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Banks' optimal implementation strategies for a risk sensitive regulatory capital rule:
a real options and signalling approach

by

Kjell Bjørn Nordal

Banks' optimal implementation strategies for a risk sensitive regulatory capital rule: a real options and signalling approach *

Kjell Bjørn Nordal[†]

Norges Bank, Bankplassen 2, 0107 Oslo, Norway

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Abstract

I evaluate a bank's incentives to implement a risk sensitive regulatory capital rule and to invest in improved risk measurement. The decision making is analyzed within a real options framework where optimal policies are derived in terms of threshold levels of risk. I also evaluate the situation where exercise or non-exercise of the options to implement or invest are signals about the underlying quality of the loan portfolio. The framework is used for a numerical evaluation of banks' decision of whether to use internal rating based models for credit risk (the IRB-approach) under the new Basel accord (Basel II), where the dynamic behavior of risk is described by an Ohrnstein-Uhlenbeck process. I discuss empirical implications of the evaluation framework.

JEL classification: G13; G21; G28; G32

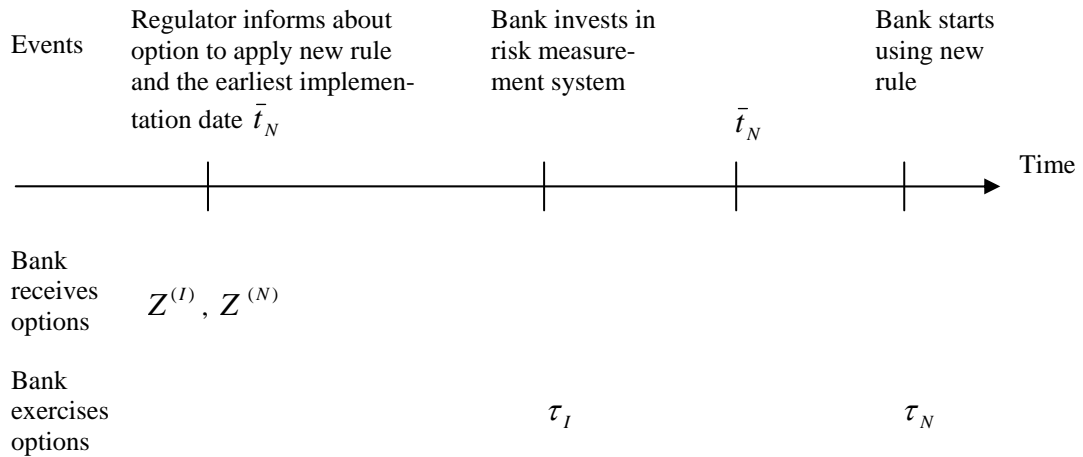Keywords: Risk measurement, capital structure, real options, Basel II

1 Introduction

Authorities require that financial institutions hold a minimum level of capital[1]. I study the situation where banks are given an option to select between two rules for the computation of regulatory capital. The banks can continue to apply the rule currently in use or switch to a new rule. The decision to switch to the new rule is irreversible, i.e., banks have to use this rule both today and in the future. The irreversibility is imposed by the authorities because they do not want that banks shift back and forth between different rules. The banks' decision situation is as outlined in Figure 1. At a random time the regulator informs about the option to apply a new risk sensitive regulatory rule. The new rule may only be applied after having received approval from the regulator. The earliest date the new rule may applied is $\bar{t}_N$. In order to get the approval from the regulator, banks must have a risk measurement system of sufficient quality. Banks that do not have such a system must develop the system in order to get the possibility to introduce the new rule[2]. It takes a time period of $\Delta t^{(I)}$ to develop an acceptable system. A bank starting to develop the system at time t will therefore be able to apply the new rule at time $t + \Delta t^{(I)}$, provided that this date does not come earlier than the earliest feasible date $\bar{t}_N$. The bank's decisions may be viewed as the exercise or non-exercise of two options. The possibility to invest in an acceptable system may be viewed as an option $Z^{(I)}$. If this option is exercised, i.e., the investment is made, the bank receives the option to introduce the new rule $Z^{(N)}$. The bank's decision problem is to select the date τ_I to exercise the investment option and the date τ_N to implement the new rule.

A rule is here a function of characteristics on the underlying loan portfolio yielding the regulatory capital. The analysis is inspired by the option given to banks in the Basel accord (Basel II), see Basel Committee on Banking Supervision (2004). Banks can choose between a standard approach and an approach based on internal rating based models (IRB) to compute regulatory capital for credit risk. The latter alternative is more sensitive to changes in risk over time than the first alternative. Within the Basel II framework, a bank using the standard approach may be seen as using the current rule, here comparable to the

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[1] I adhere to the convention in the banking literature and use the term capital. For non-banks it is customary to use the term equity.

[2] A risk measurement system is, of course, not only used for computing regulatory capital. The system may also be used to improve earnings by discriminatory price setting based on differences in risk levels.

Figure 1: Decision sequence - implementation of new rule

Events — Regulator informs about option to apply new rule and the earliest implementation date $\bar{t}_N$ — Bank invests in risk measurement system — Bank starts using new rule — $\bar{t}_N$ — Time

Bank receives options — $Z^{(I)}, Z^{(N)}$

Bank exercises options — τ_I — τ_N

non-rating based rule.

The basic premise underlying the analysis is that there is a unique optimal capital ratio. The optimal capital ratio is the highest of either regulatory capital or the capital requirement imposed by outside stake holders. The concept of such a capital requirement is similar to the one found on page 395 in Berger et al. (1995), where a bank's market capital "requirement" is described as

> ".. the capital ratio that maximizes the value of the bank in the absence of regulatory capital requirements (and all the regulatory mechanisms that are used to enforce them), but in the presence of the rest of the regulatory structure that protects the safety and soundness of banks".

The implication is that the market value of the bank will decline if the the bank has too much or too little capital. The introduction of the risk sensitive rule may influence the bank in two ways. The first effect is *reduced regulatory capital*. A reduction in regulatory capital will reduce the optimal capital ratio, provided that the bank is constrained by the current rule. Because the decision to use the new rule is irreversible, the bank must not only take into consideration

the immediate change in optimal capital from the new rule, but also the future development in the difference in regulatory capital between the old and the new rule. The second effect is the *signalling effect.* In short, the signalling effect reflects changes in the capital requirement. The capital requirement is changed by the market, possibly upon observing whether the bank switches to the new rule. Under the Basel II rules the internal models are to be approved by regulators. Regulators will only approve models if they are of a sufficient standard. Such an approval may be a signal about the portfolio quality and the quality of the management of the bank. If the bank does not introduce the risk sensitive rule, i.e., does not get the regulator's approval, the external stake holders may suspect that they in the future will face negative surprises concerning losses on the bank's loan portfolio. If the bank does not introduce the new rule, the optimal capital ratio may therefore increase.

The basic premise of a unique optimal capital ratio is not trivial from a theoretical perspective. After all, if it was all the same which capital ratio the owners of the bank decided on, the owners would be indifferent when selecting the regulatory capital and the level of the capital in general. All capital ratios would be optimal. According to the result of Miller and Modigliani (M & M), see Modigliani and Miller (1958), the choice of level of capital does not influence the market value of the company ("the size of the pie"). Any change in the level of capital will only cause a redistribution of value between equity and bond holders (reflecting changing "shares of the pie"). There will be no gain to the shareholders from engaging in the activity of changing the capital ratio. If one makes other assumptions than those in the M & M, there may be an optimal capital ratio. Changing the capital ratio from a non-optimal to an optimal level will then cause the value of the shareholders' holding to increase. This increase may again be caused by an increase in the market value of the company, by a redistribution of wealth from bond holders, or a combination of the two effects.

I will briefly go through factors found in the literature causing the existence of an optimal capital ratio. *Costs of financial distress* make it optimal to avoid holding low levels of capital. Examples of such costs are bankruptcy costs and the costs of foregone business opportunities due to outsiders' unwillingness to conduct business with a company that may fail. Deadweight losses due to bankruptcy and reorganization were mentioned by Modigliani and Miller (1958). *Taxes* favor the use of debt. Interest payments are deductible in the company's taxable income. Increasing the level of debt will therefore reduce the authorities share of profit and leave more to the shareholders, see, e.g., Miller (1976). *Trans-*

action costs are costs of raising capital. In the presence of transactions cost, the arbitrage argument causing the M & M argument to hold may no longer be strictly valid. Transaction costs also form the basis for the pecking-order model of debt, see Myers (1984). According to this model, retained earnings are the "cheapest" form of capital, followed by new debt and new equity. The capital ratio will then vary over time with the difference between necessary investment and internally generated funds. Several explanations for an optimal capital ratio are based on the argument of *asymmetric information*. As an example, managers of the bank may use the level of capital as a signal to financial markets about the quality of banks' assets. In Ross (1977) there are two types of companies. One company will have a higher final value than the other. The actual type of a company is not known by the market. If the manager has information about the true type, and with an appropriate incentive structure, the manager will take on relatively more debt in the best type of company in order to maximize his own reward. The market will then price the two types of companies differently. This signal causes an increase in the value of equity for the good company. Another example of asymmetric information is the agency cost argument that increased debt will lead to increased operational efficiency, see Jensen (1986). A requirement to service debt will discipline the managers and induce a more efficient operation of the firm. One argument applying specifically to banks is the presence of a *safety net* for banks' depositors. The safety net refers to the guarantee that authorities give to depositors for the safety of their bank deposits. If the price that banks pay to the authorities for this guarantee is too low relative to the actual risk, there is an incentive for banks to accept too much deposits. For discussions of the capital ratio related to financial institutions in particular, see, e.g. Berger et al. (1995) or Miller (1995). All the reasons mentioned above may, more or less, be present when a given bank is analyzed. An optimal capital ratio may therefore be the result of a trade-off between several factors. Such a mixture of explanatory factors may therefore be present in the analysis. This was, e.g., the approach taken by Fama and French (2002) when they tested the pecking-order model against what they named a trade-off model of debt.

The evaluation framework presented in this paper is appropriate when analyzing the decision making in banks that have the option to measure risk by statistical methods. All major banks, as measured by size, belongs to this category. The model is less relevant for banks that mainly perform relationship lending. Relationship lending is based on "soft information", see, e.g., Berger

and Udell (2002). Soft information is information that is hard to communicate to others (Petersen and Rajan (2002)) or information that cannot be credibly transmitted to others (Stein (2002)). Risk assessments based on soft information are difficult to quantify and to verify for a regulator.

This work adds to the literature covering the application of real options theory in different industries. Textbook treatment of optimal investment timing for irreversible investments may, e.g., be found in Dixit and Pindyck (1994) and Amram and Kulatilaka (1999). My work also contributes to the literature concerning the consequences of a risk sensitive regulatory capital regime. In particular, I describe how banks optimal policies depend on whether they are constrained by the current rule, the reduction in regulatory capital obtained by applying the new rule, and on possible changes in the market's capital requirement. Different assumptions regarding this capital requirement influences the optimal policies. I also explain how different assumptions leads to different predictions regarding changes in capital for implementing and non-implementing banks. The micro perspective, where I focus on individual banks, complements the macro perspective usually applied when analyzing financial stability issues related to such regimes.

The paper is organized as follows. In the next section I present the model. In section three I provide a numerical example. The main points are summarized in the final section.

2 The model

Optimal capital ratio

The bank optimizes its capital under regulatory regime i according to the rule

$$\gamma_t^{(i)*} = \max(\varepsilon_t, \underline{\gamma}_t^{(i)} + b), \quad i \in \{C, N\}, \tag{1}$$

where $\gamma_t^{(i)*}$ is the optimal capital, ε_t is the optimal capital ratio in absence of capital regulation, $\underline{\gamma}_t^{(i)}$ is the regulatory capital under rule i, and b is the constant buffer capital held above the regulatory capital. The bank can either use the current regulatory rule C or the new rule N. If the bank is *constrained* by the regulatory rule, it will according to equation (1) select a capital ratio equal to the regulatory capital with the addition of necessary buffer capital. In this case the bank would be willing to hold less capital in absence of capital regulation

$(\varepsilon_t < \underline{\gamma}_t^{(i)} + b)$. If the bank is *unconstrained* by the regulatory rule, the optimal capital is equal to the unregulated optimal capital ε_t.

A regulatory capital rule i specifies the lowest allowable capital ratio in a bank at time t as a function of a state variable μ_t, and characteristics β of the bank, i.e.,

$$\underline{\gamma}_t^{(i)} = f^i(\mu_t, \beta), \ i \in \{C, N\}, \ \beta \in \{\beta^L, \beta^H\}. \tag{2}$$

The state variable may be thought of as the level of risk in the portfolio, e.g., the expected percentage credit losses on the underlying portfolio over a given future time period. The bank's type β may be defined by whether or not the bank is characterized by a high (H) or low (L) level of risk of large sudden losses (event risk) in the loan portfolio.

With regulatory rule i it is optimal for the bank to hold capital equal to $\gamma_t^{(i)*}$. Any deviations from this optimal capital ratio will lead to a reduction in the flow of benefits originating from the choice of capital ratio. This flow may be interpreted as cash flow. When comparing optimal capital ratios under the old and the new rule, I assume that the flow of benefits is higher for the rule giving the lowest optimal capital ratio. The change in accumulated flow of benefits B from a reduction in the actual capital ratio is given by

$$dB_t = \mathrm{h}\left(\gamma_t^{(C)*}\right) dt - \mathrm{h}\left(\gamma_t^{(N)*}\right) dt, \tag{3}$$

where $\mathrm{h}(.)$ is the rate of cash flow influenced by the change in optimal capital ratio. If the capital ratio is not changed, there will be no changes in cash flow ($\mathrm{h}(x) - \mathrm{h}(x) = 0$) and a decrease in the capital ratio will increase the cash flow rate ($\mathrm{h}(x) - \mathrm{h}(x') > 0$, $x > x'$).

Unregulated optimal capital

The unregulated optimal capital may be determined by the outside stake holders in the bank and this capital requirement may also depend on the outsiders' perception of the bank's type. Rating agencies and bond holders are examples of outside stake holders that may influence the level of capital. The lowest bound on capital in a bank of type β a rating agency is willing to accept to keep the bank in a rating class, or, alternatively, the lowest level of capital a bond lender is willing to accept before increasing the default risk component in the bond rate, may actually correspond to the capital a bank would choose to hold in absence of capital regulation. In this setting the bank will always prefer to hold

7

as little capital as possible while maintaining the rating category or the default component on the bond rate. I show a graphical exposition of this argument in Figure 2.

I simplify the analysis by assuming that the outsiders may choose between three different capital levels at time t,

$$\varepsilon_t \in \{\varepsilon^L, \varepsilon^M, \varepsilon^H\}, \ \varepsilon^L \leq \varepsilon^M \leq \varepsilon^H \ . \tag{4}$$

The outsiders will select the required capital that maximizes the expected utility,

$$\varepsilon_t^* = \arg\max_{\varepsilon_t} \sum_{\beta} p_t(\beta) u(\varepsilon_t, \beta), \ \beta \in \{\beta^L, \beta^H\}, \tag{5}$$

where $p_t(\beta)$ is the probability that the bank is of a specific type and $u(\cdot, \cdot)$ is the utility function. One intuitive solution of (5) is that it is optimal to select the highest level of capital if the outsiders believe the bank is of type β^H ($p_t(\beta = \beta^H) = 1$) and a low level of capital if it is of type β^L ($p_t(\beta = \beta^L) = 1$). If the outsiders are uncertain about the bank's type ($p_t(\beta) \neq 1, \ \beta \in \{\beta^L, \beta^H\}$), they will require a medium level of capital.

A banks' optimal exercise strategies

The payoff from the option to implement the new rule at the exercise date equals the value of increased cash flows, i.e.,

$$Z_{\tau_N}^{(N)} = \mathrm{E}_t^Q \left(\int_t^\infty e^{-\int_t^s r_u du} dB_s \right) \ , \ \tau_N = t \ , \tag{6}$$

where $\mathrm{E}_t^Q(\cdot)$ is the expectation operator under the equivalent martingale measure Q conditioned on information at time t, r_u is the instantaneous risk free interest rate at time u, and, dB_s is the increase in cash flow given by equation (3)[3].

The market value of option i at time t when exercised at a future time τ_i, $Z_t^{(i),\tau_i}$ is then equal to

$$Z_t^{(i),\tau_i} = \mathrm{E}_t^Q \left(e^{-\int_t^{\tau_i} r_s ds} Z_{\tau_i}^{(i)} \right), \ i \in \{I, N\}, \ t \leq \tau_i \ , \tag{7}$$

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[3]Equation (6) is the standard way of writing pricing equations in the derivatives and contingent claims literature. I assume that technical conditions hold. Another commonly used term for the probability measure Q is a *risk neutral* probability measure.

Figure 2: Unregulated optimal capital

The picture shows an example of how the outside capital requirement will determine the unregulated optimal capital. The marginal benefit rate from reducing the capital ratio is shown as the line AA. Bond holders require a compensation for event risk dependent on the perception of the type of bank and the actual capital ratio. For a type β bank ($\beta \in \{\beta^L, \beta^H\}$), the bond holders will require a compensation equal to c_2 if the capital ratio is in the interval $\left[\underline{\varepsilon}^i, \overline{\varepsilon}^i\right\rangle$, $i \in \{L, H\}$. In the pictured case the lower bound determines the unregulated optimal capital for the bank.
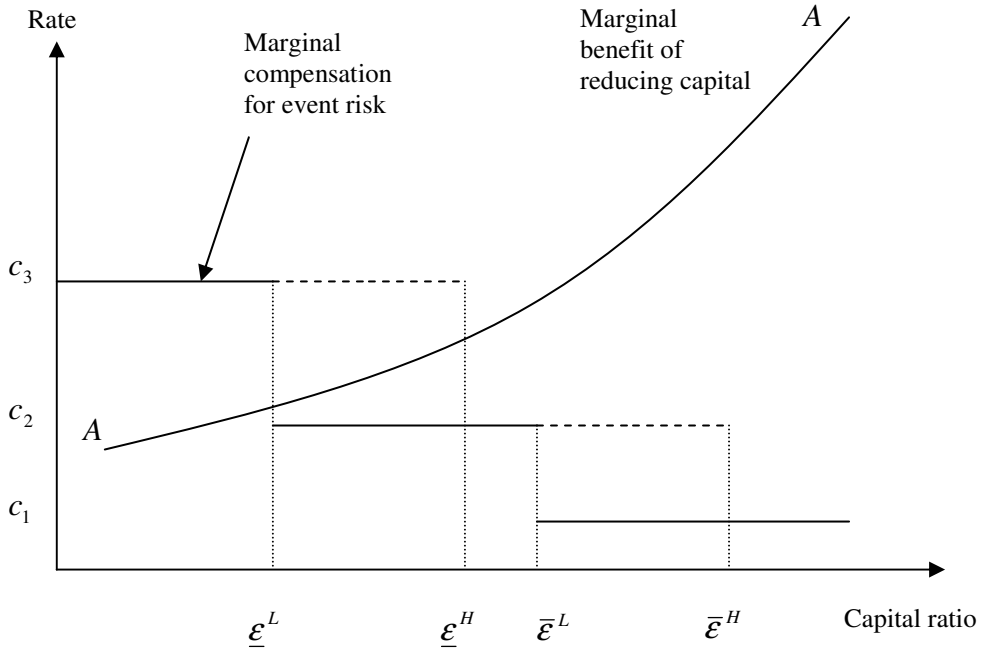
where $Z_{\tau_i}^{(i)}$ is the payoff from option i when exercised. With admissible exercise dates $\mathcal{T}_i$, the market value at time t with the optimal exercise strategy is given by

$$Z_t^{(i)*} = \sup_{\tau_i \in \mathcal{T}_i} Z_t^{(i), \tau_i}, \quad i \in \{I, N\}, \ t \leq \tau_i \ . \tag{8}$$

The payoff from the option to invest when exercised, i.e., when the investment is made, is equal to the market value of the option to implement the new rule, i.e., $Z_{\tau_I}^{(I)*} = Z_t^{(N)*}$, where $\tau_I = t$.

It is customary to express exercise policies in terms of threshold levels for

the state variable μ_t. It is reasonable to assume that policies are such that the exercise of the options are optimal when the risk level μ_t is equal to or lower than a threshold level. Waiting, or non-exercise, will then be optimal at higher risk levels. Lemma 1 gives sufficient conditions for the existence of such a threshold for the implementation option in what I have termed the benchmark case. In the benchmark case the unregulated optimal capital is a constant ε, the bank's type β is known, and there are no implementation costs.

Lemma 1 (Threshold level for implementation). *In order for the exercise of the option in the benchmark example to be optimal for $\mu_t \leq \mu^{**}$, and waiting to be optimal otherwise, it is sufficient that*

$$-h(\gamma^{(C)*}(\mu_t)) + h(\gamma^{(N)*}(\mu_t))$$

is a monotone increasing function of μ_t and that the cumulative distribution function for

$$Z^{(I)*}(\mu_{t+dt}) - Z^{(I)}(\mu_{\tau_I=t+dt})$$

shifts to the right when the state variable μ_t increases (first order stochastic dominance).

Proof. See appendix. □

Note that if $h(\cdot)$ is a non-decreasing function, the first condition of Lemma 1 will be satisfied if the optimal capital ratio with rule C is a constant independent of the risk level and if the capital with rule N is an increasing function of the risk level.

Note that it will not be optimal for a bank in the benchmark example that is constrained by rule C to implement the new rule if this leads to an immediate increase in regulatory capital.

Proposition 1 (Condition of non-increasing regulatory capital for implementation). *If the bank in the benchmark case is constrained by regulatory rule C, it will only implement the new rule N if the new rule does not give an immediate increase in regulatory capital.*

Proof. See appendix. □

Perfect Bayesian equilibrium in pure strategies

In order to analyze how banks may incorporate possible changes in outsiders' capital requirement in its exercise strategies, I consider strategies satisfying a

perfect Bayesian equilibrium in pure strategies[4]. I start with the implementation game. In the implementation game it seems reasonable to assume that the bank's true type is revealed if it uses the new regulatory rule. This means that for a bank that has implemented the rule, the outsiders can observe the implementation date τ_N and its type β from observing the minimum regulatory capital (given by equation (2)). For a bank not having implemented the new rule, the outsiders can observe at a given time t only that it has not exercised the implementation option ($\tau_N > t$).

Definition 1. *A perfect Bayesian equilibrium of the signalling game for the implementation of rule N is the triple $\{\tau_N, \varepsilon_t, \overline{p}(\cdot \mid \tau_N, \underline{\gamma}_t^{(N)})\}$ such that*

$$\tau_N^*(\beta) \in \sup_{\tau_N \in \mathcal{T}_N} Z_t^{(N),\tau_N}(\varepsilon_t^*, \beta), \ \beta \in \{\beta^L, \beta^H\}, \tag{9}$$

$$\varepsilon_t^*(\tau_N, \underline{\gamma}_t^{(N)}) \in \arg\max_{\varepsilon_t} \sum_{\beta} \overline{p}(\beta \mid \tau_N, \underline{\gamma}_t^{(N)}) u(\varepsilon_t, \beta), \ \beta \in \{\beta^L, \beta^H\}, \tag{10}$$

and

$$\overline{p}_t(\beta \mid \tau_N, \underline{\gamma}_t^{(N)}) = p_t(\beta) p(\tau_N, \underline{\gamma}_t^{(N)} \mid \beta) / \sum_{\beta' \in \beta} p(\beta') p(\tau_N, \underline{\gamma}_t^{(N)} \mid \beta') \tag{11}$$

if

$$\sum_{\beta' \in \beta} p(\beta') p(\tau_N, \underline{\gamma}_t^{(N)} \mid \beta') > 0. \tag{12}$$

The equilibrium is such that the strategies are optimal for the the bank and the outsiders given the beliefs. The beliefs are updated by using Bayes rule, whenever possible, based on what the outsiders can observe.

I make the following assumptions:

a1 Existence of threshold levels. It is optimal for a bank of type β facing a no-implementation capital requirement of ε^i to implement the new rule the first time s that $\mu_s \leq \mu_s^{i|\beta}, i \in \{L, M, H\}, \beta \in \{\beta^L, \beta^H\}, s \geq \hat{t}$. Note that $\hat{t}$ is the earliest date the bank may introduce the new regulatory rule.

a2 Threshold levels are not decreasing as a function of increasing capital requirement in case of no implementation. The threshold levels are such that $\mu_s^{L|\beta} \leq \mu_s^{M|\beta} \leq \mu_s^{H|\beta}, \ i \in \{L, H\}, \ s \geq \hat{t}$.

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[4]With pure strategies the pricing equation (6) may be used directly. In the case of mixed strategies it is necessary to specify whether the uncertainty related to the strategies requires risk compensation.

a3 The threshold level with a no-implementation capital requirement of ϵ^H is not lower for a type β^L bank than for a type β^H bank, i.e., $\mu_s^{H|\beta^H} \leq \mu_s^{H|\beta^L}, s \geq \hat{t}$.

Note that according to assumption a1 the bank faces a capital requirement of ε^i at all future dates if it does not implement rule N. This influences the optimal capital under rule C according to equation (1) and thereby also the benefits of introducing rule N according to equation (3). Assumption a2 is based on the argument that the new rule is risk sensitive and that the regulatory capital is lower for lower risk levels. This means that the reducion in capital, and thereby the value of cost savings, is increasing at lower risk levels.

Proposition 2 (Equilibrium for the implementation game). *Assume that a1-a3 holds. Let $t_1 = \min\{s : \mu_s \leq \mu_s^{H|\beta^L}\}$ and $t_2 = \min\{s : \mu_s \leq \mu_s^{H|\beta^H}\}, s > \hat{t}$. Then i)-iii) is a Perfect Bayesian equilibrium of the signalling game covering implementation. If $\hat{t} < t_2$ and $t_1 < t_2$, there will be a separating equilibrium at time t_1 when bank of type β^L will implement and the bank of type β^H will not implement the new rule.*

i) *Banks' optimal exercise policy:*

$$\tau_N^*(\beta) = \min\{s : \mu_s \leq \mu_s^{H|\beta}\}, \ \beta \in \{\beta^L, \beta^H\}$$

ii) *Outsiders' optimal capital requirement:*

$$\varepsilon_t^*(\tau_N, \underline{\gamma}_t^{(N)}) = \begin{cases} \epsilon^M & \text{if } \tau_N > t, t < t_1 \\ \varepsilon^H & \text{if } \tau_N > t, t \geq t_1 \\ \varepsilon^L & \text{if } \tau_N \geq t, t \geq t_1 \text{ and } f^{(N)}(\cdot, \beta^L) \\ \varepsilon^H & \text{if } \tau_N \geq t, t \geq t_1 \text{ and } f^{(N)}(\cdot, \beta^H) \end{cases} \tag{13}$$

iii) *Beliefs: The ex ante probability that the bank is of type β^H is q, $0 < q < 1$. Probabilities of the type of bank conditioned on observing τ_N and $\underline{\gamma}_t^{(N)}$:*

$$p_t(\beta^H \mid \tau_N, \underline{\gamma}_t^{(N)}) = \begin{cases} \varnothing & \text{if } \tau_N > t, t < t_1 \\ 1 & \text{if } \tau_N > t, t \geq t_1 \\ 0 & \text{if } \tau_N \geq t, t \geq t_1 \text{ and } f^{(N)}(\cdot, \beta^L) \\ 1 & \text{if } \tau_N \geq t, t \geq t_1 \text{ and } f^{(N)}(\cdot, \beta^H) \end{cases} \tag{14}$$

and

$$p_t(\beta^L \mid \tau_N, \underline{\gamma}_t^{(N)}) = \begin{cases} \varnothing & \text{if } \tau_N > t, t < t_1 \\ 1 & \text{if } \tau_N > t, t \geq t_1 \\ 1 & \text{if } \tau_N \geq t, t \geq t_1 \text{ and } f^{(N)}(\cdot, \beta^L) \\ 0 & \text{if } \tau_N \geq t, t \geq t_1 \text{ and } f^{(N)}(\cdot, \beta^H) \end{cases} \tag{15}$$

Proof. See the appendix. $\qquad\qquad\square$

The intuition behind the separating equilibrium for the implementation game is the following. It is optimal for the type β^L bank to implement the new rule at a higher risk level than the β^H bank under the highest capital requirement ε^H. The beliefs are such that a non-implementing bank is classified as a type β^H bank the first time that it is optimal <u>only</u> for the type β^L bank to implement. This will, of course, be a separating equilibrium. Figure 3 shows the the two types' optimal decisions for different paths of the risk level. Proposition 2 offers an explanation for why the capital may increase in banks that are not implementing a new regulatory rule.

I now turn to the investment game. In this game the outsiders can only observe the risk level and whether or not an investment has been made[5].

Definition 2. *A perfect Bayesian equilibrium of the signalling game for the investment in improved risk measurement is the triple $\{\tau_I, \varepsilon_t, \overline{p}(\cdot \mid \tau_I)\}$ such that*

$$\tau_I^*(\beta) \in \sup_{\tau_I \in \mathcal{T}_I} Z_t^{(N), \tau_I}(\varepsilon_t^*, \beta), \ \beta \in \{\beta^L, \beta^H\}, \tag{16}$$

$$\varepsilon_t^*(\tau_I) \in \arg\max_{\varepsilon_t} \sum_{\beta} \overline{p}(\beta \mid \tau_I) u(\varepsilon_t, \beta), \ \beta \in \{\beta^L, \beta^H\}, \tag{17}$$

and

$$\overline{p}(\beta \mid \tau_I) = p(\beta) p(\tau_I \mid \beta) / \sum_{\beta' \in \beta} p(\beta') p(\tau_I \mid \beta') \tag{18}$$

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[5]Banks may release press statements or give information in the annual accounts regarding investments made to improve the risk measurement in order to satisfy the new regulatory rule. Another indication of whether investments in improved risk measurement have taken place, is if the bank starts giving detailed information about the risk in their loan portfolio. Implementation of the new rule will be directly observable through the description of regulatory capital in the bank's accounting reports

Figure 3: The implementation decision

The bank of type β ($\beta \in \{\beta^L, \beta^H\}$) will implement the new rule provided that the state variable is lower or equal to the threshold $\mu^{H,\beta}$ at time t, $t \geq \hat{t}_N$. The figure shows three paths of the state variable μ_t, each with a different starting point. In the situation described by path A, neither type of bank will invest at time $\hat{t}_N$. At time t' banks of type β^L will invest. The market will then know that the banks that did not invest are of type β^H, and the capital requirement for such banks is assigned to them. At time t''', the banks of type β^H will also implement the new rule. In the situation described by path B, banks of type β^L will immediately implement at time $\hat{t}_N$. The non-implementing banks belongs to type β^H. For path C, both types of banks will implement at the earliest possible time.
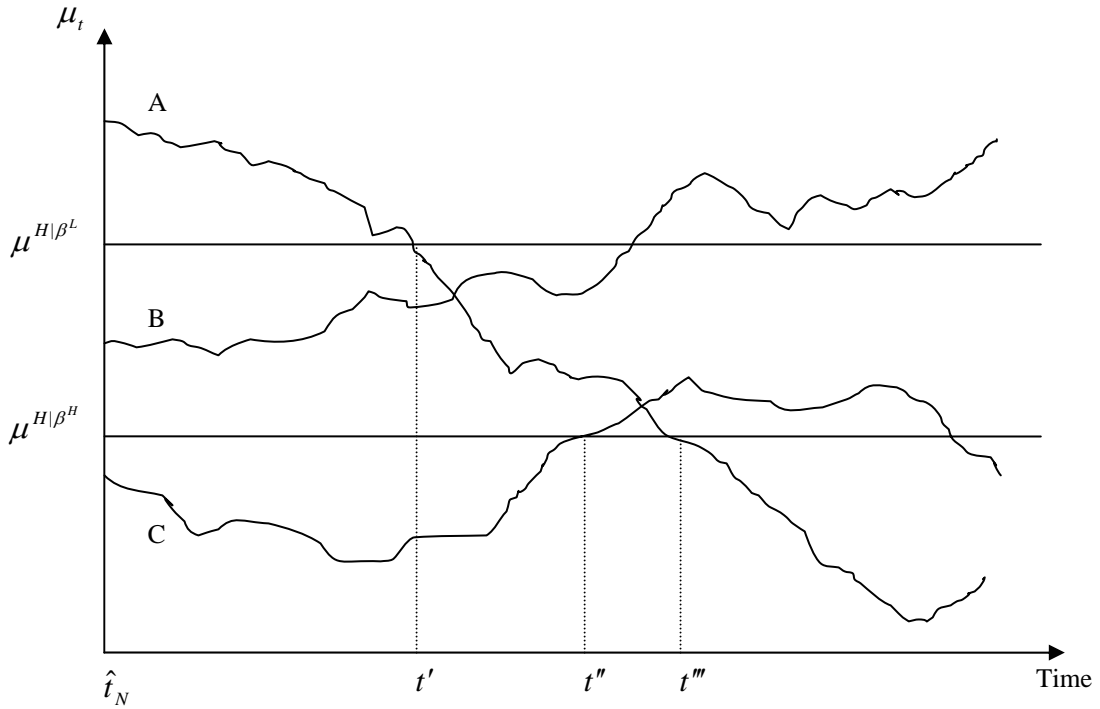
if

$$\sum_{\beta' \in \beta} p(\beta')p(\tau_I \mid \beta') > 0. \tag{19}$$

In the implementation game the bank's type was revealed through the disclosure of the regulatory capital after the new rule had been implemented. A bank would therefore get the capital requirement corresponding to its type after

the exercise date. This is not necessarily the case for the investment game. I make the following assumptions regarding the investment game, corresponding to assumptions a1-a3 in the implementation game:

a4 Existence of threshold levels. For a bank facing a no-investment capital requirement of ε^i if it does not invest and a capital requirement of ε^k if it invests, it is optimal to invest the first time s that $\mu_s \leq \mu_s^{\{k,i\}|\beta}$, $\{i,k\} \in \{L,M,H\}$, $\beta \in \{\beta^L, \beta^H\}$, $s \geq \hat{t}$. Note that the capital requirement that is achieved if the bank invests may be changed in the following implementation game.

a5 Threshold levels are not decreasing as a function of increasing capital requirement in case of no investment. The threshold levels are such that $\mu_s^{\{i,L\}|\beta} \leq \mu_s^{\{i,M\}|\beta} \leq \mu_s^{\{i,H\}|\beta}$, $i \in \{L,M,H\}$, $\beta \in \{\beta^L, \beta^H\}$, $s \geq \hat{t}$.

a6 The threshold level when banks face a no-investment capital requirement of ε^H and a investment capital requirement of ε^L is higher for a type β^L bank than for a type β^H bank, i.e., $\mu_s^{\{L,H\}|\beta^H} < \mu_s^{\{L,H\}|\beta^L}$.

Proposition 3 (Equilibrium for the investment game). *Assume that a4-a6 holds. Let $t_1 = \min\{s : \mu_s \leq \mu_s^{\{L,H\}|\beta^L}\}$ and $t_2 = \min\{s : \mu_s \leq \mu_s^{\{L,H\}|\beta^H}\}, s \geq \hat{t}$. If $\hat{t} < t_2$ then i)-iii) is a Perfect Bayesian equilibrium for the investment game. It is a separating equilibrium at time t_1 where only the type β^L bank invests.*

i) *Banks' optimal exercise policy:*

$$\tau_I^*(\beta) = \min\{s : \mu_s \leq \mu_s^{\{L,H\}|\beta}\}, \ \beta \in \{\beta^L, \beta^H\}$$

ii) *The outsiders' optimal capital requirement:*

$$\varepsilon_t^*(\tau_I) = \begin{cases} \varepsilon^M & \text{if } \tau_I > t, t < t_1 \\ \varepsilon^H & \text{if } \tau_I > t, t \geq t_1 \\ \varepsilon^L & \text{if } \tau_I \geq t, \tau_I = t_1 \\ \varepsilon^H & \text{if } \tau_I \geq t, \tau_I > t_1 \end{cases} \qquad (20)$$

iii) *Beliefs: The ex-ante probability that the bank is of type β^H is q, $0 < q <$*

1. *Probabilities of the type of bank conditioned on observing τ_I:*

$$p_t(\beta^H \mid \tau_I) = \begin{cases} \varnothing & \text{if } \tau_I > t, t < t_1 \\ 1 & \text{if } \tau_I > t, t \geq t_1 \\ 0 & \text{if } \tau_I \geq t, \tau_I = t_1 \\ 1 & \text{if } \tau_I \geq t, \tau_I > t_1 \end{cases} \quad . \tag{21}$$

Proof. See the appendix. $\square$

The intuition behind this equilibrium is the same as for equilibrium in the implementation game. The bank not investing the first time it will be optimal only for a type β^L bank to invest is, according to the beliefs, a type β^H bank. Note that the investment threshold is such that the bank at this date will get the highest capital requirement if it does not invest and the lowest if it invests. If the risk level $\mu_{\hat{t}}$ is so low that both banks would invest in order to get an immediate low capital requirement, there would be no separating equilibrium[6]. This corresponds to the situation in Figure 4 where the risk level is below point b. However, when the risk level is initially high (higher than point b in Figure 4), Proposition 3 does explain why the capital requirement may increase for non-investing banks. Note also that the proposition applies for the important cases where only the bank of type β^L will invest, i.e., when $t_2 = \infty$.

3 Internal models for credit risk under Basel II - numerical computations

Internal models and credit risk

Basel II allows for the use of internal models to measure credit risk and to use the measured risk when computing regulatory capital. A short summary of the Basel II rule is given in Appendix B. The rule requires that the probability of default (PD) and loss given default (LGD) are specified in order to compute regulatory capital for a specific loan under the IRB approach.

In order to simplify the exposition I base my computation only on expected percentage credit losses μ_t on an underlying portfolio. I extract the average probability of default at time t from expected losses by dividing by the parameter

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[6]A separating equilibrium may, however, exist in mixed strategies.

Figure 4: Investment decision

The figure depicts the threshold levels for banks in terms of the state variable μ_t at time t. The threshold investment level $\mu_t^{\{i,j\}|\beta}$ applies for a bank of type β facing a capital requirement of i if it invests at time t and j if not. When $\mu_t > a$ neither type of bank will invest. When $b < \mu_t \leq a$, the type β^L-bank will invest. When $c < \mu_t \leq b$, both types of banks would invest if they believed this would shift the capital requirement from the high to the low one. If the bank believed that the investment decision would not change the capital requirement, it would not invest. When $d < \mu_t \leq c$, only the type β^L-bank would invest if it believed that this would not change the capital requirement.
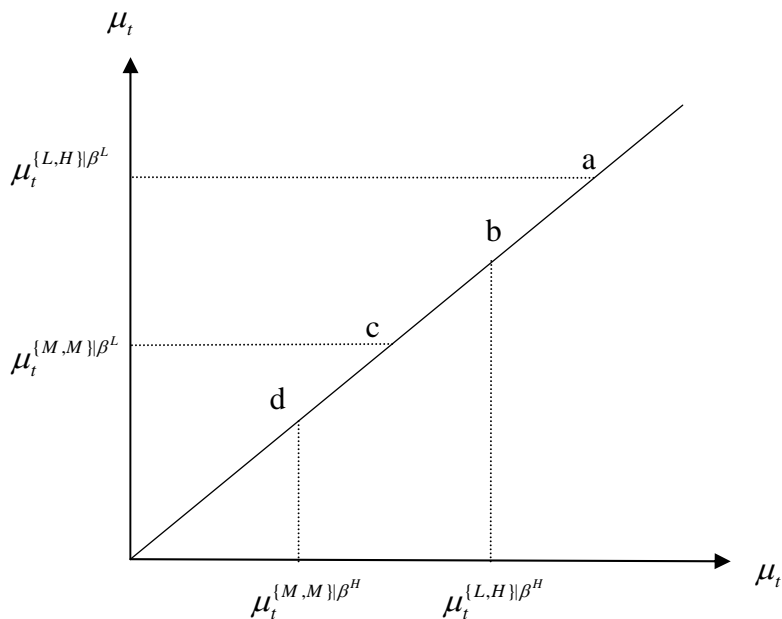
LGD, i.e.,

$$PD_t = \max(0, \min(1, \mu_t/LGD)), \; 0 < LGD \leq 1 \,. \tag{22}$$

I consider the case where the instantaneous change in expected percentage credit losses on the portfolio develops according to an Ohrnstein-Uhlenbeck process,

$$d\mu_t = \kappa(\theta - \mu_t)dt + \sigma dW_t, \tag{23}$$

where θ, κ, and σ are nonnegative constants and where dW_t is the increment of a standard Brownian motion. We interpret θ as the long term mean of expected losses. The speed of reversion to the long run mean is captured by the parameter

κ. The process (23) corrected for the price of risk is

$$d\mu_t = \kappa(\theta - \frac{\lambda\sigma}{\kappa} - \mu_t)dt + \sigma dW_t^*, \tag{24}$$

where W_t^* is a Brownian motion under an equivalent martingale measure, and λ is the price of risk related to unexpected changes in expected losses. The price of risk equals the required compensation beyond the risk free interest rate $(\eta - r)$ per unit of risk σ, i.e.,

$$\lambda = \frac{\eta - r}{\sigma}. \tag{25}$$

One procedure to determine the price of risk is to compute the required expected return (η) on holding an asset influenced by the specific risk (σ) by applying the CAPM, see, e.g., p. 115 in Dixit and Pindyck (1994) or Nordal (2001). I would expect the prices of risk in (25) to be negative because it is likely that expected losses (losses are measured as a positive number) are negatively correlated with the return on the market portfolio. I may alternatively write (24) as

$$d\mu_t = \kappa(\theta^* - \mu_t)dt + \sigma dW_t^*, \tag{26}$$

where $\theta^* = (\theta - \lambda\sigma/\kappa)$, see, e.g., Bjerksund and Ekern (1995) or Schwartz (1997). The effect of the correction for the price of risk is therefore to reduce (increase) the "long term mean" if the price of risk is positive (negative).

Value and threshold levels

I build a trinomial tree for the state variable μ_t as in Hull and White (1994). Table 1 shows the assumptions for the benchmark example. In the benchmark example I do not differ between types of banks. The unregulated optimal capital is a constant ε equal to 5 per cent. I have used a risk free interest rate of 4 per cent, a price λ per unit of risk in the development in expected losses of -1 per cent, and a volatility of expected losses σ equal to 0.5 per cent. I have further used a time step of 0.25 (quarters). The long run mean of expected losses in the risky portfolio is 0.5 per cent. The long run mean under the risk neutral probability is 0.51 per cent. The value of the new rule is found by using an evaluation period of 50 years. For every node in the tree the present value of the advantage of using the new rule instead of the old during the next time period

Table 1: Parameter values used in the numerical example

Parameter	Numerical value	Description
Δt	0.25	time step
r	0.04	risk free interest rate
λ	-0.01	price of risk
T_N	10	time horizon for option to implement
T_I	0	time horizon for option to invest
$\Delta t^{(I)}$	0	lead time investment
I	0.15	investment in risk measurement
θ	0.005	long run mean
κ	0.23	speed of mean reversion
σ	0.005	volatility
LGD	0.45	loss given default
b	0.02	buffer capital
ε	0.05	unregulated optimal capital

Δt is computed as[7]

$$V_t \left(\int_t^{t+\Delta t} e^{-r(s-t)} \left(\gamma^{(C)*}(\mu_s) - \gamma^{(N)*}(\mu_s) \right) ds \right) \approx \frac{(1 - e^{-r\Delta t}}{r}) \left(\gamma^{(C)*}(\mu_t) - \gamma^{(N)*}(\mu_t) \right).$$

(27)

The regulatory capital is 8.0 per cent for loans under the current rule C. The regulatory capital under the new rule is derived by using the IRB model in Basel II with no reduction for small- and medium-sized entities, see Appendix B. Figure 5 shows the regulatory capital for different levels of expected losses. The break even level of expected losses μ_t making the regulatory capital with rule C equal to the regulatory capital with rule N is approximately 0.6 per cent. This corresponds to a PD of approximately 1.3 per cent.

Figure 6 shows the value of the "immediate implementation" and the "option to implement"- alternatives for different levels of current expected losses μ_t. The values decrease when current expected losses increase. I have also shown the values for different levels of the parameter representing the force of mean reversion κ. For expected loss levels above 1 per cent, a longer half life (weaker force of mean reversion) means that the values of the "implement now"-alternatives are reduced. The curves showing the values are becoming less sensitive to the

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[7]This means that equation (3) is given by $dB_t = 1 \times \gamma_t^{(C)*} dt - 1 \times \gamma_t^{(N)*} dt$.

Figure 5: Current and new regulatory rule
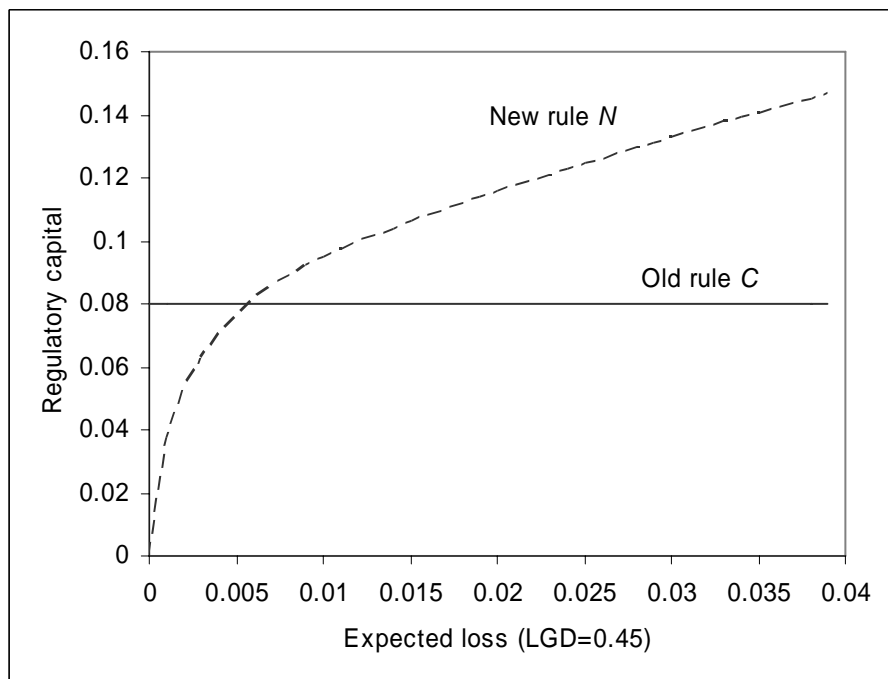
current level expected losses when the half life is reduced (i.e., the force of mean reversion increases). With a strong force of mean reversion, the current risk level means less for the value. The curves for the option to implement are generally not so steep as the value of the immediate implementation alternative. With a half life of 3 years the value of immediate implementation range from approximately 0.3 when expected losses are zero to approximately -0.2 when expected losses are equal to eight per cent. The value of the option to implement range from approximately 0.3 to 0.15. Figure 7 shows the value of immediate investment and the value of the option to invest. The schedule showing the value of the "invest-now" alternative in Figure 7 is equal to the schedule showing the "option to implement"-alternative in Figure 6 after the deduction of the investment expenditure of 0.15. The value of the "option to invest"-alternative is approximately equal to the value of the "invest-now"-alternative, but the value of the option is not negative.

In Figure 8 I show how the decision thresholds for implementation and investment are influenced by the force of mean reversion κ, volatility σ, and the unregulated optimal capital ε. The options to invest or implement are exercised if the risk level μ_t is equal to or lower than the threshold levels. The strength

20

of mean reversion influences more the threshold level for investment than the threshold level for implementation. For both decisions an increase in the half life of the process decreases the decision threshold. Increased volatility reduces the implementation threshold but increases the investment threshold. An increase in volatility increases the long term mean θ^* in equation (26) when the price of risk is negative. This makes, ceteris paribus, the new rule less valuable. At low levels of risk, where regulatory capital is bounded below by zero, a higher variance in expected losses may increase the future expected regulatory capital. At higher level of risk, the opposite will generally be true, i.e., that higher variance reduces the expected regulatory capital[8]. A higher level of unregulated optimal capital makes the new rule less valuable. A reduction in regulatory capital cannot be exploited because the bank is bounded by the outsiders' capital requirement and not by regulatory capital. When the outsiders' capital requirement becomes sufficiently high, the value of the new rule will be negative. This will be the case when the bank is not bounded by regulatory capital under rule C. By changing to rule N, the bank may risk to hold even more capital if it becomes constrained by rule N.

To see how the presence of two types of banks may influence the investment thresholds, I study the case where the bank in the benchmark example is the β^L bank. The β^H bank will get 1 per cent higher regulatory capital with the new rule compared to the β^L bank. The implementation threshold for the β^L bank for different assumptions about the capital requirement for the β^H bank, ε^H, is shown in the first part of Figure 9. The capital requirement will only influence the implementation threshold when the classification as a β^H bank will increase the optimal capital under rule C. This happens when ε^H is higher than 10 per cent.

The β^H bank will not invest in improved risk measurement[9]. The investment threshold for the type β^L bank will not be influenced when ε^H is equal to or lower than 10 per cent. When the required capital is higher than 10 per cent, however, the bank may avoid an increase in capital by investing. Consider the case when ε^H is equal to 11 per cent. If the bank procrastinates and invests later, it will anyway need to live with a capital requirement of 11 per cent (Proposition 3) until it implements rule N and reveals its true type (Proposition 2). The value of

---

[8]By Jensen's inequality we know that $\mathrm{E}(f(x)) \leq f(\mathrm{E}(x))$ if $f(\cdot)$ is a concave function. The regulatory rule N is a concave function of the risk level μ_t.

[9]Computations not reported here show that the value of investing in risk measurement is negative

this reduced capital requirement until implementation[10] is shown in the second part of Figure 9. When taking into account the value of avoiding an increase in required capital, the type β^L bank will always prefer to invest.

Figure 6: Implementation decision: Exercise and option values for the benchmark example

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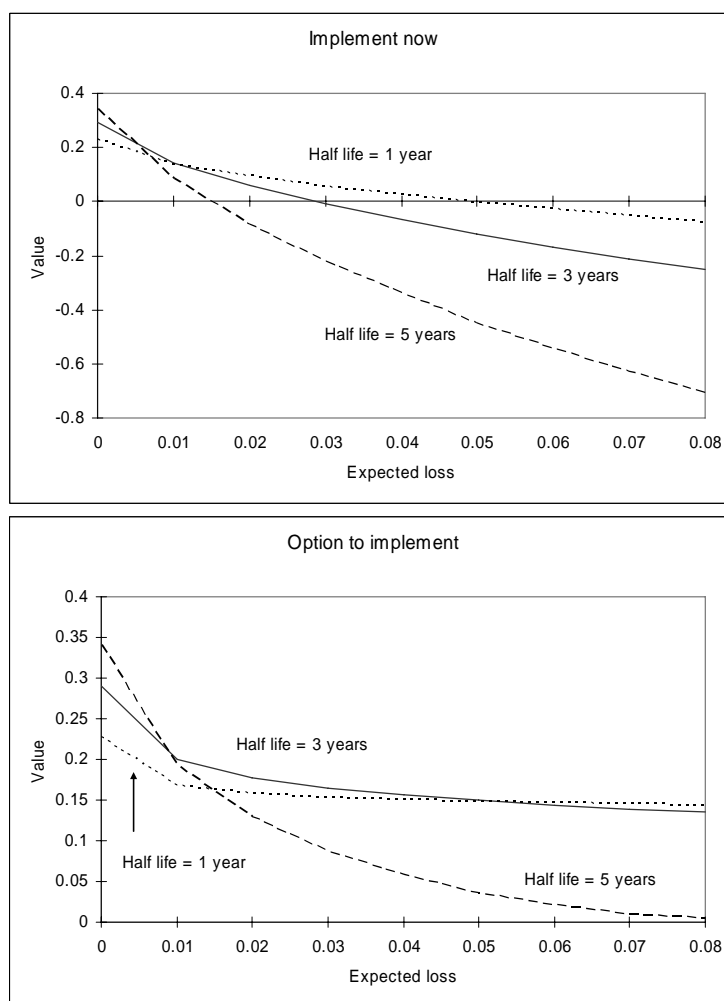
[10]The value is computed by discounting the flow following an increase in capital of one per cent with the risk free interest rate under the Q measure. This flow will continue for the investment period of 1 year. It will also continue after one year as long as the state variable μ_t has not been lower than the implementation threshold of approximately 0.5 per cent. The pricing of this flow following the temporary increase in capital is comparable to the pricing of a "down-and-out" option.

Figure 7: Investment decision: Exercise and option values for the benchmark example
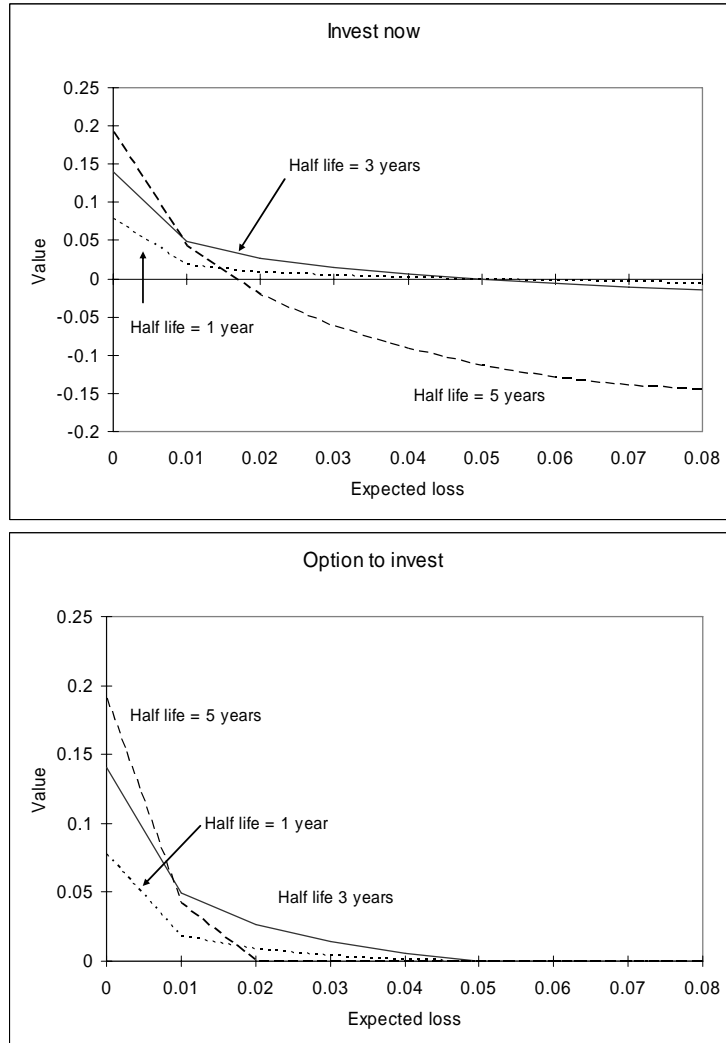
Figure 8: Decision thresholds for the benchmark example
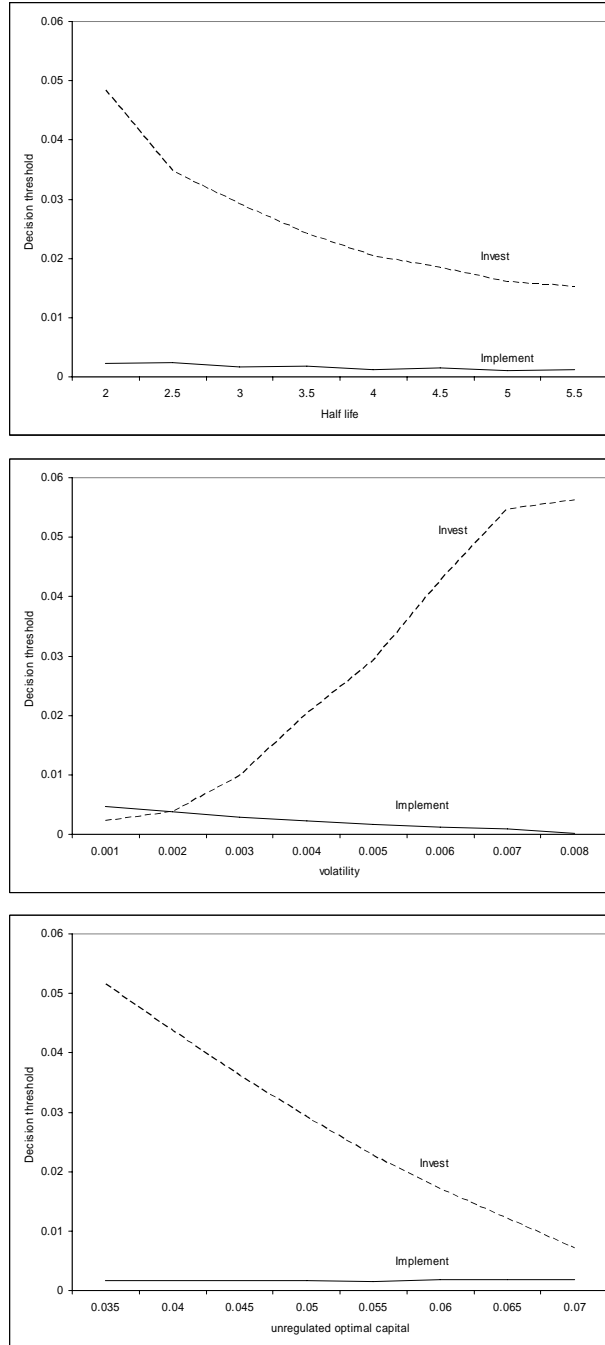
Figure 9: Two types of banks: Threshold levels and value

The first picture shows the investment and implementation thresholds for a type β^L bank as a function of the unregulated optimal capital ε^H that will apply when, respectively, the investment is not made or the rule is not implemented. When ε^H is higher than 0.1, the bank will always prefer investing to waiting. Investing leads to the avoidance of an increase in unregulated optimal capital for the time period from investment until implementation of the new rule. The value of avoiding this temporary increase in capital is shown in the second picture for the case when ε^H is equal to 0.11.
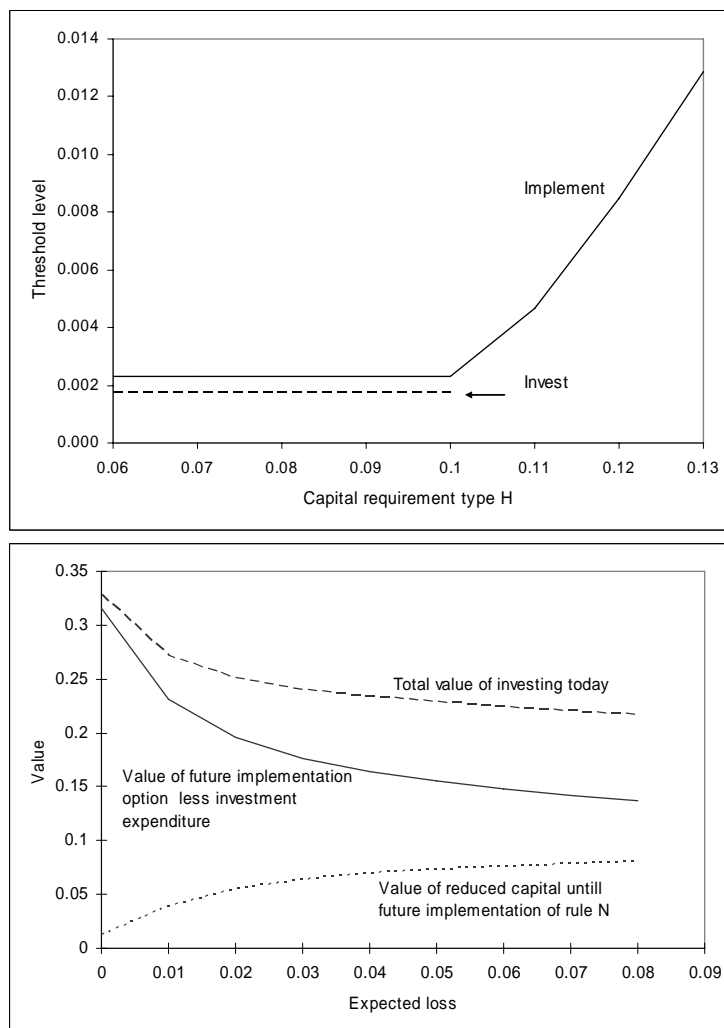
4 Summary

I have analyzed banks' incentives to introduce a risk sensitive rule for regulatory capital. The underlying premise is that banks prefer to have as little capital as possible, but that markets, or outside stake holders, put requirements on how much capital the banks should hold. Banks' optimal policies will depend on whether they are constrained by the current rule, the reduction in regulatory capital obtained by applying the new rule, and on possible changes in the market's capital requirement. I explain how different assumptions regarding this capital requirement influences the optimal policies. I also explain how different assumptions leads to different predictions regarding changes in capital for implementing and non-implementing banks. The framework presented here may be used to evaluate banks' decision making in situations where banks are given an option to irreversibly select between a set of regulatory rules.

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A Proofs

Lemma 1

Proof. The proof follows the same arguments as those on page 128 in Dixit and Pindyck (1994). The optimal policy is derived from the Bellman equation

$$F(\mu_t) = \max\left(B(\mu_t), \mathrm{E}_t^Q\left(F(\mu_{t+dt})\right)e^{-r_t dt}\right), \tag{28}$$

where the value function $F(\mu_t)$ is equal to the market value of the implementation option. The present value of the cost reduction if rule N is implemented at time t is $B(\mu_t)$. If the rule is not implemented, the bank gets the present value of the option value at time $t + dt$. Exercise is optimal if the exercise value is larger than the continuation value. We may rewrite (28) as

$$
\begin{aligned}
F(\mu_t) - B(\mu_t) \quad = \quad & \max(0, -h(\gamma^{(C)*}(\mu_t)dt + h(\gamma^{(N)*}(\mu_t)dt + \\
& \mathrm{E}_t^Q\left(F(\mu_{t+dt}) - B(\mu_{t+dt})\right)e^{-r_t dt}).
\end{aligned}
\tag{29}
$$

The LHS of (29) is a nonnegative number. In order for the implementation of rule N to be optimal only if $\mu_t \leq \mu^{**}$, it is sufficient that

1. $-h(\gamma^{(C)*}(\mu_t) + h(\gamma^{(N)*}(\mu_t)$ is an increasing function of μ_t, and

2. the cumulative probability distribution of $(F(\mu_{t+dt}) + B(\mu_{t+dt}))$ shifts to the right when μ_t increases. This means that there is first order stochastic dominance.

$\square$

Proposition 1

Proof. The result follows directly from equation (29). Continuation is only optimal if the continuation value is larger than zero. Because

$$\mathrm{E}_t^Q\left(F(\mu_{t+dt}) - B(\mu_{t+dt})\right)e^{-r_t dt})$$

is nonnegative, the optimal capital ratio with rule N cannot be larger than with rule C if stopping (exercise of the option) is optimal. Because of the assumptions in the benchmark example and the assumption that the bank is constrained by rule C, it is only an increase in regulatory capital that may cause an increase in the optimal capital ratio if rule N is implemented. $\square$

Proposition 2

Proof. Consider the case when the state variable has been below the threshold level for the type β^L-bank, but the bank has not implemented the new rule. The probability that the bank is a type β^H-bank is then

$$
\begin{aligned}
\overline{p}_t(\beta^H \mid \tau_N > t, t \geq t_1) &= \frac{p(\beta^H)p(\tau_N > t, t \geq t_1 \mid \beta^H)}{p(\beta^H)p(\tau_N > t, t \geq t_1 \mid \beta^H) + p(\beta^L)p(\tau_N > t, t \geq t_1 \mid \beta^L)} \\
&= \frac{q \times 1}{q \times 1 + (1-q) \times 0} = 1.
\end{aligned}
$$

Note that the regulatory capital under rule N did not enter the Bayesian updating formula in this case, because it is unobservable for the outsiders. Part iii) of the proposition contains all the conditional probabilities necessary to compute the ex-post probabilities, wherever this is possible.

The outsiders' payoff function is, by assumption, maximized by selecting ε^H if the probability of a type β^H-bank is equal to one, by selecting ε^L if the probability of a type β^L-bank is equal to one, and, by selecting ε^M otherwise.

According to assumption a1, it is optimal for the bank to implement the new rule the first time that the state variable is below the threshold level. By using assumption a2 and the beliefs, the highest threshold level $\mu_t^{H|\beta}$ will apply. By assumption a3, it will never be optimal for bank of type β^H to implement before the type β^L-bank (this implies that $t_1 \leq t_2$). If $t_1 < t_2$, it will be optimal for bank β^L only (and not for bank β^H to implement the rule at time t_1. This is a separating equilibrium. Note that $t_1 < t_2$ only if the threshold level is strictly lower for the β^H-bank and $\hat{t} < t_2$.

$\square$

Proposition 3

Proof. Part *iii*) of the proposition contains the conditional probabilities necessary to compute the ex-post probabilities for the type of bank, whenever this is possible. By assumption, the outsiders' payoff function is maximized by selecting ε^H if the probability of a type β^H-bank is equal to one, by selecting ε^L if the probability of a type β^L-bank is equal to one, and, by selecting ε^M otherwise.

By assumption a4 it is optimal for the bank to invest the first time the risk level is equal to or below the threshold level. Assumption a5 secures that the highest threshold level is for the case when investment leads to ε^L and no

30

investment leads to ε^H. According to assumption a6, this threshold level is strictly higher for type β^L banks than for type β^H banks. If the state variable μ_t is higher than the threshold level for the type β^H bank at the start of the game ($\hat{t} < t_2$), the bank's optimal investment policy follows directly from the threshold levels.

$\square$

B   Capital adequacy regulation

The weighted capital ratio at time t, γ_t, is given by

$$\gamma_t = \frac{S_t}{\sum_i w_t^i A_t^{(i)}}, \tag{30}$$

where S_t is capital, $A_t^{(i)}$ is the loan in category i, w_t^i is a weight for loan in asset category i. Under the Basel II rules, see Basel Committee on Banking Supervision (2004) a bank can choose between three alternatives regarding credit risk; the standardized approach, the Internal rating based (IRB) foundation approach, and the IRB advanced approach. Under the standardized approach the weights are constants. Under the IRB approaches the weights are computed based on estimates of probabilities of default (PD) and expected loss given default (LGD). With the IRB foundation approach it is assumed that the loss given default is 45 per cent of the exposure. Under the IRB advanced approach, the bank also uses own estimates for LGD.

The procedure to compute the necessary capital under the IRB approach involves several steps. First the required capital S per unit of currency (corresponding to the risk weight per one unit of a loan) is computed according to the formula

$$S = [LGD \; \mathrm{N}(d) - LGD \; PD]\frac{1 + (M - 2.5)b}{1 - 1.5b}, \tag{31}$$

where

$$d = \left(\frac{1}{1 - R}\right)^{0.5} \mathrm{N}^{-1}(PD) + \left(\frac{R}{1 - R}\right)^{0.5} \mathrm{N}^{-1}(0.999), \tag{32}$$

M is the effective maturity, and $\mathrm{N}(\cdot)$ is the cumulative normal standard distribution. The maturity adjustment factor b is

$$b = (0.11852 - 0.05478 \ln(PD))^2. \tag{33}$$

The correlation factor R is[11]

$$R = 0.12 \left(\frac{1 - e^{-50PD}}{1 - e^{-50}} \right) + 0.24 \left(1 - \frac{1 - e^{-50PD}}{1 - e^{-50PD}} \right). \qquad (34)$$

The correlation factor R^{SME} for small- and medium-sized entities (SME) is given by the formula

$$R^{SME} = R - 0.04 \left(1 - \frac{s - 5}{45} \right), \ 5 \le s \le 50, \qquad (35)$$

where R is given by (34) and s is total annual sales.

Risk weighted assets RWA are computed according to the formula

$$RWA = S12.5EAD, \qquad (36)$$

where EAD is exposure at default measured in units of currency.

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[11] For residential mortgage exposures $R = 0.15$ and

$$S = [LGD \ \mathrm{N}(d) - LGD \ PD)],$$

where d is given by (32).

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