Economic implications of copulas and extremes

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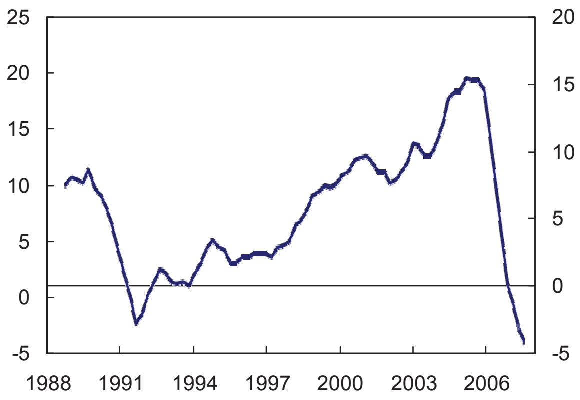
Recent events in financial markets and in nature have made it clear that it is vital to understand extremes. Such 'tail events' occur in many aspects of economic life. As suggested by the subprime market spillover of 2007, the effects of which are still being felt, extreme events can spin out of control, so it is valuable to investigate how to characterise them. When extremes occur across several instruments or variables at the same time, the copula approach is one method of analysis. This article introduces and illustrates recent ideas on copulas and tail events. We also give examples of the relation of these concepts to investor choice and the potential implications for regulatory policy.

1 Introduction and motivation

Extreme events have been with humanity through the ages. Some classic examples of extreme occurrences include the eruption of Mount Vesuvius in AD 79, which exploded 20 miles into the atmosphere and buried the inhabitants of Pompeii in volcanic ash, and the Black Death of the 1340s, which killed around 75 million people worldwide and destroyed more than one third of Europe's population. More recently, extreme events include the stock market crash of 1929 when on Monday October 28 the US Dow Jones Index lost 13 % of its value in a single day; and the destruction of the World Trade Center on September 11, 2001.

Why do we care about extremes and copulas? These concepts are very important for us to understand for two reasons. The first reason concerns the increasing interconnectedness of the world. This interconnectedness is a result of globalization as well as technological advances such as the internet. An interconnected world has many advantages. However, interconnectedness may also be a disadvantage in that we become more dependent on each other, since we can affect each other's welfare quickly and directly. Dependence seems to be particularly pronounced at extremes, for example during times of economic crisis. In economic markets, assets become more dependent in the lower tail during extremes, as documented by Ang and Chen (2002), Cappiello, Engle, and Sheppard (2006) and Hartmann, Straetmans, and de Vries (2003), among others. The standard way of measuring dependence is the Pearson correlation.[1] This measure does not work well for data with a substantial amount of extremes, as discussed in Section 2 below. Copulas are general measures of dependence, and their parameters can help us to estimate the effect of our behaviour and our markets on others.[2]

Chart 1 Percentage Change in US House Prices



The chart shows the percentage change in the Case-Schiller US House Price Index, relative to the previous year.

Source: Standard and Poors

The second reason concerns the recent prevalence of extreme events. For example, as shown in Chart 1, the percentage change in US house prices reached record highs and lows during the period 2005 to 2007. Moreover, in the context of recent subprime-related scares, the price of interbank borrowing in the UK reached decade-record levels in the fall of 2007, as shown in Chart 2. It is common to discuss extremes as exogenous, for example Barro (2006) and Friedman and Laibson (1989). However, since at least the time of Fisher (1933), it has been acknowledged that some extreme periods are

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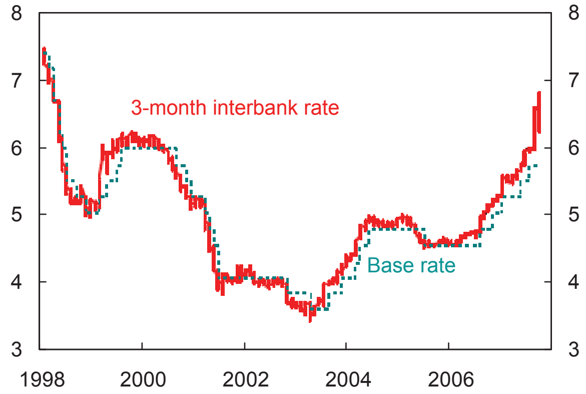
1 The Pearson correlation is the standard correlation measure used in economics. As mentioned below, there are other correlation and comovement measures, one of which is the copula function.

2 In keeping with research literature, throughout this paper we use the terms dependence and comovement interchangeably.

endogenous. An endogenous extreme event is one that depends on or is amplified by the behaviour of economic agents. Understanding the origin and patterns of such extreme events is very important for investors and central bankers alike.

An important aspect of the financial sector is relevant here, namely liquidity. Interestingly, during extreme events, liquidity often dries up. For example, Charts 3 and 4 show that during the 1987 and 1998 market events, liquidity displayed a sharp drop in the US. Similar findings have been documented in the Norwegian stock market by Chollete, Naes, and Skjeltorp (2007) during the burst of the dotcom bubble. Moreover, recent work by Chollete (2008) suggests that liquidity might be a channel for endogenous extremes and a potential predictor of extremes.

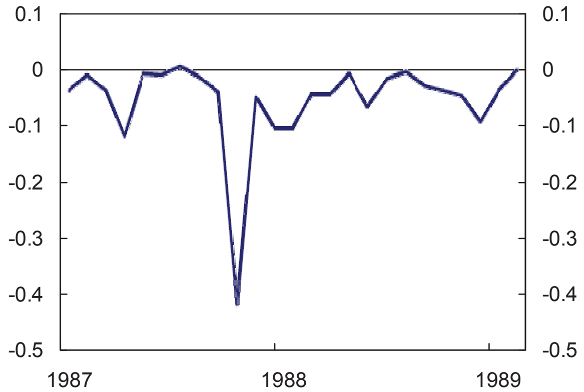
Chart 2 Price of interbank borrowing in UK



The chart shows the price of interbank borrowing in UK.

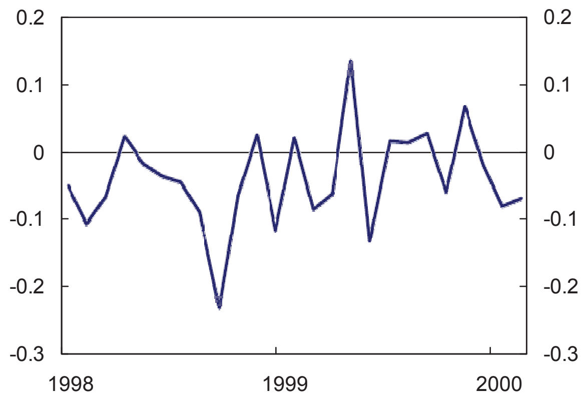
Source: DataStream

Chart 3 Marketwide Liquidity during the 1987 Crash



The chart shows the level of the Pastor and Stambaugh (2003) liquidity measure in the period around the US stock market crash in 1987.

Chart 4 Marketwide Liquidity during the LTCM Event in 1998



The chart shows the level of the Pastor and Stambaugh (2003) liquidity measure around the time of the LTCM Event in summer1998.

2 How do copulas relate to what we know?

Before we discuss copulas, it is important to relate them to what is already known, namely correlations. For two generations the financial and academic communities have used some form of correlation or other second moment to summarize risk or diversification opportunities. The central insight is that we seek assets that do not comove with each other, in order to protect our investment portfolios. That is, we demand higher returns to compensate for increased comovement, since we do not like to put all our eggs in one basket. A classic example in financial economics is the CAPM approach. Under some conditions, the CAPM says that for any stock i, its return R_i depends on its covariance with the market return R_m:

$$(1) \quad E(R_i) - R_f = \beta_i[E(R_m) - R_f]$$

where $\beta = \mathrm{Cov}(R_m, R_i)/\mathrm{Var}(R_m)$. Therefore, the more a stock is correlated with the market return, the higher its own return needs to be.

If properly specified, correlations tell us about average diversification opportunities over the entire distribution. Given two random variables X and Y, the standard correlation coefficient $\rho_{x,y}$ is the covariance divided by the product of the standard deviations:

$$(2) \quad \rho_{x,y} = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\cdot\mathrm{Var}(Y)}}$$

The main advantage of Pearson's correlation is tractability. There are, however, several disadvantages of using correlations in finance, many of which are discussed by Embrechts, McNeil, and Straumann (2001). Three major shortcomings relate to heavy tails, estimation biases, and linearity. First, there is much evidence of heavy tails in financial data, which are associated with infinite variance. From Equation (2), if either X or Y has infinite variance, the estimated correlation may give little information on comovement, since it will be undefined or close to zero. Second, with regard to estimation bias, an important issue is noted by Forbes and Rigobon (2002), who show theoretically that, by definition, conditional correlation must increase during volatile periods. After adjusting for such bias, Forbes and Rigobon (2002) document that prior findings of international comovement are reversed. Third, correlation is linear and therefore may overlook important nonlinear comovements, for example differential comovement during up and down markets.[3]

Thus far, we have used the term correlation quite loosely. In fact, there are numerous correlation measures, and the Pearson correlation we discussed above is only one of them. An alternative measure is the *rank (*or *Spearman) correlation,* ρ_S. This is more robust than the traditional correlation. Rank correlation measures comovement of ranks, and is expressed as

$$(3) \quad \rho_S = \frac{\mathrm{Cov}(F_X(x), F_Y(y))}{\sqrt{\mathrm{Var}(F_X(x)) \cdot \mathrm{Var}(F_Y(y))}}$$

where $F_X(x)$ and $F_Y(y)$ are the distribution functions of X and Y, respectively.[4] The rank correlation is useful when analysing data with many extreme observations, since it is independent of the levels of the variables, and therefore less sensitive to outliers. For example, if we denote the Norwegian and UK stock markets by X and Y, and examine the traditional correlation between these two markets, it will be highly affected by periods of booms and crises, which yield very large and very small stock prices, respectively. The reason is that the traditional correlation in (2) depends on the level of X and Y. However, as seen in expression (3) the rank correlation between Norway and UK stock markets will only change if there is a change in the distribution of the stock returns.

2.1 Copulas

Unlike correlations, copulas can help us to uncover both linear and nonlinear diversification opportunities. Consider a portfolio of two assets with returns X and Y. All the relevant comovement in the portfolio is contained in the joint density $f_{X,Y}(x, y)$. However, this information is often unavailable for large portfolios, because there might be no simple, single parametric joint density to depict the relationship among all the securities.

An alternative for measuring dependence in this setting is the *copula function c(u,v)*. In Norwegian, it is called a 'koblingsfunksjon', which means a 'joining function'. This is exactly what a copula does: it joins the marginal distributions together, to form the full, joint, distribution. For example, in the case of two returns X and Y as above, the copula would be expressed as

$$(4) \quad f(x, y) = c(F_X(x), F_Y(y)) \cdot f_X(x) \cdot f_Y(y)$$

Why is Equation (4) interesting? One important reason is that it empowers us to separate out the joint distribution from the marginals. For example, if we are interested in determining the source of increased risk in a Norwegian-UK portfolio during extreme periods, this could come from either the fact that the marginals f are heavy-tailed, or their dependence c is heavy-tailed, or both.

There are a number of parametric copula specifications. We focus on three types: the normal, the student-*t*, and the Gumbel copulas.[5] The normal specification is a natural benchmark, as the most common distributional assumption in finance, with zero extreme comovement.[6] The student-*t* is useful since it has symmetric but non-zero extreme comovement and nests the normal copula. The Gumbel copula is useful because it has nonlinear comovement and asymmetric extreme comovement – the mass in its right tail greatly exceeds the mass in its left tail. Moreover, the Gumbel copula is a member of two important families, archimedean copulas and extreme value copulas.[7] In addition to these single copulas, we use a 'mixed' copula, which combines a normal, Gumbel and Rotated Gumbel copula. In terms of practicality, these copulas are a subset of those most frequently used in recent empirical papers, for example, Embrechts, McNeil, and Straumann (2001), Patton (2005) and Rosenberg and Schuermann (2006). Table 1 provides functional forms of the copulas. In this table, the parameters ρ, α and β measure dependence. They are therefore similar to the traditional correlation, although they can allow for nonlinear dependence.

Intuitively, the normal and student-*t* copulas are just multivariate versions of familiar distributions like the univariate normal and student-*t* distributions. The Gumbel is a multivariate version of the Gumbel density. For the purposes of risk management and financial stability, these copulas are useful mainly because of their shapes: some are symmetric, some are skewed, and oth-

[3] Such nonlinearity is documented by a number of researchers, including Ang and Chen (2002).

[4] Since the distribution functions are monotonic, they preserve the ranks of the original data. Therefore the Spearman correlation defined above is based on ranks, see Cherubini, Luciano, and Vecchiato (2004) page 100.

[5] Since we wish to investigate left comovement or downside risk, we also utilize the survivor function of the Gumbel copula, denoted the Rotated Gumbel.

[6] In the absence of a consensus in the literature, we use the term extreme comovement to mean comovement at the extreme quantiles of the distribution.

[7] Archimedean copulas represent a convenient bridge to gaussian copulas since the former have dependence parameters that can be defined through a correlation measure, Kendall's tau. Extreme value copulas are important since they can be used to model joint behaviour of the distribution's extremes.

Table 1: Distribution of various copulas

Copula	Distribution	Parameter range	Complete dependence	Independence
Normal	$C_N(u,v;\rho)=\Phi_\rho(\Phi^{-1}(u),\Phi^{-1}(v))$	$\rho\in(-1,1)$	$\rho=1, \text{or} -1$	$\rho=0$
Student-*t*	$C_t(u,v;\rho,d)=t_{d,\rho}(t_d^{-1}(u),t_d^{-1}(v))$	$\rho\in(-1,1)$	$\rho=1, \text{or} -1$	$\rho=0$
Gumbel	$C_G(u,v;\beta)=\exp\{-[(-\ln(u))^{1/\beta}+(-\ln(v))^{1/\beta}]^\beta\}$	$\beta\in(0,1)$	$\beta=0$	$\beta=1$
RG	$C_{RG}(u,v;\alpha)=u+v-1+C_G(1-u,1-v;\alpha)$	$\alpha\in(0,1)$	$\beta=0$	$\beta=1$

RG denotes the Rotated Gumbel copula. The symbols $\Phi\rho(x, y)$ and $t_{v,\rho}(x, y)$ denote the standard bivariate normal and Student-*t* cumulative distributions, respectively:

$$\Phi_\rho(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}\frac{1}{2\pi|\Sigma|}\exp\left\{-\frac{1}{2}(x\ y)\sum^{-1}(x\ y)'\right\}dx\,dy \text{, and}$$

$$t_{v,\rho}(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}\frac{\Gamma\left(\frac{v+2}{2}\right)}{\Gamma(v/2)(v\pi)|\Sigma|^{1/2}}\left\{1+(s\ t)\Sigma^{-1}(s\ t)'/v\right\}^{\frac{-(v+2)}{2}}ds\,dt\ .$$

The correlation matrix is given by $\Sigma=\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}$.

ers are asymmetric. These shapes correspond to types of risk. For example, if a bank's assets are fit by copula with a heavy left tail, this means that the bank is prone to have low returns across all assets at the same time.

2.2 The case for and against copulas

There are three main advantages of using copulas in finance. First, and perhaps most relevant for central banks, copulas can aggregate portfolio risk from disparate sources, such as credit and operational risk. This is possible even for risk distributions that are subjective and objective, as in Rosenberg and Schuermann (2006). Second, copulas are a convenient choice for modelling potentially nonlinear portfolio comovement, such as correlated defaults. A third advantage is invariance. Since the copula is based on ranks, it is invariant under strictly increasing transforms.[8] That is, the copula extracts the way in which x and y comove, regardless of the scale used to measure them.[9] There are two disadvantages to using copulas. First, from a finance perspective, a disadvantage is that many copulas do not have moments that are directly related to Pearson correlation. It is therefore sometimes difficult to compare copula results to those of financial models based on correlations or variances. Second, from a statistical perspective, it is sometimes not easy to say which parametric copula best fits a given dataset, since some copulas may fit better near the center and others near the tails. It is possible to overcome this issue by focusing on different shapes of copulas that are important from a finance perspective, and by using several specification checks, namely AIC, BIC, a mixture model, and the econometric test of Chen and Fan (2006).[10]

[8] The copula is based on ranks since it is a function of the marginal distribution functions $F_X(x)$ and $F_Y(y)$. The marginal distributions are monotonic and preserve the ranks of the original data. Therefore the copula is based on ranks, see Cherubini, Luciano, and Vecchiato (2004).

[9] See Schweizer and Wolff (1981). For more details on copula properties, see Nelsen (1998).

[10] AIC and BIC denote Akaike Information Criterion and Bayes Information Criterion, respectively. Consider a sample with size equal to T, and the number of estimated parameters (θ) equal to q. Then the AIC and BIC are defined as

$$\text{AIC}(q)=-2\ln\left[\hat{L}(\theta)\right]+2q$$

$$\text{BIC}(q)=-2\ln\left[\hat{L}(\theta)\right]+q\ln(T)$$

where $\ln\left[\hat{L}(\theta)\right]$ is the maximized log likelihood. The AIC is based on maximum entropy considerations, and the BIC modifies AIC with a stricter penalty for overfitting a model. The best model is selected to be the one that minimizes AIC or BIC.

2.3 Extreme comovement

As mentioned in the introduction, extreme events often coincide across different asset classes, especially during down markets. Since copulas measure dependence of ranks, they can be used to estimate comovement at the tail u (highest and lowest ranks). However, copulas can be restrictive since they apply to the whole distribution, both centre and tails. In some instances, a bank or risk manager is interested in downside risk only, for example. Therefore we would like to have a number that just measures tail or extreme comovement. Such extreme risk is often assessed using the concept of upside or downside risk, denoted $\lambda(u)$. This function measures, for example, the probability of a *joint* extreme event in the Norwegian and UK exchanges:

$$\lambda(u) \equiv \Pr(F(O) \geq u \mid F(L) \geq u)$$

where F(L) and F(O) refer to the distribution functions of average returns on the London and Oslo exchanges, respectively. $\lambda(u)$ measures upside risk. Its counterpart downside risk, $\lambda(l)$, is obtained by reversing the inequalities above. Formally, extreme comovement is measured by the limit of upside and downside risk.[11]

How do we assess extreme comovement? In practice, we use a measure, χ, defined as

$$(5) \quad \chi \equiv \lim_{u \uparrow 1} \lambda(u) \quad ^{12}$$

In terms of the above example, if the London and Oslo stock markets have significant extreme comovement as measured by χ, then they are likely to experience joint booms or crashes.[13] We will see a simple application of such measures of downside risk in section 4.

3 Examples of correlations and copulas in financial economics

To give an indication of the difference between copulas and correlations, we now show some examples using international stock market data. We use three sets of countries, G5, East Asia and Latin America. For further details, see Chollete, de la Pena, and Lu (2006), which is a source of the results presented here.

3.1 Correlation estimates

Table 2 presents the correlations and rank correlations for stock return indices from the G5, East Asian and Latin American countries. These indices are available from MSCI. We also present ranges in order to indicate the scope of diversification opportunities for an international investor. We first consider G5 countries. Panel A shows results for the entire sample. The average correlation for the G5 countries is 0.545, with a range of 0.519. Panel B shows results for the first part of the sample, from 1990 to 2001. The average correlation in this part of the sample is slightly lower, at 0.487. The range is smaller than for the full sample, at 0.480. Panel C displays the correlations from the latter part of the sample, 2001 to 2006. In all cases the average correlations are larger. The average correlation is 0.637, with a range of 0.545. To summarize, there are three main findings for the G5 countries. First, average comovement increased for every country pair from the earlier to latter part of the sample. Second, the countries affording maximal and minimal diversification benefits were the same over the sample period. This is true whether we measure comovement with Pearson or rank correlation. Third, the range of diversification opportunities was larger in the latter period, although the very best (lowest) comovement was in the earlier part of the sample.

Now let us consider the East Asian economies. For the entire sample, in Panel A, the average Pearson correlation of 0.406 is somewhat lower than for the G5 economies. The range of 0.273 is also much lower. The maximal and minimal correlations are for Hong Kong-Singapore and Taiwan-Thailand, respectively. Panel B shows results for the first part of the sample. Here, the average correlation is slightly lower than for the full sample, at 0.379. The maximum is also smaller than for the full sample, although the diversification range increases to 0.340. Panel C shows the latter part of the sample. In this case, the average correlation increases

[11] Specifically, left extreme comovement is the limit of $\lambda(l)$ and right extreme comovement is the limit of $\lambda(u)$.

[12] In some instances, χ is supplemented by a second measure, $\bar{\chi}$

$$(6) \quad \bar{\chi} \equiv \lim_{u \uparrow 1} \frac{2 \log(1-u)}{\log \bar{C}(u,u)} - 1$$

where $\bar{C}(u, u)$ is the survivor copula of $C(u, u)$. This second measure is useful when χ is undefined.

[13] It is outside the scope of this article to develop the analysis of extreme comovement. For more details, see Poon, Rockinger, and Tawn (2004) and Berliant, Goegebeur, Segers, and Teugels (2005), Chapter 9.

Table 2: Country correlations

	G5				East Asia				Latin America			
Panel A: 1990–2006												
	Avg	Max	Min	Range	Avg	Max	Min	Range	Avg	Max	Min	Range
Pearson's ρ	0.545	0.822	0.303	0.519	0.406	0.588	0.315	0.273	0.414	0.506	0.355	0.151
		(FR-DE)	(JP-US)			(HK-SI)	(TW-TH)			(BR-ME)	(AR-CH)	
Rank Corr	0.523	0.772	0.304	0.468	0.373	0.539	0.271	0.268	0.376	0.447	0.299	0.148
		(FR-DE)	(JP-US)			(HK-SI)	(TW-TH)			(AR-ME)	(AR-CH)	
Panel B: 1990–2001												
	Avg	Max	Min	Range	Avg	Max	Min	Range	Avg	Max	Min	Range
Pearson's ρ	0.487	0.762	0.281	0.480	0.379	0.577	0.237	0.340	0.416	0.493	0.359	0.134
		(FR-DE)	(JP-US)			(HK-SI)	(KR-TW)			(BR-ME)	(AR-BR)	
Rank Corr	0.471	0.709	0.267	0.442	0.322	0.511	0.176	0.335	0.366	0.480	0.307	0.173
		(FR-DE)	(JP-US)			(HK-SI)	(KR-TW)			(AR-ME)	(BR-CH)	
Panel C: 2001–2006												
	Avg	Max	Min	Range	Avg	Max	Min	Range	Avg	Max	Min	Range
Pearson's ρ	0.637	0.901	0.355	0.545	0.511	0.639	0.353	0.286	0.423	0.561	0.310	0.251
		(FR-DE)	(JP-US)			(HK-SI)	(HK-TH)			(BR-ME)	(AR-CH)	
Rank Corr	0.624	0.887	0.389	0.499	0.512	0.641	0.376	0.265	0.405	0.520	0.266	0.254
		(FR-DE)	(JP-US)			(HK-SI)	(TW-TH)			(BR-ME)	(AR-CH)	

Rank Corr denotes the rank correlation ρ_S , defined in Section 2.1. G5 countries are France (FR), Germany (DE), Japan (JP), United Kingdom (UK) and United States (US). East Asian countries are Hong Kong (HK), Taiwan (TW), Thailand (TH), Malaysia (M) and Singapore (SI). Latin American countries are Argentina (AR), Brazil (BR), Chile (CH) and Mexico (ME). Although the maximum (Max) and minimum (Min) correlations are presented for completeness, we focus our comments on the Range and Average (Avg) of the data presented above, when we discuss the results in the text.

to 0.511, while the range falls to 0.286. Throughout the samples, the country pair with maximal correlation is that of Hong Kong-Singapore. Interestingly, the minimal correlation switches from Korea-Taiwan in the first half to Hong Kong-Thailand in the latter half, and is Taiwan-Thailand for the entire sample. This suggests that the best diversification opportunities would differ depending on the investor's holding period. To summarize, for the East Asian economies, the average correlations increased over time, the two-country portfolios affording best diversification opportunities were not stable, and the diversification range fell over time.

Finally, we consider the Latin American economies. Panel A shows the full sample estimates, which feature average correlations of 0.414 with a range of 0.151. The maximum and minimum correlations are for Brazil–Mexico and Argentina–Chile, respectively. Interestingly, the rank correlation picks a different country pair for maximal correlation, namely Argentina–Mexico. This suggests that in this sample, the usual correlation does not capture the full story for diversification. Panel B shows the first part of the sample, 1990-2001, with an average correlation of 0.416, and a diversification range of 0.134. Once again, the Pearson and rank correlation disagree on the same country pairs regarding the maximal comovement. Panel C shows the latter part of the sample, 2001-2006. Here, the average correlation and diversification range both rise, to 0.423 and 0.251 respectively. To summarize, for the Latin American countries, both average comovement and diversification range have increased over time. Moreover, disagreement of the Pearson and rank correlations indicates potential nonlinear diversification benefits, for example during up versus down markets.

In terms of general comparison, in all sample periods the G5 countries have the highest average and maximum correlations of the three regions, as well as the largest range of diversification opportunities. The lowest minima for the full sample and first period are in East Asia, but in Latin America for the latter sample period. In economic terms, our correlation results suggest that an investor who invests solely in the G5 countries has a wider array of diversification possibilities, relative to an investor who invests solely in the East Asian or Latin American economies.

3.2 Copula estimates

We now continue our discussion with a presentation of estimates from our empirical copula methodology.[14] We consider four copulas, namely the normal, student-*t*, Gumbel, and Rotated Gumbel.[15] The diagnostic methods we consider are AIC, BIC, and a mixture model.[16] The copulas are estimated by maximum likelihood.

3.2.1 Model selection

Table 3 presents goodness of fit tests from AIC and BIC. We will first discuss the AIC results. For the G5 countries, the best model (lowest AIC) is the mixed copula, which has an average AIC of –318.18 across countries, closely followed by the student-*t*. This is evidence of heavy joint tails. For the East Asian economies, the lowest AIC of –139.43 corresponds to the Rotated Gumbel, followed by the student-*t*. Finally, for the Latin American countries, the lowest AIC of –183.97 is for the Rotated Gumbel model, followed by the mixed copula. Turning to the BIC results for the G5 countries, the best model on average is the Rotated Gumbel, with an average BIC of –307.64, closely followed by the student-*t* copula. Similarly, for both the East Asian and Latin American countries, the best model on average is also the Rotated Gumbel, closely followed by the student-*t*.

Table 3: Basic goodness of fit for copulas

Panel A: G5		
Models	AIC	BIC
Gumbel	–269.17	–264.44
Rotated Gumbel	–312.37	–307.64
Normal	–302.82	–298.10
Student-*t*	–316.20	–306.75
Mixed Copula	–318.18	–294.57
Panel B: East Asia		
Models	AIC	BIC
Gumbel	–111.25	–106.53
Rotated Gumbel	–139.43	–134.71
Normal	–132.38	–127.66
Student-*t*	–138.47	–129.02
Mixed Copula	–138.98	–115.36
Panel C: Latin America		
Models	Average AIC	Average BIC
Gumbel	–121.23	–116.51
Rotated Gumbel	–183.97	–179.25
Normal	–153.02	–148.30
Student-*t*	–167.56	–158.12
Mixed Copula	–179.22	–155.61

The terms AIC and BIC denote the Akaike and and Bayesian Information Criteria, as defined in Section 2.3 of the text. The numbers presented are averages for each region.

Single copulas are restrictive in that they assume a single dependence structure. In order to address this restriction, we examine more closely the mixed copula, which has normal, Gumbel and Rotated Gumbel components. The results are presented in Table 4.[17] Since the weights on each copula in the mixture reflect the proportion of the data consistent with that copula shape, a large weight on the Gumbel suggests large upside comovement (systemic booms) while a large weight on the Rotated Gumbel copula suggests large downside comovement. First, consider the G5 estimates. The largest average weight of 0.517 is on the Rotated Gumbel copula, and the smallest weight of 0.097 is on the Gumbel copula. This suggests that there are generally heavy tails in the G5, and that these tails are highly asymmetric, with substantial downside dependence. Now we consider results for the East Asian models. In this case, the weights are closer than for the G5. The largest average weight of 0.471 is on the normal copula, suggesting that relatively skinny joint tails are common for these countries. Finally, we consider results for the Latin American countries. For this region, the Rotated Gumbel copula is again dominant, with an average weight of 0.787.

Table 4: Mixed copula estimates

Weights	**G5**	**East Asia**	**Latin America**
W_{Gumbel}	0.097	0.145	0.099
	(0.085)	(0.102)	(0.084)
$W_{Rotated\ Gumbel}$	0.517	0.384	0.787
	(0.170)	(0.147)	(0.160)
W_{Normal}	0.386	0.471	0.114
	(0.177)	(0.196)	(0.161)

The terms AIC and BIC denote the Akaike and and Bayesian Information C riteria, as defined in Section 2.3 of the text. The numbers presented are averages for each region.

To summarize the mixed copula results, there is evidence of asymmetric heavy tails (downside risk) in all

[14] Copulas deliver information on the entire portfolio distribution. However, for clarity of comparison, we focus only on the copula dependence parameters, which reflect potentially nonlinear comovement, or in financial terms, nonlinear diversification benefits.

[15] There are many other copulas available. However, we choose these copulas because they have all been used in a number of recent finance studies, and because they represent four important "portfolio shapes" for finance, namely symmetric skinny tails, symmetric heavy tails, heavy upper tail, and heavy lower tail respectively.

[16] The AIC and BIC are not formal statistical tests, although it is customary to use them as they give a rough sense of goodness of fit. Since they are employed in this literature by many researchers, such as Dias and Embrechts (2004) and Frees and Valdez (1997), we include them. Further tests are described in Chollete, de la Pena, and Lu (2006).

[17] The mixed copula is also useful since the weights can provide information on another aspect of diversification, namely downside risk. The mixed copula is estimated by iterative maximum likelihood, as is standard in mixture model research. For details on mixture model estimation, see McLachlan and Peel (2000).

the countries, particularly in the G5 and Latin American regions. These latter two regions are also the ones with increased diversification ranges, according to our correlation estimates from before. The greatest downside risk is in Latin America, which has nearly 80 % of the average weight on the Rotated Gumbel.[18]

4 Further implications for finance

4.1 Relationship between returns and diversification

What do our empirical findings above imply for global returns? As we discussed in Section 2, a central tenet of finance is that investors require higher returns for lower diversification (see Equation (1)). It is therefore natural to explore which of our diversification measures is most closely related to returns over our sample period. Table 5 presents information on the relationship between average returns and average diversification measures in each region. We are particularly interested in patterns in the β, corresponding to equation 1. For a simple comparison, each variable is ranked from low (L) to high (H). Panel A shows the results for the full sample. The ranking of returns from lowest to highest is East Asia, then G5, then Latin America. Even though the Latin American region has more than double the returns of the others, its world market β is not the largest. This indicates that a world CAPM will not tell the full story. The only diversification measure that has the same relation across the regions is the left $\bar{\chi}$, which measures downside risk in extreme periods.[19] Panel B shows the first half of the sample, which has the same pattern. Panel C shows the second half, where none of the diversification measures has the same pattern as returns, although $\bar{\chi}$ still has its highest rankings for the region with highest returns.

To summarise, the only diversification measure for which there is a monotonic relation to returns, is left extreme comovement. This monotonic relation exists for our sample as a whole, and for the bigger part of our sample, although not for the shorter, more recent sample. Left comovement is also the only measure that is always largest for the region with the largest returns. This is true even when the world beta goes in the opposite direction. In economic terms, this finding is plausible if investors are averse to (and therefore demand returns for) exposure to downside risk during extreme periods.

Table 5: Regional returns and diversification measures

	Return	World beta	ρ	ρ_{copula}	Left χ	Left $\bar{\chi}$	Right χ	Right $\bar{\chi}$
Panel A: Full sample								
East Asia	2.68 (*L*)	0.416 (*L*)	0.406 (*L*)	0.385 (*L*)	0.461 (*M*)	0.671 (*L*)		0.510 (*H*)
G5	5.35 (*M*)	0.739 (*H*)	0.545 (*H*)	0.525 (*H*)	0.515 (*H*)	0.750 (*M*)	0.547	0.499 (*M*)
Latin	13.24 (*H*)	0.426 (*M*)	0.414 (*M*)	0.414 (*M*)	0.441 (*L*)	0.834 (*H*)		0.478 (*L*)
Panel B: 1990–2001								
East Asia	–1.00 (*L*)	0.358 (*L*)	0.379 (*L*)	0.324 (*L*)	0.454 (*M*)	0.652 (*L*)	0.432	0.517 (*H*)
G5	6.31 (*M*)	0.701 (*H*)	0.487 (*H*)	0.469 (*H*)	0.504 (*H*)	0.709 (*M*)		0.443 (*L*)
Latin	13.15 (*H*)	0.370 (*M*)	0.416 (*M*)	0.398 (*M*)	0.443 (*L*)	0.812 (*H*)		0.499 (*M*)
Panel C: 2001–2006								
East Asia	10.19 (*M*)	0.537 (*L*)	0.511 (*M*)	0.530 (*M*)	0.467 (*M*)	0.742 (*L*)	0.448 (*M*)	0.568 (*M*)
G5	3.38 (*L*)	0.812 (*H*)	0.637 (*H*)	0.641 (*H*)	0.535 (*H*)	0.768 (*M*)	0.556 (*H*)	0.606 (*H*)
Latin	13.43 (*H*)	0.544 (*M*)	0.423 (*L*)	0.447 (*L*)	0.443 (*L*)	0.862 (*H*)	0.412 (*L*)	0.456 (*L*)

The table presents average returns and average comovement for different regions. The world beta is computed on filtered returns. *L, M* and *H* denote the lowest, middle and highest returns or comovement, compared across regions. ρ_{copula} denotes the comovement parameter estimated for the student t copula. χ and $\bar{\chi}$ denote extreme comovement, as defined in Section 2. For example, Left χ and Left $\bar{\chi}$ are measures of extreme comovement in the left tail, while Right χ and Right $\bar{\chi}$ are measures of extreme comovement in the right tail. The main distinction that we focus on in the text is that these are all different measures of comovement or diversification: ρ is the familiar correlation; ρ_{copula} is a copula based measure of comovement, and χ and $\bar{\chi}$ are extreme based measures of comovement. For each of these measures we discuss (in the text) whether their lowest (*L*) and highest (*H*) values correspond to the lowest and highest returns. Whenever this happens, it suggests a possible risk-return relationship, which is the basis for our discussion of this table in the text.

[18] These results are all in-sample, and may not hold out of sample. In order to assess such predictive power, we would have to use conditional copulas, as in Patton (2005).

[19] For more detailed definitions of downside comovement, see Chollete, de la Pena, and Lu (2006).

Therefore, if we believe in a simple international CAPM model where returns depend on the world market beta, as in Equation (1), we might consider augmenting the model to account for a risk factor related to extreme comovement. In economic terms, this reflects *joint* downside risk (Karni, 1979), and differs from previous studies that focus on univariate downside risk. Our findings, while suggestive and related to theoretical work on loss aversion, are evidently preliminary and may merit further study in a conditional setting with a wider group of countries.

4.2 Implications for international portfolio choice

While the diversification range indicates available diversification opportunities, it is useful to work with explicit financial models of asset allocation, which quantify the attractiveness of various risk-reward combinations to investors, and therefore allow us to compute portfolio weights. A simple model is the mean-variance framework.[20] In this setting, investors like high returns but dislike high volatilities. This framework is used by Pastor (2000) and Lewis (1999), in the context of international portfolio choice. The utility function of an investor with wealth W_1 is given by:

$$(7)\quad U = U(EW_1, \mathrm{Var}(W_1))$$

where the partial derivatives of the first and second arguments satisfy $U_1 > 0$, and $U_2 < 0$, respectively. Define the home and foreign portfolio weights as ω_h and ω_f, and let $\omega \equiv (\omega_h, \omega_f)'$, with $\omega_h + \omega_f = 1$. Then if the return vector is defined as $r \equiv (r^h, r^f)'$, the mean and variance of wealth can be written as: $EW_1 = W_0\,(1 + \omega' Er)$ and $\mathrm{Var}(W_1) = W_0^2\,\mathrm{Var}(\omega' r) = W_0^2 \omega' \mathrm{Var}(r)\omega$, where W_0 is the investor's initial wealth. When utility is maximized subject to a wealth constraint, the first order conditions give optimal foreign portfolio shares as

$$(8)\quad \omega_f = \frac{(Er^f - Er^h)/\gamma}{\mathrm{Var}(r^f - r^h)} + \frac{\sigma_h^2 - \rho_{hf}}{\mathrm{Var}(r^f - r^h)}$$

where γ is the coefficient of relative risk aversion, σ_h^2 is the variance of the home asset returns, and ρ_{hf} is the traditional Pearson correlation of the home and foreign returns. We will now use our various estimates of comovement in Equation (8), to compute optimal portfolio weights.

A useful illustration of the potential contribution of copulas and extreme value techniques is in the context of the simple portfolio choice problem presented above. In particular, we consider the optimal portfolio choice of a mean-variance US investor who can choose between investing at home and abroad. The investor can invest in the US and in an index of the major non-North American countries, EAFE.[21] The investor is allowed to go *short* on each asset. In similar fashion to Lewis (1999), we calculate the optimal portfolio weights from Equation (8), using the average regional ρ as an estimate of the Pearson correlation ρ_{hf}. In addition, we calculate the weights with two alternative measures of diversification, the *t* copula's correlation and extreme comovement. Given the different nature of the various diversification measures, and the fact that the mean-variance paradigm is established mainly for the traditional correlation, we merely wish to determine whether there is a qualitative difference in optimal choice during the 1990s and after, that might reflect the shifts in diversification scope documented above.[22] In sum, we are making a rough qualitative assessment of an investor's optimal choice, if she follows the signal from each of our diversification measures.

The results are in Table 6. During the 1990s the US dominated many foreign portfolios. Therefore, we should expect a negative home bias, at least for the first part of the sample.[23] Panel A shows the full sample results. Let us focus on the first column, which gives weights based on the familiar correlation. The most important finding is that the maximal portfolio weight outside of Latin America, is always less than 40 %, regardless of the value of the risk aversion. This is in stark contrast with Lewis (1999), who finds the minimum weight to be 43.1 %, in a sample from 1970 to 1996. Therefore, according to all the diversification measures, there is a sort of reverse home bias in East Asia and the EAFE. Further, there is substantial heterogeneity. In particular, for the G5 and East Asia, a US investor actually

[20] We use mean-variance because it provides a well understood benchmark. There are many other possibilities, such as CARA-normal and CRRA-lognormal settings, and the safety-first approach of Roy (1952). However, the first two approaches restrict the distribution of returns, and the latter approach does not readily yield closed form solutions. Moreover, mean-variance is often used in international portfolio choice research. We therefore use the mean-variance approach for our admittedly preliminary illustrations.

[21] The acronym EAFE denotes Europe, Australia and the Far East.

[22] Related empirical work using CAPM-like settings with downside risk and other related risk measures include Berkelaar, Kouwenberg, and Post (2004) and Post and van Vliet (2004).

[23] Moreover, because the US returns dominated returns for other countries, the first term in equation (8) may be negative. This, coupled with low volatility, results in foreign portfolio weights that can increase with risk aversion, which occurs for many countries in the first sample, but never in the second sample.

Table 6: Foreign portfolio weights for a US investor, using different diversification measures

Panel A: Full sample

	Correlation-based weights				Copula-based weights				Extreme value-based weights			
$\gamma =$	2	5	7	10	2	5	7	10	2	5	7	10
G5 – US	–49.4	4.8	15.2	22.9	–45.9	6.4	16.4	23.9	–169.2	–49.6	–26.8	–9.7
East Asia	–40.4	–4.3	2.6	7.8	–38.5	–3.1	3.7	8.7	–133.3	–60.0	–46.0	–35.5
Latin America	55.5	28.1	22.8	18.9	55.8	26.9	21.4	17.2	56.8	23.2	16.8	12.0
EAFE	1.5	29.7	35.0	39.0	2.3	30.0	35.3	39.2	–15.5	22.5	29.8	35.2

Panel B:1990–2001

	Correlation-based weights				Copula-based weights				Extreme value-based weights			
$\gamma =$	2	5	7	10	2	5	7	10	2	5	7	10
G5 – US	–131.1	–25.7	–5.6	9.5	–132.6	–26.3	–6.0	9.2	–296.2	–94.6	–56.2	–27.4
East Asia	–111.1	–35.8	–21.4	–10.6	–100.3	–30.0	–16.6	–6.6	–273.5	–122.2	–93.4	–71.8
Latin America	13.3	8.1	7.1	6.3	12.2	6.8	5.8	5.0	–3.8	–11.5	–13.0	–14.1
EAFE	–56.2	5.7	17.5	26.4	–56.5	5.6	17.4	26.3	–95.4	–10.6	5.5	17.7

Panel C:2001–2006

	Correlation-based weights				Copula-based weights				Extreme value-based weights			
$\gamma =$	2	5	7	10	2	5	7	10	2	5	7	10
G5 – US	157.4	81.6	67.2	56.4	154.4	80.7	66.7	56.2	872.4	292.2	181.7	98.8
East Asia	204.2	104.5	85.5	71.3	212.7	107.5	87.5	72.5	489.2	205.3	151.2	110.7
Latin American	195.6	94.1	74.8	60.3	217.9	100.9	78.6	61.9	275.8	118.4	88.4	66.0
EAFE	101.3	70.9	65.1	60.8	99.3	70.1	64.5	60.3	144.6	88.5	77.9	69.8

Percent portfolio weights are calculated from (8), representing ρ_{hf} with average comovement measures in the region: correlation ρ, copula parameter ρ_{copula}, and left comovement $\bar{\chi}$. G5–US are G5 economies excluding the US. γ is the coefficient of relative risk aversion. The terms East Asia, Latin America and EAFE refer to stock indices from East Asia, Latin America, and a broad group of non-US countries, 'Europe, Australia and Far East', respectively. We use this table in the text by going from left to right for each set of portfolio weights. That is, we focus on what happens to the desired portfolio weight as we move from lowest risk aversion ($\gamma = 2$) to the highest ($\gamma = 10$). For example, if we want to compare portfolio weights for East Asia, for the full sample, Panel A, we would go from left to right under "Correlation-based weights", then do the same for "Copula-based weights", then for "Extreme value-based weights". This is the basis of our discussion in the text.

wishes to short sell foreign stocks, if risk aversion is low enough. By contrast, for Latin America and the EAFE, most foreign portfolio weights are positive. This pattern holds, in general, for all the diversification measures during the full sample. Now, in Panel B, let us examine the first half of the sample. The negative home bias is even more pronounced here, especially for the East Asian economies, where it is always optimal to go very short. Once again this is true for all of our diversification measures, and once again the investment in all regions falls short of the benchmark (Lewis, 1999) value of 43.1 %. Finally, we examine the latter sample, in Panel C. In accordance with the reduced dominance of US returns after the 1990s, the foreign portfolio weights are very large and positive for all regions. Using the standard correlation, the optimal foreign investment is always well in excess of 40 %: the minimal portfolio weight is 56.4 %, and the maximum is as high as 204.2 %. Interestingly, for low levels of risk aversion, the optimal foreign weight exceeds 100 %. This amounts to short selling the US index, just the opposite pattern from that of the 1990s. This pattern is qualitatively the same using other diversification measures.

To summarise, when we estimate optimal portfolio weights we find substantial heterogeneity over the sample period. For the 1990s US returns dominated many foreign markets (especially East Asia, and EAFE), and the home bias phenomenon was not relevant. A notable exception was the Latin American economies, where it was optimal to hold some stocks. By contrast, since the turn of the century, US returns are not so dominant, and it is once again optimal for a US investor to place in excess of 50 % of her wealth overseas. These patterns hold qualitatively when we use any of our diversification measures. In economic terms, our findings are reassuring because they indicate that the large time variation in diversification benefits that we documented in Section 3.1 may manifest itself in dramatic shifts of portfolio weights for an international investor.

A final important aspect is that there is some disagreement of the proposed investment strategy using various dependence measures. For example, in Table 6 Panel

A, a US investor with risk aversion of 2 would hold 1.5 % of her wealth in foreign stocks (EAFE) using the correlation based approach, but would actually go short –15.5 % using the extreme comovement approach. This disagreement, coupled with the inconsistency of Pearson and rank correlation estimates for Latin America in Section 3.1, suggest that it may be important to consider both traditional methods and extreme value methods in portfolio choice for investors. The above results can be seen as quantitative evidence on the possible financial relevance of copulas in risk management and asset allocation.

5 Stability: A possible link between borrowing and extremes

While a formal analysis is outside the scope of this paper, we provide a brief introduction to the empirical properties of extremes. Extreme events can be exogenous or endogenous. Exogenous events are effectively acts of nature, while endogenous extremes depend on the behaviour of economic agents. Why is this distinction relevant? Let us denote the likelihood of an extreme event in financial markets at period t as p_t. If this variable is predictable by some economic quantity, then central banks can, in principle, ameliorate the effects of extremes *ex ante*, instead of ex post.

In a recent paper, Chollete (2008) develops a simple setting where, due to externalities, excess borrowing can cause an inefficiently high likelihood of extremes. Building on this logic, we can consider a test to show whether borrowing can predict extreme probabilities. Specifically, Table 7 shows the results of estimating the effect of past borrowing on extreme probabilities. The extreme probabilities are calculated by computing the proportion of days in each month the Dow Jones Industrial Average exceeded k standard deviations from the previous year's median, k = 1, 2, 3.[24] We find that for 1- and 2-σ events, borrowing (REALLOAN) has a significant effect. Moreover, both liquidity and an interaction between liquidity and borrowing have significant effects for 1-σ events. Similar findings arise for various reference periods, including 2, 5 and 10 years. Consequently, there is evidence that extremes are endogenous via the channel of borrowing. This result shares the same spirit as that of Borio (2007) and Goodhart

Table 7: The effect of borrowing on extremes

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN *LIQ0	REALLOAN *LIQ
Panel A: 1-σ events									
Coefficient	5.3675	4.5980	–21.5218	0.1487	–2.3985	–5.7905	–1.5121	26.0566	9.5101
	(0.0011)	(0.0047)	(0.0052)	(0.2759)	(0.6978)	(0.0179)	(0.4771)	(0.0197)	(0.3332)
Tests of Overall Fit (p-values):		LR	0.0186						
		Score	0.0175						
		Wald	0.0409						
Panel B: 2-σ events									
Coefficient	8.6457	7.9444	–45.0513	–0.0339	–9.3293	–8.0929	–2.9394	39.1506	15.3235
	(0.0112)	(0.0195)	(0.0091)	(0.8263)	(0.1551)	(0.0443)	(0.4729)	(0.0472)	(0.4558)
Tests of Overall Fit (p-values):		LR	0.0011						
		Score	0.0094						
		Wald	0.0275						
Panel C: 3-σ events									
Coefficient	8.7419	7.6638	–57.7413	0.2965	–9.3413	–15.1531	–6.2688	68.5561	34.5645
	(0.3130)	(0.3765)	(0.2006)	(0.4033)	(0.4732)	(0.1723)	(0.5143)	(0.2016)	(0.4852)
Tests of Overall Fit (p-values):		LR	0.3148						
		Score	0.4609						
		Wald	0.5756						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent variable is the likelihood of extremes, $p_t(\omega)$, ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. A chi square statistic is computed as the squared ratio of each parameter to its standard error, and the corresponding p-values are in parentheses.

[24] For more details, see Chollete (2008)

and Tsomocos (2007), suggesting new, micro-based policy approaches for central banks in search of optimal financial stability. These approaches would have fiscal elements, instead of the pure monetary methods which have been more popular over the last three decades.

6 Conclusions

Dependence is at the heart of modern financial economics. We naturally encounter and try to understand dependence in modern economic life, whether we seek to price an asset, invest in a diversified portfolio, or assess spillover effects from one market to another. There has been a recent flurry of research seeking to understand dependence in economic settings. Such research covers econometrics (Patton (2005)) as well as finance and banking (Rosenberg and Schuermann (2006). Partly in response to this diversity of research, the current paper seeks to outline a few key, common ideas in the literature in a simple way, in a manner similar to Dorfman (1969). We illustrate these ideas in simple empirical settings, implementing both the relatively new techniques from copulas as well as traditional correlation-based methods.

Our two most important findings are as follows. First, measures of comovement based on ranks may disagree from those based on traditional correlation, for example in Latin American markets. This disagreement is mirrored in the inconsistency of portfolio choices made by investors who decide based on correlation instead of properly considering extreme comovement, as shown at the end of Section 4.2. Second, and perhaps of special interest to central bankers, in the end of Section 5 we find that the likelihood of extreme events is related to individual´s previous real estate borrowing behaviour. This latter finding suggests that extremes may be endogenous and potentially amenable to regulatory policy analysis, since the propagation of extremes is related to observable data on borrowing. Such information is obviously valuable during the current, turbulent international financial markets of 2008.

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