STAFF MEMO

Optimal variable bank capital requirements

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OPTIMAL VARIABLE BANK CAPITAL REQUIREMENTS*

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Abstract

The purpose of the "counter-cyclical capital buffer" (buffer) is to dampen procyclicality in the financial system, absorb capital losses and prevent a credit crunch during recessions. In this paper, a stylized analytical expression for optimal buffer policy is presented. Results are derived within a minimalist model framework, useful for a transparent presentation of how authorities' preferences and structural parameters have implications for optimal buffer policy. In the model, there is a risk of a financial crisis, and an ex ante higher buffer may counteract the effects of it on the economy. The buffer also affects output in the short term, and here the difference between banks' actual capital coverage ratio and capital requirements plays a role. Under quite general conditions, authorities will want to lower the buffer and allow banks to be less well capitalized today if the output gap is negative. The model illustrates that unless authorities care also about a stable bank capital coverage ratio (in addition to output stabilization and the costs of crisis), optimal policy may prescribe a very volatile buffer. In particular, that is the case if the effect of the buffer on output is weak. This result highlights the importance of learning more about the effects of buffer policy in order to achieve good policy design.

Keywords: *CCyB, Optimal policy, Crisis.* **JEL Classification:** *E32, E44, E61, G21, G28.*

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1 INTRODUCTION

In this paper, the optimal implementation of variable capital requirements is discussed. We will for simplicity refer to the "counter-cyclical capital buffer" (CCyB, or buffer), although different authorities may in practice use different measures to implement variable capital requirements. The contribution of this paper is to establish an analytical solution for optimal policy in stripped down and stylized model. The emphasis is on establishing a very simple framework that may aid intuition and provide an overview.

The purpose of the CCyB, expressed by e.g. the Basel Committee, is to dampen procyclicality in the financial system and prevent a credit crunch during recessions. In this paper, we pose a loss function for the policymaker that captures these purposes. As a novelty in the context of small stylized macromodels, authorities in this paper also explicitly care about the bank capital coverage ratio not deviating too much from its optimal value. It is shown that under plausible parameter values, the buffer should ensure that banks are relatively well capitalized when the output gap is positive, but authorities should tolerate less well capitalized banks when the output gap is negative. Optimal policy depends on how strongly authorities weigh the objective of stabilizing output, relative to their potential dislike of bank capital deviating from a given socially optimal value, as well as their emphasis on having a high buffer available when the risk of a deep crisis is large. An expression for this trade-off is the first main result of this paper.

The existence of some optimal steady state capital coverage ratio (and steady state total capital requirements) is taken as given here. It may depend on a trade-off between banks' incentives to take on risk and the associated need for taxpayer protection versus banks' ability to efficiently supply credit. Such optimal steady state capital requirements are not calibrated or modelled here. Instead, this paper is about optimal variations in the capital requirement around a given optimal level.

The macro model in this paper is inspired by microfounded models where financial frictions amplify recessions. In the model, output is affected by how well capitalized banks are, and bank capital itself depends on the activity level in the economy. There is a risk that the economy may enter a crisis, and the crisis will be more costly if vulnerabilities are high. The buffer affects the economy in three direct ways: It makes banks build more capital, it affects the difference between the actual capital level of banks and total capital requirements, and it dampens the costs of a crisis, in case a crisis happens. Given the stylized model, optimal policy is expressed analytically as a simple buffer response to three exogenous variables: An output shock, a bank capital shock, and a reduced form (exogenous) indicator for financial vulnerabilities. The analytical optimal buffer response to disturbances is the second main result of this paper. Emphasis is not on time dynamics, but on the instantaneous trade-off that the policymaker faces.

It is illustrated here that if authorities only care about financial vulnerabilities (e.g. motivated by the downside risk to output that they signal) when determining the buffer, a higher buffer is always better. A finite optimal level exists, however, if authorities for example also are concerned about output stability in the short term (non-crisis times).

Furthermore, it is shown that optimal policy is sensitive to how strongly and in which way the buffer affects output and the actual bank capital coverage ratio. If the effect of the buffer on output is weak, it is necessary to directly impose also a concern for the stability of bank capital coverage in order to avoid very volatile capital requirements.

The rest of this paper proceeds as follows: Section 2 presents some relevant literature, and describes how it relates to this paper. Section 3 defines the stylized bank capital ratio concepts that are used in the model specification. Section 4 describes the main optimization problem of authorities and the implied target criterion for optimal buffer policy. Section 5 presents a small model and discusses optimal buffer policy given the economic mechanisms present in that model. Section 6 provides concluding remarks.

2 Related literature

2.1 The measurement of financial vulnerabilities and the challenges of establishing a "CCyB-rule"

The early literature on variable capital requirements was empirically focused and sought to establish a decision framework for the CCyB based on early warning indicators for financial crises, see e.g. Drehmann, Borio, and Tsatsaronis (2011) and Gerdrup, Kvinlog, and Schaanning (2013). Early warning indicators are often referred to as financial vulnerabilities, and these concepts are also typically closely related also to the financial cycle. These three concepts will be used interchangeably here. A natural point of reference for this literature is the Basel "buffer guide", where the credit-to-GDP gap is used as a reduced form indicator of financial vulnerabilities. The buffer guide constitutes a simple (univariate) "buffer rule" for the build-up phase of capital requirments.[1]

Wezel (2019) provides an overview of the literature on challenges with the measurement of financial vulnerabilities, including with the filtering of the credit-to-GDP gap. The author establishes a concept of "necessary buffer" related to excess loan losses to be expected based on excess financial vulnerabilities (credit gaps) during boom times. He thus introduces a way of measuring the extent to which the buffer may be either higher or lower than necessary to cover those expected excess losses, given the credit gap. In this way, an optimal level for the buffer that depends on preferences may be established. A related approach is presented in Brave and Lopez (2017). In their work, financial stability indicators are transferred into an estimate of the probability of crisis in a Markov-switching framework. The focus is on the optimal timing of activating the buffer, given that a high buffer is costly for banks in normal times, while the calibration is simple (the buffer is on or off).

Estimates of "growth at risk" (see Adrian, Boyarchenko, and Giannone (2019)) enable policymakers to assess possible output shortfalls as a function of financial vulnerabilities. Chavleishvili, Fahr, Kremer, Manganelli, and Schwaab (2021) build on the growth at risk

[1] See BIS (2010) and https://www.bis.org/bcbs/ccyb/. A description of the buffer guide seen from the perspective of the model in this paper is provided in Appendix A on page 19

approach and formulate a loss function that trades off the growth shortfalls during crises against mean output loss during expansions. The authors suggest that their framework may be useful as a communication device for macroprudential policy, while they also note that estimating the effects of macroprudential instruments on the financial cycle is a remaining issue.

2.2 A concern for sufficient capacity to use the buffer in a crises

Given the explicit goal of using the buffer to prevent a credit crunch during recessions, some practitioners have opted for using stress-testing to aid the calibration of buffer policy, see Andersen, Gerdrup, Johansen, and Krogh (2019) and Bennani, Couaillier, Devulder, Gabrieli, Idier, Lopez, Piquard, and Scalone (2017). We may think of this as using the buffer to dampen the implications of financial vulnerabilities, as opposed to (or as a complement to) a potential goal of dampening the vulnerabilities themselves directly.

The assessment of what a large enough policy space (available buffer) is, will depend on the structure of the economy and the nature of shocks. It will for example depend on the extent to which banks actually may be expected to utilize an opportunity to increase lending if the capital requirement is lowered, and it will depend on the existence of credit demand that remains for banks to satisfy in a crisis (that is, the actual effect of using the buffer). This approach therefore requires a structural model for the economy, as opposed to the more empirical approach described in the previous subsection. It also requires a goal for what the buffer is to achieve during crisis times - for example a goal for acceptable credit growth or output growth during crisis. With such a goal for the buffer, a well defined optimal buffer level can be established with the stess testing approach.

The present paper is inspired by the practical macroprudential stress-test approach. The expected loss in case of crisis is here expected to be higher with higher vulnerabilities, and then a larger buffer is always helpful. Other concerns (in non-crisis times) are added in order to establish a finite optimal buffer level in this paper.

Authorities in many countries have a policy of keeping a positive buffer during normal times (see Arbatli-Saxegaard and Muneer (2020)). The specification of the possible policy space differs in the literature. For example, Brave and Lopez (2017) assume that the buffer is either on or off. In this paper, where the emphasis is not on the timing of action, but instead on the calibration of action and tradeoffs between various concerns, it is convenient to assume a continuous possible range for the buffer. Therefore, in this paper we assume that the steady state buffer level is positive, and a buffer change can be positive or negative and take on any value within a range around the steady state.

2.3 The effects on the economy of changing buffer capital requirements

There are at least three lines of work in the literature on the effects of variable capital requirements on the economy: (i) Papers with structural DSGE models and theoretical mechanisms, estimated or calibrated, (ii) reduced form empirical studies, as well as (iii) regulatory studies and institutional studies of the interplay between various types of bank regulations.

Regarding (i) and microfounded structural models: Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015) find that for time-varying capital requirements to be effective at stabilizing output, the steady state capital requirement level needs to be quite high. Otherwise, banks may become vulnerable when requirements are lowered, and lowering capital requirements in a crisis may then be counterproductive. Such nonlinear features are not captured in the simple model of this paper. One should think of the model of this paper instead as a linearization around a steady state where reducing the buffer is not counterproductive. Schroth (2021) suggest that time-varying capital requirements should be kept low for a while after a crisis. The intuition is that the expectation of dividend payouts (that are made possible if buffer requirements are low) after a crisis may help secure banks' access to funding during the crisis, if the policy is known ahead of the crisis.

Regarding (ii), empirical research on the effects of lowering the CCyB is limited due to short times series. In BIS (2021), an assessment of the effects of the release of buffer requirements across jurisdictions in the spring of 2020 is presented. BIS concludes that there are indications that CCyB releases had positive effects on loan growth, but results are subject to caveats - such as several policy measures being implemented at the same time. Using a sample of 14 countries, Avezum, Oliveira, and Serra (2021) find evidence that macroprudential buffer releases in the spring of 2020 contributed, on average, to mitigate the procyclicality of credit to households. Compared to countries that did not release buffers, credit growth to households was 0.99 percentage point higher in countries where there was a buffer release. However, they find that the effect on consumption was muted. The empirical effects of reduced capital requirements on lending by Norwegian banks is studied in Arbatli-Saxegaard and Juelsrud (2020). They look at the lending response of a subset of Norwegian banks during their transition from Basel I to Basel II in 2008. They find that lower capital requirements lead to higher bank lending, consistent with the aims of the CCyB. Furthermore, they find that the increased lending has significant and positive real economic effects. Regarding the effects of higher capital requirements, there is a broader range of studies, see e.g. the literature review in BIS (2019), and also Roulet (2018).

Regarding (iii) and regulations, Danmarks Nationalbank (2020) explains how interaction between different types of capital requirements and liquidity regulations may influence the effectiveness of the CCyB. Andersen, Haugen, Johnsen, Turtveit, and Vale (2021) discuss how capital requirements interact in particular under stress.

Parameters in the model of the present paper capture the extent to which a change in the buffer actually has effects on the economy, and optimal policy is (not surprisingly) very sensitive to these parameters. It is thus of high importance to extend our knowledge of how the buffer affects banks and the economy, given other regulations, in order to arrive at optimal buffer policy.

2.4 Optimal buffer policy in structural models

Aikman, Giese, Kapadia, and McLeay (2019) is close in spirit to the present paper, in the sense that a stylized semi-structural framework is used in order to derive optimal buffer policy. The authors investigate the interaction between monetary policy and buffer policy. In this paper, instead, the focus is on the buffer only, and the capital coverage ratio of banks is an endogenous variable that authorities may care about. With an even more stripped down setup, the focus in this paper is on analytical expressions for optimal buffer policy only, and where the capital coverage ratio of banks is one of the endogenous variables. Similar to the present paper, Bennani et al. (2017) specify an ad-hoc loss function for the policymaker, but they go on to analyze an optimal simple rule for the buffer in a relatively large applied model (instead of applying optimal control, which is done in this paper). The same is the case in the estimated DSGE-model in Mendicino, Nikolov, Suarez, and Supera (2018).

Also Kockerolls, Kravik, and Mimir (2021) study the use of capital requirements in a large-scale empirically relevant DSGE-model. They find that a buffer requirement is more useful than monetary policy for the purpose of "leaning against the wind" in order to reduce the welfare loss associated with financial crises. However, they find that variation in the buffer requirement does not contribute much to lowering the crisis loss. It is the long-term capital requirement level that is of importance according to their results.

3 Capital concepts

The definitions of capital concepts used in later sections are established in this section. One may conceptually imagine a normal (steady state) level of the buffer ($\overline{c}^b$). The steady state level for the buffer plus other capital requirements can then add up to some optimal level for steady state aggregate capital requirements c^*, as illustrated by the upper horizontal line in Figure 1. For simplicity, we assume that banks' steady state actual capital coverage ratio is equal to the steady state total capital requirements, c^*. One could alternatively assume that banks adjusted with a certain add-on buffer above total requirements in the steady state, but results regarding the optimal variable buffer policy in this paper would not be affected. Total capital requirements (including the time-varying buffer) is equal to c_t^{b*}. Hence the gap between the buffer and its steady state level, the "buffer gap" is:

$$c_t^b \equiv c_t^{b*} - c^*. \tag{1}$$

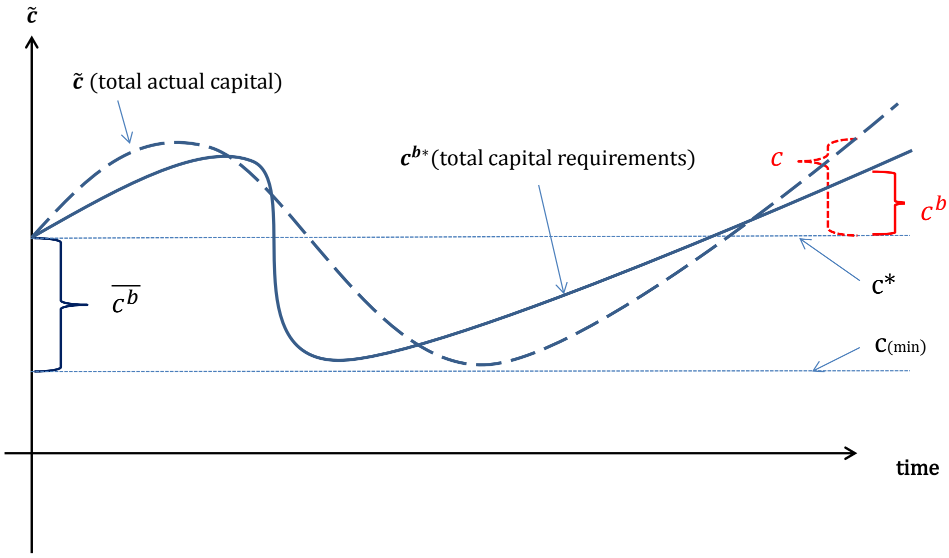
The policy decision in this paper, will be do determine the response of the buffer gap c_t^b (also referred to as the buffer) to shocks. The absolute level of the buffer, given a certain buffer policy, is then:

$$b_t \equiv \overline{c}^b + c_t^b. \tag{2}$$

This setup means that there is already by assumption "room for manouvre" for the buffer, and no "zero lower bound" constraint for policy will bind in this paper (c_t^b can be both positive and negative).[2]

[2]The joint distribution of shocks, and preferences, will in this paper determine whether a high steady state normal level for the buffer is required in order for policy not to be constrained by the zero lower bound that

Figure 1: *Capital concepts*



The gap between the *actual* capital level $\tilde{c}_t$ and the optimal steady state capital level (the capital gap) is defined as:

$$c_t \equiv \tilde{c}_t - c^*. \tag{3}$$

Again, see example illustrated in Figure 1. An optimal steady state (long term) level for capital (and capital requirements) could in principle be derived from a trade off between e.g. banks' incentives to take on too much risk due to limited liability on the one hand, and the effectiveness of credit supply in the economy, see e.g. Clerc et al. (2015) and Cline (2016). This trade off may be regarded as structural and a concern for capital requirements that are time-invariant. The trade-off will for example involve a consideration of how frequently one would be willing to accept that a banking crisis could occur. It is arguably not a trade off that one would reconsider very frequently, and we view that trade off as distinct from the purpose of the variable buffer.

4 A criterion for optimal capital buffer policy

4.1 A loss function relevant for capital buffer policy

Authorities seek to minimize expected loss across normal times and crisis times. In order to facilitate analytical results, we assume that each period ex ante is equal - the expectation is that normal times will prevail, or a crisis will occur. Authorities can observe the current output gap as well as the current capital coverage ratio in banks, and they expect that if

the buffer is subject to. A high steady state level for the buffer in turn requires other capital requirements to be somewhat lower, given that there is some optimal steady state capital level and steady state capital requirements.

normal times prevail the gaps will close by the next period. But policy today can affect gaps today and thus increase welfare. Furthermore, they observe financial vulnerabilities (growth at risk). Thereby, authorities have a view on how deep a crisis may turn out to be (if it occurs) and the associated welfare loss. The only uncertainty at the beginning of period t is wether a crisis will happen or not, and this setup means that it is enough to minimize the following simple loss function:

$$\min_{c_t^b} \left[(1-p) \cdot \frac{1}{2}\{c_t^2 + \lambda \cdot y_t^2\} + p \cdot \lambda_i \cdot L_t \right], \tag{4}$$

The first part of the loss function captures the loss in normal times, which is relevant with a probability $(1-p)$. A concern by authorities for the optimal capitalization of banks is captured by the first quadratic term in the loss function. There is some socially optimal capitalization level c^* that authorities try to target, as discussed in section 3 (recall that the capital gap is $c_t \equiv \tilde{c}_t - c^*$). Furthermore, authorities are concerned about the (quadratic) output gap today, with a weight λ.

At the beginning of each period, there is a certain probability p of entering a crisis.[3] The last term in the loss function captures the resulting loss should a crisis happen before the end of the period. The extent to which policymakers are concerned about this loss is captured by the parameter λ_i. The loss L_t is endogenous, and may for example depend on c^b and financial vulnerabilities (which is what we will assume later).[4] We assume that authorities observe exogenous shocks and financial vulnerabilities in period t and determine the level of the buffer based on this knowledge, but before they know wether a crisis will happen or not.

4.2 A TARGET CRITERION (FIRST ORDER CONDITION) FOR THE OPTIMAL BUFFER

We here consider the target criterion (first order condition) for optimal policy, which can be expressed based on the loss function. We can disregard time subscripts in the following. Differentiating the loss function with respect to c^b, assuming that c, y and L are all endogenous functions of c^b, gives

$$(1-p)(c \cdot \partial c/\partial c^b + \lambda \cdot y \cdot \partial y/\partial c^b) + p \cdot \lambda_i \cdot \partial L/\partial c^b = 0. \tag{6}$$

[3] The probability of crisis is assumed to be unaffected by policy to a first order. The steady state level of capital is assumed to be at its optimal level at the beginning of the period. A perturbation of policy around optimal policy should only have a second order effect on the probability of crisis. This may justify the present specification.

[4] One could also interpret L_t as the extra loss that affects the economy if a crisis occurs, in addition to the "normal times" loss that always is present (in the spirit of the approach in Svensson (2017)). This reformulation would not affect the results in this paper, but it would mean that the period loss function should be written as

$$\min_{c_t^b} \left[\frac{1}{2}\{c_t^2 + \lambda \cdot y_t^2\} + p \cdot \lambda_i \cdot L_t \right], \tag{5}$$

and in the algebra in this paper, $\frac{p}{(1-p)}$ would then be replaced by p. The latter is immediately clear when we consider the fact that the solution to the optimization problem in (4) is unchanged if we divide through by $(1-p)$ since p is a fixed parameter.

According to this equation, when considering changing the buffer, the policymaker faces a trade-off between the capital gap c and the output gap y. The concern about the crisis loss L will affect the trade off as a wedge represented by the term $p \cdot \lambda_i \cdot \partial L/\partial c^b$. The expression is quite general and does not depend on any particular macroeconomic model.[5]

We may as a benchmark assume that the economy behaves in such a way that output decreases when the buffer is increased ($\partial y/\partial c^b \equiv y' < 0$) while capital increases when the buffer is increased ($\partial c/\partial c^b \equiv c' > 0$) and the loss if a crisis happens will be smaller with a higher buffer available ($\partial L/\partial c^b \equiv L' < 0$. Rearranging equation (6), and using the simplified notation for derivatives just introduced here, gives:

$$c \cdot c' = \lambda \cdot y \cdot (-y)' + p/(1-p) \cdot \lambda_i \cdot (-L)'. \tag{7}$$

With the assumed signs for the derivatives, the terms $(-y)'$, c' and $(-L)'$, are all positive. If they are constant or exogneous as well (as they will be in our simple model), the capital gap will under optimal policy be proportional to the output gap plus a term that increases in crisis probability:

$$c \propto [y + p/(1-p)]. \tag{8}$$

Equation (7) and the simplified version (8) is the first main result of this paper. It tells us that a condition for optimal buffer policy under the benchmark assumptions regarding the behavior of the economy, is that a positive output gap y should be associated with a relatively high capital coverage ratio c in banks. The capital coverage ratio should be higher still if the probability of a costly crisis is high (since $p/(1-p)$ is increasing in p in the relevant range). In recessionary times, or in recovery after a crisis (typically a situation with negative y) or when the risk of a crisis is low (low p), authorities will if necessary want to live with relatively poorly capitalized banks (negative c), rather than increasing the buffer too much: A higher buffer could improve the capital coverage ratio (make c approach zero from below), but might have a too negative effect on the economy (lower y further). A microfounded liquidity-oriented reason for the acceptance of relatively low capitalization of banks during recoveries after recessions is provided in Schroth (2021): The argument is that dividend payouts during the recovery may ease banks' access to funding during crises.

5 Optimal buffer policy in a stylized model with financial frictions

In this section, a simple model economy is presented. Optimal policy, which implements the target criterion described in the previous section then also necessarily takes a very simple analytical form.

5.1 A stylized model

Consider:

$$y_t = \rho \cdot (c_t - \theta \cdot c_t^b) + \varepsilon_t^y, \tag{9}$$

[5] The main limitation is that the model economy must define periods such that they are all equal ex ante. Dynamics play not role here.

$$c_t = \tau \cdot c_t^b + \gamma \cdot y_t + \varepsilon_t^c. \quad (10)$$

$$L_t = i_t \cdot (1 - l \cdot c_t^b), \quad (11)$$

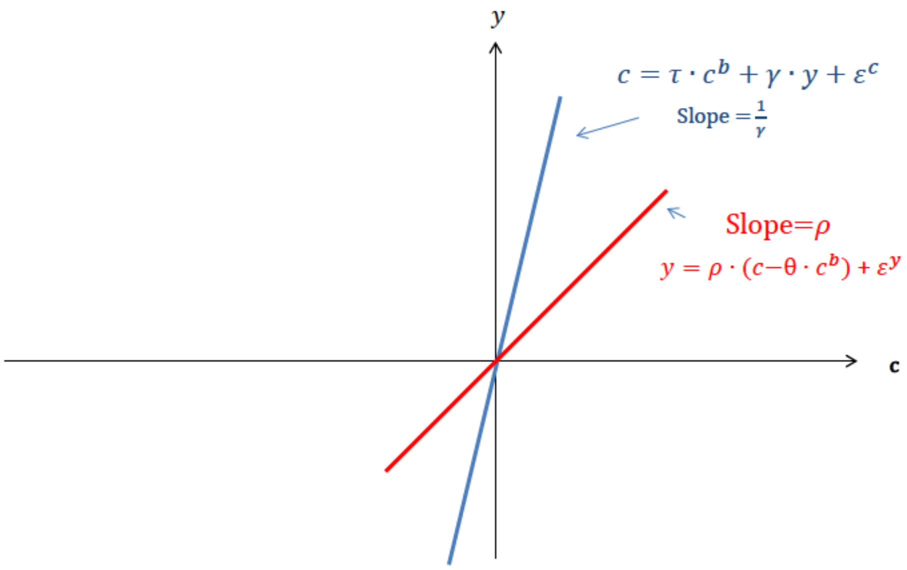
Equation (9) describes a reduced form relationship between output y_t and financial frictions. It may be natural to assume that $\rho > 0$: Financial frictions (e.g. risk premia) fall as bank capital (the banks' equity ratio) c increases, and therefore y is expected to be increasing in c.[6] A high ρ will mean that capital shocks have strong effects on output, and can be interpreted as capturing strong procyclicality in the financial system. The difference between actual capital and capital requirements, rather than the capital level only, may matter for output. If $\theta > 0$, this is the case, and the buffer then has a direct effect on output - not only via the capital accumulation of banks. A θ equal to zero will instead mean that a lower buffer has zero direct effects on output, the effect then only goes via capital accumulation in banks.

Equation (10) describes a relationship between the bank equity ratio c, time-varying capital requirements c^b and economic activity y. The parameter γ captures the response in banks' capitalization to output. One reason that banks' capitalization may depend on output, is the observation that banks often prefer to smooth their dividend payments. When the economy is strong, they will then accumulate capital, and when the economy is weaker, they will prefer to let their equity ratio fall. The desired equity ratio may for the same reason be low for a while after unusually large losses. A high γ, like a high ρ, will serve to exacerbate the effect of shocks and it thus represents procyclicality or vulnerability in the financial system. The equation also expresses that banks are inclined to increase their capital ratio when the buffer requirement increases, but given that they typically also hold voluntary buffers, the increase may be assumed to be less than one-for-one. Thus it is reasonable that $0 < \tau < 1$. If the increase in the capital requirement is expected to be persistent, banks may respond more strongly. But if different types of bank regulations interact, making the buffer requirement less binding, τ may be zero and changing the buffer may have small effects on banks and the economy.

In equation (11), the crisis loss in period t, L_t, is described as a function of (reduced form) financial vulnerabilities i_t and the buffer c_t^b. The variable i_t may represent a summary measure of financial vulnerabilities, or also represent "growth at risk". Financial vulnerabilities i_t are in the present model assumed to develop exogenously from e.g the output gap and the capital coverage ratio in period t. They are to be considered predetermined, in what we may call the short term, and they come in addition to the structural financial vulnerability (procyclicality) otherwise captured by the parameters ρ and γ. According to the model, the impact of reduced form vulnerabilities can be counteracted by an appropriate buffer level. The buffer is more effective at alleviating the crisis loss when l is positive and high. It will not be optimal for authorities to respond to reduced form financial vulnerabilities unless l is positive. For example, if credit supply is not expected

[6] Woodford (2010), building on Curdia and Woodford (2010), presents a New-Keynesian model where short-term equilibrium output decreases as financial frictions (risk premia which depend on banks' leverage) increase. Output in equation (9) may thus be regarded as capturing an equilibrium from a simple New-Keynesian model setup in a reduced form, where output here depends on the level of frictions only. Behind each combination of output and capital traced out by equation (9), there is then one particular solution for inflation and the interest rate, for a given monetary policy specification.

Figure 2: *Equilibrium in the stylized model*



to be a constraint in a crisis, one should assume $l = 0$.

The solutions for the capital coverage ratio, the output gap and the crisis loss L in terms of exogenous variables and the policy variable c^b are given by

$$c = \frac{(\tau - \gamma\rho\theta)}{(1-\gamma\rho)} \cdot c^b + \frac{\gamma}{(1-\gamma\rho)}\varepsilon^y + \frac{1}{(1-\gamma\rho)}\varepsilon^c, \tag{12}$$

$$y = \rho(\frac{\tau - \theta}{(1-\gamma\rho)}) \cdot c^b + \frac{1}{(1-\gamma\rho)}\varepsilon^y + \frac{\rho}{(1-\gamma\rho)}\varepsilon^c, \tag{13}$$

and

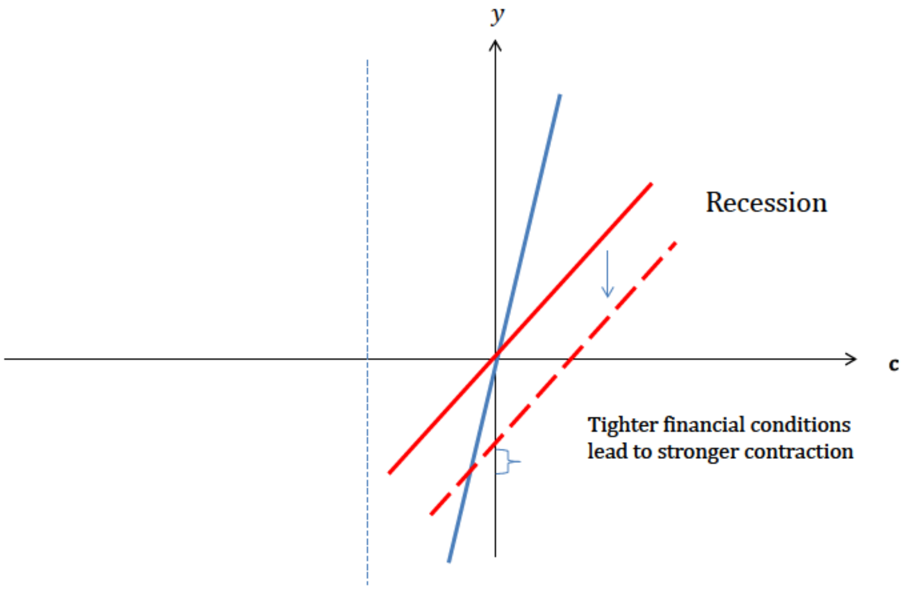
$$L = i \cdot (1 - l \cdot c^b), \tag{14}$$

where the extra loss L, should a crisis happen, is recursively determined since it only depends on the buffer and exogenous financial vulnerabilities. Again, with $\tau, \theta = 0$ there will be no effect of the buffer on the economy, and with $\gamma, \rho = 0$ there will be no structural procyclicality via the financial system.

It is reasonable to assume that parameters are such that a positive capital shock (ε^c) increases the capital coverage ratio. This means that $(1-\gamma\rho) > 1$ and $\frac{1}{\gamma} > \rho$. Graphically, the equilibrium solutions for c and y are illustrated in figure 2, where the steepness of the lines satisfy the parameter constraints just described. Furthermore, we assume that $\tau - \theta > 0$ and $\tau - \gamma\rho\theta > 0$. Finally, we assume that the buffer will serve to dampen the crisis loss, and thus we assume $l > 0$. The blue line in Figure 2 illustrates that banks' desired capital level depends on output developments. A capital-loss (shift left in the blue line, negative ε^c) is then self-reinforcing via second-round negative output effects. Both ρ and γ contribute to financial vulnerability by exacerbating shocks via the financial system. Any reduction in the buffer would also shift the blue line left if $\tau > 0$.

Furthermore, a negative output shock (negative ε^y) shifts the red curve down. The fall is reinforced via capital loss and increased financial frictions. This situation is illustrated in Figure 3, where the red line shifts down (negative output shock) and the eventual output loss is larger than the initial shock. Any possible reduction in the buffer would shift the red curve up, if and only if $\theta > 0$.

Figure 3: *A recession (negative ε^y-shock)*



The role of exogenous shocks and parameters are summarized in Table 1 below.

Table 1: *Parameters and systemic risk*

		Parameters
	Determinants of the effectiveness of the buffer*)	θ, τ, l
+	Determinants of vulnerabilities/procyclicality:	ρ, γ, p, i
+	Policymaker's preferences	λ, λ_i
+	Policy response c^b	
=	Systemic risk**)	

*) The parameters τ and θ capture the way that the buffer affects the economy. l captures the degree to which the buffer can counteract the effect of high (reduced form) vulnerabilities in a crisis. It determines size of the benefit (in terms of reduced loss L) of having a high buffer should a crisis happen.
**) The assessment of systemic risk will depend on policy preferences. The optimal policy response and thus the best contribution to lowering systemic risk that buffer policy can make, is derived in the next subsection and depends on loss function parameters.

5.2 Optimal buffer policy given the stylized model

Plugging the expressions for output and capital in terms of the capital buffer and structural shocks, as well as their derivatives, and the derivative of the loss L, into the first order condition (target criterion) for optimal policy given by equation (7), we can now derive the optimal buffer policy given this particular model. For that purpose, it is useful to note that the derivatives referred to in equation (7) - the sensitivity of endogenous variables to policy - are given by (where the benchmark sign assumptions are indicated as well):

$$c' = \frac{(\tau - \gamma\rho\theta)}{(1-\gamma\rho)} > 0,$$

$$y' = \rho(\frac{\tau - \theta}{1-\gamma\rho}) < 0,$$

and

$$L' = -l \cdot i < 0.$$

Also, it is helpful to define

$$\beta \equiv \frac{1}{(1-\gamma\rho)} > 0.$$

Now, inserting the derivatives, and expressions (12) and (13) into the target criterion (7), and rearranging, we arrive at

$$c^b = \frac{p/(1-p) \cdot \lambda_i \cdot l}{(c')^2 + (y')^2 \cdot \lambda} \cdot i - \beta \cdot \{\frac{\gamma \cdot c' + \lambda \cdot y'}{(c')^2 + (y')^2 \cdot \lambda}\} \cdot \varepsilon^y - \beta \cdot \{\frac{c' + \lambda \cdot \rho \cdot y'}{(c')^2 + (y')^2 \cdot \lambda}\} \cdot \varepsilon^c. \quad (15)$$

This equation is the second main result of this paper. There are three terms, describing buffer responses to the exogenous disturbances i, ε^y and ε^c, respectively. The coefficients in front of each exogenous variable are further discussed below.

5.2.1 THE RESPONSE TO AN OUTPUT SHOCK

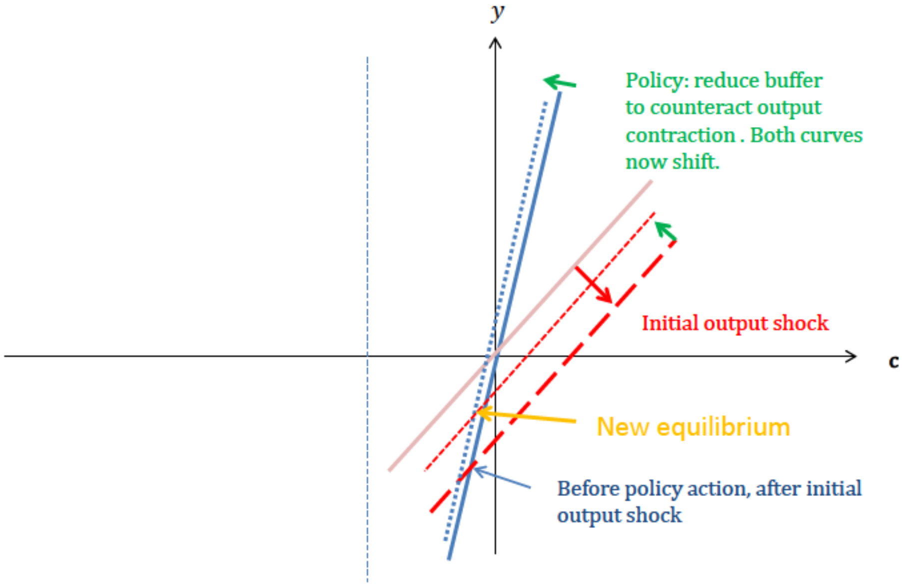
Here we look more closely at the optimal response in the buffer to an output shock, the second term on the right hand side of equation (15) above.

$$c^b = -\beta \cdot \{\frac{\gamma \cdot c' + \lambda \cdot y'}{(c')^2 + (y')^2 \cdot \lambda}\} \cdot \varepsilon^y$$

The denominator is positive and $-\beta$ is negative. Thus, a negative output shock will be met with a lower buffer if the numerator of the fraction is negative. The first term in the numerator is a multiple of the marginal effect of the buffer on capital, which by assumption is positive, and the second term is the marginal effect of the buffer on output, which by assumption is negative, weighted by λ.

The sign of the response of the buffer to output shocks thus depends on the relative strength of the effect of the buffer on endogenous variables: A strong effect on output counts in favor of a lowering of the buffer. The concern for negative output effects will in addition be stronger when λ is higher. If that concern dominates, the numerator bracket is negative and a negative output shock will be met with a lower buffer, as illustrated in Figure (4). The figure illustrates an initial negative output shock that is exacerbated by financial frictions. To counteract the shock, authorities lower the buffer if their λ is high enough, and the lowering of the buffer shifts output somewhat up again (red curve). The

Figure 4: *Using the buffer to counteract negative output shock*



change in the buffer also shifts the capital equation left (blue dotted curve) and weakens the capital coverage ratio. How much authorities choose to use the buffer depends on preferences and structural parameters. One might imagine that authorities wanted to completely stabilize output by lowering the buffer more, but that would at the same time further weaken the capital coverage ratio of banks.

From equation (15) we know that the response to financial vulnerabilities comes in addition to any response to output shocks, and may thus either exacerbate or counteract the response to output shocks. If financial vulnerabilities i are measured to be relatively high at the same time as a negative output shock arrives, authorities may according to equation (15) choose not to lower the buffer in this situation. A strong concern for, or likelihood of, a possible crisis will pull stronger in the same direction.

5.2.2 The response to a bank capital loss

The response to capital shocks is given by

$$c^b = -\beta \cdot \{\frac{c' + \lambda \cdot \rho \cdot y'}{(c')^2 + (y')^2 \cdot \lambda}\} \cdot \varepsilon^c$$

This expression is a differently weighted sum of the marginal effects of the buffer on capital and output. Now the second term in the numerator is weighted by ρ. Intuitively, authorities would like to reestablish the optimal capital coverage ratio if it is not too costly in terms of output. If λ (or ρ, the real effect of the buffer on output) are not very high, the first (positive) term in the numerator will dominate, and the response to a negative capital shock will be to raise the buffer. If financial vulnerabilities at the same time are relatively high, authorities will be even more encouraged to increase the buffer, in accordance with equation (15).

5.2.3 THE RESPONSE TO FINANCIAL VULNERABILITIES i

The optimal response to financial vulnerabilities is

$$c^b = \frac{p/(1-p) \cdot \lambda_i \cdot l}{(c')^2 + (y')^2 \cdot \lambda} \cdot i$$

The response to i increases with the probability of a crisis, p (since $p/(1-p)$ increases in p for the relevant range of p). If the concern for financial vulnerabilities (given by λ_i) is high, or the usefulness of the buffer in a crisis is high (l is high), the response will likewise be strong. Hence, authorities may want to increase the buffer in response to higher financial vulnerabilities in order to smooth out losses associated with suboptimal capital levels (and low output), even though they may technically have enough room for manouvre without such an ex ante increase in the buffer.

Also, policymakers might behave according to $\lambda_i = 0$ if $i \leq 0$. This would correspond to a non-linear response to financial vulnerabilities: Increase the buffer when vulnerabilities are higher than usual, but leave the buffer at its normal level when they are lower than usual, everything else equal. Such a specification could capture a preference for a precautionary use of the buffer.

5.2.4 THE SPECIAL CASE OF UNLIMITED (OR SINGULAR) CONCERN FOR LOSS IN CASE OF A CRISIS

We have already noted that a singular concern for loss in crisis, and a linear crisis loss, will not deliver a bounded solution for an optimal buffer. It will always be optimal, in normal times, to increase the buffer. Technically, this can be shown by investigating the case of $\lambda_i \to \infty$. Dividing both the numerator and the denominator of the first term on the right hand side of equation (15) by λ_i, and imposing $\lambda_i \to \infty$ gives the following optimal response to financial vulnerabilities i (while the response to the other terms are unchanged and not repeated here):[7]

$$c^b = \frac{p/(1-p) \cdot l}{(c')^2/\lambda_i + (y')^2 \cdot \lambda/\lambda_i} \cdot i. \tag{16}$$

But in this case the denominator approaches zero and the response to financial vulnerabilities measured by i explodes and dominates any response to output or capital shocks. Intuitively, there is no limit to how high buffer authorities would like to have if they observe financial vulnerabilities: The loss falls without limit when the buffer is increased due to the high weight on crisis loss (while costs related to a high buffer in normal times in terms of output and capital disturbances are bounded and disappear in relative terms). Authorities may in practice impose an ad hoc ceiling on the buffer in order to arrive at a bounded buffer level, should they have preferences as described here.

[7]This point can also be made looking at equation (7) on page 9. We may note that without any weight on crisis loss L ($\lambda_i = 0$), or with both λ_i and $\lambda = 0$, the solution for a finite optimal buffer level would still be well defined. One could also leave the capital coverage ratio alone in the loss function. The optimality condition would then reduce to either close the capital coverage gap or the output gap (one goal and one instrument), or it would be a trade-off between closing those two gaps. If only L remains in the loss function, however, there may be a problem. Then the optimality condition requires $L' = 0$. If crisis loss is monotonously lower when the buffer is increased, this condition is never satisfied, and there is no bounded solution for the optimal buffer.

5.2.5 THE SPECIAL CASE OF CONCERN BOTH FOR OUTPUT IN NORMAL TIMES AND LOSS IN CASE OF A CRISIS, BUT NOT FOR BANK CAPITAL c

The problem discussed in the previous subsection can be solved by extending the loss function to other concern, as the solution for optimal policy in equation (15) shows. One possible choice, would be for authorities to only worry about output losses in normal times in addition to the crisis loss (and not worry about the stability of the capital coverage ratio of banks). But we will see here, that optimal buffer policy then may become very volatile. Technically, in order to investigate this case, we may consider the case where $\lambda_i, \lambda \to \infty$. We now get a simplified version of the equation:

$$c^b = \frac{p/(1-p) \cdot l}{(y')^2} \cdot i - (\beta/y') \cdot (\varepsilon^y + \rho \cdot \varepsilon^c). \tag{17}$$

Also in this case, the use of the buffer is bounded. The intuitive reason is that increasing the buffer distorts output in normal times, and authorities face a trade-off. But notably, authorities under these preferences still may end up using the buffer very actively. This will be the case if the buffer has weak effects on output. To see this, consider equation (17) in the case of $y' = \rho(\frac{\tau-\theta}{1-\gamma\rho}) \to 0$. In this case, the use of the buffer explodes. Intuitively, now the buffer disturbs output just a little bit in normal times, but the ability to counteract the effect of vulnerabilities in a crisis is intact, and hence this will pull in the direction of responding very strongly to vulnerabilities, as described in the first term. Also, in response to output shocks or capital shocks, authorities will want to use the buffer very actively since it takes a large buffer change to impact output and counteract the effect of the shocks. The capital shock appears here although authorities are not concerned about capital directly, because capital also affects output via γ (procyclicality of the financial system).

The very active use of the buffer, given a weak effect of the buffer on output, disappears if authorities also are concerned about the capital level in banks. In that case, the general equation (15) applies, and we may again consider the case of $y' = \rho(\frac{\tau-\theta}{1-\gamma\rho}) \to 0$. Optimal buffer policy is now:

$$c^b = \frac{p/(1-p) \cdot \lambda_i \cdot l}{(c')^2} \cdot i - \beta \cdot \{\frac{\gamma}{c'}\} \cdot \varepsilon^y - \beta \cdot \{\frac{1}{c'}\} \cdot \varepsilon^c. \tag{18}$$

With zero effect of the buffer on output, authorities will now furthermore respond to negative output shocks with an *increase* in the buffer; Negative output shocks reduce the capital level as long as $\gamma > 0$, and the buffer will be used to counteract this effect. This is in strong contrast to the response to output shocks in equation (17), which calls for an unbounded lowering of the buffer when $y' = \rho(\frac{\tau-\theta}{1-\gamma\rho}) \to 0$. A positive concern for the capital level in banks makes a big difference in terms of optimal buffer policy. The same moderation of the volatility of the optimal buffer when the estimated size of the buffer-effect on the economy is small, could be achieved if authorities directly were concerned also about buffer smoothing in the loss function.

6 CONCLUDING REMARKS

The model in this paper differs from typical small-scale macromodels in that it includes the bank capital coverage ratio as an endogenous variable. One result is that the optimal buffer level may be very volatile. This is the case if the buffer has a very weak effect on output. But if authorities have a concern for a stable capital coverage ratio in banks, the optimal buffer response to output shocks may on the other hand become very moderate and also switch signs. This is the case if the effect of the buffer on output is weak. Such a concern for the capital coverage ratio seems reasonable to assume in practice.

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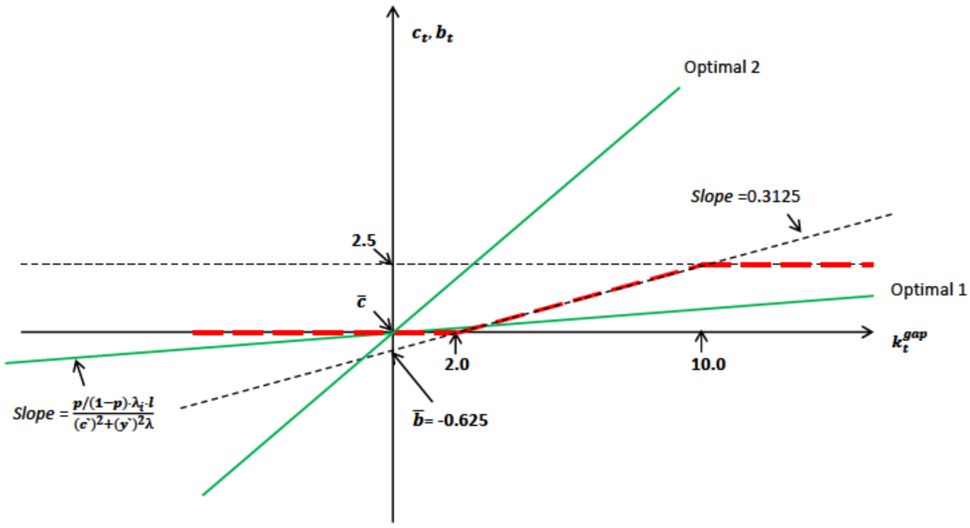
Figure A.1: *The Basel Buffer guide (red dashes) and two alternative optimal responses to financial vulnerabilities (green solid). Vulnerabilities/credit gap on x-axis, total buffer-level on y-axis.*



APPENDIX

A THE TARGET CRITERION AND THE BASEL BUFFER GUIDE

The Basel Buffer guide may be expressed as (see BIS (2010)):

$$b = \begin{cases} 0 & \text{if } k_t^{gap} \leq 2 \\ 0.3125 \cdot (k_t^{gap} - 2) & \text{if } 2 < k_t^{gap} \leq 10 \\ 2.5\% & \text{if } k_t^{gap} > 10 \end{cases}, \tag{A.1}$$

where b corresponds to our $\bar{c}^b + c^b$ (the total buffer, not in deviation from its normal level, see figure below), and k^{gap} (the credit-to-GDP-gap) corresponds to financial vulnerabilities i in the model of this paper. We now mark the steady state normal level for the buffer $\bar{c}^b$ on the vertical axis as indicated in Figure A.1, and the total level of the buffer is now measured on the vertical axis ($\bar{c}^b + c^b$). The "buffer guide" represents an asymmetric response to financial vulnerabilities. The vulnerabilities are measured on the horizontal axis. The slope coefficient for the optimal rule (green lines in the graph) depends on structural parameters and is given by $\frac{p/(1-p)\cdot\lambda_i\cdot l}{(c')^2+(y')^2\cdot\lambda}$ (see equation (15) on page 13). Note that in a case where there is no concern for the output gap in normal times or the capital coverage ratio of banks, the slope explodes (the denominator of the slope coefficient approaches zero).