

Working Paper

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Norges Bank Research

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Keywords

Monetary policy, firm

heterogeneity, financial frictions,

regime switching

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ISSN 1502-8143 (online)

ISBN 978-82-8379-333-8 (online)

Monetary Policy Shock, Financial Frictions and Heterogeneous Firms*

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June 28, 2024

Abstract

This paper examines the influence of financial constraints on the transmission of monetary policy shocks across heterogeneous firms. To this end, we develop a Dynamic Stochastic General Equilibrium (DSGE) model incorporating firm heterogeneity, nominal rigidity, and financial frictions. Financial constraints hinder firms from expanding production, even under expansionary monetary policy shocks. This dynamic discourages the production of competitive firms and exerts downward pressure on factor prices, leading to the proliferation and entry of less efficient firms. The prevalence of these inefficient firms becomes more significant in economies with higher granularity, where the withdrawal of large firms from the market opens up space for less productive smaller non-producers.

Keywords: *Monetary policy, firm heterogeneity, financial friction, regime switching*

JEL classification: E32; E52; L51; O47.

1 Introduction

Understanding how monetary policy shock propagates in the economy is a central concern for central banks. Since the Great Recession, debates have focused on whether accommodative policy contributes to the existence of “zombie” firms (Banerjee and Hofmann, 2018; Acharya et al., 2019; Albuquerque and Mao, 2023). Yet, little is understood about how monetary policy shock impacts heterogeneous firms facing varying degrees of financial stress and how it propagates within the

*We extend our sincere gratitude to the anonymous external referee for his/her invaluable contribution in the publication of this paper as part of the Norges Bank working paper series. We also appreciate the valuable feedback received from seminar participants at the first RISE workshop and ICMAIF 2024. This paper should not be reported as representing the views of Norges Bank. The views expressed herein are those of the authors and do not necessarily reflect those of Norges Bank. This project was supported by JSPS KAKENHI Grant Number 19KK0338. Any remaining errors are our own.

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economy. In the presence of financial frictions, an innovative firm may be unable to secure a loan, forcing a reduction in production or market exit. Conversely, an inefficient firm might remain in the market by successfully borrowing for operations. In both scenarios, common monetary policy shocks could have divergent impacts depending on firm characteristics and their capacity to raise necessary business loans. Therefore, understanding the transmission of monetary policy shock among heterogeneous firms, potentially exposed to financial frictions, is crucial to addressing this question.

The current paper addresses these issues through a stylized model. In our framework, firms exhibit heterogeneity in terms of their specific productivities and require loans for production. Firms prefer external financing (bonds) over internal financing (equities) because lenders (workers) are more patient than investors (shareholders). Furthermore, working capital is assumed to be essential for production. However, borrowing for working capital is limited as lenders, cautious of default, require collateral and impose a maximum lending value. Large and efficient firms, due to their extensive production scale, need more loans and thus exert a significant economic impact when facing financial difficulties. An expansionary monetary policy shock can improve their balance sheet by reducing debt repayment and enhancing collateral value. However, the positive effects of such a policy are constrained when borrowing limits are binding. In these situations, firms may be compelled to reduce production, as extending loans is not an option.

The standard transmission of monetary policy shock is thus dampened under the possibility of binding financial constraints. Due to financial constraints, firms fail to expand production even in the presence of an expansionary monetary policy shock. This, in itself, discouraging the production of competitive firms, puts downward pressure on factor prices, fostering the production and entry of less efficient firms. Consequently, accommodative monetary policy shock contributes to the creation of “zombie” firms, which are less prevalent in the absence of financial friction (Caballero and Hammour, 2005; Caballero, Hoshi, and Kashyap, 2008). The zombification of the economy tends to be more pronounced when there is greater granularity in the economy, as the cessation of operations by large firms creates more opportunities for unproductive firms.

Furthermore, we assert that firm entry plays a pivotal role in determining the dynamics of aggregate efficiency in the economy. Massive firm entry following an expansionary shock leads to market congestion and exerts upward pressure on factor prices. This pressure, in turn, acts as a selective mechanism, allowing only a subset of efficient firms to remain in the market. This, to an extent, actually improves aggregate productivity in the transitory dynamics following the expansionary monetary policy shock.

Our theoretical model is a Dynamic Stochastic General Equilibrium (DSGE) model featuring heterogeneous firms that face financial frictions. We construct a model incorporating firm entry and exit, following the frameworks of Bilbiie, Gironi, and Melitz (2012) and Hamano and

Zanetti (2017). To effectively capture the impact of monetary policy, we introduce wage nominal rigidity, as outlined in Hamano and Zanetti (2022). Specifically, financial frictions are modeled as enforcement constraints on working capital, in line with the approaches of Bergin, Feng, and Lin (2017) and Jermann and Quadrini (2012). In these models, firms prefer external borrowing because workers are less patient than investors. However, our model differs in that only a subset of producing firms may be financially constrained due to their insufficient collateral value. In our analysis, smaller firms are constrained since the binding constraint of the average producer is assumed to be binned with some probability. Importantly, we model financial frictions as an occasionally binding constraint, treated as a regime-switching problem, akin to the approach by Binning and Maih (2017).

Financial frictions have been recognized as key amplifiers of macroeconomic shocks (Kiyotaki and Moore (1997); Bernanke, Gertler, and Gilchrist (1999); Iacoviello (2005); Andrés, Óscar Arce, and Thomas (2013)). In the context of monetary policy shocks, this amplification often occurs when financing conditions are relaxed, improving debt repayment conditions through higher collateral values for constrained firms. However, the same type of accommodative monetary policy shock can also create contractionary side effects, as in our model, by ultimately reducing factor prices under financial constraints. In line with this perspective, Carlstrom, Fuerst, and Paustian (2010), Fiore and Tristani (2013), and Manea (2020) find no significant impact from monetary policy shocks, as the dampening effect tends to dominate the amplification channel. Specifically, our model aligns with the arguments of Manea (2020), Gilchrist et al. (2017), and Ottonello and Winberry (2020), who explore monetary policy transmission in setups involving both financially constrained and non-constrained firms. Unlike these studies, which focus on investment and/or price dynamics in the presence of financial frictions, our paper emphasizes the selection of firms for production and their impact within this framework.

Moll (2014) focuses not only on the change in the cutoff level of productivity but also on the shift of the distribution over time among heterogeneous firms. Additionally, González et al. (2021) explore the interplay between financial friction and firm heterogeneity, finding that an expansionary monetary policy results in an increase in aggregate productivity. In their study, expansionary monetary policy disproportionately boosts the investment of more productive firms, thereby channeling resources towards high-productivity constrained firms. Our theoretical model features increasing aggregate productivity depending on the extent of competition with entrants. Congestion in entry, which increases factor prices, can contribute to a more significant upward adjustment in productivity under lax monetary conditions.

The paper is structured as follows: Section 2 introduces the benchmark model. In Section 3, we calibrate the theoretical model, drawing on existing literature. Section 4 specifically analyzes the impact of an expansionary monetary policy shock. The paper concludes with Section 5.

2 The Model

Our model features firms heterogeneous in their specific productivities, endogenous entry, and selection for production. The goods markets are monopolistically competitive, with each firm producing its own product variety. Due to operational fixed costs, only a subset of firms are actively producing. The economy features two types of households: workers and investors. As per the framework in [Jermann and Quadrini \(2012\)](#), workers are more patient than investors, which motivates firms (represented by investors) to borrow from workers to finance production. Furthermore, production requires loans (working capital) whose amount is limited by the effective collateral value. Additionally, wages are assumed to adjust sluggishly, highlighting the role of monetary policy in this framework.

2.1 Production Decision and Pricing

The consumption basket of household j of type k ($k = w$, workers or $k = I$, investors) is defined as

$$C_{k,t}(j) = \left(\int_{\zeta \in \Omega} c_{k,t}(j, \zeta)^{1-\frac{1}{\sigma}} d\zeta \right)^{\frac{1}{1-\frac{1}{\sigma}}}.$$

Here, only a subset of goods from the total universe of goods, Ω is available. The term $c_{k,t}(j, \zeta)$ represents the demand addressed for each product variety, indexed by ζ . The parameter σ (greater than 1) denotes the elasticity of substitution among the differentiated product varieties. Given the above consumption basket, the optimal consumption demand for a typical variety is determined as:

$$c_{k,t}(j, \zeta) = \left(\frac{p_t(\zeta)}{P_t} \right)^{-\sigma} C_{k,t}(j).$$

The price index that minimizes nominal expenditure is: $P_t = \left(\int_{\zeta \in \Omega} p_t(\zeta)^{1-\sigma} d\zeta \right)^{\frac{1}{1-\sigma}}$. In this context, the welfare-consistent consumer price index P_t is chosen as a numéraire. Specifically, we define the real individual price as $\rho_t(z) \equiv \frac{p_t(z)}{P_t}$.

In our model, firms operate in a monopolistically competitive market, producing differentiated product varieties. The production level for each incumbent firm is thus determined by the demand it receives. Firms are assumed to be heterogeneous in terms of specific productivities denoted as z , which they randomly draw upon entry. Considering a productivity distribution $G(z)$, there exists a mass N_t of incumbent firms and a mass H_t of new entrants with a range of productivity levels over $[z_{\min}, \infty)$.

Firms generally prefer external financing (via bonds) over internal financing (equities). This preference is attributed to the comparatively higher patience of lenders (workers) compared to owners (investors). Further, production necessitates working capital. Importantly, this working

capital is considered an intra-period loan and, thus, does not accrue interest.

A firm with productivity level z requires $l_t(z)$ units of labor to produce $y_t(z)$ units of goods. The production function for a firm at this productivity level is given by:

$$l_t(z) = \frac{y_t(z)}{Z_t z} + f.$$

Here, f represents the fixed operational costs for production. Both variable and fixed costs are comprised of labor, which is imperfectly substitutable, similar to the labor used for entry costs. The labor demand $l_t(z)$ is thus defined as

$$l_t(z) = \left(\int_0^1 l_t(z, j)^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}.$$

In this equation, θ (greater than 1) is the elasticity of substitution among differentiated labor services.

Producing firms generate profits, denoted as $\pi_t(z)$, utilizing the externally raised intertemporal debt $b_{t+1}(z)$. These firms are responsible for honoring matured debt $b_t(z)$ along with the contracted interest \tilde{r}_t which is common for all firms due to the arbitrage, and subsequently distribute the remaining amount of real cash as dividends $d_t(z)$. The dividends of firms with productivity level z are thus defined as:

$$d_t(z) = \pi_t(z) - (1 + \tilde{r}_t) b_t(z) + b_{t+1}(z). \quad (1)$$

Given the demand, the profit function is given by:

$$\pi_t(z) = \frac{1}{\sigma} \rho_t(z)^{1-\sigma} C_t - f w_t.$$

In equation (1), $b_{t+1}(z)$ represents new corporate debt issued at time t . This model assumes that debt matures after one period. Accordingly, a firm repays its previous period's debt $b_t(z)$ along with real interest \tilde{r}_t , as determined by the mutual fund.

The firm aims to maximize its value at the beginning of each period by deciding on the amount of new debt, $b_{t+1}(z)$ to issue. The value of a firm with productivity level z at the beginning of a period, $v_t(z)$, is defined as:

$$v_t(z) = d_t(z) + E_t [m_{I_t, t+1} v_{t+1}(z)]. \quad (2)$$

Here, $m_{t, t+1}$ represents the discount factor of shareholders, which is defined in a later section. The term $E_t [m_{I_t, t+1} v_{t+1}(z)]$ denotes the end-of-period value of the firm after paying out dividends, which is equivalent to the firm's stock price. Production necessitates working capital for wage payments $w_t l_t(z)$ within the same period, thus constraining the incumbent firm's optimization problem.

As in the model by [Bergin, Feng, and Lin \(2017\)](#), the end-of-period value of the firm, $E_t [m_{t,t+1}v_{t+1}(z)]$ is utilized as collateral and considered to be liquidated only partially in the event of default. As a result, the enforcement constraint is formulated as follows.¹

$$\zeta_t E_t [m_{I_t,t+1}v_{t+1}(z)] \geq w_t l_t(z). \quad (3)$$

Here, ζ_t represents the rate of expropriation in the event of default. This rate is assumed to be uniform across firms and is known as the "financial shock".

The firm aims to maximize equation (2) under the aforementioned constraint by choosing $b_{t+1}(z)$. This optimization results in the following equation and defines the shadow value of the constraint, which is found to be the same across firms:

$$\eta_t(z) = \tilde{\eta}_t = \frac{1 - (1 + \tilde{r}_{t+1}) E_t [m_{I_t,t+1}]}{\zeta_t (1 + \tilde{r}_{t+1}) E_t [m_{I_t,t+1}]} \quad (4)$$

Intuitively, by issuing bonds, firms balance the marginal gain in the current period against the marginal costs related to debt repayment and the constraints imposed in the future period. When the constraint is binding (equation (3) holds with equality), this equation provides the level of debt.

When the constraint is not binding, we assume that firms can issue debt as much as they want for working capital as follows

$$\frac{\tilde{b}_{S,t}}{\tilde{b}_S} - \frac{w_t \tilde{l}_{S,t}}{w \tilde{l}_S} = 0.$$

Additionally, the first-order condition with respect to $\rho_t(z)$ under the previously derived demand function yields:

$$\rho_t(z) = \frac{\sigma}{\sigma - 1} (1 + \tilde{\eta}_t) \frac{w_t}{Z_t z}.$$

When the constraint binds as $\zeta_t E_t [m_{I_t,t+1}v_{t+1}(z)] = w_t l_t(z)$, and thus $\tilde{\eta}_t > 0$, it implies that workers become less productive since to produce one unit of goods, it requires a higher $(1 + \tilde{\eta}_t) / Z_t z$ number of workers for the producer. Put differently, when the constraint binds, it works as a negative labor productivity shock by increasing the marginal cost of production.

Firms earn varying profits depending on their idiosyncratic characteristics and the macroeconomic environment. Firms with negative profits ($\pi_t(z) < 0$) cease operations, thus not employing

¹[Jermann and Quadrini \(2012\)](#) argue that liquidity can be easily diverted, and physical capital is the only available asset for liquidation in the event of default. However, we assume that liquidity can also be available for liquidation. Specifically, the enforcement constraint is represented as:

$$E_t [m_{I_t,t+1}v_{t+1}(z)] \geq w_t l_t(z) + E_t [m_{I_t,t+1}v_{t+1}(z)] - \zeta_t (E_t [m_{I_t,t+1}v_{t+1}(z)])$$

In this expression, the right-hand side represents the expected value of defaulting, and the left-hand side denotes the value of not defaulting. By rearranging this, we obtain equation (3).

any resources or relying on external financing. A firm with negative profits repays its matured debt $b_t(z)$, which reduces dividends $d_t(z)$ at the beginning of the period. Consequently, the productivity level $z_{S,t}$ at which a firm earns zero profits is determined by:

$$\pi_t(z_{S,t}) = 0.$$

2.2 Firm Entry and Exit

Upon entering the market, entrants face sunk entry costs and must hire $f_E = l_{E,t}$ amounts of composite labor. This labor is composed of imperfectly differentiated labor services (indexed by j), similar to production labor, and is expressed as:

$$l_{E,t} = \left(\int_0^1 l_{E,t}(j)^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{\frac{1}{\theta}}}$$

The variable f_E is assumed to be exogenous and common for all firms. [Bergin, Feng, and Lin \(2017\)](#) postulate immediate production upon entry, significantly simplifying the model and making the problem faced by entrants isomorphic to that of incumbent firms. We adopt this assumption, positing that entrants produce within the same period of entry. As elaborated in the Appendix, this results in entrants' first-order conditions mirroring those of incumbents. Notably, entrants do not have debt repayment obligations from the previous period. Thus, following [Melitz \(2003\)](#), the "value of entry" is defined as:

$$v_t^E(z) = \pi_t(z) + b_{t+1}(z) + E_t \left[m_{I,t,t+1} v_{t+1}^E(z) \right] - w_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau.$$

This represents the sum of profits, new debt issuance, and the end-of-period value of entry, net of entry costs. The adjustment cost in the entry process is modeled following [Lewis \(2009\)](#) and [Lewis and Poilly \(2012\)](#) and [Bergin, Feng, and Lin \(2017\)](#), where τ is a parameter determining congestion in entry.

2.3 Firm Averages, Binding Constraint, and Free Entry

Following [Melitz \(2003\)](#), the average productivity level \tilde{z} and $\tilde{z}_{S,t}$ are defined for firms and active producers as follows

$$\tilde{z} \equiv \left[\int_{z_{min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{S,t} \equiv \left[\frac{1}{1 - G(z_{S,t})} \int_{z_{S,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

Given these definitions, we denote the average value of any variable x for all firms as x as $\tilde{x}_t \equiv x_t(\tilde{z})$ and for the subset of firms that are actively producing as $\tilde{x}_{S,t} \equiv x_t(\tilde{z}_{S,t})$, respectively.

We assume that when the enforcement constraint is binding, it binds for this particular average producer. We reformulate the financial constraint (3) for the average producer as the slackness condition. Denoting the labor demand of this average producer as $l_t(\tilde{z}_{S,t}) \equiv \tilde{l}_{S,t}$, this implies that²

$$\tilde{\eta}_t (\zeta_t E_t [m_{I_t,t+1} \tilde{v}_{S,t+1}] - w_t \tilde{l}_{S,t}) = 0,$$

with

$$\tilde{\eta}_t > 0, \text{ and } \zeta_t E_t [m_{I_t,t+1} \tilde{v}_{S,t+1}] - w_t \tilde{l}_{S,t} = 0,$$

or

$$\tilde{\eta}_t = 0, \text{ and } \zeta_t E_t [m_{I_t,t+1} \tilde{v}_{S,t+1}] > w_t \tilde{l}_{S,t}.$$

Given the production function, the labor demand of the average producer is derived as follows:

$$\tilde{l}_{S,t} = \frac{\tilde{y}_{S,t}}{\tilde{z}_{S,t}} + f,$$

where $\tilde{y}_{S,t} = \tilde{\rho}_{S,t}^{-\sigma} C_t$ and $\tilde{\rho}_{S,t} = S_t^{\frac{1}{\sigma-1}}$ with the definition of the price index.

Following [Binning and Maih \(2017\)](#), we model the above slackness condition with a switching parameter ϕ which is dependent on the state of the economy,

$$S_t = BIN, NON.$$

It is assumed that in the binding state, $\phi(BIN) = 0$, and non-binding state $\phi(NON) = 1$. Given this, the slackness condition is rewritten as

$$\phi(S_t) \tilde{\eta}_t - (1 - \phi(S_t)) [\zeta_t E_t [m_{I_t,t+1} \tilde{v}_{S,t+1}] - w_t \tilde{l}_{S,t}] = 0.$$

Markov process is governed by the following transition matrix:

$$\mathcal{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{BIN-NON,t} & p_{BIN-NON,t} \\ p_{NON-BIN,t} & 1 - p_{NON-BIN,t} \end{bmatrix},$$

where $p_{BIN-NON,t}$ stands for the probability of transition from the binding state at t to the non-binding state at $t+1$ while $p_{NON-BIN,t}$ stands for the transition probability from non-binding state at t to the binding-state at $t+1$.

The optimal pricing for the average producer becomes:

²Note that the collateral value (the end-of-period value) is not indexed with $\tilde{z}_{S,t}$ because of the selection for production at the beginning of the next period.

$$\tilde{\rho}_{S,t} = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \tilde{z}_{S,t}} (1 + \tilde{\eta}_t).$$

Further, we can define the following average profits, dividends, and value of producers:

$$\tilde{\pi}_{S,t} = \frac{1}{\sigma} \frac{C_t}{S_t} - f w_t,$$

$$\tilde{d}_{S,t} = \tilde{\pi}_{S,t} - (1 + \tilde{r}_t) \tilde{b}_{S,t} + \tilde{b}_{S,t+1},$$

and

$$\tilde{v}_{S,t} = \tilde{d}_{S,t} + E_t [m_{I,t,t+1} \tilde{v}_{S,t+1}].$$

Also, the average of profits, dividends, value, and debt of firms are computed as $\tilde{\pi}_t = \frac{S_t}{N_t + H_t} \tilde{\pi}_{S,t}$, $\tilde{d}_t = \frac{S_t}{N_t + H_t} \tilde{d}_{S,t}$, $\tilde{v}_t = \frac{S_t}{N_t + H_t} \tilde{v}_{S,t}$, and $\tilde{b}_t = \frac{S_t}{N_t + H_t} \tilde{b}_{S,t}$.

Free entry occurs until the following condition is met:

$$\tilde{v}_t^E = 0.$$

As discussed by [Bergin, Feng, and Lin \(2017\)](#), entry costs $w_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau$ are financed by three sources: current period profits, new debt issuance, and investment value by shareholders (the expected value of entry or the share price).

A mass of H_t entrants is endogenously determined through the above condition. Firms are assumed to remain in the market until they encounter an exit-inducing shock δ at the end of the period. Therefore, the motion of the firm count is specified as

$$N_{t+1} = (1 - \delta) (N_t + H_t).$$

2.4 Parametrization of Productivity Draws

The following Pareto distribution for $G(z)$ is considered:

$$G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^\kappa,$$

where z_{\min} stands for the minimum productivity level, and $\kappa (> \sigma - 1)$ is a shape parameter of the distribution. With the above distribution, the productivity of average producers $\tilde{z}_{S,t}$ is shown as

$$\tilde{z} = z_{\min} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{S,t} = z_{S,t} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}}.$$

As noted previously, there exists a firm with a specific productivity cutoff $z_{S,t}$ with which she

earns zero profits: $\pi_t(z_{S,t}) = 0$. Combined with the above Pareto distribution, this implies the following zero-cutoff profit condition:³

$$\tilde{\pi}_{S,t} = \frac{\sigma - 1}{\kappa - (\sigma - 1)} f w_t. \quad (5)$$

Among all incumbent firms at time t , a subset $S_t = [1 - G(z_{S,t})] (N_t + H_t)$ number of firms produces. Also, with the above Pareto distribution, the share of producing firms is given by⁴

$$\frac{S_t}{N_t + H_t} = \left(\frac{\bar{z}}{\bar{z}_{S,t}} \right)^\kappa$$

2.5 Household's Intertemporal Choices

The representative household of each type k maximizes her life time utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_{k,t}$, where β ($0 < \beta_k < 1$) is the exogenous discount factor.

2.5.1 Workers

The utility of individual household j of workers at time t depends on consumption $C_{w,t}(j)$ and labor supply $L_t(j)$ as follows

$$U_{w,t}(j) = \frac{C_{w,t}(j)^{1-\gamma_w}}{1-\gamma_w} - \chi \frac{L_t(j)^{1+\varphi}}{1+\varphi}.$$

where the parameter γ_w represents the relative risk aversion of workers ($\gamma_w > 1$). The parameter χ represents the degree of (un)satisfaction in supplying labor while φ measures the inverse of the Frisch elasticity of labor supply.

Workers finance all incumbent firms, including entrants, by purchasing corporate bonds through mutual funds. They also hold bonds issued by the government, which are considered risk-free. The budget constraint for a worker j is thus given by

³The zero-cutoff profit implies that

$$\pi_t(z_{S,t}) = 0.$$

We have $\tilde{\pi}_{S,t} = \frac{1}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{w_t}{\bar{z}_{S,t}} (1 + \tilde{\eta}_t) \right]^{1-\sigma} C_t - f w_t$. As a result, we get

$$\pi_t(z_{S,t}) = \frac{\kappa - (\sigma - 1)}{\kappa} \tilde{\pi}_{S,t} - \frac{(\sigma - 1)}{\kappa} f w_t = 0.$$

With the Pareto distribution. Plugging the expression, we get the zero-cutoff profit condition (5).

⁴Note that with the Pareto distribution, we have

$$1 - G(z_{S,t}) = \left(\frac{z_{\min}}{z_{S,t}} \right)^\kappa = \left(\frac{z_{\min} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}}}{\bar{z}_{S,t}} \right)^\kappa = \left(\frac{\bar{z}}{\bar{z}_{S,t}} \right)^\kappa.$$

$$C_{w,t}(j) + b_{t+1}(j)S_t + b_{f,t+1}(j) = (1 + \nu) w_t(j)L_t(j) + (1 + \tilde{r}_t) b_t(j)S_{t-1} + (1 + r_{f,t}) b_{f,t}(j) + T_t,$$

where $b_{t+1}(j)$ and $b_{f,t+1}(j)$ represent the purchases of corporate and government bonds, respectively. ν is the subsidy rate on real labor income, $w_t(j)L_t(j)$. The subsidy is set such that $1 + \nu = (\theta - 1) / \theta$ and it is financed by a lump-sum transfer, T_t . The subsidy is used to undo the steady state distortion due to the wage markup. Finally, in the above expression, the real interest rate $r_{f,t}$ is defined as

$$1 + r_{f,t} \equiv \frac{1 + i_{t-1}}{1 + \pi_t},$$

where i_t represents nominal interest rate and π_t represents inflation.

Wages are set à la [Calvo \(1983\)](#): only a fraction of $1 - \vartheta$ household can re-optimize their wages $W_t(j)$ knowing the following labor demand:

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} L_t,$$

where W_t is the wage index, defined as:

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$

The first-order condition with respect to wage setting, which is derived in Appendix B, yields:

$$\left(\frac{W'_t(j)}{W_t} \right)^{1+\varphi\theta} = \frac{\frac{\chi^\theta}{(\theta-1)(1+\nu)} \sum_{k=0}^{\infty} (\beta_w \vartheta)^k E_t \left[\left(\frac{W_{t+k}}{W_t} \right)^{\theta(1+\varphi)} L_{t+k}^{1+\varphi} \right]}{\sum_{k=0}^{\infty} (\beta_w \vartheta)^k E_t \left[\lambda_{W,t+k} \frac{W_{t+k}}{P_{t+k}} \left(\frac{W_{t+k}}{W_t} \right)^{\theta-1} L_{t+k} \right]}, \quad (6)$$

where $W'_t(j)$ represents the optimally set wage rate.

Additionally, the wage markup $\mu_{w,t}(j)$ is defined by

$$w_t(j) \lambda_{W,t}(j) = \mu_{w,t}(j) \chi L_t^\varphi(j).$$

where $\lambda_{W,t}(j) = C_{w,t}(j)^{-\gamma_w}$ represents the marginal utility of consumption for worker j .

The first-order condition with respect to corporate bond holdings $b_{t+1}(j)$ is

$$1 = \beta_w (1 + \tilde{r}_{t+1}) E_t \left[\frac{\lambda_{W,t+1}(j)}{\lambda_{W,t}(j)} \right].$$

Finally, the first-order condition with respect to risk-free bond holdings $b_{f,t+1}(j)$ is

$$1 = \beta_w (1 + r_{f,t+1}) E_t \left[\frac{\lambda_{W,t+1}(j)}{\lambda_{W,t}(j)} \right].$$

2.5.2 Investors

The utility of the representative investors j is defined as

$$U_{I,t}(j) = \frac{C_{I,t}(j)^{1-\gamma_I}}{1-\gamma_I}.$$

The parameter γ_I represents the relative risk aversion of investors ($\gamma_I > 1$). The investor holds all firms including entrants by purchasing a share $x_{t+1}(j)$ with the price \tilde{q}_t proposed by the mutual funds. The budget constraint in real terms is thus given by

$$C_{I,t}(j) + x_{t+1}(j) (N_t + H_t) \tilde{q}_t = x_t(j) N_t (\tilde{q}_t + \tilde{d}_t).$$

The first-order condition with respect to shareholdings $x_{t+1}(j)$ is given by

$$\tilde{q}_t = E_t [m_{I,t,t+1}(j) (\tilde{q}_{t+1} + \tilde{d}_{t+1})].$$

where $m_{I,t,t+1}(j) = \beta_I (1 - \delta) \frac{\lambda_{I,t+1}(j)}{\lambda_{I,t}(j)}$ is the stochastic discount factor in which $\lambda_{I,t}(j) = C_{I,t}(j)^{1-\gamma_I}$ stands for the marginal utility in consumption of the investor.

2.6 General Equilibrium and Larger Producers

In equilibrium, there is a symmetry between workers and investors. As a result, we drop all household-specific index j . Specifically, by the law of large numbers, the wage is determined as

$$W_t^{1-\theta} = \vartheta W_{t-1}^{1-\theta} + (1 - \vartheta) W_t'^{1-\theta}.$$

Expressed in terms of wage inflation, it becomes⁵

Further, there exists a link between wage inflation $\pi_{w,t}$ and inflation π_t as

$$\frac{w_t}{w_{t-1}} = \frac{1 + \pi_{w,t}}{1 + \pi_t}.$$

In general equilibrium, the labor market clears as

$$L_t = S_t \tilde{I}_{S,t} + H_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau.$$

⁵This provides the following wage Phillips curve in the first order approximation:

$$\pi_t^w = \beta E_t [\pi_{t+1}^w] - \frac{(1 - \beta\vartheta)(1 - \vartheta)}{(1 + \vartheta\varphi)\vartheta} \hat{\mu}_t^w,$$

where $\hat{\mu}_t^w$ represents the deviation of wage markup μ_t^w from its steady state value.

Finally, we specify the following simple Taylor rule:

$$1 + i_t = (1 + i_{t-1})^\rho \left[(1 + i) \left(\frac{P_t^e}{P_{t-1}^e} \right)^{\phi_\pi} \left(\frac{Y_t^e}{Y_{t-1}^e} \right)^{\phi_Y} \right]^{1-\rho} v_t, \quad (7)$$

where v_t stands for a monetary policy shock that is specified below. ρ is the persistence of the previous nominal rate. ϕ_π and ϕ_Y are the weight on nominal inflation and fluctuations in nominal GDP, respectively. Given the inability of the statistical agency in capturing all fluctuations in product turnover (Broda and Weinstein (2006, 2004)), we assume that monetary authority conducts policy based on imperfectly observed price P_t^e and its inflation π_t^e that capture fluctuations in nominal prices only. Specifically, π_t^e is defined as

$$1 + \pi_t^e \equiv (1 + \pi_t) \left(\frac{S_t}{S_{t-1}} \right)^{\frac{1}{\sigma-1}}.$$

Further, we define GDP is defined from the spending side as $Y_t \equiv C_t + H_t \tilde{q}_t$. Based on the observed price, it defines the observed GDP as

$$Y_t^e \equiv \frac{P_t Y_t}{P_t^e} = \frac{Y_t}{S_t^{\frac{1}{\sigma-1}}}.$$

The whole system is summarized in Table 1 and Table 2.

Furthermore, we characterize the larger producers which have a higher productivity than the average producer, $\tilde{z}_{S,t}$, and define the average level of productivity of the larger producers as

$$\tilde{z}_{M,t} \equiv \left[\frac{1}{1 - G(\tilde{z}_{S,t})} \int_{\tilde{z}_{S,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

We can characterize the average of larger producers based on the above average. Specifically, we have nine new variables: the average price of larger producers ($\tilde{\rho}_{M,t}$), their average value ($\tilde{\vartheta}_{M,t}$), their average dividends ($\tilde{d}_{M,t}$), their average production size ($\tilde{y}_{M,t}$), their average debt level ($\tilde{b}_{M,t}$), their mass ($\tilde{\pi}_{M,t}$), their average labor demand ($\tilde{l}_{M,t}$), their mass (M_t), and $\tilde{z}_{M,t}$. The equations that define these variables are found in Table 5 in the Appendix.

3 Calibration

The calibration is conducted on a quarterly basis and is summarized in Table 3. The discount factors for workers and investors, β_W and β_I , are set at 0.995 and 0.985, respectively. The coefficients of risk aversion for workers and investors, γ_W and γ_I are set at 2 and 1, respectively. The inverse of the Frisch elasticity of labor supply, φ , is set at 0.5. The elasticity of substitution among varieties, σ , is set at 6. The exogenous exit shock, δ , is set at 0.025. The entry adjustment costs, τ , are set at 2.42. These parameter values are consistent with those reported by Bergin, Feng, and

Table 1: The Model

Price index	$1 = S_t^{-\frac{1}{\sigma-1}} \tilde{\rho}_{S,t}$
Av price of producers	$\tilde{\rho}_{S,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \tilde{z}_{S,t}} (1 + \tilde{\eta}_t),$
Financial friction	$\tilde{\eta}_t = \frac{1 - (1 + \tilde{r}_{t+1}) E_t[m_{I,t,t+1}]}{\tilde{c}_t (1 + \tilde{r}_{t+1}) E_t[m_{I,t,t+1}]}$
Complementary slackness conditions	$\tilde{\eta}_t (\tilde{c}_t E_t[m_{I,t,t+1} \tilde{v}_{S,t+1}] - w_t \tilde{l}_{S,t}) = 0$
Av debt of producers when $\tilde{\eta}_t = 0$	$\frac{\tilde{b}_{S,t}}{\tilde{b}_S} - \frac{w_t \tilde{l}_{S,t}}{w \tilde{l}_S} = 0$
Av value of producers	$\tilde{v}_{S,t} = \tilde{d}_{S,t} + E_t[m_{I,t,t+1} \tilde{v}_{S,t+1}]$
Av labor demand of producers	$\tilde{l}_{S,t} = \frac{\tilde{y}_{S,t}}{\tilde{z}_{S,t} Z_t} (1 + \tilde{\eta}_t) + f$
Av output of producers	$\tilde{y}_{S,t} = \tilde{\rho}_{S,t}^{-\sigma} C_t$
Av dividends of producers	$\tilde{d}_{S,t} = \tilde{\pi}_{S,t} - (1 + \tilde{r}_t) \tilde{b}_{S,t} + \tilde{b}_{S,t+1}$
Av profits of producers	$\tilde{\pi}_{S,t} = \frac{1}{\sigma} \frac{C_t}{S_t} - f w_t$
Zero-cutoff-profits condition	$\tilde{\pi}_{S,t} = \frac{\sigma-1}{\kappa - (\sigma-1)} f w_t$
Free entry	$\tilde{v}_t^E = 0$
Value of entry	$\tilde{v}_t^E = \tilde{\pi}_t + \tilde{b}_{t+1} - w_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau + E_t[m_{I,t,t+1} \tilde{v}_{t+1}^E]$
Av profits of firms	$\tilde{\pi}_t = \frac{S_t}{N_t + H_t} \tilde{\pi}_{S,t}$
Av dividends of firms	$\tilde{d}_t = \frac{S_t}{N_t + H_t} \tilde{d}_{S,t}$
Av value of firms	$\tilde{v}_t = \frac{S_t}{N_t + H_t} \tilde{v}_{S,t}$
Av debt of firms	$\tilde{b}_t = \frac{S_t}{N_t + H_t} \tilde{b}_{S,t}$
Labor market clearing	$L_t = S_t \tilde{l}_{S,t} + H_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau$
Av productivity of firms	$\tilde{z} = z \min \left[\frac{\kappa}{\kappa - (\sigma-1)} \right]^{\frac{1}{\sigma-1}}$
Nb of producers	$\tilde{z}_{S,t} = \tilde{z} \left(\frac{S_t}{N_t + H_t} \right)^{-\frac{1}{\kappa}}$
Law of motion	$N_{t+1} = (1 - \delta) (N_t + H_t)$
Marginal utility of consumption of investors	$C_{I,t}^{-\gamma_I} = \lambda_{I,t}$
Euler equation for share holdings	$\tilde{q}_t = E_t[m_{I,t,t+1} (\tilde{q}_{t+1} + \tilde{d}_{t+1})]$
Subjective discount factor of investors	$m_{I,t,t+1} = \beta_I (1 - \delta)^{\frac{\lambda_{I,t+1}}{\lambda_{I,t}}}$
Budget constraint of Investors	$C_{I,t} + H_t \tilde{q}_t = N_t \tilde{d}_t$
Marginal utility of consumption of workers	$C_{w,t}^{-\gamma_w} = \lambda_{W,t}$
Euler equation for corporate bonds	$1 = \beta_w (1 + \tilde{r}_{t+1}) E_t \left[\frac{\lambda_{W,t+1}}{\lambda_{W,t}} \right]$
Euler equation for government bonds	$1 = \beta_w (1 + r_{f,t+1}) E_t \left[\frac{\lambda_{W,t+1}}{\lambda_{W,t}} \right]$
Aggregate consumption	$C_t = C_{I,t} + C_{w,t}$
GDP definition	$Y_t = C_t + H_t \tilde{q}_t$

Table 2: The Model (con't)

Risk-free rate	$1 + r_{t+1}^f \equiv \frac{1+i_t}{1+\pi_{t+1}}$
Wage markup	$w_t = \mu_t^w \chi L_t^\varphi C_{w,t}^{\gamma_w}$
Wage setting	$\left(\frac{W_t'}{W_t}\right)^{1+\varphi\theta} = \frac{\frac{\chi^\theta}{(\theta-1)(1+\nu)} \sum_{k=0}^{\infty} (\beta_w \vartheta)^k E_t \left[\left(\frac{W_{t+k}}{W_t}\right)^{\theta(1+\varphi)} L_{t+k}^{1+\varphi} \right]}{\sum_{k=0}^{\infty} (\beta_w \vartheta)^k E_t \left[\lambda_{W,t+k} \frac{W_{t+k}}{P_{t+k}} \left(\frac{W_{t+k}}{W_t}\right)^{\theta-1} L_{t+k} \right]}$
Wage dynamics	$\vartheta (1 + \pi_{w,t})^{\theta-1} + (1 - \vartheta) \left(\frac{W_t'}{W_t}\right)^{1-\theta} = 1$
Identity	$\frac{w_t}{w_{t-1}} = \frac{1+\pi_{w,t}}{1+\pi_t}$
Empirical inflation	$1 + \pi_t^e = (1 + \pi_t) \left(\frac{S_t}{S_{t-1}}\right)^{\frac{1}{\sigma-1}}$
Empirical GDP	$Y_t^e = \frac{Y_t}{S_t^{\sigma-1}}$
Monetary Policy	$1 + i_t = (1 + i_{t-1})^\rho \left[(1 + i) \left(\frac{P_t^e}{P_{t-1}^e}\right)^{\phi_\pi} \left(\frac{Y_t^e}{Y_{t-1}^e}\right)^{\phi_Y} \right]^{1-\rho} v_t$

Lin (2017). The Pareto distribution parameter, κ , is set at 6, satisfying the restriction $\kappa > \sigma - 1$.⁶ The parameters determining nominal wage stickiness, λ , and the elasticity of substitution among differentiated labor services, θ , are set at 0.64 and 3.5, respectively, as documented by Christiano, Eichenbaum, and Evans (2005). The coefficients in the Taylor rule ($\rho = 0.7$, $\phi_\pi = 1.7$, and $\phi_Y = 0.1$) align with Berger, Herkenhoff, and Mongey (2020). These calibration values are based on US data. The monetary policy shock process is assumed to feature zero persistence.

Following Hamano and Zanetti (2017), we establish the steady state level of fixed production costs f at 0.0156 to ensure the ratio of producers S/N in both the binding and non-binding steady states remains at 0.98. We also adjust the steady state debt-to-wage ratio \tilde{b}/w to maintain a debt-to-GDP ratio of 1.6 in both steady states, identical to Bergin, Feng, and Lin (2017), resulting in $\tilde{b}/w = 0.92553397$. Consequently, the value of the financial shock ξ is calibrated at 0.2729. Furthermore, we set the steady state labor supply L to unity by adjusting χ , requiring $\chi = 0.7486$ in the binding steady state and $\chi = 0.6612$ in the non-binding steady state. Finally, we set the transition probability from the binding to the non-binding state as $p_{BIN-NON} = 0.05$ and that from the non-binding to the binding state as $p_{NON-BIN} = 0.01$. These transition probabilities do not influence the value of the steady state according to the perturbation method described in Chang, Maih, and Tan (2021). The details of the steady state are elaborated in the following subsection.

⁶This value is higher than that used by Chironi and Melitz (2005) ($\kappa = 3.4$) and lower than that used by Hamano and Zanetti (2017) ($\kappa = 11$).

Table 3: Calibration of the model

β_w	Discount factor of workers	0.995
β_I	Discount factor of investors	0.985
φ	Inverse of Frisch elasticity of labor supply	0.5
σ	Elasticity of substitution among varieties	6
γ_w	Risk aversion of workers	2
γ_I	Risk aversion of investors	1
δ	Exogenous death shock	0.025
κ	Pareto shape	6
λ	Calvo wage revision	0.64
θ	Elasticity of substitution among workers	3.5
τ	Entry adjustment cost	2.42
f	Fixed cost for production	0.0152
f_E	Fixed cost for entry	1
ρ	Interest smoothing on previous rate	0.7
ϕ_y	Output gap target	0.1
ϕ_π	Inflation target	1.7
ξ	Financial shock	0.2729
χ	Dis-utility in labor supply (BIN)	0.7486
χ	Dis-utility in labor supply (NON)	0.6612
$p_{BIN-NON}$	Transition probability from BIN to NON	0.05
$p_{NON-BIN}$	Transition probability from NON to BIN	0.01

3.1 The Binding and Non-Binding Steady States

We characterize the steady states with zero inflation as $\pi = \pi_w = \pi^e = 0$ in both binding and non-binding steady states. First, at the binding steady state, we have⁷

$$\tilde{\eta} = \left(\frac{\beta_w}{\beta_I(1-\delta)} - 1 \right) \frac{1}{\xi} \quad (8)$$

This expression implies that $\tilde{\eta}$ can be strictly positive, and thus the enforcement constraint is binding at the steady state as long as $\beta_w > \beta_I(1-\delta)$, given a finite positive value of the financial shock such that $\xi < \infty$.⁸

The risk-free rate and return on corporate bonds and equity returns are found to be the same across regimes, a direct consequence of constant discount factors and depreciation rates

⁷From the Euler equation of workers for corporate bonds at the steady state, we have

$$1 + \tilde{r} = 1 + r_f = \frac{1}{\beta_w}.$$

Also, the discount factor of investors at the steady state is $m_I = \beta_I(1-\delta)$. Plugging the relations found in the shadow price on the renouncement constraint, we have (8).

⁸Note that this holds even when the same discount factor is imposed between investors and workers, as $\beta_w = \beta_I$. Although [Jermann and Quadrini \(2012\)](#) and [Bergin, Feng, and Lin \(2017\)](#) argue that the presence of more patient workers than investors is necessary to motivate external financing for firms, in our setup, this is no longer the case due to firm heterogeneity. Intuitively, investors are less patient than workers, holding shares of startup firms H , while workers lend only to all incumbent firms N .

Table 4: Steady State

Variables	$\tilde{\eta}$	$\tilde{b} (N + H) / Y$	\tilde{b}_S	\tilde{r}	$\frac{\tilde{q}+d}{\tilde{q}}$	S/N	L	w
Binding	0.1321	1.6000	1.1765	0.005	0.0413	0.98	1	1.2146
Non-binding	0	1.6000	1.3320	0.005	0.0413	0.98	1	1.3751

across regimes. Because of the imposition of the same level of debt-to-GDP ratio, the debt level in the non-binding steady state is higher ($\tilde{b}_S = 1.3320$) than that in the binding steady state ($\tilde{b}_S = 1.1765$). The steady state values of the economy are shown in Table 4.

4 Quantitative Analysis

In this section, we explore the dampening of monetary policy shock in conjunction with the occasionally binding constraint. Initially, we present the impulse response functions for pivotal economic variables within both non-binding and binding regimes, aiming to elucidate the role of the binding constraint following an expansionary monetary policy shock.

Subsequently, our focus shifts to documenting the role played by various parameters in the model, utilizing the generalized impulse response functions. First, we explore the role of exogenous transition probabilities across two distinct regimes. Explicitly, we modify the transition probability from the non-binding to the binding regime to observe the impact of the potential for binding regimes. Second, we discuss the role played by firm heterogeneity in our economy. Third, we describe the role played by entry adjustment costs, which have implications on the average productivity of producers following an expansionary monetary policy shock.

4.1 Dampening Effect of Monetary Policy Shock under Financial Constraints

Monetary policy plays a role within an economy, predominantly through its influence on interest rates. We represent equation (4) to elucidate the impact of policy on firms' external financing costs:

$$\tilde{\eta}_t = \frac{1 - (1 + \tilde{r}_{t+1}) E_t [m_{It,t+1}]}{\tilde{\zeta}_t (1 + \tilde{r}_{t+1}) E_t [m_{It,t+1}]}.$$

In the context of an expansionary monetary action, characterized by a drop in the nominal rate and assuming subdued inflationary pressures, a subsequent decline in the real rate is observed. This is facilitated by arbitrage activities between corporate and risk-free bonds, with the latter's interest payment being anchored to the nominal policy rate. Consequently, this alteration affects firms' anticipated external financing costs by reducing future debt repayment (as indicated by the negative term in the numerator) and enhancing the value of collateral due to a decrease in future debt repayments (denoted by the term in the denominator).

In typical circumstances where the constraint is not binding, such a monetary pivot would encourage firms to accrue more inter-temporal debt. However, this paradigm shifts in a binding economy. In this situation, the multiplier, $\tilde{\eta}_t$, must increase, negating the incentives derived from the enhanced financial conditions and potentially causing a decline in inter-period loans.

Figure 1 shows the impulse response functions of major economic variables for both binding (solid lines) and non-binding economies (dashed lines) following an accommodative monetary shock. Specifically, v_t is assumed to be purely transitory and decreases by one percentage point without persistence. To see the role of the binding constraint, let's focus first on the case of the non-binding economy. In the non-binding economy, the expansionary shock creates an expansionary impact increasing real GDP Y_t and the number of entrants H_t . The latter is especially triggered by a sharp rise in the average share price of firms \tilde{q}_t . Given the expected increase in future labor demand due to the boom, workers set wages w_t higher, which results in a gradual increase in wage inflation $\pi_{w,t}$ (not shown). The latter is reflected in an increase in π_t^e . According to the Taylor rule described in equation (7), the fall in v_t induces a fall in the policy rate, i on impact despite such a sharp increase in observed price inflation π_t^e in general equilibrium. As real wages increase gradually, producers that need working capital increase their debt level. This is highlighted by the rise in general debt level \tilde{b}_t in the economy. The number of producers S_t increases accordingly with the higher entry H_t and the increasing number of incumbents N_t . They become less efficient on impact due to a lux demand condition which allows less efficient incumbents to produce. This is reflected in the fall of $\tilde{z}_{S,t}$. However, as wages increase gradually over time, the average productivity of producers resumes its increase in transitory dynamics.

Contrastingly, the above dynamics under the non-binding constraint alter significantly for the binding economy. When the constraint is binding for the average producer and thus for all producers smaller than this average producer, the multiplier, $\tilde{\eta}_t$, is positive and they borrow less. As a result, the average debt level among firms, \tilde{b}_t , is smaller compared to that observed in the non-binding regime. Furthermore, it would be interesting to see the hidden heterogeneity in this dynamic of the aggregate average debt. Figure 2 presents the average debt and value in both the non-binding and binding regimes for the average producer (top panel) and that of the larger producers (bottom panel). In the non-binding regime, both firms expand debt along with the rise in their values (left panels in the figure). However, in the binding regime, the debt issuance of the average producer contracts and falls, while that of the average of the large producers increases despite a limited increase in their value (right panels in the figure). The negligible rise in the value of large producers is a consequence of the dampened transmission of the expansionary monetary policy shock. The binding producers are required to cut production and hence labor demand, while the large producers are not.

The financial constraint that binds in the binding regime thus acts to diminish the real wages w_t and consequently impacts not only real GDP Y_t , which now expands more modestly. The ensuing sharp inflation of the observed price π_t^e renders the accommodative policy less expan-

sionary (a smaller fall in i_t). However, an amplification is observed in the number of entrants H_t , and consequently, a higher number of potential producers N_t and producers S_t in subsequent periods compared to the non-binding economy. This occurs as a fall in real wage bolsters new entry due to lower sunk entry costs, notwithstanding the decline in current profits, and stock price \tilde{q}_t in the binding economy. In both binding and non-binding economies, an expansionary policy shock allows the perseverance of less efficient firms $\tilde{z}_{S,t}$, with the divergence being quantitatively akin in this benchmark parameterization. In summation, binding constraints act to impede the transmission of monetary policy while concurrently generating an excessive number of new entrants.

4.2 Role of parameters

In this subsection, we document the role played by various parameters in the model, utilizing the generalized impulse response functions (GIRFs) which are the outcome of the interaction of two regimes. These parameters include exogenous transition probabilities from the non-binding to the binding regime ($p_{NON-BIN,t}$), firm heterogeneity (κ), and entry adjustment costs (ω).

4.2.1 Role of the Transition Probability

Figure 3 displays the GIRFs following an expansionary monetary policy shock with distinct transition probabilities from the non-binding to the binding state ($p_{NON-BIN,t}$), while keeping the benchmark calibration. The solid lines represent the GIRFs following an expansionary monetary policy shock, where the transition probability from the non-binding to binding state is zero. In contrast, the dashed and dotted lines represent those obtained with transition probabilities of 0.01 (the benchmark case) and 0.05, respectively.

It is evident that as the economy exhibits a higher likelihood of transitioning from the non-binding to the binding state, the impulse response functions increasingly resemble those observed under a perpetually binding economy, as previously discussed.

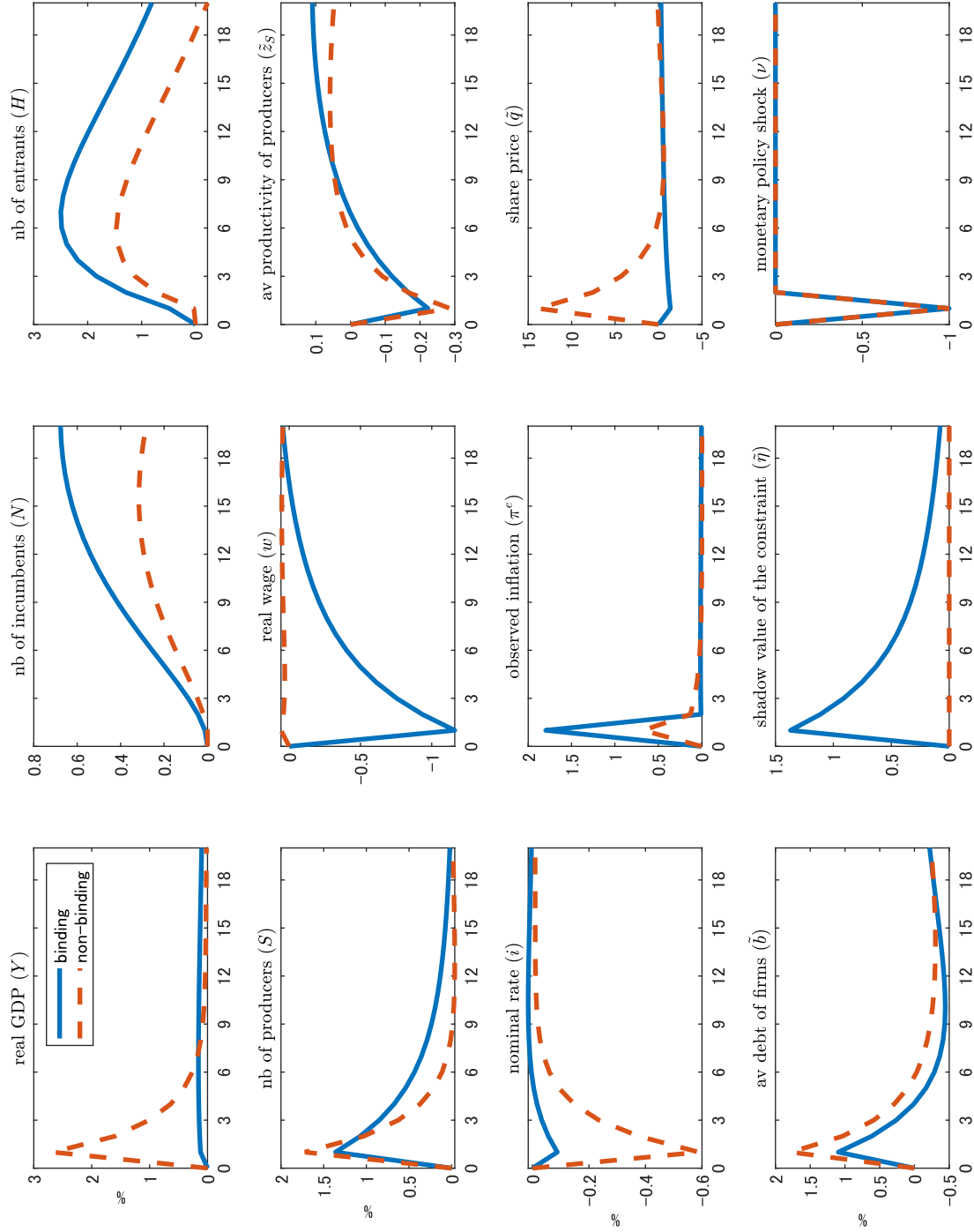
4.2.2 Role of Firm Heterogeneity

In Figure 4, we present the GIRFs for the benchmark parameterization and economies with lower granularities ($\kappa = 10$ and $\kappa = 14$).⁹ In the benchmark economy (represented by solid lines), after the same expansionary monetary policy shock, we observe the most pronounced downward adjustment in the average productivity of producers $\tilde{z}_{S,t}$ compared to other less granular economies.

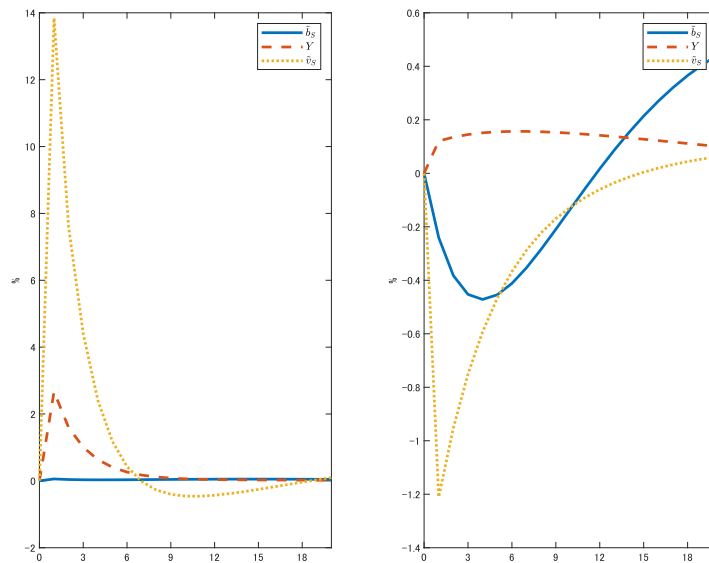
Why is this the case? Since the economies have the potential for binding constraints, the dampening effect of the monetary policy shock is in operation. As a result, producers reduce their

⁹Details on the steady state with different values of κ and the GIRFs of other variables are available upon request.

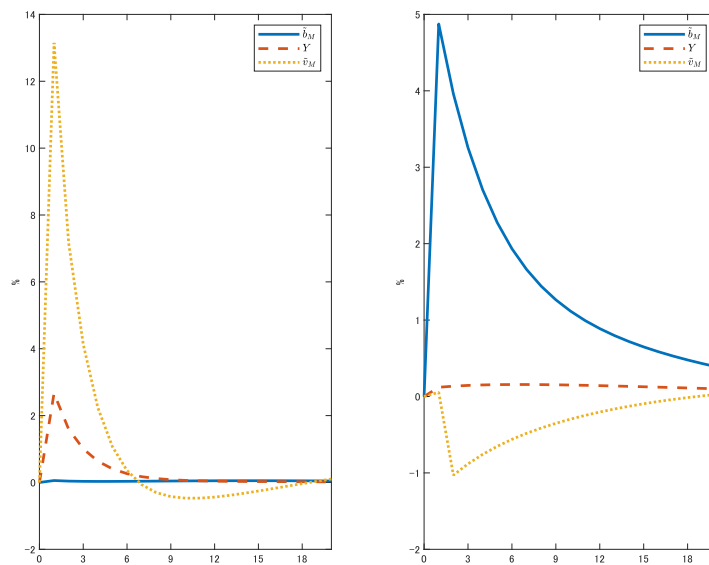
Figure 1: Monetary Policy Shock



Note: The figure shows the impulse response functions of the major economic variables following an expansionary monetary policy shock for those under the binding (solid lines) and non-binding regime (dashed lines). Calibration is on a quarterly basis.



(a) Debt and value of the average producers in non-binding and binding economy

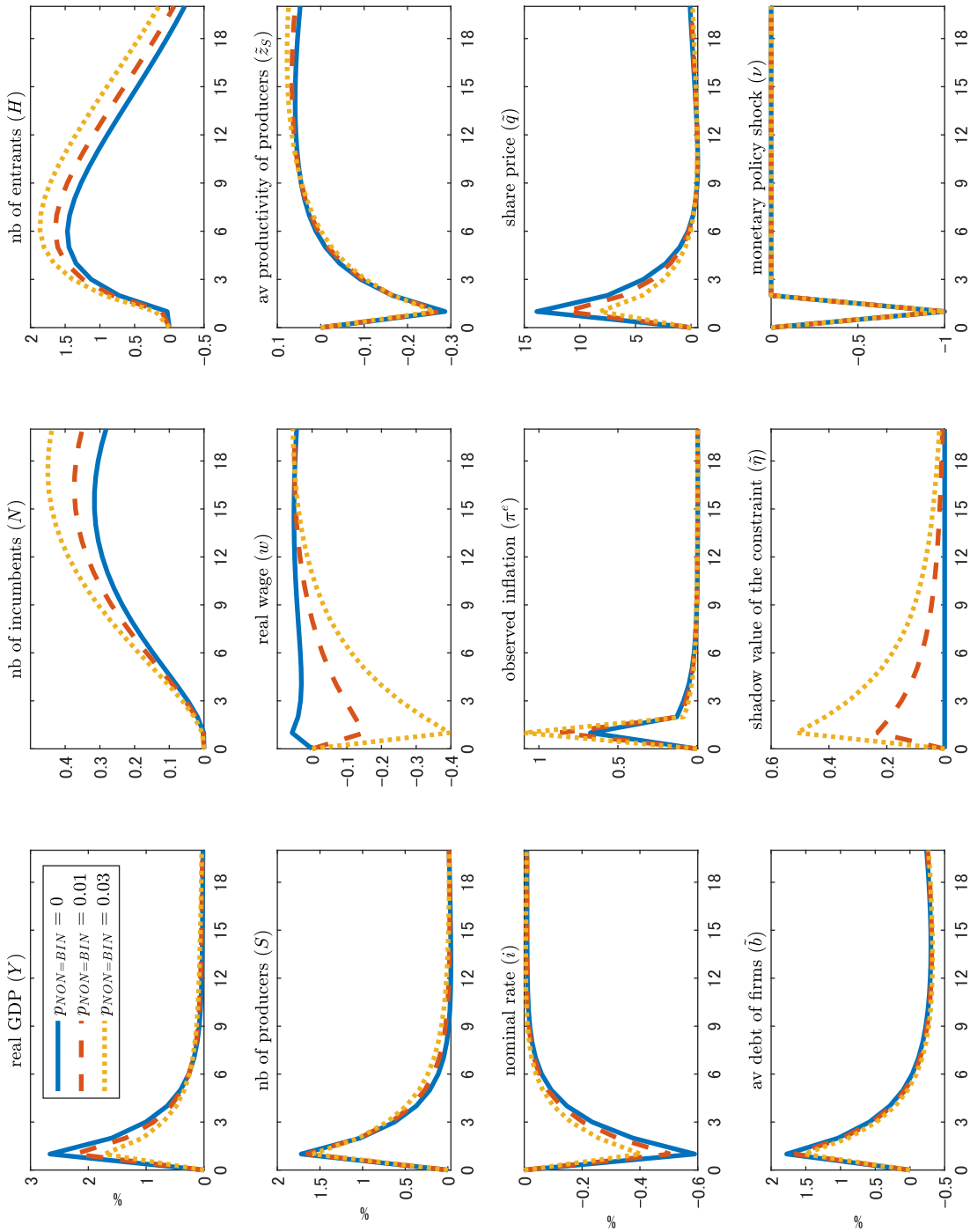


(b) Debt and value of large producers in non-binding and binding economy

Figure 2: Debt and value in non-binding and binding regime

Note: The figure shows the impulse response functions of the average debt and value of the producers and the larger producers, together with GDP, following an expansionary monetary policy shock. In both (a) and (b), the left panels show the case for the non-binding regime while the right panels show the case for the binding regime.

Figure 3: Monetary Policy Shock



Note: The figure shows the generalized impulse response functions of the major economic variables following an expansionary monetary policy shock under different probabilities from the non-binding to the binding regime. Calibration is on a quarterly basis.

production levels, resulting in decreased labor demand (not shown). Consequently, wages w_t decrease, but this reduction tends to be more significant in the economy with higher granularity.

The reason is as follows: When the financial constraint binds, it binds for the average producer by assumption. Since larger firms tend to face more significant financial constraints despite their increased need for inputs and working capital, the impact of the financial constraint is more significant for the economy with higher granularity. However, this challenging situation for larger firms brings good news for smaller non-producers, as the decrease in input prices w_t enables them to participate in the market and produce. This is why, following the same expansionary shock, the average productivity $\bar{z}_{S,t}$ in the economy with higher granularity experiences a sharper decline. Note also that the cheaper costs boost the entry H_t .

4.2.3 Role of Entry Adjustment Costs

We now explore the influence of entry adjustment costs, denoted as τ , on our analysis. Entry adjustment costs, which regulate congestion within the economy, are crucial in shaping the overall efficiency of the economic landscape, particularly when combined with the presence of financial frictions, as in our model. Figure 5 illustrates the GIRFs with the benchmark calibration ($\tau = 2.4$) and with its smaller values ($\tau = 1$ and $\tau = 0.1$).

When congestion in entry is sufficiently low due to the small value of entry adjustment costs, we observe a greater level of entry following an expansionary monetary policy shock of the same magnitude. This influx exerts upward pressure on factor prices w_t and leads to the displacement of less efficient firms from production, particularly in transitory dynamics. Consequently, this dynamic promptly reverses the trajectory of average productivity among producers $\bar{z}_{S,t}$. In other words, the dampening effect of monetary policy is alleviated when entry adjustment costs are low, leading to higher input prices that favor only a subset of efficient producers, specifically in transitory dynamics.¹⁰

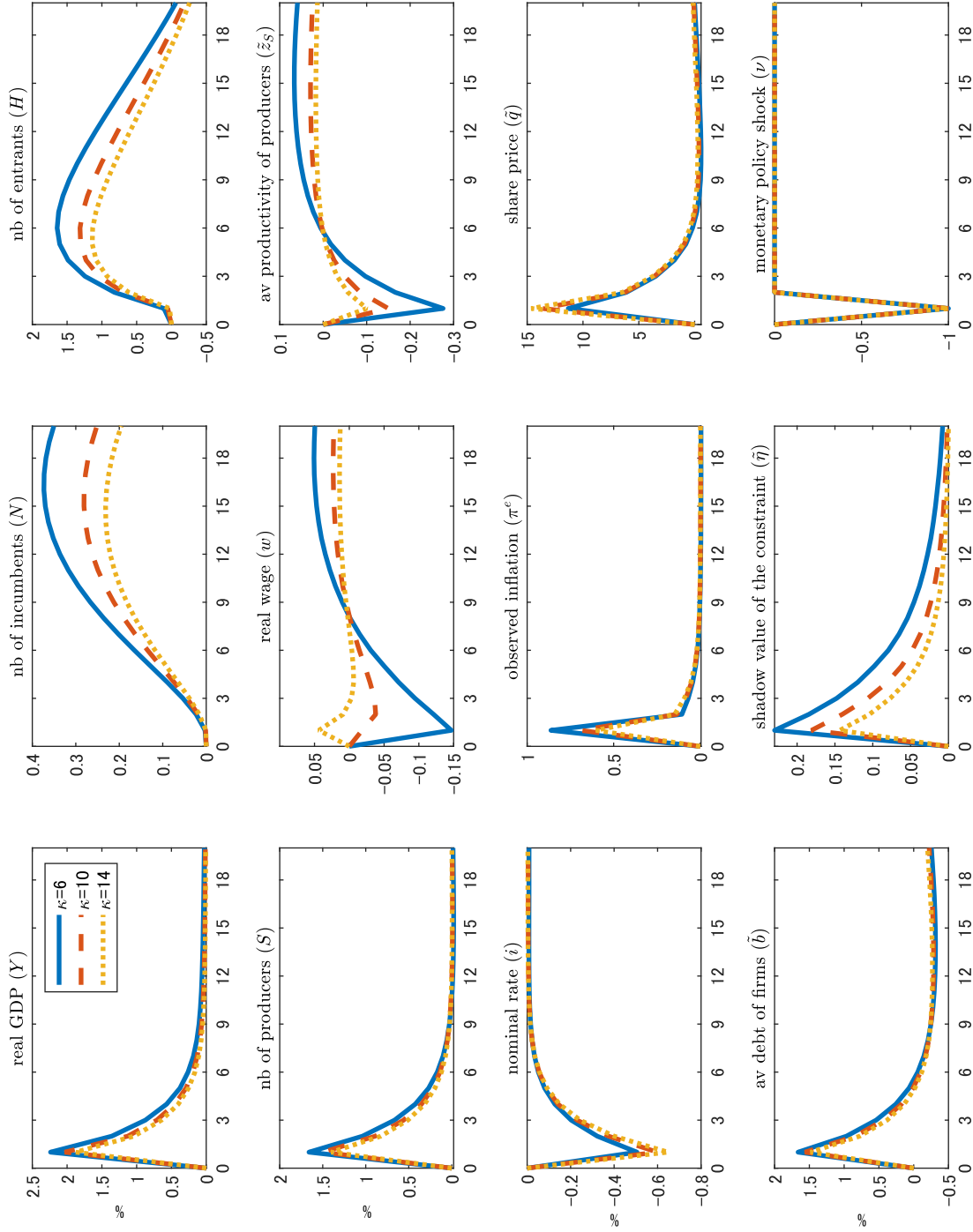
5 Conclusion

This paper investigates the effects of monetary policy on heterogeneous firms facing financial constraints. We develop a Dynamic Stochastic General Equilibrium (DSGE) model incorporating firm heterogeneity, nominal rigidity, and financial frictions. Financial frictions are specifically modeled as enforcement constraints that are occasionally binding. Using generalized impulse response functions, we demonstrate that a higher probability of binding works to dampen the expansionary monetary policy shock.

Additionally, an expansionary monetary policy shock leads to the entry of less efficient, non-producing firms, particularly when the granularity of the economy increases. Also we emphasize

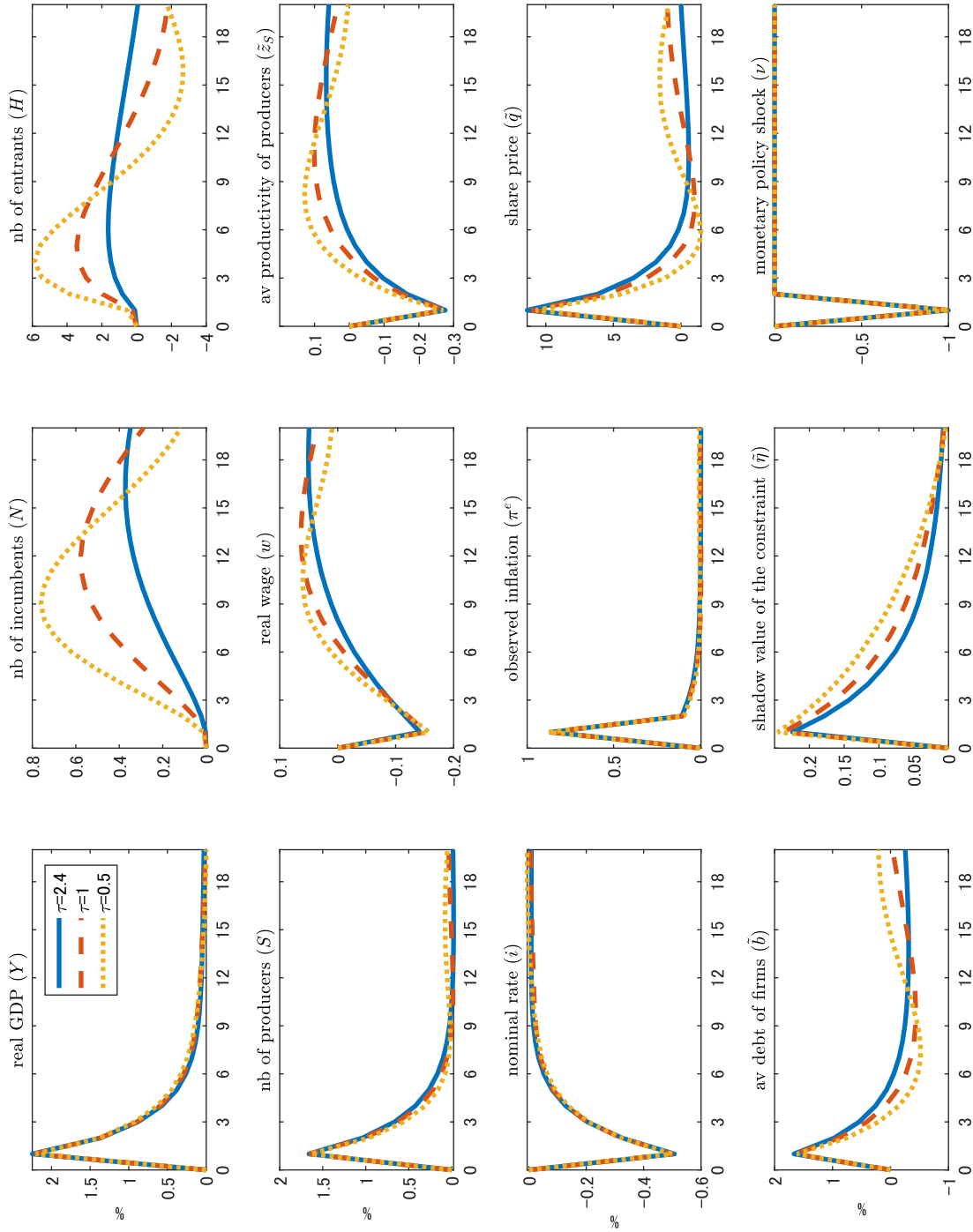
¹⁰Hamano and Zanetti (2017) argue for the pivotal role played by entry adjustment costs in reshaping productivity dynamics without the existence of financial frictions.

Figure 4: Role of Firm Heterogeneity



Note: The figure shows the generalized impulse response functions of the major economic variables following an expansionary monetary policy shock under different degrees of firm heterogeneity. Calibration is on a quarterly basis.

Figure 5: Role of Entry Adjustment Costs



Note: The figure shows the generalized impulse response functions of the major economic variables following an expansionary monetary policy shock under different values of entry adjustment costs. Calibration is on a quarterly basis.

the crucial role of firm entry in shaping the dynamics of aggregate efficiency in the economy. With sufficiently low entry adjustment costs, there is a marked increase in firm entries, leading to significant market congestion and enhancing aggregate productivity during the transient phase following the expansionary monetary policy shock.

For future research, it is essential to compare the predictions of the theoretical model with actual data. Additionally, a simulation-based analysis with endogenous transition probability would further illuminate the underlying mechanisms.

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A Bond Issuance of Incumbents and Entrants

We characterize the optimal choices of a particular firm z . After the realization of a shock, namely the state of the monetary policy shock v_t , the firm decides to ask for an intra-period loan (working capital) when $\pi_t(z) > 0$ and make an inter-period borrowing decision, $b_{t+1}(z)$, together with the pricing decision. In case of zero or negative profits, the firm stops operating. If it was an incumbent producer in the previous period, it honors the debt from the previous period with negative dividends. Specifically, the firm maximizes the following objective,

$$v_t(z) = d_t(z) + E_t [m_{I,t,t+1}v_{t+1}(z)],$$

with

$$d_t(z) = \pi_t(z) - (1 + \tilde{r}_t) b_t(z) + b_{t+1}(z),$$

and

$$\zeta_t E_t [m_{I,t,t+1}v_{t+1}(z)] \geq w_t l_t(z).$$

The Lagrangian is found to be

$$\begin{aligned} \mathcal{L}(z) = & \pi_t(z) - (1 + \tilde{r}_t) b_t(z) + b_{t+1}(z) \\ & + E_t [m_{I,t,t+1} \{ \pi_{t+1}(z) - (1 + \tilde{r}_{t+1}) b_{t+1}(z) + b_{t+2}(z) + E_t [m_{I,t+1,t+2}v_{t+2}(z)] \}] \\ & + \eta_t(z) \{ \zeta_t E_t [m_{I,t,t+1} \{ \pi_{t+1}(z) - (1 + \tilde{r}_{t+1}) b_{t+1}(z) + b_{t+2}(z) + E_t [m_{I,t+1,t+2}v_{t+2}(z)] \}] - w_t l_t(z) \}. \end{aligned}$$

The first-order condition with respect to $b_{t+1}(z)$ is

$$1 - (1 + \tilde{r}_{t+1}) E_t [m_{I,t,t+1}] - \eta_t(z) \zeta_t (1 + \tilde{r}_{t+1}) E_t [m_{I,t,t+1}] = 0.$$

Rearranging the equation, we get

$$\eta_t(z) = \frac{1 - (1 + \tilde{r}_{t+1}) E_t [m_{I,t,t+1}]}{\zeta_t (1 + \tilde{r}_{t+1}) E_t [m_{I,t,t+1}]}.$$

The first-order condition with respect to $\rho_t(z)$ is given by

$$\rho_t(z) = \frac{\sigma}{\sigma - 1} \frac{w_t}{z} (1 + \eta_t(z)).$$

Note that not only incumbents but also entrants are facing the same constraints and solve the same optimization problem with the assumption of instantaneous production as the entry. Specifically, the entrant (who has no debt from the previous period by definition) with a produc-

tivity level z maximizes the following objective:

$$v_t^E(z) = d_t^E(z) + E_t \left[m_{I,t,t+1} v_{t+1}^E(z) \right]$$

with

$$d_t^E(z) = \pi_t(z) + b_{t+1}^E(z) - w_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau,$$

and

$$\xi_t E_t \left[m_{I,t,t+1} v_{t+1}^E(z) \right] \geq w_t l_t(z).$$

The Lagrangian of the entrant is found to be

$$\begin{aligned} \mathcal{L}^E(z) &= \pi_t(z) + b_{t+1}^E(z) - w_t f_E \left(\frac{H_t}{H_{t-1}} \right)^\tau \\ &+ E_t \left[m_{I,t,t+1} \left\{ \pi_{t+1}(z) - (1 + \tilde{r}_{t+1}) b_{t+1}^E(z) + b_{t+2}^E(z) - w_{t+1} f_E \left(\frac{H_{t+1}}{H_t} \right)^\tau + E_t \left[m_{I,t+1,t+2} v_{t+2}^E(z) \right] \right\} \right] \\ &+ \eta_t(z) \xi_t E_t \left[m_{I,t,t+1} \left\{ \pi_{t+1}(z) - (1 + \tilde{r}_{t+1}) b_{t+1}^E(z) + b_{t+2}^E(z) - w_{t+1} f_E \left(\frac{H_{t+1}}{H_t} \right)^\tau + E_t \left[m_{I,t+1,t+2} v_{t+2}^E(z) \right] \right\} \right] \\ &\quad - \eta_t(z) w_t l_t(z) \end{aligned}$$

The first order condition with respect to $b_{t+1}^E(z)$ is

$$1 - (1 + \tilde{r}_{t+1}) E_t [m_{I,t,t+1}] - \eta_t(z) \xi_t (1 + \tilde{r}_{t+1}) E_t [m_{I,t,t+1}] = 0.$$

Also the first order condition with respect to $\rho_t^E(z)$ is given by

$$\rho_t^E(z) = \frac{\sigma}{\sigma - 1} \frac{w_t}{z} (1 + \eta_t(z)).$$

Thus it is shown that incumbents and entrants make the same decision such that $b_{t+1}^E(z) = b_{t+1}(z)$ and $\rho_t^E(z) = \rho_t(z)$.

B Wage Setting

The representative worker maximizes the following utility by setting $W_t'(j)$.

$$E_t \sum_{k=0}^{\infty} (\beta \vartheta)^k U_t (C_{w,t+k}(j), L_{t+k|t}(j)),$$

where $L_{t+k|t}(j)$ are the consumption and labor supply at $t+k$ under the preset wage rate $W_t'(j)$.

The first order condition yields

Table 5: Non-bidding producers

Av price of non-binding producers	$\tilde{\rho}_{M,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \tilde{z}_{M,t}}$
Av value of non-binding producers	$\tilde{v}_{M,t} = \tilde{d}_{M,t} + E_t [m_{I_t,t+1} \tilde{v}_{M,t+1}]$
Av debt of non-binding producers	$\frac{\tilde{b}_{M,t}}{\tilde{b}_M} - \frac{w_t \tilde{l}_{M,t}}{w \tilde{l}_M} = 0$
Av labor demand of non-binding producers	$\tilde{l}_{M,t} = \frac{\tilde{y}_{M,t}}{\tilde{z}_{M,t} Z_t} + f$
Av output of non-binding producers	$\tilde{y}_{M,t} = \tilde{\rho}_{M,t}^\sigma C_t$
Av dividends of non-binding producers	$\tilde{d}_{M,t} = \pi_{M,t} - (1 + \tilde{r}_t) \tilde{b}_{M,t} + \tilde{b}_{M,t+1}$
Av profits of non-binding producers	$\tilde{\pi}_{M,t} = \frac{1}{\sigma} \tilde{\rho}_{M,t} \tilde{y}_{M,t} - f w_t$
Nb of non-binding producers	$\frac{M_t}{N_t + H_t} = \left(\frac{\tilde{z}_N}{\tilde{z}_{M,t}} \right)^\kappa$
Av productivity of non-binding producers	$M_t = \tilde{z}_{S,t} \left[\frac{\kappa}{\kappa - (\sigma - 1)} \right]^{\frac{1}{\sigma - 1}}$

$$W'_t(j) = \frac{\frac{\eta\theta}{(\theta-1)(1+\nu)} \sum_{k=0}^{\infty} (\beta\vartheta)^k E_t [L_{t+k|t}^{1+\varphi}(j)]}{\sum_{k=0}^{\infty} (\beta\vartheta)^k E_t \left[\lambda_{w,t} \frac{1}{P_{t+k}} L_{t+k|t}(j) \right]}.$$

Further, using

$$L_{t+k|t}(j) = \left(\frac{W'_t(j)}{W_{t+k}} \right)^{-\theta} L_{t+k},$$

we have (6)

$$\left(\frac{W'_t(j)}{W_t} \right)^{1+\varphi\theta} = \frac{\frac{\chi\theta}{(\theta-1)(1+\nu)} \sum_{k=0}^{\infty} (\beta_w \vartheta)^k E_t \left[\left(\frac{W_{t+k}}{W_t} \right)^{\theta(1+\varphi)} L_{t+k}^{1+\varphi} \right]}{\sum_{k=0}^{\infty} (\beta_w \vartheta)^k E_t \left[\lambda_{w,t} \frac{W_{t+k}}{P_{t+k}} \left(\frac{W_{t+k}}{W_t} \right)^{\theta-1} L_{t+k} \right]}.$$

C Average of Non-Binding Producers

Table 5 show the system of equations that determine the average of non-binding producers. In addition to the system of equations presented in the main text we have nine new variables: $\tilde{\rho}_{M,t}$, $\tilde{v}_{M,t}$, $\tilde{d}_{M,t}$, $\tilde{y}_{M,t}$, $\tilde{b}_{M,t}$, $\tilde{\pi}_{M,t}$, $\tilde{l}_{M,t}$, M_t and $\tilde{z}_{M,t}$.