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R&D heterogeneity and its implications for growth*

Sigurd Mølster Galaasen† and Alfonso Irarrazabal‡

Abstract

This paper quantifies the determinants of heterogeneity in R&D investment and its implications for growth. Using a panel of Norwegian manufacturing firms we document a negative correlation between R&D intensity and firm size, driven mainly by small firms with high R&D intensity. We estimate a Schumpeterian growth model with heterogeneous firms, that differ with respect to innovation efficiency. The estimated model fits the shape of the R&D investment distribution as well as the negative correlation between R&D intensity and firm size. A larger selection effect contribution to aggregate growth is found when we include R&D moments in the estimation. Finally, we study the link between firm heterogeneity and R&D subsidies, and show that the growth effects of subsidies depend crucially on how the policy influences the equilibrium distribution of firms.

JEL Classification: L11, O3,O4 *Keywords*: R&D, Heterogeneous Firms, Subsidies, Growth.

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1 Introduction

Surveys report substantial and persistent firm heterogeneity in R&D intensity.[1] In fact, most firms report zero R&D, some firms report moderate R&D investment, and a few firms report large investment in R&D relative to size. What is the source of this heterogeneity? How important is it to allow innovative firms to grow at the expense of less innovative firms? And finally, in an environment characterized by firm heterogeneity, how does R&D policy (e.g. innovation subsidies) affect economy growth?

In this paper we address these questions by estimating an equilibrium model of firm-level innovation and growth. We adopt the creative-destruction framework of Klette and Kortum (2004) extended by Lentz and Mortensen (2008). In this model firms grow through product innovation that results from innovation made by incumbent and new firms. In the model the two key forces that generate heterogeneity in R&D intensity (R&D expenditures relative to value-added) at the firm level are demand shocks and firm type heterogeneity. A firm invests in R&D that may lead to an innovation. Demand shocks are generated by letting consumer expenditure shares vary across products, implying that the firm's revenue associated with a new product is uncertain. Firm-type heterogeneity arises because incumbent firms differ with respect to the quality improvement associated with an innovation, i.e. some firms produce higher quality innovations than others. This heterogeneity is exogenous and realized upon entry. Absent demand shocks and type heterogeneity every firm would have the same R&D intensity.[2] Firm-type heterogeneity generates variations in R&D intensity since high types (those producing high-quality improvements) have higher expected returns to R&D and thus invest more than low types, both in terms of R&D levels and relative to firm size. Thus, high type firms have higher R&D intensity and grow faster than low type firms.

The model provides a rich, yet tractable, framework that links firm-level dynamics to micro-level data. Using observations on size, productivity and R&D expenditures from a panel of Norwegian manufacturing firms, we estimate the model and quantify the relative importance of different sources of R&D heterogeneity. Thus, we contribute to the recent literature that estimates variants of the Klette-Kortum model

[1] R&D intensity (RI) is a measure of R&D expenditures relative to size. Firm size is measured in terms of value-added.

[2] The R&D production function has the property that a firm's optimal R&D investment is proportional to its size (measured as number of products). Absent demand shocks, value-added is proportional to the firm's product size.

using micro data on R&D.[3]

The model's fit maps well to both R&D and non-R&D moments. In particular, the estimated model fits the observed distribution of R&D expenditures and intensity (mean, dispersion and skewness) as well as the negative correlation between R&D intensity and size. In our dataset, which is obtained from a survey of all firms above 50 workers and a sample of firms between 10 and 50 workers, this correlation is driven by small firms. In contrast, most previous studies have used large firms (for survey limitations) and found a zero correlation in their sample (Cohen and Klepper 1996; Klette and Grilliches 2000; and others). In our framework, the negative correlation results from firms becoming larger not only due to innovation activity, but also due to persistent random shocks to the demand for their products. In the estimated model, the group of small firms tends to be dominated by those experiencing negative demand shocks. We also find that this shock is key for generating substantial cross-sectional dispersion in R&D intensity, while firm-type heterogeneity is important for cross-sectional variation in R&D expenditures.

Firm-type differences imply that reallocating workers from less to more productive firms generates aggregate productivity gains. In the model, creative destruction induces such gains by generating a reallocation of product shares across types. More productive firms innovate more intensely and crowd out less productive firms in steady state. Using data from Danish firms, Lentz and Mortensen (2008) find that this selection effect accounts for around 53 percent of aggregate growth. We use Norwegian manufacturing firms and infer the importance of selection for growth in our sample. Crucially, we have R&D information, which we use to discipline the model along the R&D dimension. To make our estimation comparable to Lentz and Mortensen (2008), we first exclude observations on R&D, and find that the selection effect accounts for 44.5 percent of aggregate growth. This magnitude is similar to the 49 percent they find for the manufacturing sector. However, we miss some key empirical R&D patterns: Research intensity is too negatively correlated with firm size and those firms engaging in R&D are too many, too small, homogeneous, and invest too little in R&D relative to the data. When re-estimating the model by adding R&D moments the new parameters imply a larger role for reallocation, which then accounts for 72 percent of aggregate growth.

We subject our model to several tests of robustness. It produces firm-level responses to R&D subsidies that are in line with micro evidence from a natural exper-

[3]Recently, several papers have used R&D information to estimate structural models similar to ours; for example, see Akcigit and Kerr (2010) and Acemoglu et al. (2013)

iment (Bøler et al. (2015)). In the short run, firms increase their R&D spending by roughly 40 percent in response to a 20 percent R&D subsidy. Using a 2002 Norwegian policy reform, aimed at firms with less than 4 million NOK in R&D spending, Bøler et al. (2015) estimates a reform-induced increase in R&D spending by 35 to 72 percent during 2003-2005. Moreover, the model also explains several cross sectional and dynamic moments for R&D, firm size and productivity when we restrict the sample to large firms. Finally, the model reproduces features of firm's life cycle over a longer horizon than we consider in the estimation.

Finally, we use the estimated model to quantitatively explore the growth effects of R&D subsidies. Since our estimation finds a strong reallocation channel, we expect substantial variation in aggregate growth effects, depending on how a subsidy policy is implemented. By studying stylized reforms, we show how failing to target the best innovators may lead to subsidies creating small, or indeed adverse, growth effects. In general, a subsidy's effect on growth depends crucially on how it influences the equilibrium distribution of firms and R&D spending. For example, a subsidy that targets small firms (in terms of R&D expenditures) results in a 0.7 percentage point reduction in the aggregate productivity growth rate relative to the decentralized equilibrium of 1.47 percent. Compared to only subsidizing incumbent firms, a subsidy to all firms (potential entrants and incumbents) reduces the growth rate from 1.83 to 1.53 percent. The reason for these adverse effects is that subsidies to small firms weaken the selection effect, and a larger share of less innovative firms is thus sustained in equilibrium.[4]

Our paper is related to several different literatures. First, it relates to the literature on R&D heterogeneity. Several papers have attempted to account for within-industry differences in firm R&D intensity. Cohen and Klepper (1992) proposed a simple mechanism to explain the dispersion in R&D intensity observed in the data. The authors developed a probabilistic model where firms partially control the outcome of their R&D effort. Cohen and Klepper (1992) also propose a mechanism that relates R&D spending to firm size. More recent papers have used a structural approach to understand the link between firm dynamics and R&D heterogeneity, for example; see Akcigit and Kerr (2010) and Acemoglu et al. (2013). Akcigit and Kerr (2010) develop a model in which firms undertake heterogeneous research activities; exploration (capture new products) and exploitation (improve exciting product

[4]Acemoglu et al. (2013) find that an optimal R&D policy involves subsidizing both entrants and high incumbent firms. The key mechanism that drives the difference in policy implications is that in Acemoglu et al. (2013) firm-type heterogeneity is transitory.

lines). Aw et al. (2011) estimate a structural model of producer decisions to invest in R&D and export. Their partial equilibrium model limits the analysis to within-plant productivity gains.

Second, our paper relates to the literature on reallocation (Petrin and Levinsohn, 2012, Foster et al., 2001, Bartelsman et al., 2013 and others). Finally, our paper contributes to the literature on R&D policy (Aghion et al. 2013; Acemoglu et al. 2013; Atkeson and Burstein 2014; Lentz and Mortensen 2016).

The paper proceeds as follows. Section 2 describes the data. Section 3 goes through the model. In particular, section 3.2 focuses on the link between Lentz and Mortensen (2008) and Klette and Kortum (2004), and section 3.4 explores the model implication for R&D patterns. In section 4 we go through the empirical implementation and estimation results, and section 5 contains the policy experiments. Section 6 concludes.

2 R&D Facts

In this section we describe the data, and discuss some stylized facts about R&D heterogeneity for the Norwegian manufacturing sector, and characterize the relationship between R&D intensity, firm size and productivity.

2.1 Data

The data consists of a panel of Norwegian manufacturing firms for the period 1997 to 2001, and gathered from two sources. First, we use balance sheet data from Statistics Norway's Capital database,[5] which is an annual unbalanced panel of all non-oil manufacturing joint-stock firms. The panel provides information about each firm's value-added, wage bill and number of workers. Second, we use the biennial R&D survey from Statistics Norway,[6] which provides information about firm-level R&D investment. The survey records R&D information for all firms with more than 50 workers. It also contains information for all firms with less than 50 employees, that have reported intramural R&D activity in the previous survey of more than NOK 1 million or extramural R&D of more than NOK 3 million. Finally, for the remaining firms with 10-49 employees, a random sample was selected with a sampling rate of roughly 35 percent. We follow Lentz and Mortensen (2008) and exclude entry firms

[5] For Capital database data documentation, see Raknerud et al. (2004)
[6] See Statistics Norway (2004)

from the sample. Consequently, we follow the 1997 cross-section of firms over the 1997-2001 period . Before we trim the data we compute the aggregate wage rate in 1997 as the ratio of the aggregate wage bill to aggregate employment, $w_t = \sum_j W_{j,t} / \sum_j N^*_{j,t}$, where $W_{j,t}$ and $N^*_{j,t}$ are total wage bill and employment (number of workers) for firm j in year t. For subsequent years, we compute the wage rate using firms that were incumbents in 1997. We also follow Lentz and Mortensen (2008) and construct the quality-adjusted employment ($N_{j,t}$) for firm j using $N_{j,t} = W_{j,t}/w_t$, which we use as our measure of firms' employment when constructing empirical moments.

We trim both tails of the employment distribution. At the bottom we eliminate firms with less than three workers. Many of these very small firms are single employee companies. At the top we exclude all firms above the highest 1 percent of the size distribution. We also exclude all firms with R&D intensity (R&D expenditures over value-added) above one in at least one year. Table 9 (appendix A) shows some descriptive statistics for our sample. We have 5290 firms, with around 7 percent of those firms reporting positive R&D activity. The mean R&D intensity of these firms is 8 percent. We also report summary statistics for firms with 10-50 workers and over 50 workers.

2.2 Stylized Facts

Now we present some stylized facts about R&D, firm size and productivity.

Distributions. Figure 1 panel (a) shows the R&D intensity distribution for all firms with positive R&D expenditure in 1997. The R&D distribution is positively skewed with a long right tail. This means that most of the R&D intensity is concentrated at low intensities but that there are a few firms with a large proportion of R&D expenditures relative to its size. The average R&D intensity for all sampled firms is around 8 percent and around 6 percent for firms with more than 200 workers.[7]

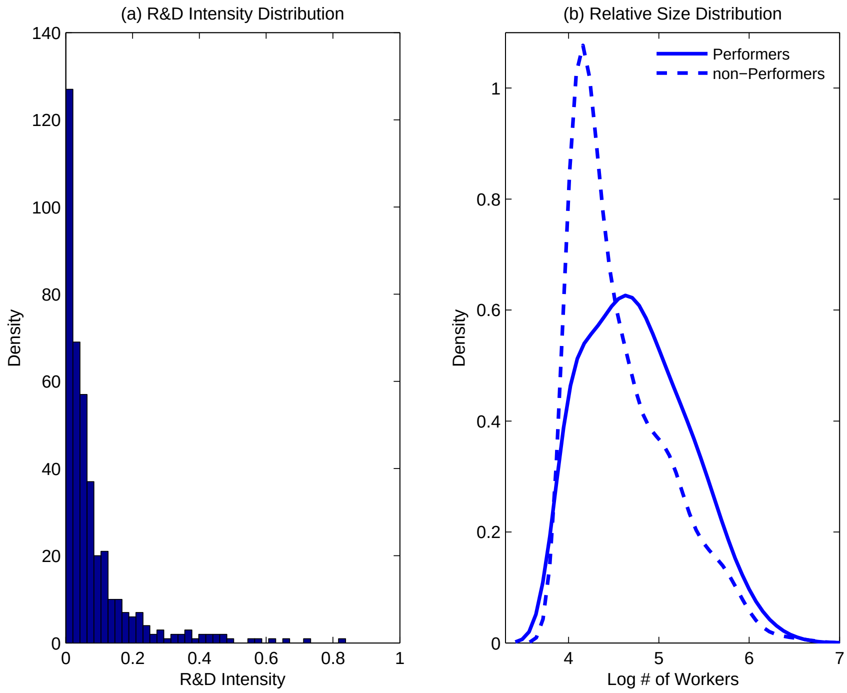
Figure 1 panel (b) depicts the employment distribution of performers (firms with positive R&D) and non-performers (firms reporting zero R&D) for 1997, for firms with more than 50 workers.[8] The size distribution for performers has more mass

[7] Doraszelski and Jaumandreu (2013) report values between 1 to 2.7 percent for Spanish manufacturing firms for a sample of firms with more than 200 workers. Acemoglu et al. (2013), using the Survey of Industrial Research and Development, report values of 9.9 for small firms and 4.2 for large firms. In their sample 32 percent of the firms have more than 500 employees.

[8] Using other datasets, it has been found that a considerable fraction of firms report zero innovation. For example, for manufacturing firms with more than 10 workers, Harrison et al. (2008) reports a fraction of non innovators ranging from 0.47 to 0.6 for four European countries. In our sample, the fraction of firms with zero R&D for firms over 50 workers is 0.65. When we include firms above three workers, this fraction rises to 0.92, which is one of the moments we target in the estimation. Notice that

to the right and performers on average are 1.22 times larger than non-performers in terms of employment.

Figure 1: The Distributions of R&D intensity and Size for Performers and Non-Performers.



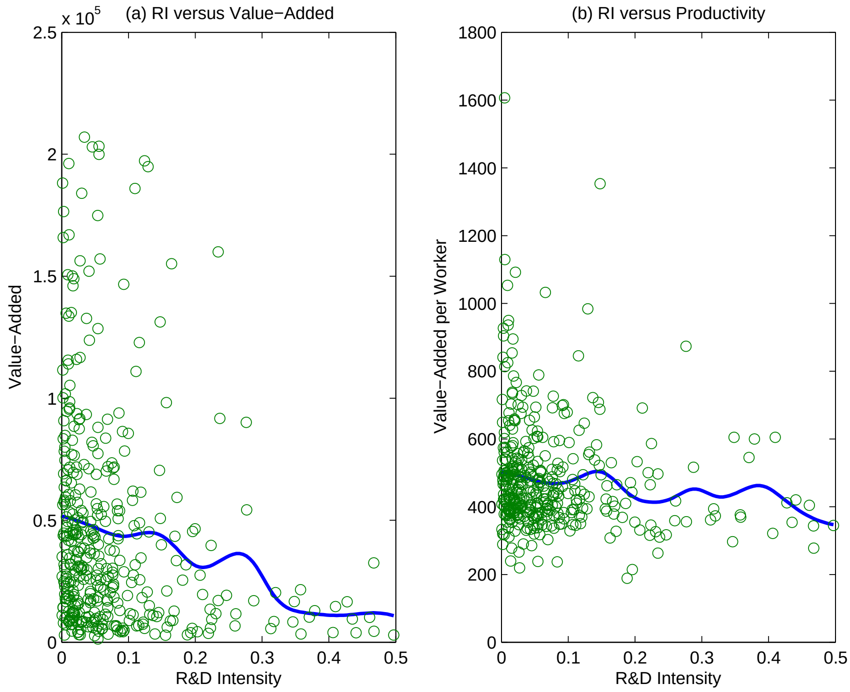
Notes: The data is from 1997. The R&D intensity histogram is computed by including all sampled firms. The size distributions considers all firms with more than 50 workers and depicts kernel densities.

Correlations. We also document negative correlations between R&D intensity with firm size and productivity. In Figure 2 panel a, we plot a kernel regression between R&D intensity and value-added for 1997. The unconditional correlation is -0.18. It is interesting that most of the correlation is driven by firms with low R&D intensity. In fact, the correlation between R&D intensity and firm size is -0.02 for firms with 50 or more workers. Our model will be able to explain this negative correlation because firms can become large not only due to innovation activity, but also due to a random shock to demand for their products. Since market demand for a product is unrelated to R&D expenditures it creates a negative correlation between

we do not target the fraction of firms above 50 workers reporting zero R&D, but the estimated model gives a fraction of 0.71.

firm size and R&D intensity. In panel b we plot a kernel regression between R&D intensity and productivity measured as value-added per worker. The unconditional correlation is -0.18

Figure 2: R&D Intensity, Value-Added and Worker Productivity.



Notes: Panel (a): Kernel regression and scatter plot of R&D intensity on value-added (panel a) and value-added per worker (panel b), in 1997.

3 The Model

This section lays out our model. We will first review the basics of the Klette-Kortum model, and then incorporate the innovations introduced by Lentz and Mortensen (2008).[9] The model is an endogenous growth model based on expanding product quality, and extends the work of Grossman and Helpman (1991) and Aghion and

[9]Readers already familiar with the these models may skip this section. Note that we assume unit price elasticity for all products. In contrast, Lentz and Mortensen (2008) allow for the product price to affect product revenue. However, when they estimate their model they impose unit demand elasticities for all product varieties.

Howitt (1992) by incorporating research conducted by incumbent firms. This approach implies that firm size and R&D distributions are endogenous. There is a fixed measure of differentiated goods and innovation improves product quality. Firms compete in a quality-ladder setting and invest in R&D to capture market shares.

3.1 The Klette-Kortum Model (2004)

Time is continuous. A representative household maximizes utility $U = \int_0^{\infty} e^{-\rho t} ln(C_t)dt$, and has Cobb-Douglas preferences over a unit continuum of differentiated goods,

$$ln(C_t) = \int_0^1 \ln[y_t(j)A_t(j)]dj, \tag{1}$$

where $y_t(j)$ and $A_t(j)$ measures the quantity and quality, respectively, of good j. Total labor supply (l) is exogenous, homogeneous, and can be used for two activities: production of goods and research. Total expenditures, $E_t = P_t C_t$ are normalized to Z for all periods (t). Given an interest rate r_t and the household's Euler equation $\dot{E}/E = r_t - \rho$, this normalization implies that $r_t = \rho$. Furthermore, it means that the consumption price P_t deflates at the rate of consumption growth. Since all goods have an equal log-preference weight, consumers spend Z on each good. The production technology is linear-in-labor and equal across all goods $y(j)$, with factor productivity normalized to 1. The unit (and marginal) cost of production is thus w, the cost per unit of labor.

The Innovating Firm. Firms are units that manufacture multiple products. Firms enter the industry with one product and they have to invest in R&D to add more products to their portfolio. The outcome of this research effort is stochastic. All firms innovate at the quality frontier and innovations occur randomly with a Poisson arrival rate I, chosen by the firm. Upon a successful innovation effort, the firm improves the quality of a random good j by a factor $q > 1$. This factor is firm-specific and applies to all its innovations, but varies across firms. The time t quality of good j is given by

$$A_t(j) = \prod_{i=0}^{J_t(j)} q_i(j), \tag{2}$$

that is, the product of all past innovations is $J_t(j)$, where $q_i(j)$ and $q_{J_t(j)}(j)$ are the quality improvement of the i^{th} and last innovation, respectively, and $q_0(j)$ is the initial quality. Consider a firm making a successful time t innovation in good j. The innovation creates a blueprint which is a multiplicative improvement q over the current producer's blueprint $A_t(j)$. The firm's product quality is then given by $A_t(j)q$. Hence, the innovating firm combines the past quality blueprints embedded in $A_t(j)$ (common knowledge) with its new blueprint to create a product with superior quality. The innovating firm receives a patent for this blueprint that lasts until a new innovation occurs. Since the innovator is the only firm that can produce the frontier quality goods (all other firms can produce at quality $A_t(j)$), under Bertrand pricing it becomes the sole supplier of that good. The price $p(j)$ is a markup q over marginal cost w, i.e. the price that makes the buyer indifferent between the highest quality version and the second highest quality version, priced at marginal cost. The innovator receives a flow of profits associated with the new product, given by

$$\begin{aligned}\Pi(j) &= p(j)y(j) - wy(j) = p(j)y(j)\left[1 - \frac{1}{q}\right] \\ &= Z\pi(q),\end{aligned} \tag{3}$$

where $\pi(q) = 1 - \frac{1}{q}$ is the profit share generated by the quality improvement q. The demand for production workers $l^w(j)$ (and quantity $y(j)$) associated with product j is then

$$l^w(j) = \frac{Z}{qw}. \tag{4}$$

Notice that the model features no social depreciation of knowledge. This assumption is apparent from the definition of $A_t(j)$ in equation (2), where we see that quality stays constant if no innovation occurs. However, there is private depreciation of knowledge, in the sense that the firm's return to innovation only lasts until its product is overtaken by a competitor.

Innovation Choice. The firm's state is the number of products k it currently produces. It invests in R&D to maximize the present value of future profits. R&D investment generates new products at a frequency γk. Moreover, any good the firm produces is overtaken by another firm at Poisson rate δ, and firms with k products will see any of these products overtaken at rate δk. The destruction intensity δ is the outcome of aggregate innovation, and thus is an equilibrium object.

Investment in R&D requires labor and knowledge capital, measured as the firm's number of products k. The total cost of R&D is $wc(\gamma)k$, where the function $c(\gamma)$ is assumed to be strictly increasing and convex.

The firm's optimal R&D investment solves the Bellman equation,

$$rV(k) = \max_{\gamma \geqslant 0} \{\pi(q)Zk - wkc(\gamma) + \gamma k [V(k+1) - V(k)] - \delta k [V(k) - V(k-1)]\}, \tag{5}$$

where the first two terms represent profit flow from the firm's current portfolio of goods and R&D expenditures, while the last two terms represent the value of gaining and losing a product, respectively. Since the R&D technology features constant returns to scale in labor and the number of products, the value and policy functions become proportional to the state variable,

$$\begin{aligned} V(k) &= vk \\ I(k) &= \gamma k. \end{aligned} \tag{6}$$

A firm's demand for researchers is thus proportional to the total number of products,

$$l^R(k) = kc(\gamma). \tag{7}$$

The innovation intensity γ and value per product v solves

$$wc'(\gamma) = v \tag{8}$$

$$v = \frac{Z\pi(q) - wc(\gamma)}{r + \delta - \gamma}. \tag{9}$$

Given the firm's innovation choice γ and the aggregate destruction rate δ, the firm's product size k follows a Poisson birth-death process. The time until a firm with k products at time t gains or loses a product is exponentially distributed with a mean of $1/(k(\gamma + \delta))$. When the transition occurs, the firm moves to state $k - 1$ with a probability of $\delta/(\gamma + \delta)$ and to state $k + 1$ with a probability of $\gamma/(\gamma + \delta)$. When the firm loses all of its products, it permanently exits the market, i.e. $k = 0$ is an absorbing state. As a consequence of the proportionality of the policy function, we can alternatively interpret a size-k firm as being a collection of k firms with one product.

3.2 Incorporating Lentz and Mortensen (2008)

Lentz and Mortensen (2008) estimate the Klette-Kortum model on Danish firm-level data. To account for firm heterogeneity, they extend the model along four dimensions.

A. Type Heterogeneity In the setup in section 3.1 productivity, measured as value-added per worker, is independent of firm size k:

$$PR = \frac{kZ}{kl^w + l^R(k)} = \frac{Z}{Z(wq)^{-1} + c(\gamma)}. \tag{10}$$

Klette and Kortum (2004) create productivity dispersion across firms through firm specific innovation steps q (and thus profit shares $\pi(q)$). However, Klette and Kortum modify the R&D cost function $c(\gamma)$ in such a way that the cost and benefit of large innovation steps are proportional, leaving the optimal creation rate γ constant across firm types. With homogeneous creation rates, firm size is unrelated to productivity.

Using Danish data, Lentz and Mortensen document a positive correlation between productivity and firm output size (value-added), and zero correlation between productivity and firm input size (workers). To account for these relationships, they introduce heterogeneity in q as in Klette and Kortum (2004), but allow this factor to generate heterogeneous innovation intensities. In particular, using the same R&D cost function $c(\gamma)$, profitable firms (high q) create larger quality improvements than less profitable firms. Given the firm's problem, it follows that $\pi(q_\tau) > \pi(q_{\tau'}) \Leftrightarrow \gamma_\tau > \gamma_{\tau'}$. Type τ firms have on average more products (and thus higher value-added) than type τ' firms, and from equation (4) the demand for production workers associated with a product is negatively related to the size of the innovation step. Consequently, firm-specific innovation's steps can accommodate a positive correlation between labor productivity and value-added, and a zero correlation between labor productivity and employment.

B. Supply Side Shocks Value-added per worker is perfectly persistent in the model's basic setup. To address this, Lentz and Mortensen (2008) relax the assumption that the firm-specific innovation step q is constant across innovations. When innovation does occur, the type-specific quality jump q_τ is drawn from a Weibull distribution. The quality jumps of a more innovative firm type dominates by (first order stochastic) those of less innovative firms. The more innovative firm-type is thus more profitable in expectation: $E\left[\pi(q_\tau)\right] > \left[\pi(q_{\tau'})\right] \Leftrightarrow \gamma_\tau > \gamma_{\tau'}$.

C. Demand Side "Shocks" In the basic Klette-Kortum model, firm growth is independent of firm size (Gibrat's law). In the Danish data, large firms tend to grow more slowly. To account for this discrepancy, Lentz and Mortensen (2008) add a random product market size by allowing the preference weight to vary across goods, with α_j as the weight on good j :

$$\ln(C_t) = \int_0^1 \alpha_j \ln[y_t(j) A_t(j)] dj.$$

Since $\int_0^1 \alpha_j dj = 1$, the expenditure share on good j is $z_j = \alpha_j Z$. Furthermore, R&D is undirected. Upon a successful innovation, the firm cannot choose which good the quality improvement applies to; each good on the unit interval is an equally likely candidate. Thus, from the innovator's viewpoint, product revenue is uncertain until the particular product variety is realized. This randomness creates a mean reversion in value-added, which potentially can help explain the violation of Gibrat's law.

D. Capital Cost Finally, Lentz and Mortensen (2008) add capital to the goods production function, using Leontief technology in labor and capital. Total factor productivity is normalized to 1, and marginal cost is given by $w + \kappa$ where w is the cost per unit of labor and κ is the capital cost per unit of output. The capital cost does not impact a firm's profitability, and is thus irrelevant for innovation choice. However, it directly impacts labor's share of value-added, and thus pins down the level of value-added per worker.

E. Firm Problem Adding these features does not substantially alter the firm's problem. We need to add the vectors of product demand realizations $z^k = (z_1, ..z_k)$ and innovation steps $q^k = (q_1, ..q_k)$ to the definition of current profit flow in the Bellman equation. However, looking forward, firms expect to realize mean revenue Z and mean profit share $E[\pi(q_\tau)]$ on future innovations. The optimal R&D investment is still proportional to the firm's product size $I_\tau = \gamma_\tau k$ and the type-specific innovation intensity γ_τ solves for

$$\begin{aligned} wc'(\gamma_\tau) &= v_\tau \\ v_\tau &= \frac{ZE[\pi(q_\tau)] - wc(\gamma_\tau)}{r + \delta - \gamma_\tau}, \end{aligned} \tag{11}$$

where v_τ now denotes the specific firm-type expected value of one product.

3.3 Entry and Equilibrium

A. Entry Rate There is a constant mass μ of potential entrants choosing an innovation intensity γ_0, that permits each firm to enter the market with one product. Aggregate innovation rate by entrants is then $\eta = \gamma_0 \mu$. Potential entrants face the same R&D cost function as an incumbent with one product, i.e. $wc(\gamma_0)$. On entry, firms learn their own type, drawn from the discrete distribution of potential firm types, where ϕ_τ denotes the fraction of type τ firms on entry. The expected value of entering with one product is thus given by $E[v_\tau] = \sum_\tau \phi_\tau v_\tau$. The free entry condition requires that

$$wc'(\gamma_0) = wc'\left(\frac{\eta}{\mu}\right) = \sum_\tau \phi_\tau v_\tau. \tag{12}$$

B. Stationary Equilibrium In a stationary equilibrium, with creation rate γ_τ and destruction rate δ both constant, the product birth-death process at the individual firm level give rise to a logarithmic distribution (with parameter $\frac{\gamma_\tau}{\delta}$) in k across firms of a particular type τ. Because firms of different types τ choose different creation rates γ_τ, the R&D distribution differs across types.

Firms choose their innovation intensity γ_τ taking as given the aggregate product destruction rate. In equilibrium, aggregate innovation must be consistent with the innovation undertaken by incumbents and entrants:

$$\delta = \eta + \sum_{\tau=1}^{n} K_\tau \gamma_\tau. \tag{13}$$

K_τ is the steady-state mass of goods produced by type τ firms, given by

$$K_\tau = \frac{\eta \phi_\tau}{\delta - \gamma_\tau}. \tag{14}$$

Since there is a total mass 1 of goods, we must have that $\sum_{\tau=1}^{n} K_\tau = 1$. Through the process of creative destruction, the equilibrium distribution of firms, denoted ϕ_τ^*, differs from the entry distribution ϕ_τ. The total steady-state mass of type τ firms is given by

$$M_\tau = \frac{\eta \phi_\tau}{\gamma_\tau} \ln\left(\frac{\delta}{\delta - \gamma_\tau}\right), \tag{15}$$

and the equilibrium fraction is then

$$\phi_\tau^* = \frac{M_\tau}{\sum_{\tau=1}^{n} M_\tau}. \tag{16}$$

The stationary equilibrium consists of a constant wage w, an aggregate destruction rate δ, entry rate η, firm type-specific creation rates $\{\gamma_\tau\}_{\tau=1}^n$ and distribution of products across types $\{K_\tau\}_{\tau=1}^n$, such that η satisfies the free entry condition in equation (12), the creation rate γ_τ solves the firm's optimization problem, aggregate destruction δ and product distribution K_τ satisfy equations (13) and (14), and the wage rate clears the labor market.[10]

C. Aggregate Growth Rate In keeping with Klette and Kortum (2005) and Acemoglu et.al. (2013), we assume that at time 0 the economy is in steady state, and we normalize the initial quality index such that $A_0(j) = q_0(j)\ \forall\ j$. With this normalization, we implicitly normalize the previous quality version of each good, $q_{-1}(j) = 1$, and assume that all goods are available in an improved $q_0(j)$ quality version at time 0. Consumption evolves according to

$$\begin{aligned}
\ln(C_t) &= \textstyle\int_0^1 \alpha_j \ln A_t(j) dj + \int_0^1 \alpha_j \ln y_t(j) dj \\
&= \textstyle\int_0^1 \alpha_j \left[\sum_{i=0}^{J_t(j)} \ln q_i(j) \right] dj + \int_0^1 \alpha_j \ln \left[\frac{\alpha_j Z}{w+\kappa} \frac{1}{q_{J_t}(j)} \right] dj \\
&= \textstyle\int_0^1 \alpha_j \left[\sum_{i=0}^{J_t(j)} \ln q_i(j) \right] dj - \int_0^1 \alpha_j \ln \left[q_{J_t}(j) \right] dj + \int_0^1 \alpha_j \ln \left[\frac{\alpha_j Z}{w+\kappa} \right] dj.
\end{aligned}$$

Along the stationary growth path new innovations arrive at the constant rate δ. We can then apply the law of large numbers to a weighted average (with weights $\alpha(j)$) to get[11]

$$\begin{aligned}
\ln(C_t) &= (\delta t + 1) E[\ln(q)] - E[\ln(q)] + \textstyle\int_0^1 \alpha_j \ln \left[\frac{\alpha_j Z}{w+\kappa} \right] dj \\
&= \delta t E[\ln(q)] + \textstyle\int_0^1 \alpha_j \ln \left[\frac{\alpha_j Z}{w+\kappa} \right] dj,
\end{aligned}$$

where δt is the expected number of innovations per product $J_t(j)$ over time length t, and the average log quality jump given by

$$E[\ln(q)] = \textstyle\sum_{\tau=1}^n \frac{\phi_\tau \eta + K_\tau \gamma_\tau}{\delta} E[\ln(q_\tau)],$$

[10] This equilibrium corresponds to the equilibrium definition in Lentz and Mortensen (2008) p. 1332. We refer to Lentz and Mortensen (2008, appendix C) for the equilibrium solution algorithm.

[11] Note that since the firm cannot direct an innovation to a particular product, $J_t(j)$ and $q_i(j)$ are i.i.d. across the unit continuum of products, and consequently not correlated with the weights α_j. To apply the law of large numbers to a weighted average, we use the Lindeberg Central Limit Theorem.

where $\frac{\phi_\tau \eta + K_\tau \gamma_\tau}{\delta}$ is the fraction of new innovations attributed to firms type τ and $E\ln(q_\tau)$ is the type-conditional average log quality jump. Consequently, aggregate consumption grows at a rate where

$$g = \delta E[\ln(q)].$$

Growth thus arises due to the arrival of new innovations at rate δ with average quality contribution of $E[ln(q)]$.

3.4 Model Implications for R&D Moments

In this section, we explain the different channels through which the model produces heterogeneity in R&D. Understanding these channels will prove useful when we interpret the estimation results in section 4.

The evolution of an individual firm's product size k is completely determined by the innovation choice γ_τ and the destruction rate δ. In steady state, the type-conditional distribution of the number of products is logarithmic, with parameter γ_τ/δ. Given this equilibrium parameter and drawing firm types from the discrete distribution ϕ_τ^* and demand and innovation step sizes from their corresponding distributions, we can produce observations of value-added $(Y_{i,t})$, wage bill $(W_{i,t})$, R&D expenditures $(RD_{i,t})$, employment $(N_{i,t})$, labor productivity (PR_i), R&D intensity (RI_i) across firms i at time t as follows:

$$\begin{aligned} Y_{i,t} &= \sum_{j=1}^{k_{i,t}} z_{i,j} \\ W_{i,t} &= w\left(\frac{1}{w+\kappa}\sum_{j=1}^{k_{i,t}} \frac{z_{i,j}}{q_{i,j}} + k_{i,t}c(\gamma_{\tau_i})\right) \\ RD_{i,t} &= wk_{i,t}c(\gamma_{\tau_i}) \\ N_{i,t} &= W_{i,t}/w \\ PR_{i,t} &= Y_{i,t}/N_{i,t}. \end{aligned} \tag{17}$$

The value-added $(Y_{i,t})$ created by a firm with $k_{i,t}$ products is the sum of its product revenues, and the wage bill $(W_{i,t})$ is the wage per worker times total labor demand (the sum of workers devoted to production and R&D). Given a product demand $z_{i,j}$ and the firm's pricing rule, the demand for production workers associated with the product is $\frac{z_{i,j}}{p_{i,j}} = \frac{z_{i,j}}{(w+\kappa)q_{i,j}}$. The optimal R&D investment requires $k_{i,t}c(\gamma_\tau)$ workers.

Labor productivity is measured as value-added per worker, and research intensity is defined as R&D spending $(RD_{i,t})$ over value-added, such that $RI_{i,t} = RD_{i,t}/Y_{i,t}$.

A. Dispersion in R&D The model generates cross-sectional heterogeneity in research intensity mainly through two channels, i) firm type-heterogeneity in innovation choices γ_{τ_i} and ii) demand shocks. If we shut down demand shocks, $RI_{i,t}$ becomes

$$RI_{i,t} = \frac{wk_{i,t}c(\gamma_{\tau_i})}{\sum_{j=1}^{k_{i,t}} z_{i,j}} = \frac{wk_{i,t}c(\gamma_{\tau_i})}{k_{i,t}Z} = \frac{w}{Z}c(\gamma_{\tau_i}). \tag{18}$$

Due to the proportionality of R&D investment and the state variable k, heterogeneity induced by the product birth-death process will not explain the cross-sectional R&D intensity distribution. Without demand shocks, $k_{i,t}$ drops out of the expression. With demand shocks, the k-distribution does affect the dispersion of R&D intensity across firms. But since R&D spending still scales with size, it will be of second order importance.

Demand shocks are irrelevant for explaining the heterogeneity in the level of R&D spending. The dispersion is entirely determined by the product birth-death process and heterogeneity across firms in innovation choice γ_τ .

B. Correlation between Research Intensity and Firm Size This correlation, which is negative in the data, is given by

$$corr(RI, VA) = cor\left(\sum_{j=1}^{k_{i,t}} z_{i,j}, \frac{wk_{i,t}c(\gamma_{\tau_i})}{\sum_{j=1}^{k_{i,t}} z_{i,j}}\right) \tag{19}$$

and is driven by two opposing forces. First, consider the pure Klette-Kortum (2004) model in which $RI_{i,t} = wc(\gamma)/Z$ is independent of firm size, given by $Y_{i,t} = k_{i,t}Z$. Firm type-heterogeneity introduced by Lentz and Mortensen (2008) produces a positive relationship. More profitable firm-types choose a higher innovation intensity γ since they expect a higher profit from a successful innovation than less profitable firms (low type-firm). On average, more profitable firms have more products and invest more in R&D relative to size. On the other hand, demand shocks work in the opposite direction. Firms with a series successful products tend to be large. Since demand shocks are unrelated to the firm's R&D choice, these firms tend to have low R&D intensity.

C. Correlation between Research Intensity and Productivity Among firms with $RI > 0$, R&D intensity and labor productivity is negatively correlated in the data. Firms with high R&D expenditures relative to size thus tend to have low productivity.

$$corr(RI, PR) = corr\left(\frac{wk_{i,t}c(\gamma_{\tau_i})}{\sum_{j=1}^{k_{i,t}} z_{i,j}}, \frac{\sum_{j=1}^{k_{i,t}} z_{i,j}}{\frac{1}{w+\kappa}\sum_{j=1}^{k_{i,t}} \frac{z_{i,j}}{q_{i,j}} + k_{i,t}c(\gamma_{\tau_i})}\right). \tag{20}$$

The model generates this pattern through demand shocks. Consider a model without demand shocks. As noted in section 3.2A, firm type-heterogeneity can accommodate a positive correlation between productivity and value-added. If large firms also tend to have high R&D intensity, this translates into a positive correlation between productivity and R&D intensity. Demand shocks will, as with $corr(RI, Y)$, work in the opposite direction. Firms with high R&D intensity tend to have experienced bad demand draws, and a bad demand draw reduces value-added per worker. The reason for this is the presence of R&D workers in the denominator of (equation 10). This implies that in response to demand shocks total employment moves less than proportionally to value-added. Finally, supply-side shocks (stochastic q) only create variation in productivity, hence pushing the correlation toward zero.

D. Remarks on Measurement Error In the estimation we allow for log-normal measurement error in the firm's value-added, wage bill, and R&D expenditures. Since employment is computed by dividing the wage bill with w, measurement error in W spills over to N. Measurement error in Y contributes both to variability in R&D intensity and a negative correlation between R&D intensity and firm's size and productivity. Measurement error in R&D creates additional R&D intensity dispersion, but pushes correlations towards zero. Wage bill measurement error drives $corr(RI, PR)$ to zero, but does not affect the dispersion in R&D intensity.

4 Empirical Implementation

We now estimate the model using a panel of Norwegian firms with data starting in 1997. We follow Lentz and Mortensen (2008) and use indirect inference methods to estimate the structural parameters on cross-sectional and dynamic moments in 1997 and 2001. We first describe the estimation procedure, then we show that the model estimated on Norwegian data (but without using the R&D moments) gives reallocation effects of the same order of magnitude to those of Lentz and Mortensen.

However, the estimated model has implications for R&D that are quite different from the observed R&D. Finally, we re-estimate the model (using the R&D moments) and find that resource reallocation across firms is more important for aggregate growth than what Lentz and Mortensen's study indicates.

4.1 Model Estimator

We parametrize the cost function as $c(\gamma) = c_0\gamma^{1+c_1}$. Product revenues are drawn from a Weibull distribution with mean Z, an origin at o_z and a shape parameter β_z. Quality improvements are drawn from a Weibull distribution with a shape parameter β_q (common across firm-types), an origin at 1 and a type-specific scale parameter ε^q_τ. We consider three types of firms in the estimation, and assume a that type 1 firm does not innovate (i.e. $\varepsilon^q_1 = 0$).[12] Finally, we allow for log-additive measurement error $(\xi^Y_{i,t}, \xi^W_{i,t}, \xi^{RD}_{i,t})$ in value-added, wage bill, and R&D expenditures, distributed log-normally, $\ln(\xi^x_{i,t}) \sim N(-\frac{\sigma^2_x}{2}, \sigma^2_x)$, $x \in \{Y, W, RD\}$.

In total, the model has 17 fundamental parameters. Two R&D cost function parameters (c_1, c_2), capital cost in goods production (κ), interest rate r, three demand parameters (Z, β_z, o_z), three innovation jump parameters $(\beta_q, \varepsilon^q_2, \varepsilon^q_3)$, the probability of being of a type 2 and 3 type firm at entry (ϕ_2, ϕ_3) and three measurement error variances $(\sigma^2_y, \sigma^2_w, \sigma^2_{rd})$. Given the exogenous labor supply l and the mass of potential entrants μ, the wage rate w and entry rate η are both equilibrium objects. However, in keeping with Lentz and Mortensen (2008), we estimate w and η, and let l and μ adjust such that the labor market clears and the free entry condition holds.

The wage rate, $w = 296.5$, is estimated directly from the data and the interest rate is set to $r = 0.05$. The remaining 15 parameters are estimated by indirect inference. Given the parameters $\Lambda = \left\{\eta, c_1, c_2, \kappa, Z, \beta_z, o_z, \beta_q, \varepsilon^q_2, \varepsilon^q_3, \phi_2, \phi_3, \sigma^2_y, \sigma^2_w, \sigma^2_{rd}\right\}$, we simulate a firm-year (i, t) panel of value-added $(\tilde{Y}_{i,t})$, wage bill $(\tilde{W}_{i,t})$, employment $(\tilde{N}_{i,t})$, productivity $(\tilde{PR}_{i,t})$ and R&D expenditures $(\tilde{RD}_{i,t})$ as follows: Solve for the optimal firm-type R&D choice γ_τ and aggregate δ creation rates. Calculate the aggregate growth rate g and equilibrium distribution of firm types ϕ^*_τ. Then simulate a five-year firm panel. First, draw the firm type from the distribution ϕ^*_τ and its initial state vector of products k, revenues z and quality jumps q. Using the creation and destruction rates γ_τ and δ, simulate the birth-death process of number of the prod-

[12]Lentz and Mortensen (2008) also assume three firm types, but estimate ε^q_1 for a firm of type 1. Their estimation produces $\varepsilon^q_1 = 0$. Moreover, this non innovating incumbent firm type accounts for 86 percent of all entry firms and 77 percent of equilibrium firms (manufacturing industry estimation p. 1366).

ucts. This exercise produces a panel of firm-year observations using the expressions in (17) and adding measurement error

$$\begin{aligned} \ln(\tilde{Y}_{i,t}) &= \ln(Y_{i,t}) + \xi^{Y}_{i,t} \\ \ln(\tilde{W}_{i,t}) &= \ln(W_{i,t}) + \xi^{W}_{i,t} \\ \ln(\tilde{RD}_{i,t}) &= \ln(RD_{i,t}) + \xi^{RD}_{i,t}. \end{aligned} \tag{21}$$

The simulated panel consists of all 5,290 incumbent firms in 1997, which we follow until 2001 assuming steady state. Due to firm exits, and the fact that we exclude firm entry (both in the model simulation and in the data), the 2001 cross-section does not reflect a steady state. The cross sectional shift from 1997 to 2001 consequently reflects the selected group of surviving firms. In addition, when computing the simulated R&D moments we sample firms as in Statistic Norway's R&D survey.

A firm's product cycle follows a continuous time birth-death process. To facilitate simulation, we follow Lentz and Mortensen and discretize the time space. A year is divided into 26 sub periods. In any given two week sub period, a firm with k products faces a probability of $1 - e^{-\frac{k\delta}{26}}$ of losing a product and a probability of $1 - e^{-\frac{k\gamma}{26}}$ of gaining a product.

We compute moments on the simulated panel, repeat the simulation 1,000 times and store the average simulated moments. In total we have 37 non-R&D moments (the same number of moments as Lentz and Mortensen, 2008) and 21 R&D moments. Tables 10 and 11 (appendix A) list the full set of empirical moments. Along the R&D dimension, we estimate the model on the distribution of R&D effort (intensity and level), correlations between R&D intensity and firm size and productivity, and the fraction of firms engaging in R&D (firms with positive R&D) and their size relative to non performers (firms with zero R&D). Both in the data and in the model we treat missing R&D observations, i.e. non sampled firms, as zeros.

Let Ω and $\widehat{\Omega}(\Lambda)$ denote the vectors of empirical and simulated moments, respectively. The parameter estimates are the solution to the minimization problem,

$$\min_{\Lambda} \left[\Omega - \widehat{\Omega}(\Lambda)\right]' A \left[\Omega - \widehat{\Omega}(\Lambda)\right], \tag{22}$$

where the weighting matrix A is the inverse of the diagonal covariance matrix of the empirical moments. The squared difference between simulated and empirical moments are consequently weighted by dividing by the variance of the empirical moment. These variances are obtained by bootstrapping the original firm sample

5000 times. Precisely estimated empirical moments are thus given more weight in the minimization. Standard errors for the parameters are estimated by bootstrap.[13]

4.2 Benchmark Estimation

Now we replicate the Lentz and Mortensen (2008) estimation on Norwegian data. Specifically, we drop R&D moments from the vector of moments $(\Omega, \widehat{\Omega})$. This provides a useful benchmark to Lentz and Mortensen (2008). They run estimations on both the entire Danish private sector and on particular industries. Since we use data on Norwegian manufacturing firms, the natural comparison is their corresponding estimation on Danish manufacturing firms (pp. 1366-1367). We report amounts in units of 1,000 NOK.

A. Non-R&D moments Table (1) shows estimated parameters, equilibrium values and a selection of targeted moments from our benchmark estimation. Table 12 in the appendix reports the full set of targeted moments.[14]

Overall, the model fits the data very well. The estimated model captures the mean, dispersion and median for productivity in 1997, as well as the cross sectional shift to 2001. Compared to the Danish manufacturing data, the correlation between the firm's value-added, size, and growth is essentially zero (-0.073 in Denmark, -0.006 in Norway) and the model is able to capture this relationship. In addition, the model matches both the persistence and mean reversion in productivity. As in the Danish data, productivity is positively correlated with output size (Y) and uncorrelated with input size (N), a feature the model fits quite well.

B. Reallocation and Growth Aggregate growth, estimated to 1.6 percent annually, arises because of the arrival of better quality products, produced with the same amount of labor. The contribution to growth varies across firm types according to

[13] We draw 500 bootstrap data samples from the original dataset. For each sample we estimate the model on the bootstrap sample. Both the data moments and simulated moments are re-centered around the corresponding moments from the full estimation.

[14] The estimation produces a large fraction of non-innovating incumbent firms, consistent with the results in Lentz and Mortensen. In contrast, however, the two R&D-performing incumbent firm types are quite different in terms of innovation intensities (γ_2, γ_3) in our estimation, whereas they are almost identical in Lentz and Mortensen (cf. table VII p 1366). But since the most innovative firm type only produces 0.4% of all goods in our estimation, the implication for reallocation is the same, i.e. the important margin is the resource reallocation of resources from non innovating firms to innovating firms.

Table 1: Benchmark Estimation

c_0/Z	c_1	η	Z	β_z	o_z	β_q	σ_Y^2
103.5	4.931	0.069	8891	0.428	3392	0.426	0.0296
σ_W^2	κ	g	l	μ	δ	$\frac{g_2}{g}$	
0.000	142.9	0.0161	19.0	0.97	0.124	0.445	
	ϕ_τ	ϕ_τ^*	K_τ	γ_τ	ε_τ	π_τ	
τ_1	0.7500	0.615	0.413	0	0	0	
τ_2	0.2499	0.384	0.578	0.0944	0.135	0.154	
τ_3	0.0002	0.0005	0.004	0.1216	1.311	0.409	

Selected moments (1997)

	model	data		model	data
$E(PR)$	471.6	477.8	$Corr(PR, N)$	0.000	-0.030
$std(PR)$	173.8	173.8	$Corr(PR, Y)$	0.117	0.124
$E(Y)$	13090	12872	$Corr(Y, \frac{\Delta Y}{Y})$	-0.026	-0.006
$Std(Y)$	23485	23183	$Corr(PR, \Delta PR)$	-0.363	-0.342

Notes: The benchmark estimation only targets non-R&D moments in the data. The minimum of objective function is 167.701.

their innovation step, q_τ, and their innovation rate, γ_τ. Moreover, high γ types innovate faster and capture market shares at the expense of low γ types. The main goal in Lentz and Mortensen (2008) is to quantify the role that this reallocation from less to more innovative firms plays in the growth process. They accomplish this by decomposing the contribution to annual growth into three parts:

$$g = \underbrace{\sum_\tau \gamma_\tau E[\ln(q_\tau)]\phi_\tau}_{g_1\text{: within types}} + \underbrace{\sum_\tau \gamma_\tau E[\ln(q_\tau)](K_\tau - \phi_\tau)}_{g_2\text{: between types}} + \underbrace{\eta \sum_\tau E[\ln(q_\tau)]\phi_\tau}_{g_3\text{: entry/exit}} . \tag{23}$$

The first term (g_1) measures the growth contribution made by continuing firms under the counterfactual that firm types are not allowed to increase their market share K_τ relative to their entry share ϕ_τ. The third term (g_3) measures the net effect of entry and exit. The key measure of reallocation emphasized in Lentz and Mortensen (2008) is captured by the second term (g_2). Consider a cohort of entry firms. Each firm enters with one product. Consequently, the distribution ϕ_τ of firm types on entry equals the distribution of products across firm types on entry. Over time, however, more innovative firm types grow faster than less innovative types, thereby gaining an increasing proportion of the cohort's market share. As a result of this selection, the steady-state distribution of products K_τ across types differs from the entry distribution ϕ_τ. This selection measures growth induced by the reallocation of market shares across types. Suppose we start out in an equilibrium with $g^* = g_1^* + g_2^* + g_3^*$. Now, if we counterfactually give all incumbent firms the same innovation rate $\gamma = g_1^*/(\sum_\tau E[\ln(q_\tau)]\phi_\tau)$, then there is no selection (i.e. $K_\tau = \phi_\tau$) and growth would decrease by g_2^*. From table (1) we see that imitators (type 1 firms) accounts for 75 percent of entrants' products, but only 41 percent of products in equilibrium. Hence, imitators lose roughly half of their market share to innovators due to selection.

Lentz and Mortensen's (2008) estimation for Danish manufacturing firms[15] implies that this selection accounts for 49 percent of aggregate growth. The process of entry and exit accounts for 25 percent, while the within-type contribution is 26 percent. In other words, if more innovative firms were not allowed to grow at the expense of less innovative firms, aggregate growth would be 49 percent lower. Our benchmark estimation we get similar results. Selection accounts for 44.5 percent, entry/exit 23.5 percent and within-type 32 percent.

C. R&D Moments Table 2 shows the model's fit along the R&D dimension. The model produces some degree of dispersion in R&D cost and R&D intensity, and produces the correct correlation signs. Firm's size and productivity are negatively correlated with research intensity across firms, and R&D intensity displays considerable persistence over time, $corr(RI, RI_{+2}) = 0.81$.

The model generates a negative correlation between R&D intensity and size that is much larger compared to the data. The positive contribution from firm-type heterogeneity is not enough to compensate for the negative impact of demand shocks on the correlation. Moreover, the model needs a substantial amount of demand variation in order to fit the firm-size distribution. Simulating the model without demand shocks shows that the median and dispersion of value-added increases by 72 percent and decreases by 42 percent, respectively. The degree of firm-type heterogeneity, which is primarily tied down by the gap between $corr(Pr, Y)$ and $corr(Pr, N)$, is not sufficient to generate the size dispersion observed in the data.

[15] p 1367. When they estimate the model on all private sector firms, selection accounts for 53 percent of growth

Table 2: Benchmark Estimation: Non targeted R&D Moments 1997

Moments	Data	Model	Moments	Data	Model
$E(RI)$	0.084	0.038	$corr(RI, Y)$	-0.170	-0.509
$std(RI)$	0.116	0.024	$corr(RI, PR)$	-0.188	-0.150
$Med(RI)$	0.044	0.036	$^{1}\frac{\#Firms_{RI=0}}{\#Firms}$	0.924	0.808
$E(RD)$	2907	698	$^{2}corr(RI, RI_{+2})$	0.688	0.810
$std(RD)$	5619	1081	$^{3}\frac{E(Y)_{RI>0}}{E(Y)_{RI=0}}$	4.44	3.53
$Med(RD)$	1163	455	$^{4}\frac{E(N)_{RI>0}}{E(N)_{RI=0}}$	4.44	3.21

Notes: The benchmark estimation only targets non-R&D moments in the data. [1]Fraction of firms with a zero R&D observation. [2]Correlation of R&D intensity between 1997 and 1999. [3]Average value-added for firms with positive R&D observations, relative to firms with zero R&D. [4]Average employment for firms with positive R&D observations, relative to firms with zero R&D.

The mean and dispersion of R&D effort (level and intensity) is too low compared to the data, and the same is true for the coefficient of variation. The aggregate R&D intensity is also too low. Among firms with positive R&D, aggregate R&D intensity in 1997 (total R&D expenditures to total value-added in 1997) is 65 percent smaller than in the data. Overall, the model produces too many firms with positive R&D, which on average are too small (in value-added terms), too homogeneous (in terms of dispersion in R&D) and invest too little in R&D compared to the data. These results indicate that if we add R&D moments to the estimation, resource reallocation between types becomes more important.

4.3 Estimation with R&D Moments

Now we turn to the estimation with R&D, to which we add the 21 R&D moments to the list of moments to match. Table (3) reports the estimated parameters.

Let us first consider how some key parameters adjust when adding R&D moments to the estimation. Recall from table (1) that the average R&D (both in terms of level and intensity) in the benchmark estimation is counterfactually low. Moreover, performers (types 2 and 3) are on average too small (in terms of value-added) relative to non performers. In order to increase firm R&D expenditure (measured as spending on R&D), the estimation procedure increases the cost of R&D. From table (3) we observe an increase in the cost parameters c_0 and c_1. To avoid reducing the incentive to innovate, we expect the gains from R&D (the expected jump in quality) to increase. Indeed, comparing tables (1) and (3) we see that the scale parameters ϵ_τ go up.[16] With higher average quality improvements, the average productivity would increase, as seen in equation 17. The reduction of firms engaging in R&D $(1 - \phi_1^*)$ compensates for this.

The annual growth rate is estimated to be 1.47 percent annually. We find that 87.8 percent of firms do not innovate (ϕ_1^*), and that these firms produce 69.5 percent of all goods (K_1). The expected life for these firms is nine years and they employ on average 22.8 workers. The main bulk of innovation comes from type 2 firms. This type accounts for 28.7 percent of goods produced in equilibrium and have an expected profit share of 23.3 percent. Type 2 firms are expected to survive for 20 years and have on average 52.3 production workers and 4.4 researchers. Type 3 firms are few but very innovative, producing only 1.7 percent of all products. They employ 265 production workers and 80 researchers on average and have a life expectancy of 41 years.

The model's fit along the non-R&D moments is still good. However, there are some trade offs. In particular, from table (3) we see that the estimation with R&D moments generates a somewhat smaller average productivity and larger dispersion in value-added and productivity relative to the benchmark estimation. In the process of matching R&D moments, the innovating firms not only become larger and more productive, but also more heterogeneous. In fact, relative to the benchmark estimation, both the standard deviation (conditional on performing R&D) of value-added and productivity increase by a factor of 1.9. The reduced share of firms performing R&D compensates for this, but not enough to avoid an increase in the unconditional standard deviation. The estimation penalizes a further reduction in the share of performers as this would make average productivity fall further below the corresponding data moment.[17]

A. R&D Heterogeneity Table 4 compares the fit of the model with R&D moments from the data in 1997.[18] The model fits both the dispersion and skewness of the R&D intensity distribution. Figure 3 depicts the data distribution with our estimation using R&D moments and the results from a counterfactual experiment in which we

[16] The coefficient of variation for q has gone up from 0.81 to 1.89 when adding R&D moments and the mean has roughly doubled from 1.38 to 2.15

[17] As a robustness check, we have estimated the model without using the fraction of non-performers as target and have obtained similar results. In particular, the model still generate a substantial fraction of firms not performing R&D in equilibrium (ϕ_1^* =0.85) .

[18] In appendix A, table 14 we present the model's fit to 2001 R&D moments

Table 3: Parameters: Estimation with R&D Moments

c_0/Z	c_1	η	Z	β_z	o_z	β_q	σ^2_Y
498.3	5.366	0.0814	10296	0.614	1499	0.369	0.0298
(126)	(0.1)	(0.02)	(268.7)	(0.02)	(258.9)	(0.017)	(0.003)
σ^2_W	σ^2_{RD}	κ	g	l	μ	δ	$\frac{g_2}{g}$
0.003	0.360	153.9	0.0147	21.7	1.47	0.1103	0.718
(0.002)	(0.07)	(1.96)	(0.0008)	(0.56)	(0.04)	(0.002)	(0.015)

	ϕ_τ	ϕ^*_τ	K_τ	γ_τ	ε_τ	π_τ
τ_1	0.942	0.878	0.696	0	0	0
	(0.003)	(0.005)	(0.01)			
τ_2	0.058	0.121	0.287	0.0938	0.274	0.233
	(0.003)	(0.005)	(0.01)	(0.003)	(0.03)	(0.01)
τ_3	0.0002	0.001	0.017	0.1091	1.827	0.442
	($4 \cdot 10^{-5}$)	(0.0001)	(0.002)	(0.002)	(0.58)	(0.03)

Selected moments (1997)

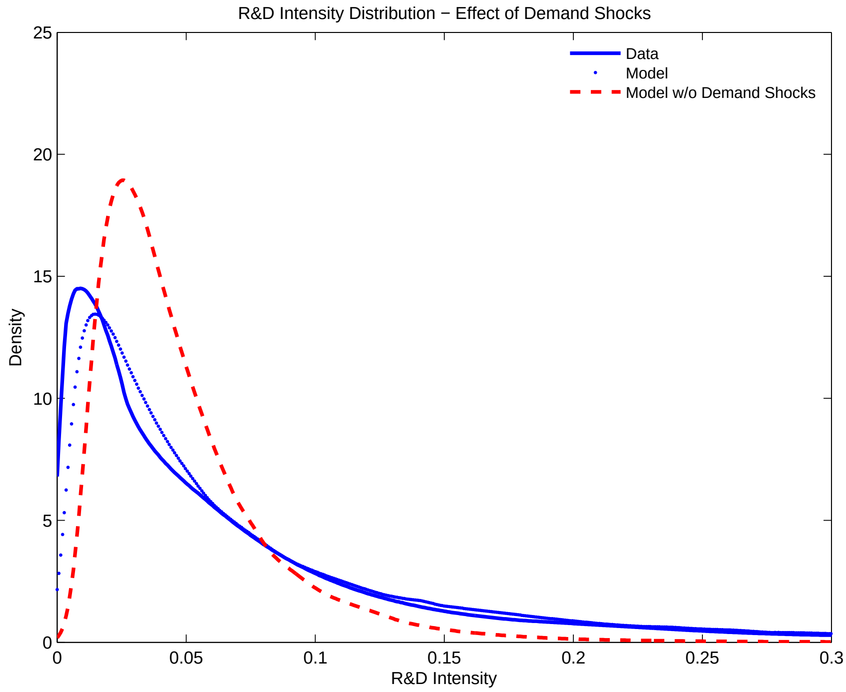
	model	data		model	data
$E(PR)$	468.6	477.8	$Corr(PR, N)$	0.023	-0.03
$std(PR)$	183.9	173.2	$Corr(PR, Y)$	0.14	0.124
$E(Y)$	12988	12872	$Corr(Y, \frac{\Delta Y}{Y})$	-0.02	-0.006
$Std(Y)$	25775	23183	$Corr(PR, \Delta PR)$	-0.365	-0.342

Notes: Estimation with R&D targets both non-R&D and R&D moments. Standard errors in parentheses. The minimum of objective function is 331.518

shut down demand shocks. Even though the estimation only targets the mean, median, and standard deviation, the model captures the shape and location of the entire distribution. In particular, it is able to match the right tail of the distribution fairly well. Figure 3 displays the R&D intensity distribution when we shut down demand shocks. Notice that the fit misses the right tail of the distribution. The model without demand shocks generates too few firms with high R&D intensity. Intuitively, firms that draw bad demand shocks will have high R&D intensity.

To further evaluate the relative importance of shocks, we shut down each of the shocks and recompute the moments. Table 5 depicts R&D moments under different scenarios. Consider what happens when measurement errors and demand shocks

Figure 3: R&D Intensity, Value-added and Worker Productivity.



Notes: The figure shows kernel densities on actual data (1997) and on simulated data, with and without demands shocks

are shut down, as shown in Table 5 column (2). All within-type R&D heterogeneity is eliminated and the type-specific R&D intensity becomes $wc(\gamma_\tau)/Z$. We end up with two R&D intensity observations with a dispersion of $std(RI) = 0.008$. Consequently, most of the dispersion is due to demand shocks and measurement error. Since R&D is proportional to the state variable k, demand shocks are important to generate heterogeneity in R&D intensity. Simulation without demand (column 3) reduces the dispersion from 0.121 to 0.036.

In contrast, demand shocks play no role in explaining the shape of the distribution of the R&D spending level. The results in table 4 show that the estimation misses the average R&D expenditures by 29 percent (2,907 in the data, 2,072 in the model) but matches both the median and dispersion quite well. Dispersion is explained by three factors: Firm-type heterogeneity in innovation intensity (γ_τ), within-type dispersion in product size (k), and R&D measurement error (σ^2_{rd}). Overall, the main contribution comes from firm-type heterogeneity and the product birth-death pro-

Table 4: Estimation including R&D Moments: Model Fit in 1997

Moments	Data	Estimation (benchmark)	Estimation (w/ R&D)	Moments	Data	Estimation (benchmark)	Estimation (w/ R&D)
$E(RI)$	0.084	0.038	0.089	$cor(RI, Y)$	-0.170	-0.509	-0.267
$std(RI)$	0.116	0.024	0.121	$cor(RI, PR)$	-0.188	-0.150	-0.148
$Med(RI)$	0.044	0.036	0.049	[1] $\frac{\#Firms_{RI=0}}{\#Firms}$	0.924	0.808	0.920
$E(RD)$	2907	698	2072	[2] $cor(RI, RI_{+2})$	0.688	0.810	0.440
$std(RD)$	5619	1081	5494	[3] $\frac{E(Y)_{RI>0}}{E(Y)_{RI=0}}$	4.44	3.53	4.40
$Med(RD)$	1163	455	1131	[4] $\frac{E(N)_{RI>0}}{E(N)_{RI=0}}$	4.44	3.21	3.68

Notes: [1]Fraction of firms with a zero R&D observation. [2]Correlation R&D intensity between 1997 and 1999. [3]Average value-added for firms with positive R&D observations, relative to firms with zero R&D. [4]Average employment for firms with positive R&D observations, relative to firms with zero R&D.

cess, while measurement error plays a minor role. Table 5 column (4) shows the simulation results when R&D measurement error is eliminated. In this case, the standard deviation of R&D spending falls by 14 percent, from $5,494$ to $4,717$. Firm-type heterogeneity, on the other hand, is important. Computing the standard deviation only on type 2 firms gives a dispersion in RD of only $2,234$. The reason why type heterogeneity is important for dispersion in RD and not for RI, is that R&D spending is proportional to the number of products. Type 3 firms invest 2.6 times as much in R&D per product than firm type 2 and has on average seven times more products. However, since RD scales with the number of products, RI is on average only 2.6 times higher. In contrast, the distribution of product size (k) is of first-order importance for the distribution of R&D spending, $RD = wc(\gamma_\tau)k$. Even though type 3 firms accounts for only 0.1 percent of firms in equilibrium, the associated type 3 logarithmic product-size distribution has a large dispersion (more than 142 times the variance of type 2), which contributes disproportionally to the overall dispersion in R&D spending.

Table 5: Effect of Shocks on Simulated Moments in 1997.

Moments	All Shocks	No Shocks	No Demand Shock	No Measurement Error	No q Shocks
	(1)	(2)	(3)	(4)	(5)
$E(RI)$	0.089	0.043	0.049	0.075	0.089
$std(RI)$	0.121	0.008	0.036	0.067	0.121
$Med(RI)$	0.049	0.042	0.039	0.052	0.049
$E(RD)$	2072	2159	2129	2035	2065
$std(RD)$	5494	4900	5535	4718	5449
$Med(RD)$	1131	1313	1170	1306	1127
$corr(RI, Y)$	-0.267	0.284	-0.023	-0.357	-0.270
$corr(RI, PR)$	-0.148	0.30	-0.040	-0.181	-0.421
$\frac{\#Firms_{RI=0}}{\#Firms}$	0.920	0.929	0.923	0.922	0.920
$corr(RI, RI_{+2})$	0.440	1	0.074	0.842	0.434
$\frac{E(Y)_{RI>0}}{E(Y)_{RI=0}}$	4.40	4.39	4.19	4.51	4.42
$\frac{E(N)_{RI>0}}{E(N)_{RI=0}}$	3.68	3.66	3.51	3.77	3.64

Notes: This table shows the simulated R&D moments when we counterfactually shut down shocks. Column (1) displays the results from the estimation. Column (3) shut down demand shocks. Column (4) shuts down measurement error. Column (5) shut down stochastic quality improvement. Column (2) combines (3)-(5), i.e. no demand shocks, no q-shocks and no measurement error

B. Non Performers From the results presented in table 4 we observe that the model assigns a value 0.92 to the fraction of firms with zero R&D. Note that this fraction is higher than the true fraction of non performers, due to the sampling of R&D observations for firms with less than 50 workers. Note that we do not target the fraction of non performers for firms above 50 workers, but the model gives a fraction of 0.71, in line with what we observe in the data (0.65). Firms reporting positive R&D are on average 4.40 times larger than firms with zero R&D. However, in terms of employment, the model under predicts the relative firm size by 17 percent. Consequently, the model predicts too high of a value-added per worker for R&D performers relative to non performers.

In the data, the average value-added per quality-adjusted worker is the same for the two groups, whereas in the model, performers have 25 percent higher worker productivity. Consider the case where we eliminate measurement error, demand

and supply shocks. Then, worker productivity is given by

$$PR = \frac{Z}{Z(w+\kappa)^{-1}q_{\tau_i}^{-1} + c(\gamma_{\tau_i})}.$$

Note that for non-performers, $q = 1$ and $\gamma = 0$. Innovating firms ($q > 1$) need fewer production workers per product than firms that do not innovate. This contributes to higher productivity PR for innovating firms. On the other hand, the presence of research workers in the denominator, $c(\gamma_{\tau_i})$, works in the opposite direction. The estimated parameters imply that the former effect dominates.[19]

C. Productivity, Firm's Size and R&D Intensity Table 4 shows that the estimated model produces a correlation between research intensity and productivity of -0.148. As explained in section 3.4C, this negative relationship can be generated by a combination of demand shocks and value-added measurement error.[20] In addition, firm-type heterogeneity in general can drive the correlation in either direction. To isolate the effect of type heterogeneity, recall that the correlation between R&D intensity and productivity can be written as follows:

$$corr\left(\frac{wc(\gamma_\tau)}{Z}, \frac{Z}{Z(w+\kappa)^{-1}q_{\tau_i}^{-1} + c(\gamma_{\tau_i})}\right),$$

where all sources of heterogeneity (demand, supply and measurement error), except firm types, are shut down. The correlation is now completely determined by firm type differences in quality jumps, q_τ, and innovation intensity, γ_τ. In this case, we see from table (5) column 2 that $corr(RI, PR)$ is positive at 0.3, which implies that type 3 firms end up with higher R&D intensity and higher productivity than type 2 firms. The labor-saving feature of larger quality jumps ($q_3 > q_2$) dominates the higher demand for research workers ($c(\gamma_3) > c(\gamma_2)$). Consequently, demand shocks and measurement error in value-added produce the negative relationship in the esti-

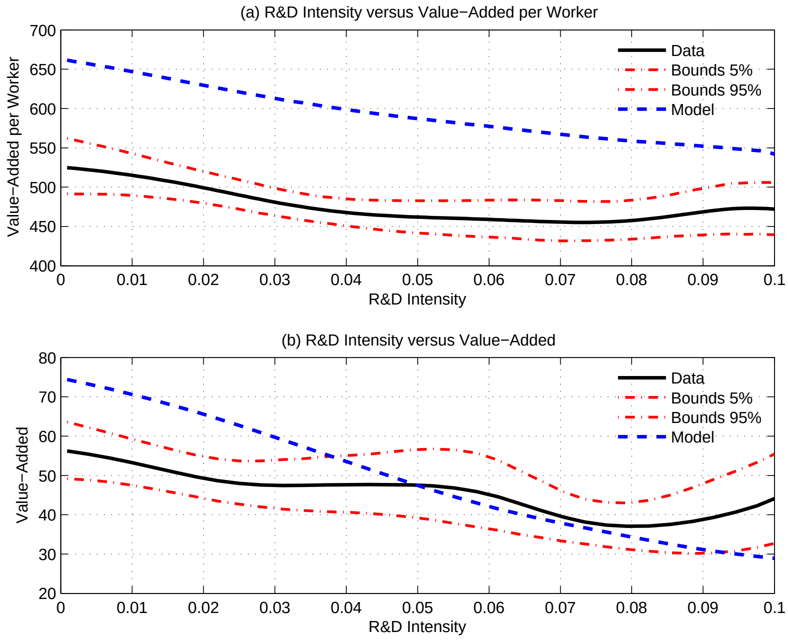
[19]There are several factors that might explain why the data shows identical worker productivity across the two groups. First, if we assume that the fraction of the firm's employees involved in R&D are the same in the data as in the model (8 percent), then firms that report positive R&D have about 9 percent higher value-added per production worker than firms with zero R&D. In addition, because of the sampling of R&D observations in the data, some firms are observed with zero R&D even if they do innovate. Finally, the equality of worker productivity between the two groups in the data is a result of the quality-adjusted measure of employment. If we instead use non quality adjusted employment, firms that report positive R&D have 11.2 percent higher value-added per worker than non performing firms.

[20]Notice that R&D measurement errors and q shocks will reduce this correlation to zero.

mated model. Simulations performed without demand variations give a correlation of -0.04 (table 5, column 3).

Regarding the relationship between R&D intensity and firm size, there is also a tension between firm-type heterogeneity (causing positive correlation) and demand and value-added measurement error (causing a negative correlation). With firm types as the only source of heterogeneity, the correlation between size and R&D intensity is 0.28. However, demand shocks and value-added measurement error drives the correlation down to -0.26, most of which is due to demand shocks. Shutting down this variation gives a correlation of -0.02.

Figure 4: R&D Intensity, Value-Added and Worker Productivity.



Notes: Kernel regressions of R&D intensity on value-added per worker (panel a) and value-added (panel b). Both panels compares using actual data (1997), with 95 percent confidence bounds, with simulated data from the estimated model.

Figure (4) shows how both firm size and worker productivity interact with different levels of R&D intensity. For the relationship between R&D intensity and productivity, shown in panel (a), the model matches the slope but under predicts the level because we overestimate the average productivity among R&D performers, as explained in section 4.3B. Regarding the relationship between R&D intensity and firm size, shown in panel (b), the model succeeds in explaining this relationship over the range between the median and mean of the R&D intensity distribution. But firms in the lower end of the R&D intensity distribution are larger in the model than in the data. In the model, firms with large demand draws tend to be large in size and have low R&D intensity relative to firms with small demand draws.

For the benchmark estimation we found that the correlation between R&D intensity and firm size was overly negative compared with the data (see table (1)). When R&D moments are added to the estimation, correlation in the model becomes less negative without resorting to smaller demand shocks.[21] This is because firms have more products on average relative to the benchmark (42 percent), which means that the impact of demand shocks falls. In addition, firm-type heterogeneity contributes more positively to this correlation in the R&D estimation than in the benchmark estimation.[22] Consequently, when adding R&D moments we get more heterogeneous innovators. Demand shocks are key to allowing for more firm-type heterogeneity and at the same time not violating the negative correlation between R&D intensity and firm size.

D. Persistence $corr(RI, RI_{+2})$ At 0.44, the R&D intensity persistence in the model is too low compared to the data (0.69), due to measurement error in R&D. The absence of shocks (shutting down demand, supply and measurement error) would imply perfect persistence (i.e. constant R&D intensity over time). Simulation without demand shocks drives the persistence down to 0.075, while simulating without measurement error gives a persistence of 0.842. To get a correlation of 0.44 we thus need product demand shocks. These shocks stay with the firm until it loses the product and thereby create persistence in value-added which carries over to research intensity.

E. Selection Effect Recall that in the benchmark estimation, the fraction of R&D performing firms is 0.19. This is over twice the fraction that we observe in the data. However, the R&D estimation correctly predicts the fraction of firms that undertakes R&D.[23] Consequently, when we also match R&D moments we end up with

[21] The standard deviation of demand shocks is 15500 and 15000 in the benchmark and R&D estimation respectively.

[22] To make this comparison we shut down all shocks, leaving firm types as the only source of heterogeneity in R&D intensity. The correlation is then 0.28 and 0.14 in the R&D estimation and benchmark, respectively.

[23] The true fraction firms performing R&D differs from the number observed, due to the sampling of R&D observations. The equilibrium fraction of R&D performing firms is 0.38 in the benchmark

fewer performers, as shown in table 6. These relatively few firms are also bigger and more research intensive than in the benchmark estimation. Relative to non performers these firms have on average 4.40 (3.53) times higher value-added and 3.68 (3.21) more workers in the R&D estimation (benchmark estimation). The aggregate research intensity is now 4.6 percent compared to 2.2 percent in the benchmark.[24] Overall, we end up with innovating firms being fewer, bigger, and more research intensive. As a result, the growth contribution of selection of firm types increases from 44.5 percent to 71.8 percent.[25]

Table 6: Model Fit II: Estimation with R&D Moments.

	Data	Estimation (w/R&D)	Estimation (Benchmark)
Firms with $RI > 0$	8%	8%	19%
$E(Y_{RD>0})/E(Y_{RD=0})$	4.44	4.40	3.53
$E(RD_{RD>0})/E(Y_{RD>0})$	6.4%	4.6%	2.2%

Notes: Estimation w/R&D targets both non-R&D and R&D moments. Minimum of objective function: 331.518

4.4 External Validity

We have shown that including R&D moments improves the model's ability to match cross-section moments. Now we perform a series of robustness checks, to assess how the model performs for moments we did not match.

A. R&D Response to Tax-Credit We start by comparing the model response to a R&D subsidy with the outcome of a 2002 R&D reform in Norway, analyzed in Bøler et al. (2015). This study exploits the introduction of a tax-credit that enabled firms to deduct 20 percent of R&D expenditures (up to a threshold of NOK 4 million) from their tax bill. Effectively, this policy reduced the marginal cost of R&D by 20 percent

estimation and 0.12 in the R&D estimation.

[24] Aggregate research intensity is measured as the ratio of average R&D spending to average value-added among firms engaging in R&D . It is not an explicit target in the estimation. However, it is implicitly targeted since we target relative value-added between performers and non-performers, average value-added and average R&D expenditures.

[25] This result is robust to leaving out the fraction of non-performing firms as a targeted moment in the estimation. In this case the selection effect is 70.6 percent

for firms with less than 4 million in R&D expenditures. Bøler et al. (2015) conclude that the reform incentivized firms that prior to the credit had positive R&D, but less than NOK 4 million, to increase R&D expenditures between 35 and 72 percent (depending on the identification strategy) during the 2003-2005 period.[26]

We use our estimated model to evaluate the impact of a similar, but simplified, reform. In the model economy, we implement a 20 percent subsidy to incumbent firms' R&D investment, but impose no upper threshold. The optimal innovation choice is still proportional to number of products, $I = \gamma_\tau k$, and we can analyze the effect on innovation intensities γ_τ using the solution to the firm problem in (11):

$$wc'(\gamma_\tau) = v_\tau$$
$$v_\tau = \frac{ZE\pi(q_\tau) - wc(\gamma_\tau)}{r + \delta - \gamma_\tau}.$$

We view at the effect of this policy reform from two perspectives. First we look at the firm's response to a subsidy (s) equal to 20 percent of R&D expenditures, ignoring equilibrium effects, i.e. we hold the wage rate w and aggregate destruction δ rate constant. Firms now face a net (of subsidy) R&D cost of $(1 - s)wc(\gamma)$. Type 2 and 3 firms respond by increasing their innovation intensity by 5.3 and 5.9 percent, respectively. Consequently, gross R&D expenditures per product increase by 39 percent (type 2) and 44 percent (type 3).

We interpret this increase in R&D as a short-run effect. With a higher innovation intensity, firms will gain more products over time and thus invest more in R&D. If we still assume a constant wage rate and a constant aggregate destruction rate, type 2 firms on average will have 27 percent more products and thus a total increase in gross R&D expenditures of 76 percent.[27] However, in equilibrium, when individual firms innovate more intensely, the aggregate destruction rate δ goes up. In the new stationary state, type 2 and 3 firms increase their innovation intensity by 4.1 and 4.2 percent respectively, and the aggregate destruction rate increases by 3.5 percent. On average, firms employ 30 percent more researchers.[28] Equilibrium effects con-

[26] Bøler et al. (2015) report an increase in R&D expenditures between 0.30 and 0.54 log points. We use the transformation $X_t/X_{t-1} = exp(logpoint) - 1$ to arrive at the percentage change.

[27] Keeping the destruction rate δ constant implies that type 3 firms choose $\gamma_3 > \delta$ and the expected number of products is +∞. In equilibrium, however, the aggregate destruction rate adjusts such that $\gamma_3 < \delta$. Notice that the number of researchers also grows by 76 percent as the wage rate is constant.

[28] The reason we report the stationary state effect of R&D subsidy on R&D workers is that the subsidy increases the economy's growth rate. Hence, in the new stationary equilibrium, the real wage and thus the research wage bill grows at a higher rate than in the initial equilibrium. Hence, it is not possible to compare the level of R&D expenditures across the two equilibria.

sequently dampen the initial increase in R&D, since innovation intensities and firm product size are decreasing in δ.

B. Large Firms Recall that in our estimation we use R&D moments from all firms. Therefore, we use the model's ability to account for moments of the large firms in our sample as an external validity test. In table (7) we display how the model's results compare with the data for firms with more than 50 workers. Indeed, the model matches quite well the level, dispersion, and skewness of the R&D intensity and R&D expenditures. It also generates a lower correlation between R&D and firm size, while at the same time it maintains a high negative correlation between R&D intensity and productivity. This result indicates that the negative correlation between R&D intensity and firm size is driven by small firms. The intuition behind this result, is that larger firms tend to have more products than smaller firms, and hence the impact of a product-specific demand shocks is lower. Moreover, the model matches the relative size (in terms of employment and value-added) of performers. Finally, the model underestimates the persistence of R&D intensity for large firms.

Table 7: The Model's Fit for Large Firms (non-targeted)

Moments	Data	Model	Moments	Data	Model
$E(RI)$	0.06	0.04	$corr(RI, Y)$	-0.02	-0.05
$std(RI)$	0.09	0.04	$corr(RI, PR)$	-0.13	-0.15
$Med(RI)$	0.03	0.03	[1] $\frac{\#Firms_{RI=0}}{\#Firms}$	0.66	0.71
$E(RD)$	2906	2071	[2] $corr(RI, RI_{+2})$	0.68	0.34
$std(RD)$	5618	5494	[3] $\frac{E(Y)_{RI>0}}{E(Y)_{RI=0}}$	1.36	1.79
$Med(RD)$	1163	1131	[4] $\frac{E(N)_{RI>0}}{E(N)_{RI=0}}$	1.34	1.50

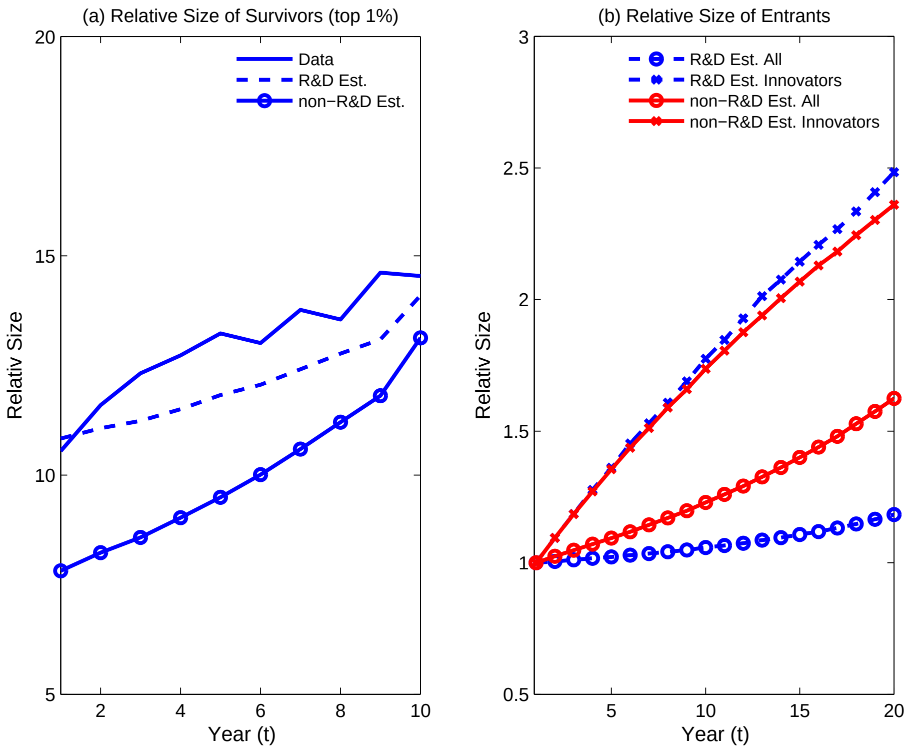
Notes: We use parameters from the estimation with R&D moments; however moments for firms with more than 50 workers were not used in the estimation. [1]Fraction of firms with a zero R&D observation. [2]Correlation of R&D intensity between 1997 and 1999. [3]Average value-added for firms with positive R&D observations, relative to firms with zero R&D. [4]Average employment for firms with positive R&D observations, relative to firms with zero R&D

C. The Firm Life Cycle Luttmer (2011) finds that firm-type heterogeneity is needed to match the size distribution and the relatively young age of large firms. We check the size history of those firms that reach the top 1 percent in the size distribution.

Again, this a feature that we have not included in our estimation.[29]

In figure (5) panel (a), we plot the relative size of firms that end up among the largest one percent of firms. Both in the data and in the model, we construct a 10-year sample and select the firms that are in the top 1 percent at the end of the sample. We exclude entrants and exitors in both the data and simulation. Then, we compare how the average value-added of these firms has grown relative to the average firm size in the economy over sample period. In the simulation we compute this ratio for both the benchmark and the R&D estimation.

Figure 5: Relative size of survivors (top 1%) and entrants.



Notes: Panel (a): graphs show the average value-added of continuing firms, conditional on being in the top 1 percent of the size distribution in period 10, relative to average value-added of incumbents over time. For the data, period 1 corresponds to 1996, and period 10 to 2005. Panel (b): Average value-added of a simulated cohort of entrants (period 1) relative to average value-added of incumbents over time (t+1 normalized to one).

We find that in the R&D estimation, both the initial size and the growth history of large firms are in line with the data. In contrast, for the benchmark estimation firms

[29]Recall that in our estimation we only match relative size after four years.

grow faster during the transition to the top 1 percent. We also see from tables (12) and (13) that the implied growth of average value-added from 1997-2001 is higher in the benchmark estimation compared to the R&D estimation. To shed some light on why firms grow faster when using the benchmark estimation we study the relative growth of a cohort of entrants. Figure (5) panel (b) shows that entry firms grow faster compared to the R&D estimation. The reason is that in the benchmark estimation, an entry cohort consists of a larger fraction of fast growing innovating firms. Recall that the entry share of type 1 firms (non performers) is 0.75 and 0.94 in the benchmark and R&D estimation, respectively, so the presence of more innovating entry-level firms in the benchmark estimation yields the higher growth rate.

5 Policy: R&D Subsidy

As in most endogenous growth models, the decentralized equilibrium growth rate in the Klette-Kortum (2004) model need not equal the welfare-maximizing growth rate. Hence, there is a potential role for policy interventions, as there are several sources of inefficiencies that can lead to too little growth in equilibrium.[30] First, all firms innovate at the technology frontier (the quality frontier is common knowledge). Consequently, the social return to a successful innovation lasts forever. In contrast, the private return to a firm only lasts until the product, and its associated profit, is lost (business stealing effect). This private depreciation of R&D investment contributes to insufficient innovation. Firm-type heterogeneity introduced in Lentz and Mortensen (2008) creates an additional source of inefficiency, caused by misallocations of R&D spending across firms and consequently too little resource reallocation. Through the process of equilibrium selection, the steady-state distribution of products K_τ across firm types differs from the entry distribution ϕ_τ. In general, the bigger share of goods K_τ produced by firms generating high quality innovations, the larger is growth. However, the amount of reallocation generated in equilibrium tends to be too low due to the business stealing effect.

Real world subsidies, however, often fail to target the most effective innovators. By studying stylized reforms, we illustrate that failing to target the best innovators may lead to subsidies creating small, or indeed adverse, growth effects. A subsidy's effectiveness depends crucially on how it influences the reallocation channel. In general policies that increase R&D and shift the composition of products to firms producing high quality innovations will stimulate growth.

[30] For example, see Acemoglu et al. (2013), Li (2001), and Atkeson and Burstein (2014).

Table 8: The Effect of R&D Subsidies on Aggregate Growth.

Reform:	g	K_1	K_2	K_3
Estimation with R&D Moments	1.47	69.6	28.7	1.7
Subsidy on:				
(I) Incumbents and Entrants	1.53	69.5	28.6	1.8
(II) Incumbents	1.82	67.0	28.5	4.4
(III) Entrants and type 2 firms	1.40	69.9	29.8	0.30
(IV) Type 3 firms	2.45	66.2	21.5	12.3

Notes: Numbers reported as percentages. All reforms are performed using parameter values from the estimation with R&D moments

We run four policy experiments, as in section 4.4A, where we subsidize R&D expenditures and evaluate the impact on the growth rate along balanced growth path. In each experiment, we set the total subsidy to 0.7 percent of the aggregate labor income of production workers (which is a proxy for GDP), financed by a non distortionary tax on consumers. The results are shown in table (8).

In experiment (I) all firms receive a R&D subsidy, while in (II) only incumbent firms are subsidized. The growth rate increases from its initial level of 1.47 percent to 1.53 when we subsidize all firms and to 1.82 percent when we only subsidize incumbents. Clearly, the growth effect is much larger when the policy reform only targets incumbent firms. The key to understanding this result is the change in the composition of products across firm types. In the second experiment, the share of products produced by the type 3 firms is 4.4 percent, compared to 1.8 percent in the first experiment. Table (8) shows that type 3 firms grow at the expense of type 1 firms (the imitators). Innovation by type 2 firms has an adverse effect on the incentive to innovate for high types (through the impact on aggregate destruction). In particular, high entry innovation reduces the incentive for incumbent firms to innovate, and thus helps sustain a large fraction of firm type 1 in equilibrium, which is bad for growth.[31]

[31] Lentz and Mortensen (2016) extends Lentz and Mortensen (2008) by solving for the planners balanced growth path allocation. Compared to the equilibrium outcome, the optimal allocation implies a twice as high growth rate.

Overall, going from reform (I) to (II) raises the growth rate by 19.0 percent. In comparison, when using parameters from the benchmark estimation, going from reform (I) to (II) raises the growth rate with only 6.6 percent. This difference is the result of the stronger selection effect we get, once we account for R&D in the estimation. A stronger selection effect means that implementing bad policies have larger consequences.

Consider the effect of implementing stylized size-dependent subsidies (size being measured in terms of R&D spending). Reform (III) provides a subsidy to incumbent type 2 firms and potential entrants, while reform (IV) targets only type 3 firms.[32] Reform (III) induces a reallocation of products from type 3 to type 2 firms. This effect is so strong that aggregate growth falls, due to the combination of two effects. First, when entrants and type 2 firms receive R&D subsidies, these firms raise their innovation intensity and gain products at the expense of type 3 firms (reallocating products away from the high quality innovator). In addition, the induced increase in the aggregate innovation rate adversely impacts the incentive for type 3 firms to innovate (through the creative destruction channel). This causes innovation intensity to fall, which induces further reallocation of products away from the high quality innovator. Reform (IV) on the other hand, creates a massive reallocation of products to type 3 firms, and hence the growth rate increases substantially.[33]

Acemoglu et al. (2013) finds that an optimal R&D policy involves subsidizing both entrants and incumbent firms that are high quality innovators. In that model there is also firm-type heterogeneity with respect to innovation ability (high and low types), realized upon entry. As in our model, subsidizing the high type encourages firms of this type at the expense of the low types that. However, our model shows that subsidizing entry-level firms has a different outcome. The reason for this is that, in Acemoglu et al. (2013)'s model all incumbent firms innovate. Moreover, the high type faces a fixed probability of becoming a low type over time. Consequently, the inflow of new firms (both high and low) helps to sustain a high type in equilibrium and an entry subsidy raises the equilibrium share of firms producing high quality innovations. Our empirical approach also differs from Acemoglu et al. (2013) as we estimate the model using observations from all firms, of which many report zero

[32]It is not a true size-dependent reform, because it targets firm types rather than firms based on size, but it highlights some important negative reallocation effects that arise in the model from subsidizing small firms. The reason is that small firms in the model are predominantly type 2 firms.

[33]It is important to emphasize that a type-dependent reform exaggerates the negative reallocation effects relative to a true size-dependent reform, since highly innovative firms will be below the size cut off (particularly young firms) and some less innovative firms will be above the cut off (especially after the reform is implemented).

R&D, whereas they focus on firms with positive R&D.

Since we only consider balanced growth path effects the growth effects in our experiments should be interpreted with caution. First, since type 3 firms constitute only a very small fraction of entry firms (0.02 percent), the transition to a new balanced growth path might be slow. More broadly, Atkeson and Burstein (2014) show, through numerical examples, that R&D subsidies in Klette-Kortum type endogenous growth models tend to have small effects in the short-to medium-term. It should be noted, however, that they consider a framework in which there is no misallocation of research activity across firms (there is no type heterogeneity). In our model, a subsidy generates more research effort in the aggregate, but it also influences the allocation of R&D (and products) across firms.

6 Conclusion

In this paper we estimate a general equilibrium model of firm-level innovation using observations on firm size, productivity, and R&D expenditures from a panel of Norwegian manufacturing firms. In particular, we estimate an extended version of the Klette-Kortum model, further developed in Lentz and Mortensen (2008), to quantify the relative importance of different sources of R&D heterogeneity and the link to resource reallocation and growth.

We find that reallocation has a larger effect on aggregate growth when the model is used to explain (in addition to other moment) firm's dispersion in R&D intensity and the negative correlation between R&D intensity and firm size observed in the data. We first replicate the study by Lentz and Mortensen (2008) using a sample of Norwegian manufacturing firms. To make our estimation comparable to Lentz and Mortensen, we first exclude observations on R&D, and find that the reallocation effect accounts for 44.5 percent of aggregate growth. This magnitude is similar to the 49 percent they find for the Danish manufacturing sector. However, our estimation misses some key empirical R&D patterns: firms doing R&D are too many, too small, and invest too little in R&D relative to data. When, we re-estimate the model by adding R&D moments, we find that the model has a good fit to both R&D and non R&D moments. The estimated model fits the empirical distribution of R&D effort (mean, dispersion and skewness) as well as the negative correlation between research intensity and firm size. More importantly, the new parameters imply a larger role for reallocation, which explains 72 percent of aggregate growth. Quantitatively, product demand shocks, measurement error, and firm differences in the ability to conduct R&D are all key factors to account for the observed heterogeneity in R&D effort.

We find that demand shocks and innovative differences are important for explaining the shape of the R&D distribution and the correlation between R&D intensity and firm size. Interestingly, when we shut down demand shocks we miss the fit on the right tail of the distribution. The model without demand shocks generates too few firms with high R&D intensity. Intuitively, firms that experience bad demand shocks will have high R&D intensity.

We also conducted several external validity tests. Our model is consistent with the firm-level response to R&D subsidies that are in line with micro evidence from a natural experiment (as in Bøler et al. (2015)). In the short run, firms increase their R&D spending by roughly 40 percent in response to a 20 percent R&D subsidy. Furthermore, the model is able to explain several cross sectional and dynamic moments for R&D, size, and productivity for large firms. The model also is able to explain features of the life cycle of firms over longer horizon.

Finally, we use the estimated model to quantitatively explore how R&D subsidies affect aggregate growth. We find that subsidies are successful in increasing growth, but that the effect depends crucially on how these influence the reallocation channel.

The model abstracts from openness and trade, which could potentially affect our results. For example, the diffusion of ideas across borders could reduce the importance of domestic R&D, while foreign competition and access to foreign markets could change the firms' incentives to innovate and the link between R&D subsidies, resource reallocation, and growth. Eaton and Kortum (2001) and Atkeson and Burstein (2010) consider innovation and growth in open economies, but abstract from product innovation by incumbent firms. An interesting extension would be to consider the Klette-Kortum/Lentz-Mortensen model in an open economy setting to understand the link between international competition, firm dynamics, and innovation.

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A Appendix

Table 9: Descriptive Statistics (1997).

Statistics	All Firms	10-50 Workers	> 50 Workers
Number of firms	5290	2087	684
Average value-added	12872	10475	57436
Average Employment	27.5	21.9	124.1
Average Productivity	482.2	479.2	467.8
Average R&D expenditure	247	75.6	1470
% R&D>0	7.6	6.9	35.1
Average RI (performers)	0.08	0.09	0.068

Notes: Value-added and R&D expenditures are reported in units of 1000,- NOK. The above 50 and 10-50 worker categories are based on quality-adjusted workers). Employment is number of quality-adjusted workers, Productivity is expressed units of 1000,- NOK per quality-adjusted worker. R&D intensity (RI) is the ratio of R&D expenditure divided by value-added.

Table 10: Non-R&D Data Moments (Standard Errors in Parentheses)

Moments	1997	2001	Moments	1997	2000
$E(Y)$	12872	14998	$Cor(PR, PR_{+1})$	0.735	0.697
	(317.8)	(479.1)		(0.017)	(0.034)
$std(Y)$	23183	28161	$Cor(PR, \Delta PR)$	-0.342	-0.371
	(796.0)	(1797.3)		(0.043)	(0.048)
$Med(Y)$	4985	6254	$Cor(PR, \frac{\Delta Y}{Y})$	-0.126	
	(105.0)	(140.4)		(0.016)	
$E(W)$	8144	10828	$Cor(PR, \frac{\Delta N}{N})$	0.037	
	(196.0)	(321.8)		(0.017)	
$std(W)$	14394	18991	$E(\frac{\Delta Y}{Y})$	-0.007	
	(472.6)	(1060.3)		(0.009)	
$Med(W)$	3157	4655	$std(\frac{\Delta Y}{Y})$	0.612	
	(67.8)	(114.2)		(0.040)	
$E(PR)$	477.8	503.6	$Cor(Y, \frac{\Delta Y}{Y})$	-0.006	
	(2.4)	(2.8)		(0.009)	
$std(PR)$	173.8	168.0	*within*	0.330	
	(7.4)	(7.8)		(0.074)	
$Med(PR)$	444.1	473.5	*between*	0.311	
	(1.68)	(2.02)		(0.071)	
$Cor(Y, W)$	0.950	0.949	*cross*	0.133	
	(0.004)	(0.007)		(0.059)	
$Cor(PR, N)$	-0.030	-0.016	*exit*	0.226	
	(0.010)	(0.015)			
$Cor(PR, Y)$	0.124	0.142			
	(0.014)	(0.019)			
survivors	5290	3564			
		(33.8)			

Notes: Average growth rate $E(\Delta Y/Y)$ includes exiting firms. They contribute with a -1 observation. The within, between, cross and exit moments are the components of a standard empirical labor productivity growth decomposition, over the period 1997-2001. See Lentz and Mortensen (2008, p. 1335)

Table 11: R&D Data Moments (Standard Errors in Parentheses)

Moments	1997	2001	Moments	1997	2001
$E(RI)$	0.084	0.090	$Cor(RI, Y)$	-0.170	-0.103
	(0.006)	(0.006)		(0.038)	(0.045)
$std(RI)$	0.117	0.117	$Cor(RI, PR)$	-0.188	-0.063
	(0.010)	(0.012)		(0.041)	(0.059)
$Med(RI)$	0.044	0.050	1 $\frac{\#Firms_{RI=0}}{\#Firms}$	0.924	0.897
	(0.004)	(0.003)		(0.004)	(0.005)
$E(RD)$	2906	3463	2 $Cor(RI, RI_{+2})$	0.688	
	(280.7)	(416.3)		(0.076)	
$std(RD)$	5619	8019	3 $\frac{E(Y)_{RI>0}}{E(Y)_{RI=0}}$	4.44	
	(880.7)	(1751)		(0.243)	
$Med(RD)$	1163	1250	4 $\frac{E(N)_{RI>0}}{E(N)_{RI=0}}$	4.44	
	(102.4)	(110.7)		(0.233)	

Notes: [1]Fraction of firms with a zero R&D observation. [2]Correlation R&D intensity between 1997 and 1999. [3]Average value-added of firms with positive R&D observation, relative to firms with zero R&D. [4]Average employment of firms with positive R&D observation, relative to firms with zero R&D.

Table 12: Model Fit Non-R&D Moments: Benchmark Estimation.

Moments	1997	2001	Moments	1997	2000
$E(Y)$	12872	14998	$Cor(PR, PR_{+1})$	0.735	0.697
	13090	15932		0.706	0.699
$std(Y)$	23183	28161	$Cor(PR, \Delta PR)$	-0.342	-0.371
	23485	27823		-0.363	-0.364
$Med(Y)$	4985	6254	$Cor(PR, \frac{\Delta Y}{Y})$	-0.126	
	5187	6209		-0.093	
$E(W)$	8144	10828	$Cor(PR, \frac{\Delta N}{N})$	0.037	
	8230	9904		0.060	
$std(W)$	14394	18991	$E(\frac{\Delta Y}{Y})$	-0.007	
	14071	16492		-0.017	
$Med(W)$	3157	4655	$std(\frac{\Delta Y}{Y})$	0.612	
	3351	4016		0.735	
$E(PR)$	477.8	503.6	$Cor(Y, \frac{\Delta Y}{Y})$	-0.006	
	471.6	507.1		-0.026	
$std(PR)$	173.8	168.0	*within*	0.330	
	173.8	186.5		0.717	
$Med(PR)$	444.1	473.5	*between*	0.311	
	442.9	475.4		0.113	
$Cor(Y, W)$	0.950	0.949	*cross*	0.133	
	0.964	0.963		0.059	
$Cor(PR, N)$	-0.030	-0.016	*exit*	0.226	
	0.000	0.006		0.110	
$Cor(PR, Y)$	0.124	0.142			
	0.117	0.126			
survivors	5290	3564			
	5290	3581			

Note: Data top row, model bottom. Benchmark estimation targets only non-R&D moments.
Minimum of objective function: 167.701

Table 13: Model Fit Non-R&D Moments: Estimation with R&D.

Moments	1997	2001	Moments	1997	2000
$E(Y)$	12872	14998	$Cor(PR, PR_{+1})$	0.735	0.697
	12988	15145		0.704	0.704
$std(Y)$	23183	28161	$Cor(PR, \Delta PR)$	-0.342	-0.371
	25775	31218		-0.365	-0.354
$Med(Y)$	4985	6254	$Cor(PR, \frac{\Delta Y}{Y})$	-0.126	
	5285	5920		-0.116	
$E(W)$	8144	10828	$Cor(PR, \frac{\Delta N}{N})$	0.037	
	8096	9292		0.061	
$std(W)$	14394	18991	$E(\frac{\Delta Y}{Y})$	-0.007	
	14373	17069		-0.0393	
$Med(W)$	3157	4655	$std(\frac{\Delta Y}{Y})$	0.612	
	3472	3869		0.621	
$E(PR)$	477.8	503.6	$Cor(Y, \frac{\Delta Y}{Y})$	-0.006	
	468.6	500.8		-0.02	
$std(PR)$	173.8	168.0	*within*	0.330	
	183.9	201.2		0.650	
$Med(PR)$	444.1	473.5	*between*	0.311	
	447.1	475.4		0.172	
$Cor(Y, W)$	0.950	0.949	*cross*	0.133	
	0.962	0.962		0.025	
$Cor(PR, N)$	-0.030	-0.016	*exit*	0.226	
	0.023	0.034		0.153	
$Cor(PR, Y)$	0.124	0.142			
	0.140	0.154			
survivors	5290	3564			
	5290	3530			

Note: Data top row, model bottom. Estimation w/R&D targets both non-R&D and R&D moments.
Minimum of objective function: 331.518

Table 14: Model Fit: R&D Moments (2001)

Moments	Estimation (w/ R&D)	Data	Estimation (Benchmark)	Moments	Estimation (w/ R&D)	Data	Estimation (benchmark)
$E(RI)$	0.087	0.090	0.037	$cor(RI, Y)$	-0.270	-0.103	-0.506
$std(RI)$	0.115	0.117	0.023	$cor(RI, PR)$	-0.153	-0.063	-0.162
$Med(RI)$	0.049	0.050	0.034	$\frac{\#Firms_{RI=0}}{\#Firms}$	0.893	0.897	0.765
$E(RD)$	2359	3463	837				
$std(RD)$	5888	8018	1242				
$Med(RD)$	1300	1250	517				