to produce oil supply goods which are exported ($M_{O*}$) or sold to the domestic rig producers ($I_{OF}$). The rig producers invest in oil rigs ($F_O$) in order to extract oil ($Y_O$) that in turn is exported in full. The revenues are invested in the Government Pension Fund Global (GPFG), named "Oil fund" in Figure 8.

Households consume ($C$), work in the intermediate goods sector ($L_I$) and in the oil sector ($L_O$), buy housing services ($H$), and interact with banks through borrowing ($B_h$) and savings through deposits ($D$).

The banking sector lends to households ($B_h$) and entrepreneurs ($B_e$), and is funded through deposits ($D$), foreign borrowing ($B^*$), and equity ($K^B$). An uncovered interest parity relationship (UIP) together with the country's net foreign debt position (private borrowing, $B^*$, minus government claims on foreigners, $B_F$) tie down the debt-elastic risk premium to ensure stationarity.[16]

## B.2 Grouping of shocks

The shocks in the historical shock decompositions in Section 4 are grouped according to Table 2.

Table 2: Categorization of estimated structural shocks

| Domestic demand | Domestic supply | Foreign |
|---|---|---|
| Consumption preference | Temporary productivity | Foreign marg. costs |
| Housing preference | Firm inv. adj. costs | Global demand |
| Government spending | Housing inv. adj. costs | Export demand pref. |
| Import demand | Price markup | Foreign interest rate |
| | Wage markup | Foreign inflation |
| | | Trading partners' output |
| **Exchange rate** | **Banking sector** | **Oil sector** |
| External risk premium | Money market risk premium | Real oil price |
| | Household LTV ratio | Oil investment |
| **Monetary policy** | Entrepreneur LTV ratio | Oil production abroad |
| Inflation target | Markup of mortgage loan rate | |
| | Markup of business loan rate | |

Note: The exchange rate shock is an external risk premium shock in the UIP condition. The monetary policy shock under the optimal policy setup is implemented as a shock to the inflation target, which is equivalent to unanticipated deviations of the policy rate from the optimal monetary policy prescription.

[16]This is one of the standard ways of solving the unit problem inherent in small open economy models with incomplete markets (see Schmitt-Grohé and Uribe (2003)).

## B.3 Difference in shock decompositions

Figure 9 shows the difference between the shock decompositions presented in Section 4 (Naive Model - True Model).

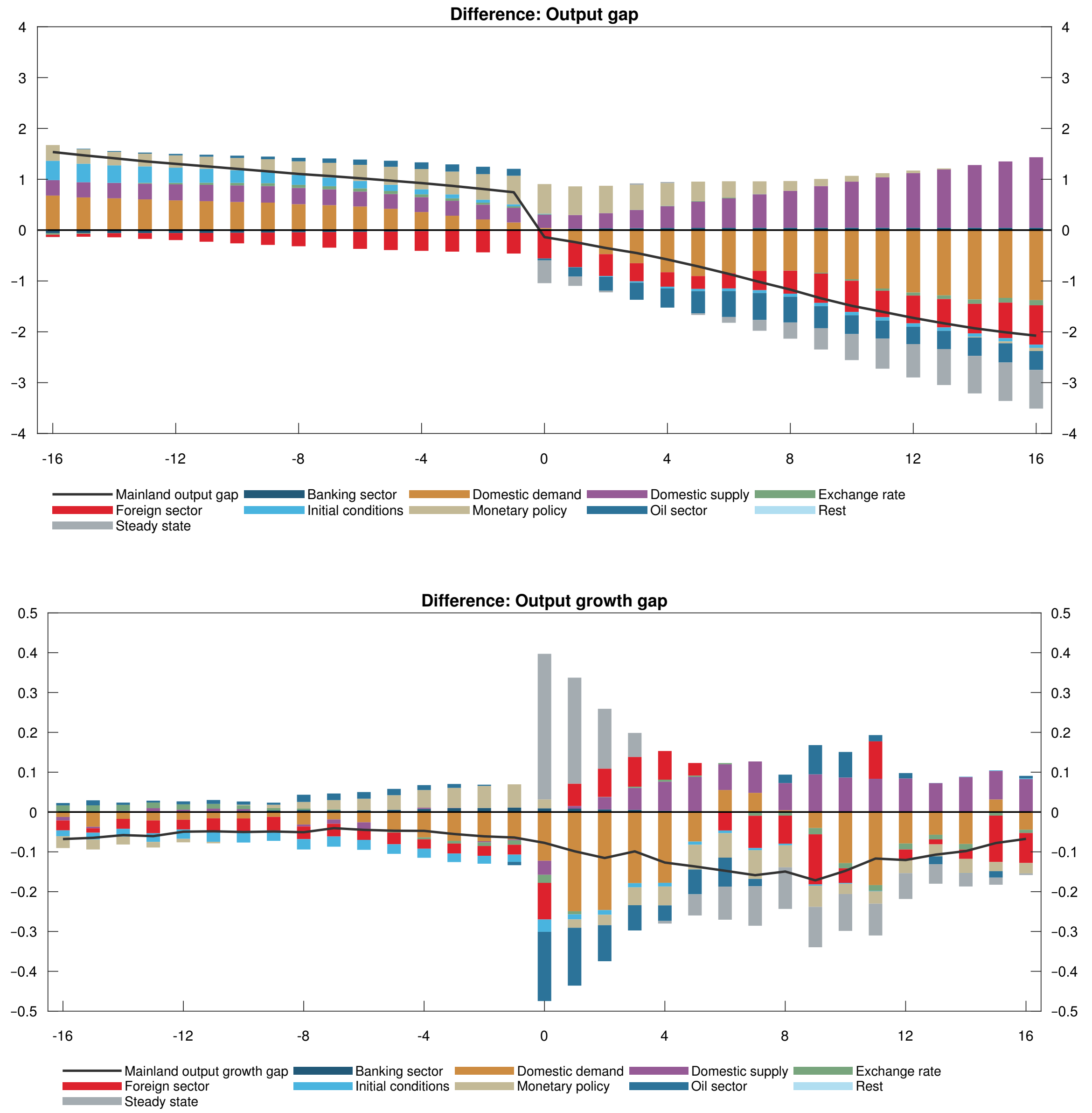


Figure 9: Difference between the Naive Model's and the True Model's historical shock decomposition of the constructed output gap and the output growth gap. Shocks are grouped according to Table 2.