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Optimality of Prompt Corrective Action in a Continuous - Time Model with Recapitalization Possibility

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Abstract

Prompt Corrective Action (PCA) is a system of predetermined capital/asset ratios that trigger supervisory actions by a banking regulator. Our paper addresses the optimality of this regulation system by adapting a dynamic model of entrepreneurial finance to banking regulation. In a dynamic moral hazard setting, we first derive the optimal contract between the banker and the regulator and then implement it by a menu of regulatory tools. Our main findings are the following: first, the insurance premium is a risk-based premium where the risk is measured by the capital level; second, our model implies a capital regulation system that shares several similarities with the US PCA. According to our proposed system, regulatory supervision should be realized in the spirit of gradual intervention and the book-value of capital is used as information to trigger intervention. Banks with high capital are not subject to any restrictions. Dividend distribution is prohibited in banks with intermediate level of capital. When banks have low capital level, a plan of recapitalization is required and in the worst case, banks are placed in liquidation.

Key words: Prompt Corrective Action, Capital Regulation, Dynamic Contracting, Recapitalization.

JEL Codes: D82, G21, G28

1 Introduction

Following the implementation of the first Basel Accord (1988), academic research has spent a lot of effort in assessing the effects of minimum capital requirement on excessive

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risk taking incentives. A conclusion derived from these works is that imposing minimum regulatory capital requirements itself does not constitute an adequate solution for reducing excessive risk taking, particularly in today's world where financial innovation has produced new markets and instruments that make it easy for banks and their employees to make huge bets easily and quickly. This thinking drives the Basel Committee to incorporate in Basel Accord II the pillar 2 - supervisory review - and the pillar 3 - market discipline - as complementary to the pillar 1 - minimum capital requirement. The Basel Committee states that the goal of the pillar 2 is to enable early supervisory intervention if the capital does not provide a sufficient buffer against risk. However, it remains silent on the way to implement this principle in practice, or in other words, it remains silent on the threshold and forms of intervention.

In the United State (US), a system of predetermined capital/asset ratios that trigger structured actions by supervisor, which is called as Prompt Corrective Action (PCA), was introduced in the 1991 Federal Deposit Insurance Corporation Improvement Act (FDICIA). PCA classifies banks in five categories depending on their capital ratios: well capitalized, adequately capitalized, undercapitalized, significantly undercapitalized and critically undercapitalized. Imposition of regulatory restraints on banks becomes more and more severe the lower their capital ratios. For instance, well capitalized and adequately capitalized banks face no restrictions. Undercapitalized banks don't have right to capital distribution (dividend or stock repurchase). Significantly undercapitalized banks must submit a recapitalization plan. Critically undercapitalized banks have to be placed in receivership within 90 days. Some positive observed effects of FDICIA in creating the appropriate incentives for banks, for deposit insurer and for prudential supervisor result in the increasing number of recommendations to introduce PCA - type provisions in other countries. Over the past years, Japan, Korea and Mexico have adopted a similar system of the US PCA. Recently, the European Shadow Financial Regulatory Committee (ESFRC) made a proposal aimed at dealing with problem banks. One of the recommendations in their proposal is to implement a PCA regime in each individual Member State. In such circumstances, a rigorous study of the optimality of PCA-type regulation seems timely and relevant.

Our paper will address this issue by adapting a standard dynamic model of entrepreneurial finance to banking regulation. We consider an infinitely repeated relationship between a banker and a Deposit Insurance Corporation (DIC). The banker runs a bank whose cumulative cashflows are assumed to follow an arithmetic Brownian motion process. The drift of this process depends on the banker's effort which is costly for the banker and unobservable to the DIC. As provider of deposit insurance service, the latter does supervise the former on behalf of depositors. The regulatory rules are specified in the contract to which both parties fully commit ex-ante. We assume that to provide the banker with appropriate incentives, the DIC can control her compensations, require her to inject more capital into the bank or force her to liquidate it. Hence, in our paper, we account for the possibility that the bank is recapitalized during its operation if necessary. The banker has to bear a positive cost for each additional unit of capital injected into the bank. Moreover, there exists some exogenous bound for the amount of capital per unit of time the banker

can contribute. This bound may be justified by some exogenous borrowing constraints due to the imperfections of capital market.

In such a framework, to derive insights about an optimal regulation system, this paper proceeds as follows: we first characterize the optimal contract between the banker and the DIC. The method we use to solve for the optimal contract is the dynamic programming technique. Specifically, we use the banker's continuation utility as a single state variable and the optimal contract will be contingent on it. In continuous-time, the incentive compatibility condition is determined by the volatility of the banker's continuation utility process. The DIC's payoff is specified through some ordinary differential equations. After the characterization, we construct a regulatory menu that can implement the optimal contract. Our menu consists of three instruments: bank chartering, capital regulation and deposit insurance premium. Bank chartering determines the condition to set up a bank. Deposit insurance premium defines the payments paid to the DIC at every time. Capital regulation is characterized by the regulatory restrictions on dividend distribution, recapitalization plan and liquidation. We find that first, the insurance premium is a risk-based premium where the risk is measured by the amount of capital. Second, the capital regulation derived from our model shares several similarities with the US PCA. According to our capital regulation, regulatory supervision should be realized in the spirit of gradual intervention and the book-value of capital is used as information to trigger intervention. Banks with high capital are not subject to any restrictions. Dividend distribution is prohibited in banks with intermediate level of capital. When banks have low capital level, a plan of recapitalization is required and in the worst case, banks are placed in liquidation.

Recently, there is a growing litterature analyzing dynamic moral hazard model. It typically consists of DeMarzo and Fishman (2007a, 2007b); DeMarzo and Sannikov (2006); Sannikov (2008); Biais, Mariotti, Plantin and Rochet (2007) (BMPR (2007)); Biais, Mariotti, Rochet and Villeneuve (2007) (BMRV(2007)) and DeMarzo and Sannikov (2007). In general, these papers study the long-term financial contract in a setting in which a risk neutral entrepreneur seeks funding from risk neutral investors to finance a project that pays stochastic cashflows over many periods. Their contracting relationship is subject to moral hazard problem which comes either from the unobservability of cashflows or from the hidden efforts. The entrepreneur is liable for payments to the investors only to the extent of current revenues. In addition to some variations in modelling, these papers propose different methods to implement the optimal contract and so, generate different insights. For example, to get implications for an optimal capital structure, DeMarzo and Sannikov (2006) consider to implement the optimal contract by a combination of equity, long-term debt and credit line. Having the same objective but BMPR (2007) study an implementation which is realized via debt, equity and cash reserves. By implementing the optimal contract through the firm's payout policy, DeMarzo and Sannikov (2007) provides an explanation for the smoothness of corporate dividends relative to earnings and cashflows.

Our paper is also based on a dynamic moral hazard model. However, compared to the above papers, we relax the limited liability of the agent (the banker) and so, allow the principal (the DIC) to require the banker to inject money during their relationship.

Moreover, in this paper, we consider the implementation through a menu of regulatory instruments available in practice of banking regulation. Therefore, we are able to discuss the issue of optimality of PCA-type regulation.

The literature on PCA is mainly empirical. Since the introduction of the US PCA, there have been several attempts to assess its functioning. Some papers recognized significant impacts of PCA in terms of raising capital ratios and reducing risk for banks. Nevertheless, Barth et al. (2004) in a study of bank regulation and supervision in 107 countries raise doubts about government policies that rely excessively on direct government regulation and supervision of banks. For the theoretical analysis of PCA, we can mention Shim (2006), Freixas and Parigi (2007). The most relevant for our work is Shim (2006). Our paper takes the same approach as that paper, i.e., applying the dynamic moral hazard model of entrepreneurial finance to banking regulation. However, while Shim (2006) uses the discrete - time model and does not account for the possibility of recapitalization by the banker, in our paper, we formulate the problem in continuous - time and take into consideration the costly recapitalization option.

The rest of paper is organized as follows. In section 2, we briefly describe the current system of Prompt Corrective Action applied in the US. In section 3, we present the model in a continuous - time setting. Section 4 is devoted to the characterization of the optimal contract. Section 5 shows how this optimal contract is implemented by regulatory instruments. Finally, section 6 concludes.

2 Description of the US PCA

Prompt Corrective Action (PCA) is part of package of measures adopted in the Federal Deposit Insurance Corporation Improvement Act (FDICIA) which was enacted in 1991 after the US banking and thrift breakdowns of the 1980s. It is a version of the Benston and Kaufman's (1988) proposal for structured early intervention and resolution (SEIR). PCA creates five categories for banks: well capitalized, adequately capitalized, undercapitalized, significantly undercapitalized and critically undercapitalized. Bank classification into these categories depends on three different capital ratios: (1) the total risk-based capital ratio; (2) the Tier 1 risk-based capital ratio and (3) the Tier 1 leverage ratio, as shown in the following table[1]

[1] Source: Table 1 in Aggarwal and Jacques (2001)

	Total risk-based capital[2] (%)	Tier 1 risk-based ratio[3] (%)	Tier 1 leverage ratio[4] (%)
Well capitalized	≥ 10	≥ 6	≥ 5
Adequately capitalized	≥ 8	≥ 4	≥ 4
Undercapitalized	< 8	< 4	< 4
Significantly undercapitalized	< 6	< 3	< 3
Critically undercapitalized		Tangible equity[5] ≥ 2	

Some mix of mandatory and discretionary restrictions is prescribed for banks in each category. Mandatory restrictions become increasingly severe as the bank's capital ratios decrease. For example, no bank may make capital distribution if it belongs to any of the three undercapitalized categories. Significantly undercapitalized banks are subject to a multitude of constraints such as required capital restoration; restrictions on transactions with affiliates and affiliated banks, on asset growth... For critically undercapitalized banks, they face not only more stringent restrictions on activities but also the appointment of a conservator (receiver) within 90 days.

The FDICIA requires each appropriate federal banking agency to take prompt corrective actions to resolve the problems of insured depository institutions at the least possible long - term loss to the deposit insurance fund. To increase the accountability of the regulators in carrying out their delegated responsibilities, the Office of the Inspector General at the relevant agencies is required to file audit reports in cases that generate material losses to the deposit insurance fund. These reports review the timeliness and cost effectiveness of corrective actions taken.

As noted by Benston and Kaufman (1997), the system of predetermined capital/asset ratios that trigger actions by the regulatory authorities serves two purposes. One is to give banks an incentive to strive for high capital levels. The second purpose is to place limits on the discretion of regulators.

3 Model

We consider here a repeated relationship between a risk-neutral banker who wants to operate a bank and the risk-neutral Deposit Insurance Corporation (DIC) who is in charge of insuring the deposits and supervising the bank.

More specifically, at the initial time, the banker has some endowment of cash. If she transfers an amount E_0 to the DIC, she can set up a bank, collects D units of deposits and invests them in a long-term risky loan portfolio. The cumulated cashflows $R =$

[2] Total capital is the sum of Tier 1 and Tier 2 capital. Tier 1 mainly comprises permanent shareholders' equity, i.e. common stock and disclosed reserves or retained earnings. Tier 2 comprises loan loss reserves, subordinated debts, asset revaluation reserves, hybrid capital instruments, etc. Total risk-based capital ratio is the ratio of total capital to risk-weighted assets.

[3] Tier 1 risk-based ratio is the ratio of Tier 1 capital to risk-weighted assets.

[4] Tier 1 leverage ratio is the ratio of Tier 1 capital to total assets.

[5] The tangible equity ratio equals the total of Tier 1 capital plus cumulative preferred stock and related surplus less intangibles except qualifying purchased mortgage servicing rights divided by the total of bank assets less intangible assets except qualifying purchased mortgage servicing rights.

$\{R_t, 0 \le t < \infty\}$ of this portfolio evolve according to the following diffusion process[6]

$$dR_t = \mu A_t dt + \sigma dZ_t^A$$

where $\mu > 0$ and σ are constants; A_t denotes the effort level of the banker at time t and $Z^A = \left\{Z_t^A, \mathcal{F}_t, 0 \le t < \infty\right\}$ is a standard Brownian motion defined on the measurable space $(\Omega, \mathcal{F})$ equipped with a probability measure $\mathbb{P}^A$ induced by the effort process $A = \{A_t, 0 \le t < \infty\}$. For simplicity, we assume that the set of feasible effort levels contains two elements $\{0, 1\}$. Effort is costly for the banker in the sense that she enjoys a private benefit B if exerting low effort $(A_t = 0)$. Denote by $v(A_t)$ the banker's private benefits associated with effort level A_t. Hence, $v(0) = B$ and $v(1) = 0$. We assume that $B < \mu$, i.e., exerting high effort is efficient.

The relationship between the banker and the DIC is subject to a moral hazard problem which comes from the unobservability of the banker's effort. That means, whereas the cashflows process $R = \{R_t, 0 \le t < \infty\}$ is publicly observable by both the DIC and the banker, the effort level A_t is private information of the latter. A contract between the banker and the DIC specifies, based on the entire history of cashflow realizations, a liquidation time $\tau\left(R_s, 0 \le s < \tau\right)$ and the payments for the banker at each time before the liquidation time. Denote by $C = \left\{C_t\left(R_s, 0 \le s \le t\right), 0 \le t < \tau\right\}$ the process describing the cumulative payments to the banker. At any time, the bank can also be closed if the banker decides not to run the bank any more and switches to her second-best business which gives her an expected utility $\tilde{W} \in \left[0, \frac{\mu}{\rho}\right)$ [7] where ρ is the discount rate of the banker. The value of the loan portfolio at the time of termination is assumed to be zero.

We here assume that the DIC possesses an option of requiring the banker to contribute capital into the bank during its operation. *The introduction of this option constitutes our major novelty compared to Shim (2006).* The supplementary capital contributed serves to reinforce the bank's balance sheet in order to avoid default. It is interpreted as recapitalization and is modeled in this paper by a negative payment to the banker. Hence, in our paper, at each time $t < \tau$, the banker can receive a positive, a zero or a negative transfer. Positive transfer corresponds to some dividend paid to the banker while negative transfer is consistent with a capital injection by the banker. To capture the fact that in general, recapitalization is costly, we assume that the banker has to bear a cost $\alpha > 0$ for each unit of capital contributed. Moreover, we also assume that the amount of capital per unit of time the banker can contribute can not exceed some quantity K, that is $dC_t \ge -Kdt$. This lower bound can be justified by some exogenous borrowing constraint due to the imperfection of capital market.

If the banker discounts the future at the rate ρ and the DIC at the riskless interest rate[8] $r < \rho$, then, given a contract (τ, C) and an effort strategy A, the total expected

[6] In our model, the choice of effort level affects the expected value of the cashflow but not its volatility. Moreover, the effort at time t affects only the distribution of the cashflows at this time.

[7] Since $\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} dR_t\right] = \int_0^{+\infty} e^{-\rho t} \mu dt = \frac{\mu}{\rho}$, if $\tilde{W} > \frac{\mu}{\rho}$ then, for the banker, running a bank is worse than outside options.

[8] The assumption that $r < \rho$ means that the DIC is more patient than the banker.

utility for the banker as of time 0, if she never quits, is given by

$$\mathbb{E}^A\left[\int_0^\tau e^{-\rho t}\left[(1+\alpha\mathbf{1}_{\{dC_t<0\}})dC_t+v(A_t)dt\right]+e^{-\rho\tau}\tilde{W}\right]$$

and for the DIC by

$$\mathbb{E}^A\left[\int_0^\tau e^{-rt}dR_t-\int_0^\tau e^{-rt}dC_t\right]=\mathbb{E}^A\left[\int_0^\tau e^{-rt}\left(\mu A_t dt-dC_t\right)\right]$$

where $\mathbb{E}^A$ denotes the expectation operator under the probability measure $\mathbb{P}^A$.

An effort strategy is defined as incentive compatible with respect to the contract (τ, C) if it maximizes the total expected utility of the banker given (τ, C). Here we focus on the contracts that induce the banker to choose the effort strategy $A^* = \{A_t = 1\ \forall 0 \le t < \tau\}$ (i.e. the banker exerts high effort every time) and if facing these contracts, the banker will never choose to quit. We label such a class of contracts as incentive compatible one. The DIC's problem is to find, among this class, the contract which provides him with highest payoff.

We denote by $\mathbb{P}$ the probability measure generated by the effort process A^* and by $\mathbb{E}$ the expectation under $\mathbb{P}$. Then, the DIC's problem can be formulated as follows

$$Max\ \mathbb{E}\left[\int_0^\tau e^{-rt}\left(\mu dt-dC_t\right)\right] \tag{1}$$

subject to the following constraints

$$A^*=\{A_t=1\ \forall 0\le t<\tau\}\ \text{ is incentive compatible w.r.t }\ (\tau,C) \tag{2}$$

$$W_0=\mathbb{E}\left[\int_0^\tau e^{-\rho t}(1+\alpha\mathbf{1}_{\{dC_t<0\}})dC_t+e^{-\rho\tau}\tilde{W}\right] \tag{3}$$

$$\mathbb{E}\left[\left(\int_t^\tau e^{-\rho(s-t)}(1+\alpha\mathbf{1}_{\{dC_s<0\}})dC_s+e^{-\rho(\tau-t)}\tilde{W}\right)\middle|\mathcal{F}_t\right]\ge\tilde{W}\qquad\forall 0\le t<\tau \tag{4}$$

$$\forall 0\le t<\tau:\ dC_t\ge -Kdt \tag{5}$$

The formulation of the constraint (3) is in line with DeMarzo and Sannikov (2006). By varying W_0, we can use this solution to consider different divisions of bargaining power between the banker and the DIC. For example, if the DIC charters a banker from a competitive pool, then W_0 is chosen such that the DIC's expected payoff as of time 0 is maximal subject to the constraint that the banker receives at least $\tilde{W}$ (i.e. $W_0 \ge \tilde{W}$).

4 Optimal contract

In this section, we present the derivation of the optimal contract. Based on the techniques introduced by Sannikov (2008), we know that instead of contingent the optimal contract on the whole history of the loan's cashflows, we can use the banker's continuation utility as state variable and write the contract as function of this variable. Therefore, the characterization of the optimal contract will proceed in three steps. First, we state a result that relates the incentive compatibility condition of the effort process A^* to the dynamic evolution of the banker's continuation utility. Next, we prove that the DIC's payoff function can be determined as solution to some ordinary differential equations. Finally, by solving this equation, we find the optimal contract.

4.1 Incentive compatibility condition

Here, we derive the incentive compatibility constraint for the banker. Given a contract (τ, C), for each $t < \tau$, denote by W_t^A the banker's continuation utility corresponding to an effort strategy $A = \{A_t, 0 \leq t < \tau\}$. It is the total expected utility the banker derives from the transfers paid to her from time t on if she follows the strategy A.

$$W_t^A = \mathbb{E}^A \left[\left(\int_t^\tau e^{-\rho(s-t)} \left[(1 + \alpha \mathbf{1}_{\{dC_s < 0\}}) dC_s + v(A_s) ds \right] + e^{-\rho(\tau - t)} \tilde{W} \right) \middle| \mathcal{F}_t \right]$$

The following lemma provides a useful representation of W_t^A

Lemma 1 *There exists a stochastic process $G^A = \left\{ G_t^A, 0 \leq t < \infty \right\}$ that represents the sensitivity of the banker's continuation utility to the cashflows, i.e.*

$$dW_t^A = \left(\rho W_t^A - v(A_t) \right) dt - (1 + \alpha \mathbf{1}_{\{dC_t < 0\}}) dC_t + G_t^A dZ_t^A \tag{6}$$

Proof. Define by V_t^A the banker's lifetime utility corresponding to the contract (τ, C) and to the effort stratety $A = \{A_t, 0 \leq t < \tau\}$ conditionally on the information available at time $t < \tau$, then

$$V_t^A = \mathbb{E}^A \left[\left(\int_0^\tau e^{-\rho s} \left[(1 + \alpha \mathbf{1}_{\{dC_s < 0\}}) dC_s + v(A_s) ds \right] + e^{-\rho \tau} \tilde{W} \right) \middle| \mathcal{F}_t \right]$$

So, we can rewrite V_t^A as follows

$$V_t^A = \int_0^t e^{-\rho s} \left[(1 + \alpha \mathbf{1}_{\{dC_s < 0\}}) dC_s + v(A_s) ds \right] + e^{-\rho t} W_t^A$$

that implies

$$dV_t^A = e^{-\rho t} \left[(1 + \alpha \mathbf{1}_{\{dC_t < 0\}}) dC_t + v(A_t) dt - \rho W_t^A dt \right] + e^{-\rho t} dW_t^A \tag{7}$$

On the other hand, by construction, we have that V_t^A is $\mathcal{F}_t - \textit{measurable}$ and that for all $s \leq t < \tau$, $\mathbb{E}^A \left[V_t^A \middle| \mathcal{F}_s \right] = V_s^A$. So, V_t^A is a $\mathcal{F}_t-$martingale. By the martingale representation theorem, there exists a progressively measurable stochastic process $G^A = \left\{ G_t^A, \mathcal{F}_t, 0 \leq t < \infty \right\}$ defined on the probability space $\left(\Omega, \mathcal{F}, \mathbb{P}^A \right)$ and satisfying

$$\mathbb{E}^A \left[\int_0^t \left(e^{-\rho s} G_s^A \right)^2 ds \right] < \infty$$

for all $0 \leq t < \infty$ such that

$$V_t^A = V_0^A + \int_0^t e^{-\rho s} G_s^A dZ_s^A \tag{8}$$

therefore,

$$dV_t^A = e^{-\rho t} G_t^A dZ_t^A \tag{9}$$

(6) is automatically derived from (7) and (9). Q.E.D. ■

The lemma 1 provides a representation of the banker's continuation utility as an Ito process. This representation is valid for any effort strategy A. The question arised now is to determine under what conditions the banker will optimally choose to follow the effort strategy A^*.

Let W_t and Z_t be correspondently defined for the effort process A^*. Applying the lemma 1 to this effort process, we have

$$dW_t = \rho W_t dt - (1 + \alpha \mathbf{1}_{\{dC_t < 0\}}) dC_t + G_t dZ_t \tag{10}$$

where $G = \{G_t, \mathcal{F}_t, 0 \leq t < \infty\}$ is defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Obviously, at each time t, to decide what level of effort should be taken, the banker will rely on how this decision affects her continuation utility. Exerting high effort at time t ($A_t = 1$) immediately causes a loss of private benefit B to the banker but it improves her continuation value in expected term by $\frac{G_t}{\sigma}\mu$. Hence, intuitively, choosing high effort is profitable to the banker as long as $\frac{G_t}{\sigma}\mu \geq B$. A formal statement of this result is the following:

Proposition 1 *The strategy of exerting high effort at any time is optimal for the banker if and only if the volatility of her continuation utility G_t is at least equal to $\frac{B}{\mu}\sigma$ for all $t \in [0, \tau)$.*

Proof. See appendix A. ■

The proposition 1 means that for the incentive provision purpose, the banker has to bear some minimum risk, which is materialized by requiring that her continuation utility must be sensitive enough to the cashflows.

4.2 DIC's continuation payoff function

The incentive compatibility condition being defined, it is the time for characterizing the DIC's payoff function. Denote it by $F(W_t)$[9]. It stands for the maximal continuation payoff the DIC can earn from all incentive compatible contracts when a continuation utility W_t is promised to the banker at time t.

Consider an infinitesimal time interval $[t, t+dt)$, given a promised utility W_t for the banker, if a transfer dC_t is paid to her during this period, the DIC immediately receives $dR_t - dC_t$ and his continuation payoff is equal to $F(W_t + dW_t)$. In other words, given the payment dC_t to the banker, the *actual* change in the DIC's payoff is measured by the sum of $dR_t - dC_t$ and $F(W_t + dW_t) - F(W_t)$. Since the DIC discounts the future at the rate r, the *expected* change of his payoff during the considered time interval is represented by the term $rF(W_t)dt$. Therefore, intuitively, the DIC's payoff function is solution to the following equation:

$$rF(W_t)dt = Max \;\; \mathbb{E}\left[dR_t - dC_t + F(W_t + dW_t) - F(W_t)\right] \tag{11}$$

equivalently,

$$rF(W_t)dt = Max\left\{(\mu dt - dC_t) + \mathbb{E}\left[dF(W_t)\right]\right\} \tag{12}$$

On the left-hand side of (12), we have the expected change in the DIC's payoff. On the right-hand side, we have the sum of the expected cashflows accruing to him and of the expected change in his continuation value. The maximization means that the current choice is managed optimally, bearing in mind not only the immediate payments but also the consequences for future payoffs.

In what follows, we will assume that the function $F(.)$ is concave, which will be checked later. We first determine the condition under which a positive transfer should be paid to the banker. To provide the banker with the utility W_t, the DIC has the option to pay a lump-sum transfer of $dC_t > 0$ and switching to the contract with promised utility $W_t - dC_t$. The DIC's payoff corresponding to this compensation structure is equal to $F(W_t - dC_t) - dC_t$. So, giving a positive compensation to the banker is optimal for the DIC if and only if

$$F(W_t) \leq F(W_t - dC_t) - dC_t$$

In other words, paying a positive transfer is optimal over the range of W_t where the function $F(W_t) + W_t$ is nonincreasing. Define W^* by

$$W^* = \inf\left\{W : F'(W) = -1\right\}$$

Owing to the concavity of the function F, we obtain

$$dC_t > 0 \text{ if and only if } W_t > W^*$$

Now, we turn to the circumstance under which the banker will be required to inject

[9] Note that by the stationnary property of the contract, the DIC's payoff function is common to all dates. Hence, we write it without any time label on the function symbol.

capital into the bank. Similarly with dividend threshold, to be able to find recapitalization threshold, we should compare the DIC's payoff between different payment structures. Given the utility W_t promised to the banker, if the DIC requires the banker to contributes $-dC_t > 0$, because of the cost of capital contribution, he has to move to the contract with promised utility $W_t - (1+\alpha)dC_t$ and gets a payoff $F\left(W_t - (1+\alpha)dC_t\right) - dC_t$. Therefore, demanding a recapitalization is optimal if and only if

$$F(W_t) \leq F\left(W_t - (1+\alpha)dC_t\right) - dC_t$$

or

$$F(W_t) + \frac{W_t}{1+\alpha} \leq F\left(W_t - (1+\alpha)dC_t\right) + \frac{W_t - (1+\alpha)dC_t}{1+\alpha}$$

Thus, a recapitalization should happen over the range of W_t where the function $F(W_t) + \frac{W_t}{1+\alpha}$ is nondecreasing. Define $\bar{W}$ by

$$\bar{W} = \inf\left\{W : F^{'}(W) = -\frac{1}{1+\alpha}\right\}$$

Since the function F is concave, it is obvious that $\bar{W}$ is less than W^*.

In summary, the banker collects a positive compensation once $W_t > W^*$. When W_t is in between $\bar{W}$ and W^*, no transfer between the DIC and the banker occurs. Finally, if W_t falls below $\bar{W}$, the banker must inject capital into the bank. Since the banker has the possibility to quit and take outside option with reservation utility $\tilde{W}$, on the equilibrium path, W_t can not fall below $\tilde{W}$ and the DIC must close the bank once W_t reaches this boundary value, which implies $F(\tilde{W}) = 0$.

Hence, over the interval $\left(\bar{W}, W^*\right]$, the dynamic evolution of the banker's continuation utility W_t becomes:

$$dW_t = \rho W_t dt + G_t dZ_t$$

Using Ito's formula to compute $dF(W_t)$, we see from (12) that over $\left(\bar{W}, W^*\right]$, the DIC's payoff function has to satisfy the following condition:

$$rF(W_t) = \underset{G_t \geq \frac{B}{\mu}\sigma}{Max} \left[\mu + F'(W_t)\rho W_t + \frac{1}{2}F''(W_t)G_t^2\right]$$

Since the function $F(.)$ is concave, it is optimal to set G_t at its minimal possible value $\frac{B}{\mu}\sigma$, which indicates that over $\left(\bar{W}, W^*\right]$, the DIC's payoff function is solution to

$$rF(W_t) = F'(W_t)\rho W_t + \frac{1}{2}F''(W_t)\frac{B^2}{\mu^2}\sigma^2 + \mu$$

Regarding the case where W_t belongs to the interval $\left(\tilde{W}, \bar{W}\right]$, we know that the bank should be recapitalized. We assume that in this case, payments to the banker are absolutely continuous with respect to time, that is $dC_t = c_t dt$ where $c_t \in [-K, 0)$, then, the banker's continuation utility evolves according to

$$dW_t = \left[\rho W_t - (1+\alpha)c_t\right] dt + G_t dZ_t$$

Again, applying Ito's formula, we obtain:

$$rF(W_t) = \underset{c_t \in [-K,0),\ G_t \geq \frac{B}{\mu}\sigma}{Max} \left[\mu - c_t + F'(W_t)\left(\rho W_t - (1+\alpha)c_t\right) + \frac{1}{2}F''(W_t)G_t^2 \right]$$

Since the function $F(.)$ is concave and $F'(W) \geq -\frac{1}{1+\alpha}$ for $W \in \left(\tilde{W}, \bar{W}\right]$, we get the result that over $\left(\tilde{W}, \bar{W}\right]$, it is optimal to let $c_t = -K$ and $G_t = \frac{B}{\mu}\sigma$, the DIC's payoff function will be solution to

$$rF(W_t) = F'(W_t)\left(\rho W_t + (1+\alpha)K\right) + \frac{1}{2}F''(W_t)\frac{B^2}{\mu^2}\sigma^2 + \mu + K$$

Summing up, the DIC's payoff function will be composed of two functions. Over the interval $\left[\tilde{W}, \bar{W}\right]$, it coincides with the function F_1 which satisfies the following ordinary differential equation

$$rF_1(W) = F_1'(W)\left(\rho W + (1+\alpha)K\right) + \frac{1}{2}F_1''(W)\frac{B^2}{\mu^2}\sigma^2 + \mu + K$$

Over the interval $\left(\bar{W}, W^*\right]$, it corresponds to the function F_2 - solution to the ordinary differential equation as follows

$$rF_2(W) = F_2'(W)\rho W + \frac{1}{2}F_2''(W)\frac{B^2}{\mu^2}\sigma^2 + \mu$$

Since these two functions are determined from second-order differential equations with two free boundaries, we need to specify 6 boundary conditions. The first boundary condition, namely $F_1(\tilde{W}) = 0$, ensures that on the equilibrium path, W_t will not fall below $\tilde{W}$. Three other conditions, namely $F_1^{'}(\bar{W}) = -\frac{1}{1+\alpha}$, $F_2^{'}(\bar{W}) = -\frac{1}{1+\alpha}$, $F_2^{'}(W^*) = -1$, come from the fact that $\bar{W}$ and W^* are respectively recapitalization and dividend thresholds. The fifth condition $F_2^{''}(W^*) = 0$ guarantees the optimality of W^*. Finally, in order to insure that the DIC's payoff function is continuous, we have to impose that the second-order derivatives of two functions F_1 and F_2 match at the boundary $\bar{W}$, i.e. $F_1^{''}(\bar{W}) = F_2^{''}(\bar{W})$.

The following proposition characterizes the optimal contract

Proposition 2 *The optimal contract is characterized through the continuation utility W of the banker whose dynamic evolution is governed by the following stochastic differential equation*

$$dW_t = \rho W_t dt + \frac{B}{\mu}\sigma dZ_t - \left(1 + \alpha \mathbf{1}_{\{dC_t < 0\}}\right) dC_t$$

The DIC's payoff function F is determined as follows

$$F(W) = \begin{cases} F_1(W) & for \;\; W \in \left[\tilde{W}, \bar{W}\right] \\ F_2(W) & for \;\; W \in \left(\bar{W}, W^*\right] \\ F_2(W^*) - (W - W^*) & for \;\; W > W^* \end{cases}$$

where F_1 and F_2 are specified by

$$\begin{cases} rF_1(W) = F_1^{'}(W)\left(\rho W + (1+\alpha)K\right) + \frac{1}{2}F_1^{''}(W)\frac{B^2}{\mu^2}\sigma^2 + \mu + K \\ rF_2(W) = F_2^{'}(W)\rho W + \frac{1}{2}F_2^{''}(W)\frac{B^2}{\mu^2}\sigma^2 + \mu \end{cases}$$

with six boundary conditions: $F_1(\tilde{W}) = 0$, $F_1'(\bar{W}) = F_2'(\bar{W}) = -\frac{1}{1+\alpha}$, $F_1''(\bar{W}) = F_2''(\bar{W})$, $F_2'(W^*) = -1$ and $F_2''(W^*) = 0$. The banker has to inject more capital into the bank when $W_t \in \left(\tilde{W}, \bar{W}\right]$ and receives positive payments if $W_t > W^*$.The bank will be closed in the first time W reaches $\tilde{W}$

$$\tau = \inf\left\{t : W_t = \tilde{W}\right\}$$

Proof. See appendix B ■

Compensations for the banker described in the above proposition can be summarized in the following figure

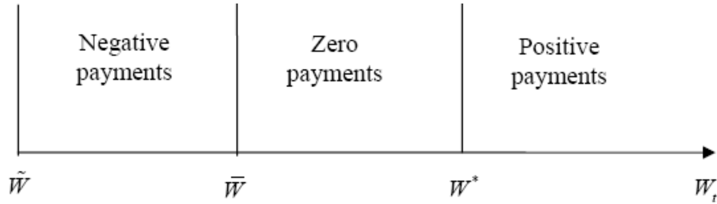

Figure 1: Compensations for the banker

We found that recapitalization threshold is decreasing with α while dividend threshold is increasing with it. *Thus, when recapitalization cost is higher, not only the recapitalization region is reduced but also, for prudent reason, it takes more time for the bank to start distributing dividend.* Two limiting cases concerning recapitalization cost are worth mentioning. When $\alpha = 0$, we see that $F_2'(\bar{W}) = -1 = F_2'(W^*)$, which means that two thresholds $\bar{W}$ and W^* coincide. The intuition is that since recapitalization is costless, it is optimal for the DIC to use this punishment device as soon as possible. For the other case, when $\alpha \to +\infty$, relying on the value of the function F_1 at the point $\tilde{W}$, we obtain $F_1'(\tilde{W}) = 0 = F_1'(\bar{W})$. Hence, if $\alpha \to +\infty$, then $\bar{W} \to \tilde{W}$, i.e. no recapitalization is involved in the optimal contract. This result is also intuitive.

Corollary 1 *In two limiting cases concerning the recapitalization cost, the optimal contract exhibits following properties*

$$\alpha \to 0 : \ \bar{W} \to W^*$$
$$\alpha \to +\infty : \ \bar{W} \to \tilde{W}$$

4.3 Determination of initial rent for the banker

Proposition 2 describes the optimal contract for a given initial promised utility W_0 for the banker. We now study how this value is determined. In the context of banking regulation, it is reasonable to think of the situation where the DIC has the right to charter

a banker from a competitive pool. Hence, the DIC retains all bargaining power. The initial rent for the banker is determined by

$$W_0^* = \arg\max_{W_0 \geq \tilde{W}} F(W_0)$$

5 Implementation of the optimal contract

5.1 Implementation results

So far, we characterized the optimal contract that induces the banker to exert high effort every time. Now, we consider to implement this optimal contract through a regulatory menu which is designed by the DIC and informed to all potential bankers from the initial time. The DIC acting as a supervisory authority commits to pursue it. Our regulatory menu consists of three tools: bank chartering, capital regulation and deposit insurance premium.

Bank chartering. It defines the initial amount of capital E_0 the banker must contribute to open the bank. Once the banker obtains the charter to set up the bank, she will collect D units of deposits. This amount of deposits is invested in the risky loan portfolio whereas the initial capital is kept as cash to meet possible future liquidity needs. The amount of cash grows at the risk - free rate r.

Deposit insurance premium. It is characterized by a sequence of payment dP_t from the banker to the DIC during each time interval $[t, t + dt)$[10].

Capital regulation. It determines the rules regarding distribution of dividends, recapitalization and liquidation policy.

Note that all provisions of the regulatory menu will be made contingent on the amount of capital E_t of the bank. In other words, in our implementation, the level of bank's capital plays the role of a record-keeping device, as W_t does in the abstract characterization of optimal contract. For implementation purpose, we define two thresholds E^*, $\bar{E}$ which respectively correspond to $W^* - \tilde{W}$, $\bar{W} - \tilde{W}$.

Proposition 3 *The following regulatory menu will implement the optimal contract*

In order to get a licence of opening a bank, the banker must contribute an amount of capital $E_0 = W_0 - \tilde{W}$. During each infinitesimal time interval $[t, t + dt)$, the banker has to pay to the DIC

$$dP_t = \begin{cases} \left[B - \rho\tilde{W} - (\rho - r)E_t\right] dt + \left(1 - \frac{B}{\mu}\right) dR_t & \text{for} \quad E_t > \bar{E} \\ \left[B - \rho\tilde{W} - (\rho - r)E_t\right] dt + \left(1 - \frac{B}{\mu}\right) dR_t - \alpha K dt & \text{for} \quad E_t \leq \bar{E} \end{cases}$$

The bank is prevented to distribute dividends as long as the amount of capital is not greater than E^*. When its level of capital is larger than this threshold, all excess capital is

[10] Our insurance premium is paid after the realization of the bank's cash-flows.

distributed as dividends. The DIC orders a recapitalization from the bank if its capital level falls below $\bar{E}$. The bank is placed into the liquidation procedure if its amount of capital is zero.

Proof. See appendix C ■

In the above implementation, the bank's capital level E_t at each time t before the liquidation time is related to the banker's continuation utility W_t by the functional relationship $E_t = W_t - \tilde{W}$ for all t. The initial amount of cash is financed by equity which is capital contributed by the banker. To ensure the voluntary participation of the banker, E_0 could not exceed $W_0 - \tilde{W}$. Since we assume that the DIC charters a banker from a competitive pool, it is optimal for the former to set $E_0 = W_0 - \tilde{W}$. This prescription regarding the minimum starting capital level for opening a bank is present in law of a number of countries.

The periodic payment to the DIC (i.e. the deposit insurance premium) is determined to coordinate the evolution of the bank's capital level with the motion of the banker's continuation utility characterized in the proposition 2. We observe that this insurance premium is decreasing with the amount of bank's capital at each time. Hence, our insurance premium can be interpreted as risk - based premium where the risk is measured by bank's capital level[11].

Relatively to the capital regulation, we see that the regulation system derived in our model and the US PCA have several similarities. Indeed, our regulation system specifies that restrictions on banks become more stringent the less capitalized banks are. According to our regulation, banks can be classified in four categories based on their capital level. Banks with high level of capital (more than E^*) would be subject to minimum prudential intervention, they can distribute dividends to shareholders. In banks with intermediate capital level (i.e. their capital level falls into the interval $\left[\bar{E}, E^*\right)$), dividend distributions are suspended but banks are still allowed to continue in normal operation, no capital restoration is required. If the bank's capital level still falls lower, the regulator will order banks to recapitalise promptly and in the worst situation, the bank's authorities resolve banks through liquidation. A remark is that when the bank has to proceed a recapitalization, the DIC will reduce the premium by αK which exactly corresponds to the total cost of recapitalization. This premium reduction can be seen as a subsidy from the DIC to the undercapitalized banks in replenishing capital.

5.2 Discussion

Banking supervisors' discretion: A key component of any regulatory arrangement is the nature, timing and form of intervention. The novelty of the US PCA is that it recommends a reduction of supervisory discretion by requiring the supervisors to take some prespecified intervention actions at some predetermined thresholds of banks' capital. Our approach to

[11] Notice that differently with Shim (2006), our insurance premium is increasing with the bank's cash-flows. The reason for this difference is that in Shim (2006), the bank's cash-flows are unobservable and so, negative relationship between the bank's return and the payment to the DIC is necessary to truthfully reporting incentive. In our model, we don't have asymmetric information concerning the bank's returns.

design the banking regulation is to implement the ex-ante optimal contract without the possibility of renegotiation. Therefore, as is done in the US PCA, in our model-implied capital regulation, all actions of the regulator is specified ex-ante by the law, which means that the regulators' discretion is limited. Another aspect introduced in the US PCA in favor of limiting regulatory forbearance is the provisions calling for timely resolution. According to these provisions, banks should be closed before the economic value of their capital becomes negative. Our liquidation policy exhibits the same property in the sense that it claims to liquidate the banks as soon as their capital is wiped out. Insolvent banks with negative capital should not be allowed to continue in operation.

Book - value vs. Market - value: One of the major issues about the effectiveness of the US PCA is related to its intervention triggering device. The triggers for prompt corrective actions in FDICIA are based on historical - cost accounting measures (i.e. on the book-value of capital), which raises the concerns about the adequacy of such indicators. Some studies (e.g. Peek and Rosengren (1996, 1997) and Jones and King (1992, 1995)) have noted that the capital ratio thresholds used in PCA are lagging indicators of a bank's financial status. In our proposed regulation system, the regulatory restrictions are contingent on the book-value of capital. However, this result should not be seen as a support for the use of book - value against the use of market - value measure. Indeed, what matters for the discrepancy between book - value and market - value is possible changes in the interest rates and default probability of loans compared to the initial situations when liabilities and assets are acquired[12]. In our model, both variables (r and σ) are assumed to be fixed over time. So, there is no interest to distinguish these two measures. A rigorous analyse of the choice between two measures requires a richer setting than ours.

6 Conclusion

In this paper, we apply the approach of designing prudential regulation of banks as a mechanism to implement the socially optimal allocation proposed by Shim (2006) to study the optimality of the current US Prompt Corrective Action. In a dynamic setting where the regulator (the DIC) can not observe the effort chosen by the banker and can require the banker to inject capital during the operation of bank, we first derive the optimal contract specifying the payments to the banker and the liquidation policy, using the banker's expected discounted utility as sate variable. Then, we show that this contract can be implemented by a combination of capital regulation and risk-based deposit insurance premium. From the implementation results, we observe that the PCA version applied in the US closely mimics properties of an optimal regulation.

[12] Book value is the amount paid for an asset or acquired upon issuance of a liability in the past, net of some accounting adjustments, such as reserving against expected losses from default. But changing interest rates affect the present value of fixed-interest obligations, and changing economic conditions affect the probability that loans will not be repaid as was expected when they were made. Consequently, the book value of equity often is discrepant from its market value.

A Appendix: Proof of proposition 1

Define by V_t the total utility the banker expects to get from the contract (τ, C) if she chooses the effort strategy A^* conditionally on the information available at time $t < \tau$

$$V_t = \mathbb{E}\left[\left(\int_0^\tau e^{-\rho s}(1+\alpha \mathbf{1}_{\{dC_s<0\}})dC_s + e^{-\rho\tau}\tilde{W}\right)\middle| \mathcal{F}_t\right]$$

and by $\tilde{V}_t$ the one the banker receives if she follows an effort strategy A up to time $t < \tau$ and then, switches to the strategy A^*

$$\tilde{V}_t = \int_0^t e^{-\rho s}\left[(1+\alpha \mathbf{1}_{\{dC_s<0\}})dC_s + v(A_s)ds\right] + \mathbb{E}\left[\left(\int_t^\tau e^{-\rho s}(1+\alpha \mathbf{1}_{\{dC_s<0\}})dC_s + e^{-\rho\tau}\tilde{W}\right)\middle| \mathcal{F}_t\right]$$

So,

$$\tilde{V}_t = V_t + \int_0^t e^{-\rho s} v(A_s)ds$$

Similarly to (8), we can represent V_t as $V_t = V_0 + \int_0^t e^{-\rho s} G_s dZ_s$. Hence, the dynamic evolution of $\tilde{V}_t$ under the probability measure $\mathbb{P}$ is the following

$$d\tilde{V}_t = e^{-\rho t} v(A_t)dt + e^{-\rho t} G_t dZ_t$$

Since Z_t and Z_t^A are related by the equality $dZ_t = dZ_t^A + \frac{1}{\sigma}(\mu A_t - \mu)\,dt$, under the probability measure $\mathbb{P}^A$, $\tilde{V}_t$ evolves according to

$$d\tilde{V}_t = e^{-\rho t}\left(v(A_t) + \frac{G_t}{\sigma}\mu A_t - \frac{G_t}{\sigma}\mu\right)dt + e^{-\rho t} G_t dZ_t^A$$

Conclusion 1 *If $\tilde{V}_t$ is $\mathbb{P}^A$ – submartingale, then the effort strategy A^* is suboptimal for the banker*

Proof. Indeed, the fact that $\tilde{V}_t$ is $\mathbb{P}^A$ – *submartingale* means for all $s \le t$,

$$\mathbb{E}^A\left(\tilde{V}_t \middle| \mathcal{F}_s\right) \ge \tilde{V}_s$$

Therefore, for all $t \ge 0$,

$$V_0 = \tilde{V}_0 \le E^A\left(\tilde{V}_t\right) \tag{13}$$

Note that $\mathbb{E}^A\left(\tilde{V}_t\right)$ represents the total utility the banker expects to get at date 0 if she follows a strategy A until the time t and then, follows the strategy A^*. Obviously, (13) implies that the strategy A^* is suboptimal compared to A. ■

Conclusion 2 *If $\tilde{V}_t$ is $\mathbb{P}^A$ – supermartingale, then the effort strategy A^* is at least as good as the strategy A for the banker*

Proof. Since $\tilde{V}_t$ is $\mathbb{P}^A - \textit{supermartingale}$, we have

$$\mathbb{E}^A \left(\tilde{V}_t \middle| \mathcal{F}_s \right) \leq \tilde{V}_s$$

for all $s \leq t$. Applying the optional sampling theorem, we get

$$\mathbb{E}^A \left(\tilde{V}_\tau \right) \leq \tilde{V}_0 = V_0 \tag{14}$$

$\mathbb{E}^A \left(\tilde{V}_\tau \right)$ accounts for the total utility the banker expects to get at date 0 if she always follows the strategy A and so, (14) concludes the proof. ■

From two conclusions above, we obtain the necessary and sufficient condition for the optimality of the strategy A^*. That is, the drift coefficient of $\tilde{V}_t$ under $\mathbb{P}^A$ is non positive:

$$v(A_t) + \frac{G_t}{\sigma}\mu A_t - \frac{G_t}{\sigma}\mu \leq 0 \text{ for all } A_t \in \{0, 1\}$$

It is equivalent to $G_t \geq \sigma \frac{B}{\mu}$. Q.E.D

B Appendix: Proof of proposition 2

For the formal proof, we have to establish the following conclusions:

1) the contract characterized in this proposition is incentive compatible

2) It is optimal among the class of incentive compatible contracts

Because of the proposition 1, the incentive compatibility of the characterized contract is derived directly from the specification of the dynamic evolution of banker's continuation utility. The proof for the optimality proceeds as follows:

B.1 Upper bound of the DIC's expected payoff

Here, we will prove that the function F - solution, if exists, to the equation (12) with boundary condition $F(\tilde{W}) = 0$ constitutes an upper bound for the expected payoff the DIC can earn from any incentive compatible contract that delivers the banker an initial expected discounted utility W_0.

Consider any incentive compatible contract (τ, C), the expected payoff of the DIC is evaluated by

$$\mathbb{E}\left[\int_0^\tau e^{-rt} dR_t - \int_0^\tau e^{-rt} dC_t\right] = \mathbb{E}\left[\int_0^\tau e^{-rt} \left(\mu dt - dC_t\right)\right]$$

Define a stochastic process $M = \{M_t\}$ by

$$M_t = \int_0^t e^{-rs} dR_s - \int_0^t e^{-rs} dC_s + e^{-rt} F(W_t) \tag{15}$$

where W_t is defined by (10) with $G_t \geq \frac{B}{\mu}\sigma$. We have

$$dM_t = e^{-rt}\left(dR_t - dC_t - rF(W_t)dt + dF(W_t)\right)$$

which implies

$$e^{rt}dM_t = \mu dt - dC_t - rF(W_t)dt + \mathbb{E}\left(dF(W_t)\right) + \left(\sigma + F^{'}(W_t)G_t\right)dZ_t$$

Since $F(W_t)$ is solution to (12), the drift coefficient of the dynamic evolution of M_t is negative, which indicates that $M = \{M_t\}$ is supermartingale. Therefore,

$$\mathbb{E}\left[\int_0^\tau e^{-rt}\left(\mu dt - dC_t\right)\right] = \mathbb{E}\left[M_\tau\right] \leq M_0 = F(W_0) \tag{16}$$

In (16), the first equality stems from $F(W_\tau) = F(\tilde{W}) = 0$; the inequality is due to the optional sampling theorem.

B.2 DIC's expected payoff from the optimal contract

Now, we show that the contract characterized in this proposition provide the DIC with expected payoff exactly equal to $F(W_0)$. Notice that if the process $M = \{M_t\}$ is martingale, then

$$\mathbb{E}\left[\int_0^\tau e^{-rt}\left(\mu dt - dC_t\right)\right] = \mathbb{E}\left[M_\tau\right] = M_0 = F(W_0)$$

Therefore, we should prove that under the contract characterized in the proposition 2, the process $M = \{M_t\}$ defined by (15) is a martingale.

B.2.1 Existence, uniqueness and concavity of the function F_2

We fixe some value $\bar{W}$. Define a function $S_2(W)$ by $S_2(W) = F_2(W) + W$. Hence, $S_2(W)$ satisfies the following differential equation

$$rS_2(W) = \mu - (\rho - r)W + S_2^{'}(W)\rho W + \frac{1}{2}S_2^{''}(W)\frac{B^2}{\mu^2}\sigma^2 \tag{17}$$

on the region $\left[\bar{W}, W^*\right]$ with boundary conditions $S_2^{'}(\bar{W}) = \frac{\alpha}{1+\alpha}$, $S_2^{'}(W^*) = 0$ and $S_2^{''}(W^*) = 0$. The homogeneous differential equation associated with (17) is written as follows

$$rS_2(W) = S_2^{'}(W)\rho W + \frac{1}{2}S_2^{''}(W)\frac{B^2}{\mu^2}\sigma^2 \tag{18}$$

It is easy to see that $\frac{\mu}{r} + W$ constitutes a particular solution to (17). Assuming that $P_0(W)$ and $P_1(W)$ are two particular solutions of the homogeneous differential equation (18) such that $P_0(\bar{W}) = 0$, $P_0^{'}(\bar{W}) = 1$ and $P_1(\bar{W}) = -1$, $P_1^{'}(\bar{W}) = 0$. Since the Wronskian $\mathcal{L}_{P_0P_1}(W)$ associated with $P_0(W)$ and $P_1(W)$ has non-zero value at the point $\bar{W}$, the two function $P_0(W)$ and $P_1(W)$ are linearly independent. Therefore, general solution to the

equation (17) will be written as

$$S_2(W) = \frac{\mu}{r} + W + d_0 P_0(W) + d_1 P_1(W)$$

Based on two conditions $S_2^{'}(\bar{W}) = \frac{\alpha}{1+\alpha}$ and $S_2^{'}(W^*) = 0$ with some value W^*, we obtain

$$\begin{aligned} d_0 &= -\frac{1}{1+\alpha} \\ d_1 &= \frac{\frac{P_0^{'}(W^*)}{1+\alpha} - 1}{P_1^{'}(W^*)} \end{aligned}$$

Thus, the general solution to the equation (17) becomes

$$S_2(W) = \frac{\mu}{r} + W - \frac{1}{1+\alpha} P_0(W) + \frac{\frac{P_0^{'}(W^*)}{1+\alpha} - 1}{P_1^{'}(W^*)} P_1(W)$$

Because of (17), we get

$$\frac{1}{2}\frac{B^2}{\mu^2}\sigma^2 S_2^{''}(W^*) = \frac{\rho W^* P_1^{'}(W^*) - r P_1(W^*) - \frac{r}{1+\alpha}\mathcal{L}_{P_0 P_1}(W^*)}{P_1^{'}(W^*)} \tag{19}$$

Define a function $\Phi(W) = \frac{\rho W P_1^{'}(W) - r P_1(W)}{\mathcal{L}_{P_0 P_1}(W)}$, the equality (19) becomes

$$\frac{1}{2}\frac{B^2}{\mu^2}\sigma^2 S_2^{''}(W^*) = \frac{\left(\Phi(W^*) - \frac{r}{1+\alpha}\right)\mathcal{L}_{P_0 P_1}(W^*)}{P_1^{'}(W^*)} \tag{20}$$

Now we will prove that the function $\Phi(W)$ is strictly decreasing and concave. Indeed, from the following expression of the Wronskian: $\mathcal{L}_{P_0 P_1}(W) = \exp\left(\frac{\rho\left(\bar{W}^2 - W^2\right)}{\frac{B^2}{\mu^2}\sigma^2}\right)$ obtained by applying the Abel's identity, we get[13]

$$\begin{aligned} \Phi^{'}(W) &= \frac{(\rho - r)\, P_1^{'}(W)}{\mathcal{L}_{P_0 P_1}(W)} \\ \Phi^{''}(W) &= \frac{2r}{\frac{B^2}{\mu^2}\sigma^2}\frac{(\rho - r)\, P_1(W)}{\mathcal{L}_{P_0 P_1}(W)} \end{aligned}$$

Therefore, showing that the function $\Phi(W)$ is strictly decreasing and concave is equivalent to prove that $P_1^{'}(W) < 0$ for all $W > \bar{W}$. Assuming by contradiction that there exists $W \in \left(\bar{W}, +\infty\right)$ such that $P_1^{'}(W) > 0$. Define $\ddot{W} = \inf\left\{W > \bar{W} : P_1^{'}(W) > 0\right\}$, then

$$\forall W \in \left(\bar{W}, \ddot{W}\right) : P_1^{'}(W) < 0 \tag{21}$$

which implies that $P_1(\ddot{W}) < P_1(\bar{W}) = -1 < 0$. Since the function $P_1(W)$ satisfies

$$\frac{1}{2}\frac{B^2}{\mu^2}\sigma^2 P_1^{''}(W) + \rho W P_1^{'}(W) - r P_1(W) = 0$$

[13] Note that $P_1(W)$ is solution to the homogeneous differential equation (18).

we get $P_1^{''}(\ddot{W}) < 0$. Hence, in the neighbourhood $\left[\ddot{W} - \varepsilon, \ddot{W}\right)$, the function $P_1^{'}(W)$ is decreasing which means that for $W \in \left[\ddot{W} - \varepsilon, \ddot{W}\right)$, $P_1^{'}(W) > P_1^{'}(\ddot{W}) > 0$, contradiction with (21). In summary, we have $P_1^{'}(W) < 0$ for all $W > \bar{W}$ and so, $\Phi(W)$ is strictly decreasing and concave function. From (20), we see that the equation $S_2^{''}(W^*) = 0$ equivalent to $\Phi(W^*) = \frac{r}{1+\alpha}$ will have unique solution W^*. Moreover, because $\Phi(\bar{W}) = r > \frac{r}{1+\alpha}$, such a solution W^* will be greater than $\bar{W}$.

Concavity: we show that solution to (17) is concave function. Differentiating the equation (17), we get

$$\frac{1}{2}S_2^{'''}(W)\frac{B^2}{\mu^2}\sigma^2 = (\rho - r)\left(1 - S_2^{'}(W)\right) - S_2^{''}(W)\rho W$$

Hence, $S_2^{'''}(W^*) > 0$, which implies that in the neighborhood $(W^* - \varepsilon, W^*)$ of W^*, $S_2^{''}(W) < 0$ and $S_2^{'}(W) > 0$. We will prove that $S_2^{'}(W) > 0$ for all $W \in [\bar{W}, W^* - \varepsilon]$. Suppose that $S_2^{'}(W) \leq 0$ for some $W < W^* - \varepsilon$. Let $\hat{W} = \sup\left\{W < W^* - \varepsilon : S_2^{'}(W) \leq 0\right\}$. So, over the interval $\left(\hat{W}, W^*\right)$, $S_2^{'}(W) > 0$ and $rS_2(W) < rS_2(W^*) = \mu - (\rho - r)W^* < \mu - (\rho - r)W$. By (17), over this interval $S_2^{''}(W) < 0$. Thus, $S_2^{'}(\hat{W}) = -\int_{\hat{W}}^{W^*} S^{''}(W)dW > 0$, contradiction. Hence, $S_2^{'}(W) > 0$ for all $W \in [\bar{W}, W^*)$. By (17), for all $W \in [\bar{W}, W^*)$, $\frac{1}{2}S_2^{''}(W)\frac{B^2}{\mu^2}\sigma^2 \leq rS_2(W) - \mu + (\rho - r)W < rS_2(W^*) - \mu + (\rho - r)W^* = 0$. Q.E.D

B.2.2 Existence, uniqueness and concavity of the function F_1

Considering a function $S_1(W)$ defined by $S_1(W) = F_1(W) + W$. $S_1(W)$ satisfies the following differential equation

$$rS_1(W) = (\mu - \alpha K) - (\rho - r)W + S_1^{'}(W)(\rho W + (1+\alpha)K) + \frac{1}{2}S_1^{''}(W)\frac{B^2}{\mu^2}\sigma^2 \qquad (22)$$

over the interval $\left[\tilde{W}, \bar{W}\right]$ with boundary conditions: $S_1(\tilde{W}) = \tilde{W}, S_1^{'}(\bar{W}) = \frac{\alpha}{1+\alpha}$. As in the previous part, the general solution to the equation (22) can be written as

$$S_1(W) = \frac{\mu + K}{r} + W + b_0 N_0(W) + b_1 N_1(W)$$

where $N_0(W)$ and $N_1(W)$ are two particular solutions to the corresponding homogeneous differential equation such that $N_0(\tilde{W}) = 1,\ N_0^{'}(\tilde{W}) = 0$ and $N_1(\tilde{W}) = 0,\ N_1^{'}(\tilde{W}) = 1$. b_0 and b_1 are calculated from two boundary conditions $S_1(\tilde{W}) = \tilde{W},\ S_1^{'}(\bar{W}) = \frac{\alpha}{1+\alpha}$ with some fixed value $\bar{W}$ as follows

$$\begin{aligned} b_0 &= -\frac{\mu + K}{r} \\ b_1 &= \frac{\frac{\mu+K}{r}N_0^{'}(\bar{W}) - \frac{1}{1+\alpha}}{N_1^{'}(\bar{W})} \end{aligned}$$

Hence

$$\frac{1}{2}\frac{B^2}{\mu^2}\sigma^2 S_1''(\bar{W}) = \frac{\left(K + \frac{\rho\bar{W}}{1+\alpha}\right) N_1'(\bar{W}) - \frac{r}{1+\alpha}N_1(\bar{W}) - (\mu+K)\,\mathcal{L}_{N_0N_1}(\bar{W})}{N_1'(\bar{W})}$$

where $\mathcal{L}_{N_0N_1}(W)$ is the Wronskian associated with N_0 and N_1. Define a function $\Pi(W)$ by

$$\Pi(W) = \frac{\left(K + \frac{\rho W}{1+\alpha}\right) N_1'(W) - \frac{r}{1+\alpha}N_1(W)}{\mathcal{L}_{N_0N_1}(W)}$$

then,

$$\frac{1}{2}\frac{B^2}{\mu^2}\sigma^2 S_1''(\bar{W}) = \frac{\left(\Pi(\bar{W}) - (\mu+K)\right)\mathcal{L}_{N_0N_1}(\bar{W})}{N_1'(\bar{W})}$$

Similarly with the previous part, we can prove that $N_1'(W) > 0$ for all $W > \tilde{W}$ and that the function $\Pi(W)$ is strictly increasing and convex. Moreover $\Pi(\tilde{W}) = K + \frac{\rho\tilde{W}}{1+\alpha} < K + \mu$. Therefore, there exists $\bar{W} > \tilde{W}$ such that $S_1''(\bar{W}) \leq 0$.

Concavity: We here prove that as long as $S_1''(\bar{W}) \leq 0$, then the solution to the equation (22) is concave function. The proof proceeds very similarly as in the precedent parts. Differentiating the equation (22), we get

$$\frac{1}{2}S_1'''(W)\frac{B^2}{\mu^2}\sigma^2 = (\rho - r)\left(1 - S_1'(W)\right) - S_1''(W)\left(\rho W + (1+\alpha)K\right)$$

Hence, $S_1'''(\bar{W}) > 0$, which implies that in the neighborhood $\left(\bar{W}-\varepsilon, \bar{W}\right)$ of $\bar{W}$, $S_1''(W) < 0$ and $S_1'(W) > \frac{\alpha}{1+\alpha}$. We will prove that $S_1'(W) > \frac{\alpha}{1+\alpha}$ for all $W \in \left[\tilde{W}, \bar{W}-\varepsilon\right]$. Suppose that $S_1'(W) \leq \frac{\alpha}{1+\alpha}$ for some $W < \bar{W} - \varepsilon$. Let $\breve{W} = \sup\left\{W < \bar{W} - \varepsilon : S_1'(W) \leq \frac{\alpha}{1+\alpha}\right\}$. So, over the interval $\left(\breve{W}, \bar{W}\right)$, $S_1'(W) > \frac{\alpha}{1+\alpha}$ and then for all $W \in \left(\breve{W}, \bar{W}\right)$:

$$rS_1(W) - \frac{r\alpha}{1+\alpha}W < rS_1(\bar{W}) - \frac{r\alpha}{1+\alpha}\bar{W} \leq \mu - \frac{\rho - r}{1+\alpha}\bar{W} < \mu - \frac{\rho - r}{1+\alpha}W \tag{23}$$

The first inequality comes from the fact that the function S_1 is strictly increasing on the interval $\left(\breve{W}, \bar{W}\right]$. The second inequality is obtained by replacing $S_1'(\bar{W}) = \frac{\alpha}{1+\alpha}$ and $S_1''(\bar{W}) \leq 0$ into the equation (22). (23) implies that $rS_1(W) < \mu + \frac{\alpha}{1+\alpha}\rho W - (\rho - r)\,W$. By (22), we have $S_1''(W) < 0$ over the interval $\left(\breve{W}, \bar{W}\right)$. Thus, $0 > \int_{\breve{W}}^{\bar{W}} S_1''(W)dW = \frac{\alpha}{1+\alpha} - S_1'(\breve{W})$ and so, $S_1'(\breve{W}) > \frac{\alpha}{1+\alpha}$ contradiction. Hence, $S_1'(W) > \frac{\alpha}{1+\alpha}$ for all $W \in \left[\tilde{W}, \bar{W}\right)$. By (22), for all $W \in \left[\tilde{W}, \bar{W}\right)$, $\frac{1}{2}S_1''(W)\frac{B^2}{\mu^2}\sigma^2 \leq rS_1(W) - \mu - \frac{\alpha}{1+\alpha}\rho W + (\rho - r)\,W < rS_1(\bar{W}) - \mu - \frac{\alpha}{1+\alpha}\rho\bar{W} + (\rho - r)\,\bar{W} \leq 0$.

So far, our proof is realized for some fixed value $\bar{W}$, the two functions F_1 and F_2 are then parameterized by $\bar{W}$. To determine this threshold, we rely on the boundary condition $F_1''(\bar{W}) = F_2''(\bar{W})$.

C Appendix: Proof of proposition 3

In order to prove this proposition, we first establish the following result

$$\mathbb{E}\left[\int_0^{\tau} e^{-rt}(r-\rho)W_t dt\right] + \mathbb{E}\left[\int_0^{\tau} e^{-rt}\left(1+\alpha\mathbf{1}_{\{dC_t<0\}}\right)dC_t\right] = W_0 - \mathbb{E}\left[e^{-r\tau}\tilde{W}\right] \quad (24)$$

Indeed, considering a stochastic process $\{Y_t\}$ defined by $Y_t = e^{-rt}W_t$. Using Ito lemma, we obtain the dynamic of $\{Y_t\}$ as follows

$$dY_t = e^{-rt}(\rho - r)W_t dt \ - e^{-rt}\left(1+\alpha\mathbf{1}_{\{dC_t<0\}}\right)dC_t + e^{-rt}\frac{B}{\mu}\sigma dZ_t$$

Hence,

$$Y_{T\wedge\tau} = Y_0 + \int_0^{T\wedge\tau} e^{-rs}(\rho - r)W_s ds \ - \int_0^{T\wedge\tau} e^{-rs}\left(1+\alpha\mathbf{1}_{\{dC_s<0\}}\right)dC_s + \int_0^{T\wedge\tau} e^{-rs}\frac{B}{\mu}\sigma dZ_s$$

Taking the expectation of both sides and then let $T \to \infty$, we get

$$\mathbb{E}\left[e^{-r\tau}\tilde{W}\right] = W_0 + \mathbb{E}\left[\int_0^{\tau} e^{-rs}(\rho - r)W_s ds\right] - \mathbb{E}\left[\int_0^{\tau} e^{-rs}\left(1+\alpha\mathbf{1}_{\{dC_s<0\}}\right)dC_s\right]$$

which is exactly the equality (24).

Owing to the above result, we can easily show that

$$\mathbb{E}\left[\int_0^{\tau} e^{-rt}dP_t\right] = \mathbb{E}\left[\int_0^{\tau} e^{-rt}(\mu dt - dC_t)\right] + E_0 = F(W_0) + E_0$$

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